Non-perturbative effects from brane couplings to bulk fluxes

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Theories of the Fundamental Interactions
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Foreword

This talk is mostly based on

- M. Billo, L. Ferro, M. Frau, F. Fucito, A. Lerda and J.F. Morales “Flux interactions on D-branes and instantons,” to appear soon.
- M. Billo, L. Ferro, M. Frau, F. Fucito, A. Lerda and J.F. Morales “Non-perturbative flux superpotentials” to appear soon.

It builds over a vast literature

- The few references scattered on the slides are of course not exhaustive. I apologize for the missing ones.

Plan of the talk

Introduction

Motivations

In studying the properties of brane-world models embedded in string compactifications two issues which are attracting much interest are the following

- The incorporation of internal bulk fluxes, which “extend” the possibilities offered by the choice of the compactification manifold with great implications, e.g., on the moduli stabilization problem

  Reviews by Grana, Douglas and Kachru, Denef et al, ...

- The exploration of the non-perturbative effects induced by instantonic (Euclidean wrapped) branes

Ordinary and stringy instantons

Let's focus on the gauge theory sector living on a stack of D-branes wrapped on some cycle c,

- Euclidean branes wrapped on c and point-like in space-time correspond to ordinary gauge instantons, reproducing the corresponding non-perturbative effects via a "stringy instanton calculus"

  Witten, 1995; Douglas, 1995-1996; Green-Gutperle 1997-, ...

- Instantonic branes wrapped on $c' \neq c$ potentially yield novel non-pert. effects (Majorana masses for neutrinos, ...)

  Blumenhagen et al 0609191; Ibanez and Uranga, 0609213; (long list)... ;

  and may play a rôle in the moduli stabilization problem. Such branes are known as "exotic" or "stringy" instantons

Fluxes for non-perturbative effects?

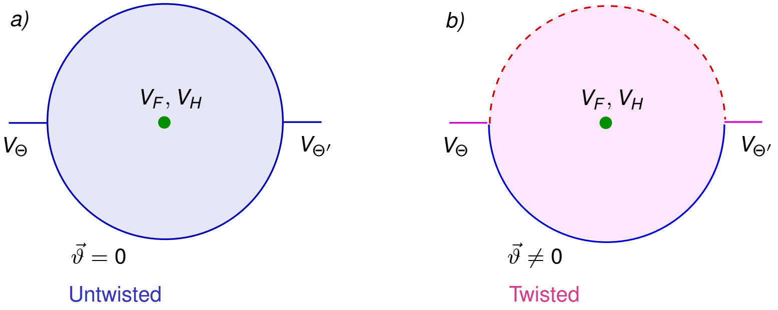
- The spectrum of moduli from the exotic instantonic branes is peculiar, and it is difficult to saturate the fermionc zero-mode integration
  Argurio et al, 0704.0262; Bianchi et al, 0704.0784; ...
- Among other mechanisms, it has been suggested that in presence of internal fluxes additional interactions can lift these fermionic zero modes allowing the actual generation of non-perturbative effects from exotic instantons
  Blumenhagen et al, 0708.0403; Petersson, 0711.1837;...
- Such non-perturbtaive effects depend crucially on the form of the flux interaction terms on the (instantonic) branes.
  - Various cases have been investigated by supergravity/κ-symmetry methods
    M. Grana, 0202118; Marolf et al, 0306066; ...
  - A systematic derivation from world-sheet methods is missing

Objectives

- Derive by world-sheet methods (disk diagrams) the interaction of bulk fluxes with open string modes
  - We focus on fermionic open string state, but we consider the generically twisted case, corresponding to various possible D-brane setups
- Use the explicit result for the flux-induced modification of the moduli action on ordinary and exotic instantons to derive flux-dependent non-perturbative effects
  - We work in an $\mathcal{N} = 1$ context, with a brane realization of SQCD
  - We find that fluxes allow effects in the exotic case, but they also generate new non-perturbative effects from ordinary instantonic branes

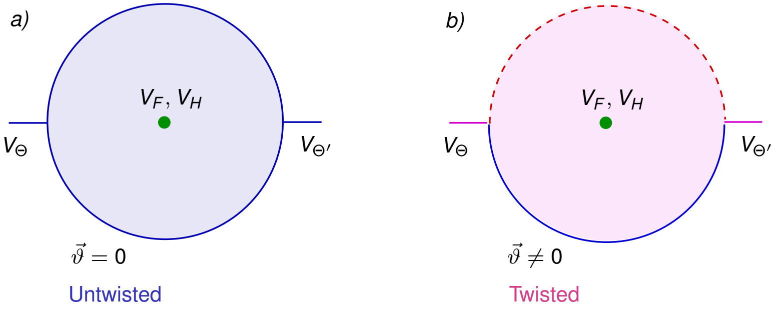
CFT derivation: flux amplitudes on disks

The disk diagrams



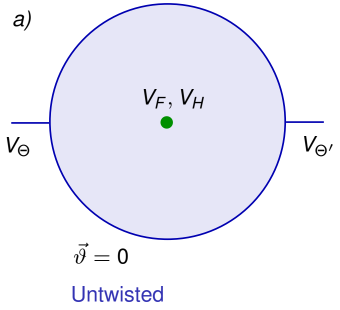
- Θ and Θ' are massless fermions from the R sector of open strings

The disk diagrams



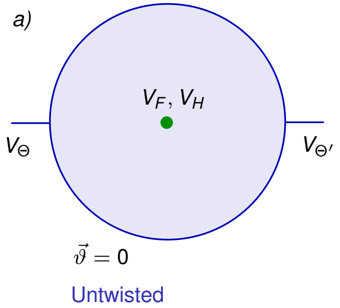
- F (H) is a closed string vertex corresponding to a RR (NS-NS) field strength

The disk diagrams



- We can treat open string with generic b.c., including both the twisted and untwisted case

The disk diagrams



- We work in a flat geometry (non-compact, toroidal or orbifolded directions)

Boundary conditions

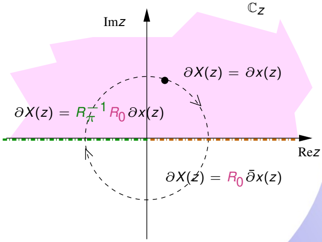
- D-branes $\leftrightarrow$ boundary conditions, e.g.

$$\bar{\partial} x^M \Big|_{\sigma=0,\pi} = (R_\sigma)^M{}_N \, \partial x^N \Big|_{\sigma=0,\pi} ,$$

with $R_\sigma = \left(1 - \mathcal{F}_\sigma\right)^{-1} \left(1 + \mathcal{F}_\sigma\right)$

