Instantons in (deformed) gauge theories from RNS open strings

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This talk is mostly based on...

M. Billo, M. Frau, I. Pesando, F. Fucito, A. Lerda and A. Liccardo, "Classical gauge instantons from open strings," JHEP **0302** (2003) 045 [arXiv:hep-th/0211250].

M. Billo, M. Frau, I. Pesando and A. Lerda, "N = 1/2 gauge theory and its instanton moduli space from open strings in R-R background," arXiv:hep-th/0402160.

Outline

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Introduction

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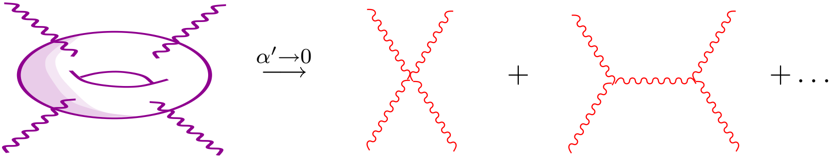
Field theory from strings

- String theory as a tool to study field theories.
- A single string scattering amplitude reproduces, for $\alpha' \to 0$, a sum of Feynman diagrams:



- Moreover,

String theory S-matrix elements $\Rightarrow$ Field theory eff. actions

String amplitudes

- A N-point string amplitude $\mathcal{A}_N$ is schematically given by

$$\mathcal{A}_N = \int_\Sigma \langle V_{\phi_1} \cdots V_{\phi_N} \rangle_\Sigma$$

  - V_{ϕ_i} is the vertex for the emission of the field ϕ_i :

$$V_{\phi_i} \equiv \phi_i \, \mathcal{V}_{\phi_i}$$

  - Σ is a Riemann surface of a given topology
  - $\langle \ldots \rangle_\Sigma$ is the v.e.v. in C.F.T. on Σ.

Gauge theories and D-branes

- In the contemporary string perspective, we can in particular study gauge theories by considering the lightest d.o.f. of open strings suspended between D-branes in a well-suited limit

$\alpha' \to 0$ with gauge quantities fixed.

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Many useful outcomes

- perturbative amplitudes (many gluons, ...) via string techniques;
- construction of "realistic" extensions of Standard model (D-brane worlds)
- AdS/CFT and its extensions to non-conformal cases;
- hints about non-perturbative aspects (Matrix models á la Dijkgraaf-Vafa, certain cases of gauge/gravity duality, ...);
- description of gauge instantons moduli space by means of D3/D(-1) systems.

Non-perturbative aspects: instantons

- We will focus mostly on the stringy description of instantons.

  [Witten, 1995, Douglas, 1995, Dorey et al, 1999], ...

- Our goal is to show how the stringy description of instantons via $D3/D(-1)$ systems is more than a convenient book-keeping for the description of instanton moduli space á la ADHM.
- The D(-1)'s represent indeed the sources responsible for the emission of the non-trivial gauge field profile in the instanton solution.

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Deformations by closed string backgrounds

- Open strings interact with closed strings. We can turn on a closed string background and still look at the massless open string d.o.f..
- In this way, deformations of the gauge theory are naturally suggested by their string realization. Such deformations are characterized by
  - new geometry in (super)space-time;
  - new mathematical structures;
  - new types of interactions and couplings.

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Non-(anti)commutative theories

- The most famous example is that of (gauge) field theories in the background of the $B^{\mu\nu}$ field of the NS-NS sector of closed string. One gets non-commutative field theories, *i.e.* theories defined on a non commutative space-time
- Another case, recently attracting attention, is that of gauge (and matter) fields in the background of a "graviphoton" field strength $C_{\mu\nu}$ from the Ramond-Ramond sector of closed strings. These turn out to be defined on a non-anticommutative superspace

[Ooguri–Vafa, 2003, de Boer et al, 2003, Seiberg, 2003], ...

Instantons from perturbative strings

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Usual string perturbation

- The lowest-order world-sheets Σ in the string perturbative expansion are

  spheres for closed strings, disks for open strings.

- Closed or open vertices have vanishing tadpoles on them:

$$\left\langle \mathcal{V}_{\phi_{\text{closed}}} \right\rangle_{\text{sphere}} = 0 \ , \qquad \left\langle \mathcal{V}_{\phi\,\text{open}} \right\rangle_{\text{disk}} = 0 \ .$$

- No tadpoles $\rightsquigarrow$ these surfaces can describe only the trivial vacua around which ordinary perturbation theory is performed, but are inadequate to describe non-pertubative backgrounds!

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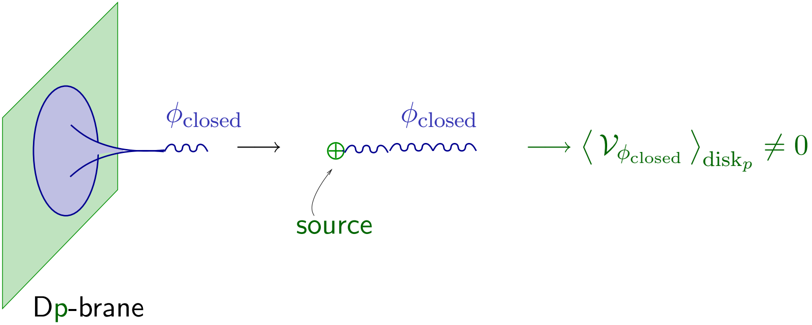
Closed string tadpoles and D-brane solutions

- The microscopic realization of supergravity p-brane solutions as Dp-branes (Polchinski) changes drastically the situation!



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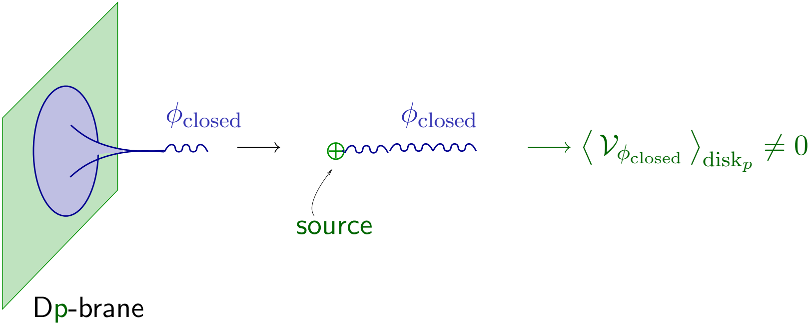

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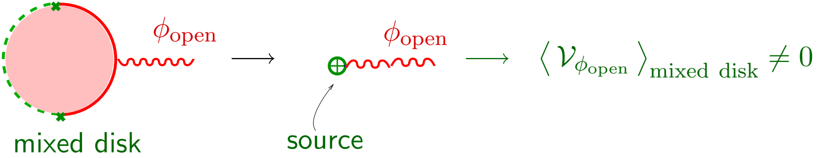
Open string tadpoles and instantons

- This approach can be be generalized to the non-perturbative sector of open strings, in particular to instantons of gauge theories.
- The world-sheets corresponding to istantonic backgrounds are mixed disks, with boundary partly on a D-instanton.

