METODI MATEMATICI della FISICA (INTRODUZIONE)
RISULTATI della prova scritta del 5 APRILE 2004

COMPITO 1

1. I esiste $\forall \alpha \ / \ \left|\alpha - \frac{1}{2}\right| \neq 1$:

$$\begin{aligned} I &= 2i\frac{\alpha - 5}{\alpha(\alpha - 1)} && \text{se } \left|\alpha - \frac{1}{2}\right| > 1 \\ I &= 2i(\alpha - 5)\left[\frac{1}{\alpha(\alpha - 1)} + \frac{\pi}{\sin \pi\alpha}\right] && \text{se } \left|\alpha - \frac{1}{2}\right| < 1 \ ; \end{aligned}$$

2.
$$u(t) = e^{2t}\left(1 - 2t + \frac{1}{2}t^2\right) \ ;$$

3.
$$\begin{aligned} f(x) &= -\frac{1}{\pi} - \frac{1}{2}\sin\frac{\pi x}{A} + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{1}{4k^2 - 1}\cos\frac{2\pi k x}{A} \\ &= -\frac{1}{\pi} - \frac{1}{2}\sin\frac{\pi x}{A} + \frac{2}{3\pi}\cos\frac{2\pi x}{A} + \cdots \end{aligned}$$

COMPITO 2

1. I esiste $\forall a \ / \ \left|a+\frac{1}{2}\right| \neq 1$:

$$\begin{aligned} I &= -2i\frac{a-3}{a(a+1)} && \text{se } \left|a+\frac{1}{2}\right| > 1 \\ I &= 2i(a-3)\left[-\frac{1}{a(a+1)}+\frac{\pi}{\sin \pi a}\right] && \text{se } \left|a+\frac{1}{2}\right| < 1 \ ; \end{aligned}$$

2.
$$y(x) = \frac{1}{2}e^{-x}\left(1+x+x^2\right) \ ;$$

3.
$$\begin{aligned} f(t) &= \frac{1}{\pi} - \frac{1}{2}\sin\frac{2\pi t}{T} + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{1}{1-4k^2}\cos\frac{4\pi k t}{T} \\ &= \frac{1}{\pi} - \frac{1}{2}\sin\frac{2\pi t}{T} - \frac{2}{3\pi}\cos\frac{4\pi t}{T} + \cdots \end{aligned}$$

COMPITO 3

1. I esiste $\forall \beta \ / \ |\beta - 1| \neq 1$:

$$\begin{aligned} I &= -2i \frac{\beta + \frac{1}{2}}{\left(\beta - \frac{1}{2}\right)\left(\beta - \frac{3}{2}\right)} && \text{se } |\beta - 1| > 1 \\ I &= 2i\left(\beta + \frac{1}{2}\right)\left[-\frac{1}{\left(\beta - \frac{1}{2}\right)\left(\beta - \frac{3}{2}\right)} + \frac{\pi}{\cos \pi\beta}\right] && \text{se } |\beta - 1| < 1 \ ; \end{aligned}$$

2.
$$x(t) = e^{t/2}\left(2 - t + \frac{1}{2}t^2\right) \ ;$$

3.
$$\begin{aligned} f(x) &= \frac{1}{\pi} + \frac{1}{2}\sin\frac{2\pi x}{L} + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{1}{1-4k^2}\cos\frac{4\pi k x}{L} \\ &= \frac{1}{\pi} + \frac{1}{2}\sin\frac{2\pi x}{L} - \frac{2}{3\pi}\cos\frac{4\pi x}{L} + \cdots \end{aligned}$$

COMPITO 4

1. I esiste $\forall b \;/\; |b+1| \neq 1$:

$$\begin{aligned} I &= 2i\frac{\frac{1}{2}-b}{\left(b+\frac{1}{2}\right)\left(b+\frac{3}{2}\right)} && \text{se } |b+1|>1 \\ I &= 2i\left(\frac{1}{2}-b\right)\left[\frac{1}{\left(b+\frac{1}{2}\right)\left(b+\frac{3}{2}\right)}-\frac{\pi}{\cos\pi b}\right] && \text{se } |b+1|<1\ ; \end{aligned}$$

2.

$$z(x) = -2e^{-x/2} + \frac{9-4\sqrt{3}}{6}e^{-(1+\sqrt{3}/2)x} + \frac{9+4\sqrt{3}}{6}e^{-(1-\sqrt{3}/2)x}\ ;$$

3.

$$\begin{aligned} f(t) &= -\frac{1}{\pi} + \frac{1}{2}\sin\frac{\pi t}{T} + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{1}{4k^2-1}\cos\frac{2\pi k t}{T} \\ &= -\frac{1}{\pi} + \frac{1}{2}\sin\frac{\pi t}{T} + \frac{2}{3\pi}\cos\frac{2\pi t}{T} + \cdots \end{aligned}$$