- For a string stretching between different branes, we get twisted fields:

$$X^M(\mathrm{e}^{2\pi \mathrm{i}} z) = R^M{}_N \, X^N(z) , \quad R = R_\pi^{-1} R_0$$



Twisted world-sheet fields

- Choose a complex basis Z^I, $I = 1, \ldots 5$, where

$$R = \mathrm{diag}\left(\mathrm{e}^{2\pi\mathrm{i}\vartheta^1}, \mathrm{e}^{-2\pi\mathrm{i}\vartheta^1}, \ldots, \mathrm{e}^{2\pi\mathrm{i}\vartheta^5}, \mathrm{e}^{-2\pi\mathrm{i}\vartheta^5}\right) \;,$$

- The Z^I fields are twisted: $\partial Z^I(\mathrm{e}^{2\pi\mathrm{i}}z) = \mathrm{e}^{2\pi\mathrm{i}\vartheta^I}\partial Z^I(z)$.
- Similarly for w.s. fermions: $\Psi^I(\mathrm{e}^{2\pi\mathrm{i}}z) = \eta\,\mathrm{e}^{2\pi\mathrm{i}\vartheta^I}\Psi^I(z)$ ($\eta = 1$ for NS, $\eta = -1$ for R sector).

General result (RR)

$$\mathcal{A}_F = -8c_F\, \Theta' \Gamma^M \Theta \left[F\mathcal{R}_0(2I_1 - I_2)\right]_M + \frac{4c_F}{3!} \Theta' \Gamma^{MNP} \Theta \left[F\mathcal{R}_0 I_2\right]_{MNP}$$

- $\Theta_{\mathcal{A}}$: polarization of the open string R vertex ▸ Details
  - $\mathcal{A} = 1, \ldots, 16$ = (antichiral) 10d spinor index labeling $\vec{\epsilon}_{\mathcal{A}} = \frac{1}{2}(\pm, \pm, \pm, \pm, \pm)$
  - However $\epsilon^I_{\mathcal{A}} = -\frac{1}{2}$ if $\vartheta^I > 0$

General result (RR)

$$\mathcal{A}_F = -8c_F\,\Theta'\Gamma^M\Theta\left[F\mathcal{R}_0(2I_1 - I_2)\right]_M + \frac{4c_F}{3!}\Theta'\Gamma^{MNP}\Theta\left[F\mathcal{R}_0 I_2\right]_{MNP}$$

- The IIB RR vertex is a bi-spinor containing the fields strengths: Details

$$F_{\mathcal{A}\mathcal{B}} = \sum_{n=1,3,5}\frac{1}{n!}F_{M_1\dots M_n}\left(\Gamma^{M_1\dots M_n}\right)_{\mathcal{A}\mathcal{B}}\ ,$$

General result (RR)

$$\mathcal{A}_F = -8c_F\,\Theta'\Gamma^M\Theta\left[F\mathcal{R}_0(2I_1 - I_2)\right]_M + \frac{4c_F}{3!}\Theta'\Gamma^{MNP}\Theta\left[F\mathcal{R}_0 I_2\right]_{MNP}$$

- l.m. and r.m. fields identification at the boundary:

$$\widetilde{X}^M(\bar{z}) = (R_0)^M{}_N X^N(\bar{z}) \quad , \quad \widetilde{s}_{\vec{\epsilon}_\mathcal{A}}(\bar{z}) = (\mathcal{R}_0)^\mathcal{A}{}_\mathcal{B}\, s_{\vec{\epsilon}_\mathcal{B}}(\bar{z})$$

where $\mathcal{R}_0$ is the spinorial reflection matrix. Thus

$$F_{\mathcal{A}\mathcal{B}} \rightarrow (F\mathcal{R}_0)_{\mathcal{A}\mathcal{B}}$$

General result (RR)

$$\mathcal{A}_F = -8c_F\,\Theta'\Gamma^M\Theta\,\left[F\mathcal{R}_0(2I_1 - I_2)\right]_M + \frac{4c_F}{3!}\Theta'\Gamma^{MNP}\Theta\,\left[F\mathcal{R}_0 I_2\right]_{MNP}$$

- I_1 and I_2 are $\vec{\vartheta}$-dependent diagonal matrices:

$$(I_1)_{\mathcal{A}_3}^{\mathcal{A}_3} = \frac{1}{2}\,\mathrm{e}^{-\frac{\mathrm{i}\pi\alpha' s}{2}}\left(\mathrm{e}^{-2\pi\mathrm{i}\left(\alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3\right)} - 1\right) B\left(\alpha' s; \alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3\right)$$

$$(I_2)_{\mathcal{A}_3}^{\mathcal{A}_3} = \frac{1}{2}\,\mathrm{e}^{-\frac{\mathrm{i}\pi\alpha' s}{2}}\left(\mathrm{e}^{-2\pi\mathrm{i}\left(\alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3\right)} - 1\right) B\left(\alpha' s + 1; \alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3\right)$$

where $\vec{\epsilon}_3$ is the spinorial weight of the r.m. part of the RR vertex

General result (RR)

$$\mathcal{A}_F = -8 c_F \, \Theta' \Gamma^M \Theta \left[F \mathcal{R}_0 (2 I_1 - I_2) \right]_M + \frac{4 c_F}{3!} \Theta' \Gamma^{MNP} \Theta \left[F \mathcal{R}_0 I_2 \right]_{MNP}$$

- c_F: factor arising from disk and vertices normalizations, that can be fixed

General result (NS-NS)

$$
\begin{aligned}
\mathcal{A}_H = & -4c_H\,\Theta'\Gamma^N\Theta\,\delta^{MP}\left[\partial B R_0(2I_1 - I_2)\right]_{[MN]P} \\
& + 2c_H\,\Theta'\Gamma^{MNP}\Theta\left[\partial B R_0 I_2\right]_{MNP}
\end{aligned}
$$

- We use an effective NS-NS vertex containing the derivatives of B

$$
\begin{aligned}
V_H(z,\bar{z}) = & \mathcal{N}_H\left(\partial_M B_{NP}\right) \mathrm{e}^{-\mathrm{i}\pi\alpha' k_{\mathrm{L}}\cdot k_{\mathrm{R}}}\left[\psi^M\psi^N \mathrm{e}^{\mathrm{i}\,k_{\mathrm{L}}\cdot X}\right](z) \\
& \times\left[\widetilde{\psi}^P\,\mathrm{e}^{-\widetilde{\phi}}\,\mathrm{e}^{\mathrm{i}\,k_{\mathrm{R}}\cdot\widetilde{X}}\right](\bar{z})
\end{aligned}
$$

General result (NS-NS)

$$\mathcal{A}_H = -4c_H\,\Theta'\Gamma^N\Theta\,\delta^{MP}\left[\partial B R_0(2I_1 - I_2)\right]_{[MN]P} + 2c_H\,\Theta'\Gamma^{MNP}\Theta\left[\partial B R_0 I_2\right]_{MNP}$$