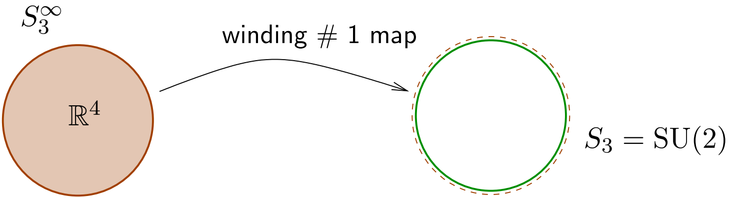


- In this case, this diagram should give the leading long-distance behaviour of the instanton solution.

Instantons & and their moduli (flashing review)

- Consider the $k=1$ instanton of $\mathrm{SU}(2)$ theory

$$A^c_\mu(x) = 2\,\frac{\eta^c_{\mu\nu}(x-x_0)^\nu}{(x-x_0)^2+\rho^2}$$



- $\eta^c_{\mu\nu}$ are the self-dual 't Hooft symbols, and $F_{\mu\nu}$ is self-dual.

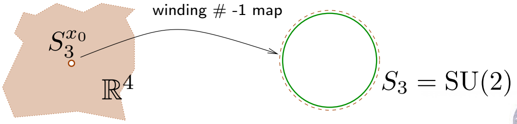
Instantons:Singular gauge

- With a singular gauge transf. $\rightarrow$ so-called singular gauge: ($F_{\mu\nu}$ still self-dual despite the $\bar{\eta}^c_{\mu\nu}$)

Return

$$A^c_\mu(x) = 2\rho^2\,\bar{\eta}^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^2\Big[(x-x_0)^2+\rho^2\Big]}$$

$$\simeq 2\rho^2\,\bar{\eta}^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^4}\left(1-\frac{\rho^2}{(x-x_0)^2}+\ldots\right)$$



Instantons: parameters

- Parameters (moduli) of $k=1$ sol. in $\mathrm{SU}(2)$ theory:

moduli	meaning	#
x_0^μ	center	4
ρ	size	1
$\vec{\theta}$	orientation$^{(*)}$	3

$^{(*)}$ from "large" gauge transf.s $A \to U(\theta) A U^\dagger(\theta)$

Instantons: parameters

- For an $\mathrm{SU}(N)$ theory, embed the $\mathrm{SU}(2)$ instanton in $\mathrm{SU}(N)$:

$$A_\mu = U \begin{pmatrix} \mathbf{0}_{N-2\times N-2} & \mathbf{0} \\ \mathbf{0} & A_\mu^{\mathrm{SU}(2)} \end{pmatrix} U^\dagger$$

  Thus, there are $4N-5$ moduli parametrizing $\frac{\mathrm{SU}(N)}{\mathrm{SU}(N-2)\times\mathrm{U}(1)}$ $\rightarrow$ total # of paramenters: $4N$.

- For instanton # k in $\mathrm{SU}(N)$: total # of moduli: $4Nk$, described by ADHM construction: moduli space as a HiperKähler quotient.

Atiyah,Drinfeld, Hitchin, Manin

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Instanton charge and D-instantons

- The world-volume action of N Dp-branes with a $U(N)$ gauge field F is

$$\mathrm{D.B.I.} \quad + \quad \int_{\mathrm{D}_p} \left[C_{p+1} + \frac{1}{2} C_{p-3} \,\mathrm{Tr}\big(F \wedge F \big) + \ldots \right]$$

- A gauge instanton ($i.e.$ $\mathrm{Tr}\big(F \wedge F\big) \neq 0$) $\rightsquigarrow$ a localized charge for the RR field C_{p-3} $\sim$ a localized D$(p-3)$-brane inside the Dp-branes.
- Instanton-charge k sol.s of 3+1 dims. $\mathrm{SU}(N)$ gauge theories k D-instantons inside N D3-branes

[Witten, 1995, Douglas, 1995, Dorey et al, 1999], ...

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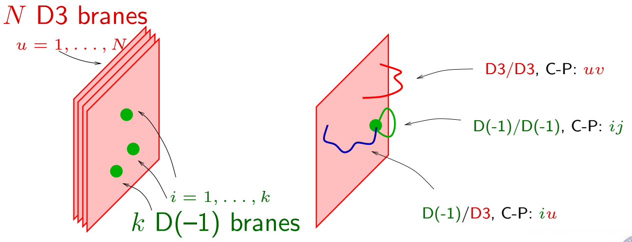
Stringy description of gauge instantons

	1	2	3	4	5	6	7	8	9	10
D3	−	−	−	−	$*$	$*$	$*$	$*$	$*$	$*$
D(−1)	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$



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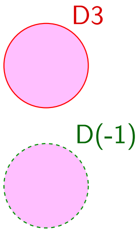
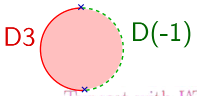
Disk amplitudes and effective actions

Usual disks:



Mixed disks:



Disk amplitudes

↓ $\alpha' \to 0$ field theory limit

Effective actions

D3/D3 → $\mathcal{N} = 4$ SYM action

D(-1)/D(-1) and mixed → ADHM measure

Plan

- We will now discuss a bit more in detail the stringy description of instantons, focusing on the case of pure $\mathrm{SU}(N)$, $\mathcal{N}=1$ SYM.
- Though for simplicity we well discuss mostly its "bosonic" part, this is the supersymmetric theory we will later deform to $\mathcal{N}=1/2$.

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The $\mathbb{C}^3/(\mathbb{Z}_2\times\mathbb{Z}_2)$ orbifold

- Type IIB string theory on target space

$$\mathbb{R}^4 \times \frac{\mathbb{R}^6}{\mathbb{Z}_2\times\mathbb{Z}_2}$$

  Decompose $x^M \to (x^\mu, x^a)$, $(\mu = 1,\ldots 4\,,\quad a = 5,\ldots,10)$.

