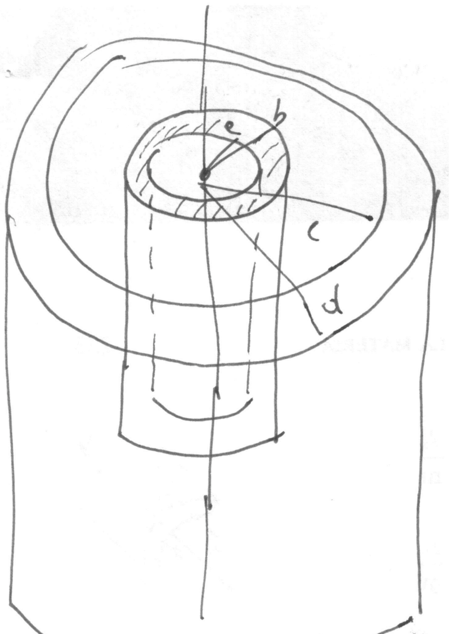

a) $$\int \vec{D}\cdot\vec{n}\, dS = q_{int}$$

$$\int \vec{E}\cdot\vec{n}\, dS = \frac{q_{int}}{\varepsilon_0}$$

per $r < a$

$$2\pi r \cdot \cancel{h}\, E = \frac{\lambda \cancel{h}}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

per $a < r < b$

$$2\pi r \cdot h \cdot D = \lambda h$$

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$$

$$D = \frac{\lambda}{2\pi r}; \quad E = \frac{\lambda}{2\pi r \varepsilon_0 \varepsilon_r}$$

per $b < r < c$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

per $c < r < d$

$$E = 0$$

per $r > d$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

$$V = \int_a^{c'} E \cdot dl = \int_a^b \frac{\lambda}{2\pi \varepsilon_0 \varepsilon_r r}\, dr + \int_b^{c'} \frac{\lambda}{2\pi \varepsilon_0 r}\, dr =$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{1}{\varepsilon_r} \ln\frac{b}{a} + \ln\frac{c'}{b}\right] =$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \cdots$$

$$= 9\times10^9 \frac{N\cdot m^2}{C^2} \cdot 2 \times 15\times10^{-6}\frac{C}{m}\left[\frac{1}{5}\ln\frac{35}{20} + \ln\frac{37}{35}\right] =$$

$$= 270\times10^3\, \cancel{Vm} \left[\frac{0.557}{5} + 0.055\right] =$$

$$= 45\times10^3\ V$$

b) $\int \vec{D}\cdot\vec{n}\, dS = q$ $\quad q$ -

$$2\pi r h \cdot D = \lambda \cdot h + \int \rho(r)\, dV$$

$$2\pi r h D = \lambda \cdot h + \int K r \, r dr\, d\theta\, dz = \lambda \cdot h + \int_a^r K r^2 dr \cdot 2\pi h =$$

$$= \lambda \cdot h + K \frac{r^3}{3}\Big]_a^r \cdot 2\pi h =$$

$$2\pi r \cancel{h} D = \lambda \cancel{h} + K\left[\frac{r^3}{3} - \frac{a^3}{3}\right]\cdot 2\pi \cancel{h}$$

$$D = \frac{\lambda}{2\pi r} + \frac{K}{3r}\left[r^3 - a^3\right]$$

$$E = \frac{1}{\varepsilon_0 \varepsilon_r}\left[\frac{\lambda}{2\pi r} + \frac{K}{3r}(r^3 - a^3)\right]$$

$$P = \varepsilon_0 \chi E = \varepsilon_0 (\varepsilon_r - 1) E$$

$$P = \frac{\varepsilon_r - 1}{\varepsilon_r}\left[\frac{\lambda}{2\pi r} + \frac{K}{3r}(r^3 - a^3)\right];$$

Densità di polarizz. volumica

$$\rho_p = \vec{\nabla}\cdot\vec{P} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\varepsilon_r - 1}{\varepsilon_r}\frac{\lambda}{2\pi} + K\frac{\varepsilon_r - 1}{\varepsilon_r}(r^3 - a^3)\right) = \frac{1}{r} K \frac{\varepsilon_r - 1}{\varepsilon_r} 3r^2 = 3K\frac{\varepsilon_r - 1}{\varepsilon_r} r$$

$$\vec{\nabla}\cdot P = \frac{1}{r}\frac{\partial}{\partial r}(r P_r) + \dots$$

Densità superficiale

$$\sigma_a = \vec{P}_a \cdot \vec{n}_1 = -\frac{\varepsilon_r - 1}{\varepsilon_r}\left[\frac{\lambda}{2\pi a}\right] = -\frac{4}{5}\,\frac{15\times 10^{-6}\ C/m}{2\pi\ 0.2\ m} = -9.55\ \mu C$$

$$\sigma_b = \vec{P}_b \cdot \vec{n}_1 = \frac{\varepsilon_r - 1}{\varepsilon_r}\left[\frac{\lambda}{2\pi b} + \frac{K}{3b}(b^3 - a^3)\right] = \frac{4}{5}\left[\frac{15\times 10^{-6}\ C/m}{2\pi\ 0.35 m} + \frac{2\ C/m^2 \times 10^{-4}}{3\times 0.35\ m}(0.35^3 - 0.2^3)\ m^3\right] =$$