- In presence of D-branes, the left-right identifications leads to

$$(\partial B) \to (\partial B R_0)$$

with the vectorial reflection matrix R_0

General result (NS-NS)

$$\mathcal{A}_H = -4c_H\,\Theta'\Gamma^N\Theta\,\delta^{MP}\left[\partial B R_0(2 I_1 - I_2)\right]_{[MN]P} + 2c_H\,\Theta'\Gamma^{MNP}\Theta\left[\partial B R_0 I_2\right]_{MNP}$$

- I_1 and I_2 are again given by:

$$(I_1)^{\mathcal{A}_3}_{\mathcal{A}_3} = \frac{1}{2}\,\mathrm{e}^{-\frac{\mathrm{i}\pi\alpha' s}{2}}\left(\mathrm{e}^{-2\pi\mathrm{i}\left(\alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3\right)} - 1\right) B(\alpha' s; \alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3)$$

$$(I_2)^{\mathcal{A}_3}_{\mathcal{A}_3} = \frac{1}{2}\,\mathrm{e}^{-\frac{\mathrm{i}\pi\alpha' s}{2}}\left(\mathrm{e}^{-2\pi\mathrm{i}\left(\alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3\right)} - 1\right) B(\alpha' s + 1; \alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3)$$

but $\vec{\epsilon}_3$ is now the vectorial weight associated to $\psi^P(z_3)$ in the r.m. part of the NS-NS vertex

Various cases

The general result can be applied to many different situations and generate various types of flux interactions

- $\vartheta^I = 0$: all fields untwisted, open strings with both ends on the same stack of D-branes
  - fields in the adjoint of a gauge theory from space-filling branes
  - neutral instanton moduli from instantonic branes
- $\vartheta^4 = \vartheta^5 = \frac{1}{2}$: ND spacetime. Open strings between a space-filling D brane and an instantonic brane
  - $\vartheta^i = 0$: twisted open strings $\leftrightarrow$ charged ADHM instanton moduli. “Ordinary” gauge instantons.
  - $\vartheta^i \neq 0$: “exotic” instantons of truly stringy nature

Various cases

The general result can be applied to many different situations and generate various types of flux interactions

- $\vartheta^4 = \vartheta^5 = 0$ (spacetime), $\vartheta^i \neq 0$ ($i = 1, 2, 3$): open strings stretching between different stacks of D branes.
  - Matter fields $\in$ bi-fundamentals. Always include massless chiral fermions.
  - In certain cases, e.g., $\sum_i \vartheta^i = 2\pi$, also massless scalars, hence $\mathcal{N} = 1$ chiral multiplets (matter content of brane-world models).

Flux-induced interactions on branes

Specializing the result

- We will concentrate here on toroidal (orbifold) compactifications to 4d and consider the interactions induced by constant internal fluxes F_3 and H on
  - space-filling branes. In this case we consider untwisted strings
  - instantonic branes. We consider untwisted strings (neutral moduli) but also also twisted ND strings (charged moduli.

Untwisted case

- The general result reduces to ($m, n \ldots$ are internal indices)

$$\mathcal{A} \equiv \mathcal{A}_F + \mathcal{A}_H = \frac{2\pi \mathrm{i}}{3} c_F \, \Theta \Gamma^{mnp} \Theta \, T_{mnp}$$

with

$$T_{mnp} = (F\mathcal{R}_0)_{mnp} + \frac{1}{g_s}[(\partial B R_0)_{mnp} + (\partial B R_0)_{npm} + (\partial B R_0)_{pmn}]$$

  - The factor of g_s is due to the relative normalizazion of RR and NS-NS vertices to account for their 10d kinetic terms in the Einstein frame

Untwisted case

- The general result reduces to ($m, n \ldots$ are internal indices)

$$\mathcal{A} \equiv \mathcal{A}_F + \mathcal{A}_H = \frac{2\pi i}{3} c_F \, \Theta \Gamma^{mnp} \Theta \, T_{mnp}$$

with

$$T_{mnp} = (F\mathcal{R}_0)_{mnp} + \frac{1}{g_s}[(\partial B R_0)_{mnp} + (\partial B R_0)_{npm} + (\partial B R_0)_{pmn}]$$

  - For unmegnetized branes,the reflection matrix R_0 is simply +1 for NN and -1 for DD directions
  - The spinorial reflection is simply $\mathcal{R}_0 = \prod_{\hat{m} \in DD} \Gamma^{\hat{m}}$

Unmagnetized branes

The coupling T_{mnp} depends on the type of brane: ▸ Back

	0-3	4	5	6	7	8	9	T_{mnp}
D3	–	×	×	×	×	×	×	$(*_6 F)_{mnp} - \frac{1}{g_s} H_{mnp}$
D5	–	–	–	×	×	×	×	$\frac{1}{g_s} H_{\hat{m}\hat{n}p}$; $-\frac{1}{2} F_{\hat{m}}{}^{qr} \epsilon_{qrnp}$; $-\frac{1}{g_s} H_{mnp}$
D7	–	–	–	–	–	×	×	$F_{\hat{m}\hat{n}}{}^{q} \epsilon_{qp} + \frac{1}{g_s} H_{\hat{m}\hat{n}p}$
D9	–	–	–	–	–	–	–	$F_{\hat{m}\hat{n}\hat{p}}$

- We neglected the H-components that would be projected out by the appropriate orientifold projections

Unmagnetized branes

The coupling T_{mnp} depends on the type of brane: Back

	0-3	4	5	6	7	8	9	T_{mnp}
D3	–	×	×	×	×	×	×	$(*_6 F)_{mnp} - \frac{1}{g_s} H_{mnp}$
D5	–	–	–	×	×	×	×	$\frac{1}{g_s} H_{\hat{m}\hat{n}p}$; $-\frac{1}{2} F_{\hat{m}}{}^{qr} \epsilon_{qrnp}$; $-\frac{1}{g_s} H_{mnp}$
D7	–	–	–	–	–	×	×	$F_{\hat{m}\hat{n}}{}^{q} \epsilon_{qp} + \frac{1}{g_s} H_{\hat{m}\hat{n}p}$
D9	–	–	–	–	–	–	–	$F_{\hat{m}\hat{n}\hat{p}}$

- Can be extended to magnetized branes, by taking general reflection matrices R_0, $\mathcal{R}_0$

Unmagnetized branes

The coupling T_{mnp} depends on the type of brane: ▸ Back

	0-3	4	5	6	7	8	9	T_{mnp}
D3	–	×	×	×	×	×	×	$(*_6 F)_{mnp} - \frac{1}{g_s} H_{mnp}$
D5	–	–	–	×	×	×	×	$\frac{1}{g_s} H_{\hat{m}\hat{n}p}$; $-\frac{1}{2} F_{\hat{m}}{}^{qr} \epsilon_{qrnp}$; $-\frac{1}{g_s} H_{mnp}$
D7	–	–	–	–	–	×	×	$F_{\hat{m}\hat{n}}{}^{q} \epsilon_{qp} + \frac{1}{g_s} H_{\hat{m}\hat{n}p}$
D9	–	–	–	–	–	–	–	$F_{\hat{m}\hat{n}\hat{p}}$

- F and H do not appear of the same footing.