- $\mathbb{Z}_2\times\mathbb{Z}_2 \subset \mathrm{SO}(6)$ is generated by
  - g_1: a rotation by π in the 7-8 and by $-\pi$ in the 9-10 plane;
  - g_1: a rotation by π in the 5-6 and by $-\pi$ in the 9-10 plane.

- The origin is a fixed point $\Rightarrow$ the orbifold is a singular, non-compact, Calabi-Yau space.

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Residual supersymmetry

- Of the 8 spinor weights of SO(6), $\vec{\lambda} = (\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2})$, only

$$\vec{\lambda}^{(+)} = (+\frac{1}{2}, +\frac{1}{2}, +\frac{1}{2}) \ , \qquad \vec{\lambda}^{(-)} = (-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})$$

  are invariant ones w.r.t. the generators $g_{1,2}$. They are the orbifold realization of the $2 (= 8/4)$ Killing spinors of the CY.
- We remain with $8 (= 32/4)$ real susies in the bulk.
- Only two spin fields survive the orbifold projection:

$$S^{(\pm)} = e^{\pm\frac{i}{2}(\varphi_1+\varphi_2+\varphi_3)} \ ,$$

  where $\varphi_{1,2,3}$ bosonize the SO(6) current algebra.

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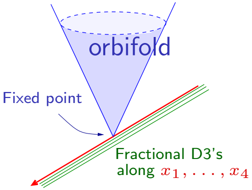
Fractional D3-branes



- Place N fractional D3 branes, localized at the orbifold fixed point. The branes preserve $4 = 8/2$ real supercharges.
- The Chan-Patons of open strings attached to fractional branes transform in an irrep of $\mathbb{Z}_2 \times \mathbb{Z}_2$.
- The fractional branes must sit at the orbifold fixed point.

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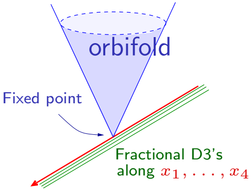

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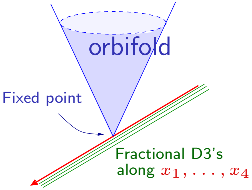

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Fractional D3 branes and pure $\mathcal{N} = 1$ gauge theory

- Spectrum of massless open strings attached to N fractional D3's of a given type corresponds to $\mathcal{N} = 1$ pure $\mathrm{U}(N)$ gauge theory. Schematically,

$$\text{NS:} \begin{cases} \psi^\mu & \rightarrow \quad A_\mu \\ \not\psi^a & \quad \text{no scalars!} \end{cases} \qquad \text{R:} \begin{cases} S^\alpha S^{(+)} & \rightarrow \quad \Lambda_\alpha \\ S^{\dot\alpha} S^{(-)} & \rightarrow \quad \Lambda_{\dot\alpha} \end{cases}$$

- The standard action is retrieved from disk amplitudes in the $\alpha' \rightarrow 0$ limit:

$$S = \frac{1}{g^2_{\mathrm{YM}}} \int d^4x \, \mathrm{Tr}\Big(\frac{1}{2} F^2_{\mu\nu} - 2\bar\Lambda_{\dot\alpha} \bar{\not{D}}^{\dot\alpha\beta} \Lambda_\beta \Big) .$$

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Auxiliary fields

- The action can be obtained from cubic diagram only introducing the (anti-selfdual) auxiliary field $H_{\mu\nu} \equiv H_c \bar{\eta}^c_{\mu\nu}$:

$$S' = \frac{1}{g^2_{\rm YM}} \int d^4x \, \mathrm{Tr}\Big\{ \big(\partial_\mu A_\nu - \partial_\nu A_\mu\big)\partial^\mu A^\nu + 2\mathrm{i}\, \partial_\mu A_\nu \big[A^\mu, A^\nu\big] - 2\bar{\Lambda}_{\dot\alpha} \bar{\not{D}}^{\dot\alpha\beta}\Lambda_\beta + H_c H^c + H_c\, \bar{\eta}^c_{\mu\nu} \big[A^\mu, A^\nu\big] \Big\} \ ,$$

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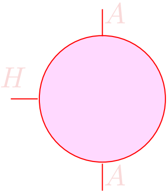
Auxiliary fields in the open string set-up

- The auxiliary field $H_{\mu\nu}$ is associated to the (non-BRST invariant) vertex

$$V_H(y;p) = (2\pi\alpha')\,\frac{H_{\mu\nu}(p)}{2}\,\psi^\nu\psi^\mu(y)\,\mathrm{e}^{i\sqrt{2\pi\alpha'}p\cdot X(y)} \ .$$



We have then, for instance,

$$\frac{1}{2}\langle\!\langle\, V_H\, V_A\, V_A\,\rangle\!\rangle \;=\; -\frac{1}{g^2_{\mathrm{YM}}}\,\mathrm{Tr}\Big(H_{\mu\nu}(p_1)A^\mu(p_2)A^\nu(p_3)\Big) \;+\; \text{other ordering}$$

$\rightsquigarrow$ last term in the previous action.

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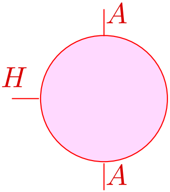

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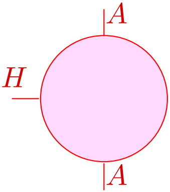

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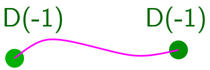
Moduli spectrum in the $\mathcal{N}=1$ case

D(-1)/D(-1) strings

With k D(-1)'s, all vertices have Chan-Paton factors in the adjoint of $\mathrm{U}(k)$.



Neveu-Schwarz sector

The vertices surviving the orbifold projection are

$$V_a(y) = (2\pi\alpha')^{\frac{1}{2}}\, g_0\, a_\mu\, \psi^\mu(y)\, \mathrm{e}^{-\phi(y)} \;.$$

Moduli spectrum in the $\mathcal{N}=1$ case

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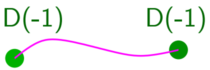

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- Here g_0 is the coupling on the D(-1) theory:

$$C_0 = \frac{1}{2\pi^2\alpha'^2}\,\frac{1}{g_0^2} = \frac{8\pi^2}{g_{\mathrm{YM}}^2}\ .$$

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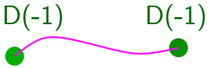

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- C_0 = normaliz. of disks with (partly) D(-1) boundary. Since g_{YM} is fixed as $\alpha' \to 0$, g_0 blows up.