$$= \frac{4}{5}\left[6.82\ \mu C + 1.9\times 10^{-4} \times 0.0348\right] =$$

$$= \frac{4}{5}\left[6.82\ \mu C + 6.6\times 10^{-6}\ C\right] =$$

$$= 10.73\ \mu C$$

c) $V_{e'} = \int_{70\,cm}^{e'} E \cdot dl = \int_{70\,cm}^{d} \frac{\lambda}{2\pi\varepsilon r} dr + \int_{c}^{b} \frac{\lambda}{2\pi\varepsilon r} dr + \int_{b}^{e} \frac{\lambda}{2\pi\varepsilon \varepsilon_r r} dr +$

$+ \int_{e}^{e'} \frac{\lambda}{2\pi\varepsilon r} dr =$

$= \frac{\lambda}{2\pi\varepsilon_0} \left(\ln \frac{d}{70\,cm} \right) + \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{b}{c} + \frac{\lambda}{2\pi\varepsilon_0\varepsilon_r} \ln \frac{e}{b} +$

$+ \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{e'}{e} =$

$= \frac{\lambda}{2\pi\varepsilon_0} \left(\ln \frac{45}{70} + \ln \frac{35}{40} + \frac{1}{\varepsilon_r} \ln \frac{20}{35} + \ln \frac{10}{20} \right) =$

$= 270 \times 10^3 V \left(-0.44 - 0.13 - \frac{1}{5} 0.56 - 0.69 \right) =$

$= 270 \times 10^3 V \times 1.372 = 370\ KV$

$E_{K,in} = \frac{1}{2} m v^2 = \frac{1}{2} 1.6 \times 10^{-27} Kg \times \left(0.001 \times 3 \times 10^8 m/s \right)^2 =$

$= \frac{1}{2} 1.6 \times 10^{-27} Kg \times 10^{-6} \times 9 \times 10^{16} m/s =$

$= 7.2 \times 10^{-17} J$

Energia spesa per $q\Delta V = 1.6 \times 10^{-19} C \times 370 \times 10^3 V = 592 \times 10^{-16} J =$

$= 5.92 \times 10^{-14} J$

Quindi ~~non~~ raggiunge il punto

$E_{K,in} = q\Delta V + E_{K,fin}$

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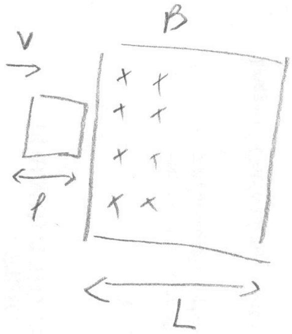
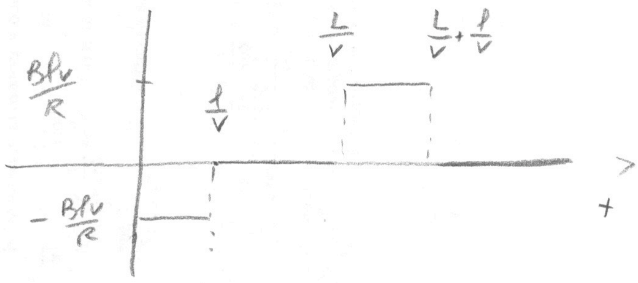
a)



fem $= -\frac{d\phi}{dt} = -\frac{d\,l x B}{dt} = -Blv$



$R = 4\rho\frac{l}{S} = 4\cdot 3\,\Omega\cdot m\,\frac{100\,mm}{1\,mm^2} =$

$= 4\cdot 3\,\Omega\cdot m\cdot\frac{0.1\,m}{10^{-6}\,m^2} = 1.2\times10^{6}\,\Omega$

$i = \frac{Blv}{R} = \frac{1.5\times 0.1\times 100\,m/s}{1.2\times10^{6}\,\Omega} =$

$= 12.5\times10^{-6}\,A$

b) Mentre entra

$F_B = ilB = \frac{Blv}{R}\,lB = l^2B^2\frac{v}{R} =$ sul filo che entra nel verso antiparallelo asse x

$= \frac{(0.1)^2\cdot 1.5^2\times 100}{1.2\times10^{6}} = 1.875\times10^{-6}\,N$

Le forze sui due pezzi di filo paralleli a v è uguale e contraria

Mentre esce ci sarà una uguale forza frenante sul lato ancora immerso nel campo

In ogni caso la forza è di attrito. Quando la spira è completamente immersa, non c'è forza

Per mantenere costante la velocità $F_{est} = -F_B$

c) $W = \int F_{est}\,dl = -\int F_B\,dl = -2\int_0^l B^2 l^2\frac{v}{R}\,dl =$

$= -2\,B^2 l^3\frac{v}{R}$

$= (-)2\,\frac{1.5^2\,(0.1)^3\,100}{\cancel{3}\,1.2\times10^{6}} =$

$= 0.375\times10^{-6}\,J$

La forza esterna produce un lavoro che viene completamente dissipato per effetto Joule, essendo servito per contrastare la forza di attrito