4d notation

- Decomposing the 10d spinors into 4+6-dimensional parts: $\Theta_{\mathcal{A}} \;\rightarrow\; (\Theta^{\alpha A}, \Theta_{\dot{\alpha}A})$, the flux coupling in 4d notation reads

$$-\mathrm{i}\,\Theta^{\alpha A}\Theta_{\alpha}{}^{B}(\overline{\Sigma}^{mnp})_{AB}\,T^{\mathrm{IASD}}_{mnp} - \mathrm{i}\,\Theta_{\dot{\alpha}A}\Theta^{\dot{\alpha}}{}_{B}(\Sigma^{mnp})^{AB}\,T^{\mathrm{ISD}}_{mnp}$$

- ISD and IASD tensors are defined as follows:

$$T^{\mathrm{ISD}}_{mnp} = \frac{1}{2}\big(T - \mathrm{i} *_6 T\big)_{mnp} \quad , \quad T^{\mathrm{IASD}}_{mnp} = \frac{1}{2}\big(T + \mathrm{i} *_6 T\big)_{mnp} \,,$$

- Decomposing the 10d spinors into 4+6-dimensional parts: $\Theta_{\mathcal{A}} \;\rightarrow\; (\Theta^{\alpha A}, \Theta_{\dot{\alpha}A})$, the flux coupling in 4d notation reads

$$-\mathrm{i}\,\Theta^{\alpha A}\Theta_{\alpha}{}^{B}\big(\overline{\Sigma}^{mnp}\big)_{AB}\,T^{\mathrm{IASD}}_{mnp} - \mathrm{i}\,\Theta_{\dot{\alpha}A}\Theta^{\dot{\alpha}}{}_{B}\big(\Sigma^{mnp}\big)^{AB}\,T^{\mathrm{ISD}}_{mnp}$$

- In a complex basis,

$$T^{\mathrm{ISD}} \rightarrow T_{(0,3)} \oplus T_{(2,1)_{\mathrm{P}}} \oplus T_{(1,2)_{\mathrm{NP}}}$$
$$T^{\mathrm{IASD}} \rightarrow T_{(3,0)} \oplus T_{(1,2)_{\mathrm{P}}} \oplus T_{(2,1)_{\mathrm{NP}}}$$

where (N)P stands for (non)-primitive

Majorana masses for gauginos

- Focusing on a stack of D3-branes,
  - the fermions $\Theta^{\alpha A}$ correspond to the chiral part of the 4 gauginos $\Lambda^{\alpha A}$ (for a toroidal compactification)
  - the coupling tensor T_{mnp} can be written as

$$T_{mnp} = (*_6 F)_{mnp} - H_{mnp}/g_s = Re\big(*_6 G - \mathrm{i}G\big)_{mnp}$$

  in terms of the complex 3-form flux $G = F - \mathrm{i}H/g_s$
- We get thus Majorana mass terms for gauginos of the form: Back

$$-\frac{2\pi\mathrm{i}c_F}{3!}\mathrm{Tr}\Big[\Lambda^{\alpha A}\Lambda_\alpha{}^B\big(\overline{\Sigma}^{mnp}\big)_{AB}\, G^{\mathrm{IASD}}_{mnp} + \bar\Lambda_{\dot\alpha A}\bar\Lambda^{\dot\alpha}{}_B\big(\Sigma^{mnp}\big)^{AB}\,\big(G^{\mathrm{IASD}}_{mnp}\big)^*\Big]$$

  - No coupling to ISD flux!, in agreement with literature based on SUGRA approach

Grana, 2002; Marolf et al, 2003; ...

Instantonic branes

- For unmagnetized Euclidean branes one can derive a table of couplings analogous to that for D-branes ▸ Recall
  - The space-time directions are now transverse, and D(-1),E1,E3 and E5 replace respectively the D3, D5, D7 and D9.
  - The couplings are similar but not identical!
- For D-instantons, $R_0 = -\mathbf{1}$ and $\mathcal{R}_0 = \mathrm{i}\Gamma^{E}_{(11)}$. One gets

$$T_{mnp} = -\mathrm{i}G_{mnp}$$

- In 4d notation, one gets flux-induced fermionic 0-modes bilinears in the moduli action of the form ▸ Back

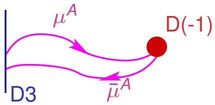
$$-\frac{2\pi \mathrm{i} c'_F}{3!}\left[\Theta^{\alpha A}\Theta_{\alpha}{}^{B}\left(\bar{\Sigma}^{mnp}\right)_{AB} G^{\mathrm{IASD}}_{mnp} + \Theta_{\dot{\alpha}A}\Theta^{\dot{\alpha}}{}_{B}\left(\Sigma^{mnp}\right)^{AB} G^{\mathrm{ISD}}_{mnp}\right]$$

Both ISD and IASD parts of G play a rôle.

Coupling to twisted instanton moduli

The only twisted case we consider here is that of D3/D(-1) strings $\leftrightarrow$ carged (or flavored) fermionic instanton moduli.
Twists and spinor weights are as follows:

- $\vec{\theta} = (0,0,0,\frac{1}{2},\frac{1}{2})$, $\epsilon_1 = (\epsilon_A, -\frac{1}{2}, -\frac{1}{2})$
- $\vec{\theta}' = -\vec{\theta} = (0,0,0,-\frac{1}{2},-\frac{1}{2})$, $\epsilon_4 = (\epsilon_A, \frac{1}{2}, \frac{1}{2})$



The flux interaction diagrams turn out to give simply

$$-\frac{4\pi \mathrm{i}}{3!}\, c_F'\, \bar{\mu}^A \mu^B \left(\overline{\Sigma}^{mnp}\right)_{AB} G^{\mathrm{IASD}}_{mnp}$$

Bulk fluxes and gauge theories

In an $\mathcal{N} = 1$ context (e.g. orbifold/orientifold) introduce a gauge-matter theory sector by (wrapped) D-branes and instantonic sectors via (wrapped) Euclidean branes.