Moduli spectrum in the $\mathcal{N}=1$ case

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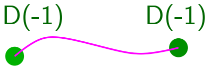

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The vertices surviving the orbifold projection are

$$V_a(y) = (2\pi\alpha')^{\frac{1}{2}}\, g_0\, a_\mu\, \psi^\mu(y)\, \mathrm{e}^{-\phi(y)} \ .$$

- The moduli a_μ are *rescaled* with powers of g_0 so that their interactions survive when $\alpha' \to 0$ with g^2_{YM} fixed.

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Moduli spectrum in the $\mathcal{N}=1$ case

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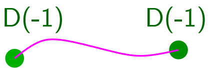

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Moduli spectrum in the $\mathcal{N}=1$ case

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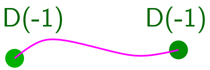

Neveu-Schwarz sector

The vertices surviving the orbifold projection are

$$V_a(y) = (2\pi\alpha')^{\frac{1}{2}}\, g_0\, a_\mu\, \psi^\mu(y)\, \mathrm{e}^{-\phi(y)} \;.$$

Moreover, we have the auxiliary vertex decoupling the quartic interactions

$$V_D(y) = (2\pi\alpha')\, \frac{D_c\, \bar{\eta}^c_{\mu\nu}}{2}\, \psi^\nu\psi^\mu(y) \;,$$

Moduli spectrum in the $\mathcal{N}=1$ case

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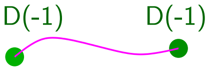


Ramond sector

The vertices surviving the orbifold projection are

$$V_M(y) = (2\pi\alpha')^{\frac{3}{4}}\,\frac{g_0}{\sqrt{2}}\,M'^{\alpha}\,S_\alpha(y)S^{(-)}(y)\,\mathrm{e}^{-\frac{1}{2}\phi(y)}\quad,$$

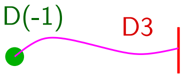
$$V_\lambda(y) = (2\pi\alpha')^{\frac{3}{4}}\,\lambda_{\dot\alpha}\,S^{\dot\alpha}(y)S^{(+)}(y)\,\mathrm{e}^{-\frac{1}{2}\phi(y)}\quad.$$

- M'^{α} has dimensions of (length)$^{\frac{1}{2}}$, $\lambda_{\dot\alpha}$ of (length)$^{-\frac{3}{2}}$.

Moduli spectrum in the $\mathcal{N}=1$ case

D(-1)/D3 strings

All vertices have Chan-Patons in the bifundamental of $\mathrm{U}(k)\times\mathrm{U}(N)$.



Neveu-Schwarz sector

The vertices surviving the orbifold projection are

$$V_w(y) = (2\pi\alpha')^{\frac{1}{2}}\,\frac{g_0}{\sqrt{2}}\,w_{\dot\alpha}\,\Delta(y)\,S^{\dot\alpha}(y)\,\mathrm{e}^{-\phi(y)} \quad ,$$

$$V_{\bar w}(y) = (2\pi\alpha')^{\frac{1}{2}}\,\frac{g_0}{\sqrt{2}}\,\bar w_{\dot\alpha}\,\bar\Delta(y)\,S^{\dot\alpha}(y)\,\mathrm{e}^{-\phi(y)} \quad ,$$

- The (anti-)twist fields $\Delta,\bar\Delta$ switch the b.c.'s on the X^μ string fields.

Moduli spectrum in the $\mathcal{N}=1$ case

D(-1)/D3 strings

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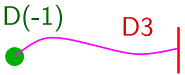

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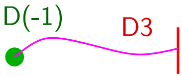
- w and w have dimensions of (length) and are related to the size of the instanton solution.

Moduli spectrum in the $\mathcal{N}=1$ case

D(-1)/D3 strings

All vertices have Chan-Patons in the bifundamental of $\mathrm{U}(k)\times\mathrm{U}(N)$.



Ramond sector

The vertices surviving the orbifold projection are

$$V_{\mu}(y) = (2\pi\alpha')^{\frac{3}{4}}\,\frac{g_0}{\sqrt{2}}\,\mu\,\Delta(y)\,S^{(-)}(y)\,\mathrm{e}^{-\frac{1}{2}\phi(y)}\quad ,$$

$$V_{\bar\mu}(y) = (2\pi\alpha')^{\frac{3}{4}}\,\frac{g_0}{\sqrt{2}}\,\bar\mu\,\bar\Delta(y)\,S^{(-)}(y)\,\mathrm{e}^{-\frac{1}{2}\phi(y)}\quad .$$

- The fermionic moduli μ,$\bar\mu$ have dimensions of $(\text{length})^{1/2}$.

The $\mathcal{N}=1$ moduli action

- (Mixed) disk diagrams with the above moduli, for $\alpha' \to 0$ yield

$$S_{\rm mod} = \mathrm{tr}\Big\{ -\,\mathrm{i} D_c \Big(W^c + \mathrm{i}\bar\eta^c_{\mu\nu}\big[a'^{\mu}, a'^{\nu}\big]\Big) \\ -\mathrm{i}\lambda^{\dot\alpha}\Big(w^u{}_{\dot\alpha}\,\bar\mu_u + \mu^u \bar w_{\dot\alpha u} + \big[a'_{\alpha\dot\alpha}, M'^{\alpha}\big]\Big)\Big\}$$

where $(W^c)_j{}^i = w^{iu}{}_{\dot\alpha}\,(\tau^c)^{\dot\alpha}{}_{\dot\beta}\,\bar w^{\dot\beta}{}_{uj}$

- D_c and $\lambda^{\dot\alpha}$ $\sim$ Lagrange multipliers for the (super)ADHM constraints

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The $\mathcal{N}=1$ ADHM constraints

- The ADHM constraints are three $k \times k$ matrix eq.s

$$W^c + \mathrm{i}\bar{\eta}^c_{\mu\nu}\left[a'^{\mu}, a'^{\nu}\right] = \mathbf{0}\ .$$

- and their fermionic counterparts

$$w^u{}_{\dot{\alpha}}\,\bar{\mu}_u + \mu^u \bar{w}_{\dot{\alpha}u} + \left[a'_{\alpha\dot{\alpha}}, M'^{\alpha}\right] = \mathbf{0}\ .$$

- Once these constraints are satisfied, the moduli action vanishes.