- Fluxes "generalize" the bulk geometry and correspond in SUGRA to GVW superpotentials for the bulk fields ▸ Details

  Gukov et al, 9906070; Taylor and Vafa, 9912152

- Via the couplings we have computed in CFT they influence directly also the gauge/matter sector
  - Induce soft-supersymmetry breaking terms in the gauge theory such as the gaugino mass we've discussed

    Grana, 0209200; Camara et al, 0311241; ...
  - Modify the couplings of the fermionic zero modes on instantonic branes. The integration over the instantonic moduli can then lead to new non-perturbative contributions to the effective action

    Blumenhagen et al, 0708.0403; Garcia-Extebarrial et al, 0805.0713

Effects on the moduli action of D-instantons

A simple laboratory: $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

To analyize the flux effects on the non-perturbative effective action of brane-world gauge theories, it is useful to focalize on a simple (yet non-trivial) example

- We consider a local model of an $\mathcal{N} = 1$ compactification given by the orbifold $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, generated by

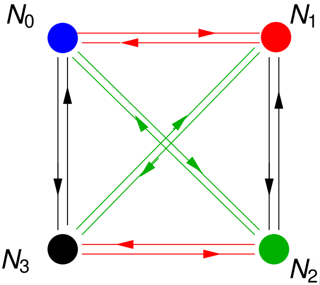
$$h_1 : \ (Z^1, Z^2, Z^3) \to (Z^1, -Z^2, -Z^3)$$
$$h_2 : \ (Z^1, Z^2, Z^3) \to (-Z^1, Z^2, -Z^3)$$

- The properties of the 4 irreducible representations, and the transformations of the string fields under this group are esaily worked out Details

The quiver

We consider fractional D3 branes transverse to the orbifold

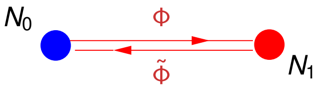
- 4 types of fD3's: the CP indices of open string endpoints attached to fD3(A) transform in the orbifold irrep R_A
- Given a system of $\{N_A\}$ fD3's, the open string massless spectrum is encoded in a quiver
  - Nodes $\leftrightarrow U(N_A)$ $\mathcal{N} = 1$ vector multiplets
  - Arrows: bifundamental chiral multiplets



A realization of SQCD

A system of N_0 (N_1) fD3's of type 0 (1) realizes SQCD ▸ Back

- $U(N_0) \times U(N_1)$ $\mathcal{N} = 1$ gauge theory
- Two chiral multiplets:



$$\Phi \in N_0 \times \bar{N}_1 \ , \quad \tilde{\Phi} \in \bar{N}_0 \times N_1$$

We're interested in the low-energy effective theory of this system

- The diagonal $U(1)$ factor is decoupled, the other $U(1)$ factor is IR free $\rightarrow$ we in fact have an $SU(N_0) \times SU(N_1)$ theory
- We focus on one the gauge factors, so we see a SQCD with

$$N_c = N_0 \ , \quad N_f = N_1$$

Incorporating flux effects

At the microspopic level, $G_{3,0}$ gives mass to the gaugino while $G_{3,0}$ induces a gravitino mass

- We want to investigate flux effects in the low energy effective theory for the massless d.o.f. in the Higgs phase, parametrizing solutions to the D-flatness eq.s ▸ Back

$$\Phi_f^u \Phi_v^{\dagger f} = \tilde{\Phi}_f^{\dagger u} \tilde{\Phi}_v^f$$

- The fluxes may modify the non-perturbative contributions which in this context are due to (fractional) D(-1) branes

“Ordinary” D-instantons

- Including k_0 fractional D(-1) of type 0 corresponds to work in the instanton # k_0 sector of the gauge theory

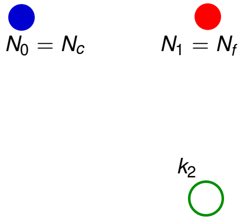
- In SQCD, the $k_0 = 1$ sector is responsible of
  - the ADS/VTY superpotential for $N_f = N_c - 1$
    Affleck et al, 1984; Taylor et al, 1983
  - Beasley-Witten F-terms for $N_f \geq N_c$
    Beasley and Witten, 0409149, 0512039
- In presence of fluxes, other effects (some of stringy nature) arise

Exotic D-instantons

D(-1)'s of type 2 or 3 give "exotic", a.k.a. "stringy" non-perturbative effects

- "Exotic" non-perturbative contributions have attracted much interest recently in brane-world constructions
- Could generate very interesting terms (neutrino masses ...)

  Blumenhagen et al 0609191; Ibanez and Uranga, 0609213; ... ;



- However, severe restrictions from integration over fermionic 0-modes: difficult to get non-vanishing results

  Argurio et al, 0704.0262; Bianchi et al, 0704.0784; ...

- To this aim, fluxes might come to the rescue!

  Blumenhagen et al, 0708.0403; Petersson, 0711.1837;...

Flux-aware stringy instanton calculus

Stringy instanton calculus

(hyper-sketchy)

Given a configuration of space-filling branes supporting the gauge/matter theory and of instantonic branes

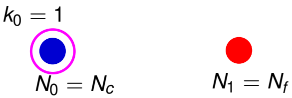
- Individuate the spectrum of open string states $\mathfrak{M}$ with at least one end on an instantonic brane: they carry no momentum, represent moduli
- Compute the disk interactions of moduli, also with insertions of gauge/matter fields parametrizing the classical low-energy theory to get $S_{mod}(\mathfrak{M}; \Phi(x))$
- integrate over the moduli to get the effective action

$$S_{eff}(\Phi) = \int d^4x \int d\widehat{\mathfrak{M}}\, \mathrm{e}^{-S_{mod}(\mathfrak{M};\Phi(x))}$$

(the position x is one of the moduli)

Ordinary instanton: spectrum

Let us focus on a single D(-1) of type 0 in the SQCD set-up



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- Neutral moduli: $\{x^\mu, D_c, \theta^\alpha, \lambda_{\dot\alpha}\}$
  - x, θ: position of the instanton + superpartner
  - D_c ($c = 1, 2, 3$): auxiliary fields (see later)

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- Charged moduli: $\{w_{\dot\alpha u}, \mu_u\}$, $\{\bar w^u_{\dot\alpha}, \mu^u\}$ from the two orientations.
  - $w_{\dot\alpha}$ bosonic, μ fermionic: effect of ND b.c.'s.
  - u= color index

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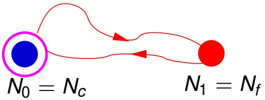

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  - $w_{\dot\alpha}$ bosonic, μ fermionic: effect of ND b.c.'s.
  - u= color index
- Flavored moduli: μ'_f, $\bar\mu'^f$ from the two orientations
  - Fermionic only! D(-1) of type 0, D3 of type 1 can be seen as branes wrapped on non-parallel (exceptional cycles): "exotic" configuration
  - f= flavor index