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Parameter counting

- E.g., for the bosonic parameters

	#
a'^{μ}	$4k^2$
$w_{\dot\alpha}, \bar{w}_{\dot\alpha}$	$4kN$
ADHM constraints	$-3k^2$
Global $\mathrm{U}(k)$ inv.	$-k^2$
True moduli	$4kN$

- **After** imposing the constraints, more or less

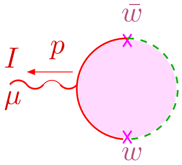
$$a'^{\mu} \rightsquigarrow \text{multi-center positions, } \ldots$$

$$w_{\dot\alpha}, \bar{w}_{\dot\alpha} \rightsquigarrow \text{size, orientation inside } \mathrm{SU}(N), \ldots$$

The instanton solution from mixed disks

- Mixed disks = sources for gauge theory fields. The amplitude for emitting a gauge field is

$$A^I_\mu(p) = \langle\, \mathcal{V}_{A^I_\mu}(-p)\,\rangle_{\mathrm{m.d}} = \langle\!\langle\, V_{\bar w}\, \mathcal{V}_{A^I_\mu}(-p)\, V_w \,\rangle\!\rangle$$
$$= \mathrm{i}\,(T^I)^v{}_u\, p^\nu\, \bar\eta^c_{\nu\mu}\big(w^u{}_{\dot\alpha}\,(\tau^c)^{\dot\alpha}{}_{\dot\beta}\,\bar w^{\dot\beta}{}_v\big)\,\mathrm{e}^{-\mathrm{i}p\cdot x_0}\ .$$



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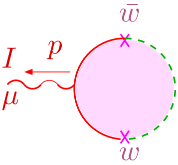

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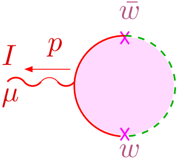
- $\mathcal{V}_{A^I_\mu}(-p)$: no polariz., outgoing p, 0-picture

$$\mathcal{V}_{A^I_\mu}(z;-p) = 2\mathrm{i}T^I\,(\partial X_\mu - \mathrm{i}p\cdot\psi\psi_\mu)\,\mathrm{e}^{-\mathrm{i}p\cdot X}(z)$$

The instanton solution from mixed disks



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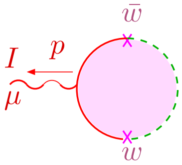
- N.B. From now on we set $k=1$, i.e. we consider instanton number 1.

The instanton solution from mixed disks



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- x_0 = pos. of the D(-1). Broken transl. invariance in the D3 world-volume $\rightsquigarrow$ "tadpole"

$$\langle\, \mathrm{e}^{-\mathrm{i}p\cdot X}\, \rangle_{\mathrm{m.d}} \propto \mathrm{e}^{\mathrm{i}p\cdot x_0}\;.$$

The classical profile

- The classical profile of the gauge field emitted by the mixed disk is obtained by attaching a free propagator and Fourier transforming:

$$A^I_\mu(x) = \int \frac{d^4p}{(2\pi)^2}\, A^I_\mu(p)\, \frac{1}{p^2}\, \mathrm{e}^{\mathrm{i}p\cdot x}$$
$$= 2\,(T^I)^v{}_u \left[(T^c)^u{}_v\right] \bar\eta^c_{\mu\nu} \frac{(x-x_0)^\nu}{(x-x_0)^4}\ ,$$

where $(T^I)^v{}_u$ are the $\mathrm{U}(N)$ generators and

$$(T^c)^u{}_v = w^u{}_{\dot\alpha}\, (\tau^c)^{\dot\alpha}{}_{\dot\beta}\, \bar w^{\dot\beta}{}_v\ .$$

The classical instanton profile

- In the above solution we still have the unconstrained moduli $\bar{w}, w$.

The classical instanton profile

- In the above solution we still have the unconstrained moduli $\bar{w}, w$.
- We must still impose the bosonic ADHM constraints

$$W^c \equiv w^u_{\dot\alpha} (\tau^c)^{\dot\alpha}{}_{\dot\beta} \bar{w}^{\dot\beta}{}_v = 0.$$

The classical instanton profile

- In the above solution we still have the unconstrained moduli $\bar{w}, w$.
- Iff $W^c = 0$, the $N \times N$ matrices

$$(t_c)^u{}_v \equiv \frac{1}{2\rho^2}\left(w_{\dot\alpha}{}^u (\tau_c)^{\dot\alpha}{}_{\dot\beta}\bar{w}^{\dot\beta}{}_v\right) ,$$

where

$$2\rho^2 = w^u{}_{\dot\alpha}\,\bar{w}^{\dot\alpha}{}_u ,$$

satisfy an $\mathrm{su}(2)$ subalgebra: $[t_c, t_d] = \mathrm{i}\epsilon_{cde}\, t_e$.

The classical instanton profile

- The gauge vector profile can be written as

$$A^I_\mu(x) = 4\rho^2 \mathrm{Tr}\,(T^I\, t_c)\bar{\eta}^c_{\mu\nu}\frac{(x-x_0)^\nu}{(x-x_0)^4}$$

The classical instanton profile

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$$A^I_\mu(x) = 4\rho^2 \mathrm{Tr}\,(T^I\, t_c)\bar{\eta}^c_{\mu\nu}\frac{(x-x_0)^\nu}{(x-x_0)^4}$$

- This is is a moduli-dependent (through t_c) embedding in $\mathrm{su}(N)$ of the $\mathrm{su}(2)$ instanton connection in
  - large-distance leading approx. $(|x-x_0| \gg \rho)$
  - singular gauge

▸ Recall the singular gauge

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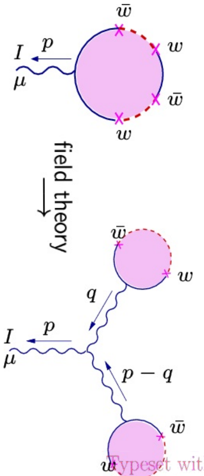
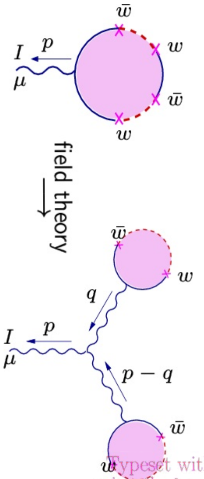
Additional remarks



- The mixed disks emit also a gaugino $\Lambda^{\alpha,I}$ $\rightsquigarrow$ account for its leading profile in the super-instanton solution.
- Subleading terms in the long-distance expansion of the solution arise from emission diagrams with more moduli insertions.
- At the field theory level, they correspond to having more source terms.

Additional remarks



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- At the field theory level, they correspond to having more source terms.

Additional remarks

Question: Why singular gauge?