Ordinary instanton: action

The disks with moduli insertions yield the action

$$S_{mod} = \frac{D_c D^c}{2g_0^2} + \mathrm{i} D_c (\bar{w}^u \tau^c w_u) + \mathrm{i}\lambda \cdot (\bar{\mu}^u w_u + \bar{w}^u \mu_u)$$

The dimensions of the moduli are chosen as follows:

x_μ	D_c	θ^α	$\lambda_{\dot\alpha}$	$w_{\dot\alpha}$	μ	μ'
M^{-1}	M^2	$M^{-1/2}$	$M^{3/2}$	M^{-1}	$M^{-1/2}$	$M^{-1/2}$

- In the field theory limit $\alpha' \to 0$, D_c and $\lambda_{\dot\alpha}$ are Lagrange multiplier for the bosonic and fermionic constraints of the ADHM construction.
  - Indeed, $1/g_0^2 \propto (2\pi\alpha')^2/g_s$ goes to 0 for g_s fixed, i.e. fixed gauge coupling

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- x^μ, θ^α have the dimensions of supercoordinates
  - They do not enter in the pure moduli action

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- The $w_{\dot\alpha u}$ are related to the size and orientation of the instanton: $\bar{w}^u \cdot w_u = \rho^2$ once the constraints are solved

Field-dependent terms

With insertions of matter fields from D3(0)/D3(1) strings ▸ Recall
the moduli action becomes ▸ Back ▸ Back2

$$S_{mod} + \bar{w}^u_{\dot\alpha}\big(\tilde\phi^\dagger(x)\tilde\Phi(x,\theta) + \Phi(x,\theta)\phi^\dagger(x)\big)\, w^{\dot\alpha}_v$$
$$+\,\bar\mu^u\tilde\phi^{\dagger f}_u(x)\mu'_f - \bar\mu'^f\phi^{\dagger u}_f(x)\mu_u + \bar{w}^{\dot\alpha u}\tilde\psi^{\dagger\dot\alpha}_u(x)\mu' - \bar\mu'\psi^{\dagger u}_{\dot\alpha}(x)w^{\dot\alpha}_u$$

- ${\Phi^u}_f$ and ${\tilde\Phi^f}_u$ are chiral multiplets:

$$\Phi(x,\theta) = \phi(x) + \theta^\alpha\psi_\alpha(x) + \theta^2 F(x)$$

  - The moduli x, θ enter in the moduli action only through this expansion

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$$+ \bar{\mu}^u\tilde{\phi}^{\dagger f}_u(x)\mu'_f - \bar{\mu}'^f\phi^{\dagger u}_f(x)\mu_u + \bar{w}^{\dot\alpha u}\tilde{\psi}^{\dagger\dot\alpha}_u(x)\mu' - \bar{\mu}'\psi^{\dagger u}_{\dot\alpha}(x)w^{\dot\alpha}_u$$

- The moduli action is not holomorphic.
  - The dependence on $\phi^\dagger(x) = \Phi^\dagger(x,\bar\theta = 0)$, is not extended (in the $\alpha' \to 0$ limit) to anti-chiral multiplets $\Phi^\dagger(x,\bar\theta)$, (same for tilded fields)

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$$S_{mod} + \bar{w}^{u}_{\dot\alpha}\big(\tilde{\phi}^{\dagger}(x)\tilde{\Phi}(x,\theta) + \Phi(x,\theta)\phi^{\dagger}(x)\big)\, w^{\dot\alpha}_{v}$$
$$+ \bar{\mu}^{u}\tilde{\phi}^{\dagger f}_{u}(x)\mu'_{f} - \bar{\mu}'^{f}\phi^{\dagger u}_{f}(x)\mu_{u} + \bar{w}^{\dot\alpha u}\tilde{\psi}^{\dagger\dot\alpha}_{u}(x)\mu' - \bar{\mu}'\psi^{\dagger u}_{\dot\alpha}(x)w^{\dot\alpha}_{u}$$

- These terms involve the “quarks”, and can be rewritten in terms of $D_{\dot\alpha}\Phi^{\dagger}(x,\bar\theta)\big|_{\bar\theta=0}$ and $D_{\dot\alpha}\tilde{\Phi}^{\dagger}(x,\bar\theta)\Big|_{\bar\theta=0}$
  - These moduli interactions are responsible of Beasley-Witten multifermion terms in the effective action (see later)

Blumenhagen et al, 0708.0403; Garcia-Extebarria, 0805.0713

Usual instanton effects

Low energy effective action in the instanton sector:

$$S_{eff} = \int d^4x\, d^2\theta\; \mathrm{e}^{2\pi\tau_{YM}(M_s)} (M_s)^{3N_c - N_f} \int d\widehat{\mathfrak{M}}\, \mathrm{e}^{-S_{mod}(\Phi,\tilde{\Phi})}$$

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- The pure disks and annuli attached to the D(-1) give the exponential of the classical instanton action with the 1-loop coupling τ_{YM} evaluated at M_s
- The dimensionality of $d\mathfrak{M}$ implies the factor $M_s^{3N_c - N_f}$
- Together, these terms reconstruct the dynamical scale $\Lambda^{3N_f - N_c}$

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$$S_{eff} = \int d^4x\, d^2\theta\, \Lambda^{3N_f - N_c} \int d\widehat{\mathfrak{M}}\, e^{-S_{mod}(\Phi,\tilde{\Phi})}$$

- S_{eff} should depend on low-energy fields only.
  - In S_{mod} we incorporated the dependence on the microscopic "quark" multiplets
  - We have to impose the D-flatness condition ▸ Recall on the fields.
  - By doing so, in the result of the integration over $d\mathfrak{M}$ only on the low-energy d.o.f. (meson fields, ...) appear

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- The integrals can be done
- The fermionic integrations impose severe restrictions: contributions to the effective action depend on N_f

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$$S_{eff} = \int d^4x\, d^2\theta\, \Lambda^{3N_f - N_c} \int d\widehat{\mathfrak{M}}\, \mathrm{e}^{-S_{mod}(\Phi,\tilde{\Phi})}$$