- Instanton produced by a point-like source, the D(-1), inside the D3 $\rightarrow$ singular at the location of the source
- In the singular gauge, rapid fall-off of the fields $\rightarrow$ eq.s of motion reduce to *free* eq.s *at large distance* $\rightarrow$ "perturbative" solution in terms of the source term
- non-trivial properties of the instanton profile from the region near the singularity through the embedding

$$S_3^{x_0} \hookrightarrow \mathrm{SU}(2) \subset \mathrm{SU}(N)$$

Deformations of gauge theories from closed strings

Non-anticommutative theories and RR backgrounds

$C_{\mu\nu}$ RR background: new geometry

- A class of "deformed" field theories, recently attracting attention, is that of gauge (and matter) fields in the background of a "graviphoton" field strength $C_{\mu\nu}$ from the Ramond-Ramond sector of closed strings.
- These turn out to be defined on a non-anticommutative superspace, where the, say, anti-chiral fermionic coordinates satisfy

$$\left\{\theta^{\dot\alpha}, \theta^{\dot\beta}\right\} \propto C^{\dot\alpha\dot\beta} \propto (\sigma^{\mu\nu})^{\dot\alpha\dot\beta} C_{\mu\nu}.$$

.

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Non-anticommutative theories and RR backgrounds

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.

Non-anticommutative theories and RR backgrounds

$C_{\mu\nu}$ RR background: new structure

- The superspace deformation can be rephrased as a modification of the product among functions, which now becomes

$$f(\theta) \star g(\theta) = f(\theta) \exp\Big(- \frac{1}{2} \overleftarrow{\frac{\partial}{\partial \theta^{\dot\alpha}}} C^{\dot\alpha\dot\beta} \overrightarrow{\frac{\partial}{\partial \theta^{\dot\beta}}} \Big) g(\theta) \ .$$

- There are also new interactions between the gauge and matter fields: see later in the talk.

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Non-anticommutative theories and RR backgrounds

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Non-anticommutative theories and RR backgrounds

Plan

- We shall analyze the deformation of $\mathcal{N} = 1$ pure gauge theory induced by a RR "graviphoton" $C_{\mu\nu}$, the so-called $\mathcal{N} = 1/2$ gauge theory.

  [Seiberg, 2003], ...

- We shall discuss how to derive explicitely the $\mathcal{N} = 1/2$ theory from string diagrams (in the traditional RNS formulation).
- Moreover we will derive from string diagrams the instantonic solutions of this theory and their ADHM moduli space.

The graviphoton background

- RR vertex in 10D, in the symmetric superghost picture:

$$\mathcal{F}_{\dot{A}\dot{B}}\, S^{\dot{A}} \mathrm{e}^{-\phi/2}(z)\, \tilde{S}^{\dot{B}} \mathrm{e}^{-\tilde{\phi}/2}(\bar{z}) \ .$$

  Bispinor $\mathcal{F}_{\dot{A}\dot{B}} \rightsquigarrow$ 1-, 3- and a.s.d. 5-form field strengths.

- On $\mathbb{R}^4 \times \frac{\mathbb{R}^6}{\mathbb{Z}_2 \times \mathbb{Z}_2}$, a surviving 4D bispinor vertex is

$$\mathcal{F}_{\dot{\alpha}\dot{\beta}}\, S^{\dot{\alpha}} S^{(+)} \mathrm{e}^{-\phi/2}(z)\, \tilde{S}^{\dot{\beta}} \tilde{S}^{(+)} \mathrm{e}^{-\tilde{\phi}/2}(\bar{z}) \ .$$

  with $\mathcal{F}_{\dot{\alpha}\dot{\beta}} = \mathcal{F}_{\dot{\beta}\dot{\alpha}}$.

- Decomposing the 5-form along the holom. 3-form of the CY $\rightsquigarrow$ an a.s.d. 2-form in 4D

$$C_{\mu\nu} \propto \mathcal{F}_{\dot{\alpha}\dot{\beta}} (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}\dot{\beta}} \ ,$$

  the graviphoton f.s. of $\mathcal{N} = 1/2$ theories.

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  using the graviphoton f.s. of $\mathcal{N} = 1/2$ theories.

The graviphoton background

- RR vertex in 10D, in the symmetric superghost picture:

$$\mathcal{F}_{\dot{A}\dot{B}}\, S^{\dot{A}} \mathrm{e}^{-\phi/2}(z)\, \tilde{S}^{\dot{B}} \mathrm{e}^{-\tilde{\phi}/2}(\bar{z}) \ .$$

  Bispinor $\mathcal{F}_{\dot{A}\dot{B}} \rightsquigarrow$ 1-, 3- and a.s.d. 5-form field strengths.
- On $\mathbb{R}^4 \times \frac{\mathbb{R}^6}{\mathbb{Z}_2\times\mathbb{Z}_2}$, a surviving 4D bispinor vertex is

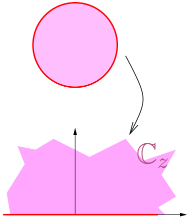
$$\mathcal{F}_{\dot{\alpha}\dot{\beta}}\, S^{\dot{\alpha}} S^{(+)} \mathrm{e}^{-\phi/2}(z)\, \tilde{S}^{\dot{\beta}} \tilde{S}^{(+)} \mathrm{e}^{-\tilde{\phi}/2}(\bar{z}) \ .$$

  with $\mathcal{F}_{\dot{\alpha}\dot{\beta}} = \mathcal{F}_{\dot{\beta}\dot{\alpha}}$.
- Decomposing the 5-form along the holom. 3-form of the CY $\rightsquigarrow$ an a.s.d. 2-form in 4D

$$C_{\mu\nu} \propto \mathcal{F}_{\dot{\alpha}\dot{\beta}} (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}\dot{\beta}} \ ,$$

  the graviphoton f.s. of $\mathcal{N}=1/2$ theories.

Inserting graviphotons in disk amplitudes



- Conformally mapping the disk to the upper half z-plane, the D3 boundary conditions on spin fields read

$$S^{\dot\alpha} S^{(+)}(z) = \tilde S^{\dot\alpha} \tilde S^{(+)}(\bar z)\Big|_{z=\bar z} \ .$$

(opposite sign for $\tilde S^{\alpha} \tilde S^{(-)}(\bar z)$).