- For $N_f = N_c - 1$ one gets

$$S_{eff} = \int d^4x\, d^2\theta\, W(\mathcal{M})$$

where $\mathcal{M}$ is the meson superfield $(\mathcal{M})_f{}^{f'} = \tilde{\Phi}_f{}^u \Phi_u{}^{f'}$ and

$$W(\mathcal{M}) = \frac{\Lambda^{2N_c+1}}{det\mathcal{M}}$$

is the ADS/TVY superpotential

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- For $N_f \geq N_c$, one brings down from the moduli action also "quark" terms $\bar{w}^{\dot{\alpha}u} \tilde{\psi}^{\dagger\dot{\alpha}}_{u}(x)\mu' + \dots$ Recall
- This generates the multifermionic F-terms of Beasley-Witten, of the schematic form

$$\Lambda^{2N_c} \int d^4x\, d^2\theta\, \frac{\bar{D}_{\dot{\alpha}}\Phi^\dagger \bar{D}^{\dot{\alpha}}\Phi^\dagger|_{\bar{\theta}=0}}{\Phi^\dagger \Phi^{2N_c-1}}$$

which are suspersymmetric, and can be written explicitly in terms of the low energy fields

Flux corrections

Applying our results for the flux interactions on D(-1)'s ▸ Recall one gets the following extra contributions to the moduli action:

$$S_{mod}^{(flux)} = \alpha'^2 G_{(0,3)} \lambda_{\dot{\alpha}} \lambda^{\dot{\alpha}} + G_{(3,0)} \theta_\alpha \theta^\alpha + G_{(3,0)} \bar{\mu}_u \mu^u$$

- Recall that , w.r.t. to the gauge thory living on the color branes,
  - $G_{(3,0)}$ appears directly as a gaugino mass m_Λ
  - $G_{(0,3)}$ appears only in the bulk as a gravitino mass $m_{3/2}$
- I will now discuss some of the effects that these extra terms induce in the non-perturbative low energy effective action (very briefly/sketchy: some are just preliminary results!)

Multifermion terms at $N_f = N_c - 1$

- If one pulls down once the term $G_{(3,0)} \bar{\mu}_u \mu^u$, the pattern of integration over the $\bar{\mu}, \mu$ and $\bar{\mu}, \mu'$ becomes similar the $N_f = N_c$ case. One gets BW multifermion terms with the structure

$$G_{(3,0)} \Lambda^{2N_c+1} \int d^4x \, d^2\theta \, \frac{\bar{D}_{\dot{\alpha}} \Phi^\dagger \bar{D}^{\dot{\alpha}} \Phi^\dagger |_{\bar{\theta}=0}}{(\Phi^\dagger)^3 \Phi^{2N}}$$

- This appears to be an effect at low energy of the soft susy breaking induced by $G_{(3,0)}$ in the microscopic theory. No explicit α', should be computable directly in field theory

Stringy effects in ordinary instantons

$G_{(0,3)}$ appears in the moduli action with an α'^2 in front. We must include other terms vanishing in the $\alpha' \to 0$ limit

- From disk diagrams one has extra terms that correspond to

$$\Phi^\dagger(x, \bar\theta = 0) \to \Phi^\dagger(x, \bar\theta = \alpha'\lambda)$$
$$\bar D_{\dot\alpha}\Phi^\dagger(x, \bar\theta = 0) \to \bar D_{\dot\alpha}\Phi^\dagger(x, \bar\theta = \alpha'\lambda)$$

in the field-dependent moduli action ▸ Recall. . .

When the λ-integration is saturated using $\bar\theta$-terms in the above superfields

- The fermionic ADHM constraint is not imposed: we loose contact with gauge instanton solutions

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When the λ-integration is saturated using $\bar\theta$-terms in the above superfields

- We get explicit α' factors in front of the corresponding contributions, which are D-terms

$$(\alpha')^2 \int d^4x\, d^2\theta\, d^2\bar\theta\, f\left(\mathcal{M}, \mathcal{M}^\dagger, \ldots\right)$$

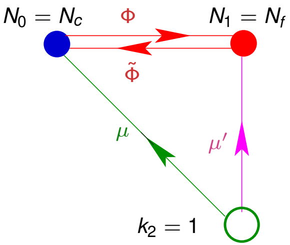
Non-holomorphic terms at $N_f = N_c$

At $N_f = N_c - 1$, by saturating the λ integration with the $G_{(0,3)}$ interaction, one gets in the end a non-holomorphic contribution of the form (e.g., for $N_c = 2$)

$$\alpha'^2 G_{(0,3)} \int d^4x\, d^2\theta \, \frac{det\left(\mathcal{M}^\dagger\right)}{Tr\left(\mathcal{M}^\dagger \mathcal{M}\right)^{\frac{1}{2}}}$$

Exotic (stringy) instantons

- Let us consider a set-up in which the instantonic brane does not correspond to a classical instanton for the gauge group
- D(-1)/D3 strings have only fermionic excitations $\mu_u, \bar{\mu}^u$ and $\mu'_f, \bar{\mu}'^f$



The field-dependent and flux-corrected moduli action is (considering only $G_{(0,3)}$ and $G_{(0,3)}$)

$$S_{mod} = (\alpha')^2 D_c D^c + \mu_u \Phi(x,\theta)^u{}_f \bar{\mu}'^f - \mu'_f \tilde{\Phi}(x,\theta)^f{}_u \bar{\mu}^u + \alpha'^2 G_{(0,3)} \lambda_{\dot{\alpha}} \lambda^{\dot{\alpha}}$$

- Without flux, the integration over the λ's kills any contribution to the effective action

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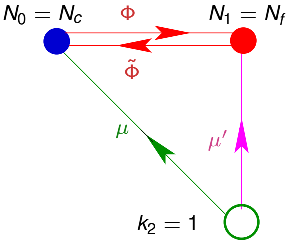

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- Notice that the field-dependent terms are now holomorphic

Exotic but Holomorphic

- Thanks to the presence of the flux, we get exotic effects. For $N_f = N_c$, we get a superpotential contribution:

$$S_{eff} \propto M_s^{2N-2} G_{(0,3)} \int d^4x \, d^2\theta \, det(\mathcal{M})$$

expressed in terms of the meson field $\mathcal{M}$

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- Thank you for your attention!

Open string R vertex

Back

Open string R massles vertex

$$V_{\Theta}(z) = \mathcal{N}_{\Theta}\, \Theta_{\mathcal{A}} \big[\sigma_{\vec{\vartheta}}\, s_{\vec{\epsilon}_{\mathcal{A}}+\vec{\vartheta}}\, \mathrm{e}^{-\frac{1}{2}\phi}\, \mathrm{e}^{\mathrm{i}\, k\cdot X}\big](z)$$

- $\mathcal{A} = 1, \ldots, 16$: (chiral) spinor index of $SO(10)$; runs over possible choices of the weight vector

$$\vec{\epsilon}_{\mathcal{A}} = \frac{1}{2}\Big(\pm, \pm, \pm, \pm, \pm\Big)$$

Open string R vertex

Back

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- $\sigma_{\vec{\vartheta}}(z)$ is the bosonic twist field
- $s_{\vec{q}}(z)$ is the fermionic one:

$$s_{\vec{q}}(z) = \mathrm{e}^{\mathrm{i} \sum_I q^I \varphi^I(z)}$$

where $\varphi^I(z)$ bosonize the world-sheet fermions: $\psi^I = \mathrm{e}^{\mathrm{i}\varphi^I}$

Open string R vertex

Back

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$$V_{\Theta}(z) = \mathcal{N}_{\Theta}\,\Theta_{\mathcal{A}}\big[\sigma_{\vec{\vartheta}}\, s_{\vec{\epsilon}_{\mathcal{A}}+\vec{\vartheta}}\, \mathrm{e}^{-\frac{1}{2}\phi}\, \mathrm{e}^{\mathrm{i}\,k\cdot X}\big](z)$$

- Conformal weight 1 restricts the allowed polarizations:

$$\Theta_{\mathcal{A}} \neq 0 \quad \text{only if} \quad \epsilon^{I}_{\mathcal{A}} = \begin{cases} \pm\frac{1}{2} & \text{for } \vartheta^{I} = 0 \\ -\frac{1}{2} & \text{for } \vartheta^{I} \neq 0 \end{cases}$$

  - All $\vartheta^I = 0$: 10D chiral spinor
  - Only $\vartheta^4 = \vartheta^5 = 0$: space-time chiral spinor
  - $\vartheta^4 = \vartheta^5 = \frac{1}{2}$ (instantonic branes): fermion w/o a spacetime spinor index (as for ADHM charged fermionic instanton moduli)

RR Vertex

Back

Closed string RR vertex (field strengths of type IIB)

$$V_F(z,\bar{z}) = \mathcal{N}_F\, F_{\mathcal{AB}}\, \mathrm{e}^{-\mathrm{i}\pi\alpha' k_{\mathrm{L}}\cdot k_{\mathrm{R}}} \left[s_{\vec{\epsilon}_{\mathcal{A}}}\, \mathrm{e}^{-\frac{1}{2}\phi}\, \mathrm{e}^{\mathrm{i}\, k_{\mathrm{L}}\cdot X}\right](z) \times \left[\widetilde{s}_{\vec{\epsilon}_{\mathcal{B}}}\, \mathrm{e}^{-\frac{1}{2}\widetilde{\phi}}\, \mathrm{e}^{\mathrm{i}\, k_{\mathrm{R}}\cdot \widetilde{X}}\right](\bar{z})$$

- Bi-spinor polarization contains the IIB RR field strengths

$$F_{\mathcal{AB}} = \sum_{n=1,3,5} \frac{1}{n!} F_{M_1\dots M_n} \left(\Gamma^{M_1\dots M_n}\right)_{\mathcal{AB}} \,,$$

RR Vertex

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- In presence of D-branes, left- and right-moving fields identified at the boundary:

$$\widetilde{X}^M(\bar{z}) = (R_0)^M{}_N\, X^N(\bar{z}) \quad , \quad \widetilde{s}_{\vec{\epsilon}_{\mathcal{A}}}(\bar{z}) = (\mathcal{R}_0)^{\mathcal{A}}{}_{\mathcal{B}}\, s_{\vec{\epsilon}_{\mathcal{B}}}(\bar{z})$$

  where $\mathcal{R}_0$ is the spinorial representative of the reflection matrix R_0. Thus

$$F_{\mathcal{AB}} \to (F\mathcal{R}_0)_{\mathcal{AB}}$$

NS-NS vertex

Back

Closed string NS-NS vertex (effective vertex for $H = DB$):

$$V_H(z,\bar{z}) = \mathcal{N}_H \left(\partial_M B_{NP}\right) \mathrm{e}^{-\mathrm{i}\pi\alpha' k_\mathrm{L}\cdot k_\mathrm{R}} \left[\psi^M \psi^N \mathrm{e}^{\mathrm{i}\, k_\mathrm{L}\cdot X}\right](z) \times \left[\widetilde{\psi}^P\, \mathrm{e}^{-\widetilde{\phi}}\, \mathrm{e}^{\mathrm{i}\, k_\mathrm{R}\cdot \widetilde{X}}\right](\bar{z})$$

- In presence of D-branes, the left-right identifications leads to

$$(\partial B) \to (\partial B R_0)$$

with the vectorial reflection matrix R_0

Bulk dependence of gauge lagrangian

Back

- Open strings interact with closed strings. In $\mathcal{N} = 1$ cases, the gauge theory on D-branes depends on the bulk moduli M through the gauge kinetic function:

$$-\frac{i}{8\pi}\int d^2\theta\ f\big(M(\theta)\big)\ \mathrm{Tr}\big(W^\alpha(\theta)W_\alpha(\theta)\big) + h.c. \qquad (1)$$

- Spurion mechanism naturally realized: θ^2 components of $f(M) \leftrightarrow$ gaugino mass. Soft susy breaking.
- The flux-induced gaugino mass fits in this scheme iff internal fluxes $\leftrightarrow$ auxiliary fields for bulk chiral multiplets

Fluxes as auxiliary fields

- This is indeed the case, as encoded in the 4d SUGRA description via the superpotential

$$W = \frac{1}{\kappa_{10}^2} \int G \wedge \Omega = \frac{4}{\kappa_4^2} G_{(0,3)}$$

where $G = F - \tau H$, and $\tau = C_0 + \mathrm{i}e^{-\phi}$. This corresponds to an auxiliary field $F^\tau \propto G_{(0,3)}$. $G_{(3,0)}$ is related to $m_{3/2}$

- In an explicit orbifold setup, e.g. $\mathcal{T}_6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, D3 gauge kinetic function $f \propto \tau$. From the mass term computed in CFT Recall we get Back

$$|m_\Lambda| = |e^{\varphi} F^\tau / 2|$$

in full agreement (including normalizations and dilaton factors) with the GVW superpotential

Details on the orbifold

Back

- Character table and Clebsh-Gordan series:

	e	h_1	h_2	h_3
R_0	1	1	1	1
R_1	1	1	-1	-1
R_2	1	-1	1	-1
R_3	1	-1	-1	1

$$R_0 \otimes R_A = R_A \ , \quad R_i \otimes R_j = \delta_{ij} R_0 + |\epsilon_{ijk}| R_k$$

- Transformations of massless string fields:

NS fields	irrep
$\partial Z^i, \psi^i$	R_i

,

chiral S^A	anti-chiral S_A	irrep
$S^0 \equiv S^{+++}$	$S_0 \equiv S_{---}$	R_0
$S^1 \equiv S^{+--}$	$S_1 \equiv S_{-++}$	R_1
$S^2 \equiv S^{-+-}$	$S_2 \equiv S_{+-+}$	R_2
$S^3 \equiv S^{--+}$	$S_3 \equiv S_{++-}$	R_3