- When closed string vertices are inserted in a D3 disk,

$$\tilde S^{\dot\alpha} \tilde S^{(+)}(\bar z) \longrightarrow S^{\dot\alpha} S^{(+)}(\bar z) \ .$$

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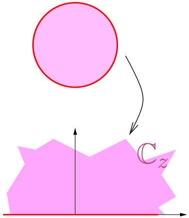

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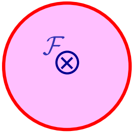
Disk amplitudes with a graviphoton



Start inserting a graviphoton vertex:

$$\langle\!\langle \, V_\Lambda \, V_\Lambda \, V_\Lambda \, V_{\mathcal{F}} \, \rangle\!\rangle$$

where

$$V_{\mathcal{F}}(z,\bar{z}) = \mathcal{F}_{\dot{\alpha}\dot{\beta}} \, S^{\dot{\alpha}} S^{(+)} \mathrm{e}^{-\phi/2}(z) \, S^{\dot{\beta}} S^{(+)} \mathrm{e}^{-\phi/2}(\bar{z}) \ .$$

$\rightsquigarrow$ we need two $S^{(-)}$ operators to "saturate the charge"

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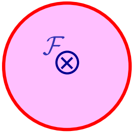
Disk amplitudes with a graviphoton



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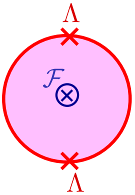


We insert therefore two chiral gauginos:

$$\langle\!\langle\, V_\Lambda\, V_\Lambda\, V_\Lambda\, V_{\mathcal{F}}\, \rangle\!\rangle$$

with vertices

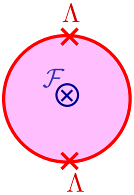
$$V_\Lambda(y;p) = (2\pi\alpha')^{\frac{3}{4}}\, \Lambda^\alpha(p)\, S_\alpha S^{(-)}\, \mathrm{e}^{-\frac{1}{2}\phi(y)}\, \mathrm{e}^{i\sqrt{2\pi\alpha'}p\cdot X(y)}\,.$$

Without other insertions, however,

$$\langle S^{\dot\alpha} S^{\dot\beta} S_\alpha S_\beta\rangle \propto \epsilon^{\dot\alpha\dot\beta}\epsilon_{\alpha\beta}$$

$\rightsquigarrow$ vanishes when contracted with $\mathcal{F}_{\dot\alpha\dot\beta}$.

Disk amplitudes with a graviphoton



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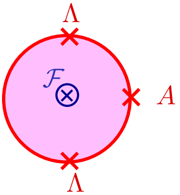

To cure this problem, insert a gauge field vertex:

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that must be in the 0 picture:

$$V_A(y;p) = 2\mathrm{i}\,(2\pi\alpha')^{\frac{1}{2}}\,A_\mu(p)\Big(\partial X^\mu(y) + \mathrm{i}\,(2\pi\alpha')^{\frac{1}{2}}\,p\cdot\psi\,\psi^\mu(y)\Big)\,\mathrm{e}^{\mathrm{i}\sqrt{2\pi\alpha'}p\cdot X(y)}$$

$\rightsquigarrow$ finally, we may get a non-zero result!

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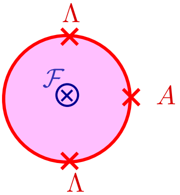

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$\rightsquigarrow$ finally, we may get a non-zero result!

Evaluation of the amplitude

- We have

$$\langle\!\langle\, V_\Lambda V_\Lambda V_A V_{\mathcal{F}} \,\rangle\!\rangle \equiv C_4 \int \frac{\prod_i dy_i dz d\bar{z}}{dV_{\mathrm{CKG}}} \left\langle V_\Lambda(y_1;p_1) V_\Lambda(y_2;p_2) V_A(y_3;p_3) V_{\mathcal{F}}(z,\bar{z}) \right\rangle$$

where the normalization for a D3 disk is

$$C_4 = \frac{1}{\pi^2\alpha'^2}\,\frac{1}{g_{\mathrm{YM}}^2}$$

and the $\mathrm{SL}(2,\mathbb{R})$-invariant volume is

$$dV_{\mathrm{CGK}} = \frac{dy_a\, dy_b\, dy_c}{(y_a - y_b)(y_b - y_c)(y_c - y_a)} \quad .$$

Explicit expression of the amplitude

- Altogether, the explicit expression is

Skip details

$$\langle\!\langle V_\Lambda V_\Lambda V_A V_{\mathcal{F}} \rangle\!\rangle = \frac{8}{g_{\mathrm{YM}}^2} (2\pi\alpha')^{\frac{1}{2}} \,\mathrm{Tr}\Big(\Lambda^\alpha(p_1)\,\Lambda^\beta(p_2)\,p_3^\nu A^\mu(p_3)\Big)\,\mathcal{F}_{\dot\alpha\dot\beta}$$

$$\times \int \frac{\prod_i dy_i dz d\bar{z}}{dV_{\mathrm{CKG}}} \Big\{ \big\langle S_\alpha(y_1)\,S_\beta(y_2)\; :\psi^\nu\psi^\mu:(y_3)\,S^{\dot\alpha}(z)\,S^{\dot\beta}(\bar z)\big\rangle$$

$$\times \big\langle S^{(-)}(y_1)\,S^{(-)}(y_2)\,S^{(+)}(z)\,S^{(+)}(\bar z)\big\rangle$$

$$\times \big\langle \mathrm{e}^{-\frac{1}{2}\phi(y_1)}\mathrm{e}^{-\frac{1}{2}\phi(y_2)}\mathrm{e}^{-\frac{1}{2}\phi(z)}\mathrm{e}^{-\frac{1}{2}\phi(\bar z)}\big\rangle$$

$$\times \big\langle \mathrm{e}^{\mathrm{i}\sqrt{2\pi\alpha'}p_1\cdot X(y_1)}\mathrm{e}^{\mathrm{i}\sqrt{2\pi\alpha'}p_2\cdot X(y_2)}\mathrm{e}^{\mathrm{i}\sqrt{2\pi\alpha'}p_3\cdot X(y_3)}\big\rangle \Big\} .$$

Evaluation of the amplitude: correlators

- The relevant correlators are:

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Evaluation of the amplitude: correlators

- The relevant correlators are:

1. Superghosts

$$\left\langle \mathrm{e}^{-\frac{1}{2}\phi(y_1)}\,\mathrm{e}^{-\frac{1}{2}\phi(y_2)}\,\mathrm{e}^{-\frac{1}{2}\phi(z)}\,\mathrm{e}^{-\frac{1}{2}\phi(\bar{z})}\right\rangle$$

$$= \Big[(y_1 - y_2)\,(y_1 - z)\,(y_1 - \bar{z})\,(y_2 - z)\,(y_2 - \bar{z})\,(z - \bar{z})\Big]^{-\frac{1}{4}} \quad .$$

Evaluation of the amplitude: correlators

- The relevant correlators are:

2. Internal spin fields

$$\left\langle S^{(-)}(y_1) S^{(-)}(y_2) S^{(+)}(z) S^{(+)}(\bar{z}) \right\rangle = (y_1 - y_2)^{\frac{3}{4}} (y_1 - z)^{-\frac{3}{4}} (y_1 - \bar{z})^{-\frac{3}{4}} (y_2 - z)^{-\frac{3}{4}} (y_2 - \bar{z})^{-\frac{3}{4}} \times (z - \bar{z})^{\frac{3}{4}} \quad .$$

Evaluation of the amplitude: correlators

- The relevant correlators are:

3. 4D spin fields

$$\left\langle S_\gamma(y_1)S_\delta(y_2) :\psi^\mu\psi^\nu:(y_3)\, S^{\dot\alpha}(z)S^{\dot\beta}(\bar z)\right\rangle$$
$$= \frac{1}{2}\,(y_1-y_2)^{-\frac{1}{2}}(z-\bar z)^{-\frac{1}{2}}$$
$$\times\left((\sigma^{\mu\nu})_{\gamma\delta}\,\varepsilon^{\dot\alpha\dot\beta}\,\frac{(y_1-y_2)}{(y_1-y_3)(y_2-y_3)}\right.$$
$$\left.+\,\varepsilon_{\gamma\delta}\,(\bar\sigma^{\mu\nu})^{\dot\alpha\dot\beta}\,\frac{(z-\bar z)}{(y_3-z)(y_3-\bar z)}\right)\quad .$$

Evaluation of the amplitude: correlators

- The relevant correlators are:

4. Momentum factors

$$\left\langle \mathrm{e}^{\mathrm{i}\sqrt{2\pi\alpha'}p_1\cdot X(y_1)}\,\mathrm{e}^{\mathrm{i}\sqrt{2\pi\alpha'}p_2\cdot X(y_2)}\,\mathrm{e}^{\mathrm{i}\sqrt{2\pi\alpha'}p_3\cdot X(y_3)}\right\rangle \xrightarrow{\text{on shell}} 1 \ .$$

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Evaluation of the amplitude: $\mathrm{SL}(2,\mathbb{R})$ fixing

- We may, for instance, choose

$$y_1 \to \infty \ , \quad z \to \mathrm{i} \ , \quad \bar{z} \to -\mathrm{i} \ .$$

- The remaining integrations turn out to be

$$\int_{-\infty}^{+\infty} dy_2 \int_{-\infty}^{y_2} dy_3 \, \frac{1}{\left(y_2^2 + 1\right)\left(y_3^2 + 1\right)} = \frac{\pi^2}{2} \ .$$

Symmetry factor $1/2$ and other ordering compensate each other.

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Final result for the amplitude

- We finally obtain for $\langle\!\langle\, V_\Lambda\, V_\Lambda\, V_A\, V_{\mathcal{F}}\,\rangle\!\rangle$ the result

$$\frac{8\pi^2}{g_{\rm YM}^2}\,(2\pi\alpha')^{\frac{1}{2}}\,{\rm Tr}\Big(\Lambda(p_1)\cdot\Lambda(p_2)\,p_3^\nu A^\mu(p_3)\Big)\,\mathcal{F}_{\dot\alpha\dot\beta}\,(\bar\sigma_{\nu\mu})^{\dot\alpha\dot\beta}\quad .$$

- This result is finite for $\alpha'\to 0$ if we keep constant

$$C_{\mu\nu}\equiv 4\pi^2\,(2\pi\alpha')^{\frac{1}{2}}\,\mathcal{F}_{\dot\alpha\dot\beta}\,(\bar\sigma_{\mu\nu})^{\dot\alpha\dot\beta}$$

- $C_{\mu\nu}$, of dimension (length) will be exactly the one of $\mathcal{N}=1/2$ theory.
- We get an extra term in the gauge theory action:

$$\frac{\rm i}{g_{\rm YM}^2}\int d^4x\,{\rm Tr}\Big(\Lambda\cdot\Lambda\,\big(\partial^\mu A^\nu-\partial^\nu A^\mu\big)\Big)C_{\mu\nu}\ .$$

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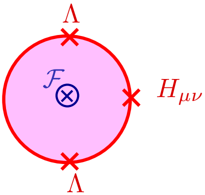
Another contribute

- Another possible diagram with a graviphoton insertion is

$$\langle\!\langle\, V_\Lambda \, V_\Lambda \, V_H \, V_{\mathcal{F}} \,\rangle\!\rangle \, .$$



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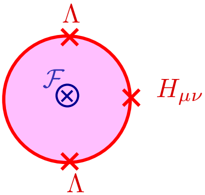

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$$\langle\!\langle\, V_\Lambda\, V_\Lambda\, V_H\, V_{\mathcal{F}}\,\rangle\!\rangle\,.$$

- Recall that the auxiliary field vertex in the 0 picture is

$$V_H(y;p) = (2\pi\alpha')\frac{H_{\mu\nu}(p)}{2}\psi^\nu\psi^\mu(y)\mathrm{e}^{i\sqrt{2\pi\alpha'}p\cdot X(y)}$$

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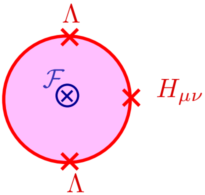

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$$\langle\!\langle\, V_\Lambda\, V_\Lambda\, V_H\, V_{\mathcal{F}} \,\rangle\!\rangle\,.$$

- The evaluation of this amplitude paralles exactly the previous one and contributes to the field theory action the term:

$$\frac{1}{2g_{\rm YM}^2}\int d^4x\ \mathrm{Tr}\left(\Lambda\cdot\Lambda\, H^{\mu\nu}\right) C_{\mu\nu}\ ,$$

having introduced $C_{\mu\nu}$ as above.

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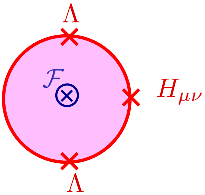
Another contribute



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- All other amplitudes involving $\mathcal{F}$ vertices either
  - vanish because of their tensor structure;
  - vanish in the $\alpha' \to 0$ limit, with $C_{\mu\nu}$ fixed.

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The deformed gauge theory action

- From disk diagrams with RR insertions we obtain, in the field theory limit

$$\alpha' \to 0 \quad \text{with } C_{\mu\nu} \equiv 4\pi^2 \, (2\pi\alpha')^{\frac{1}{2}} \, \mathcal{F}_{\dot\alpha\dot\beta} \, (\bar\sigma_{\mu\nu})^{\dot\alpha\dot\beta} \quad \text{fixed}$$

the action

$$\tilde{S}' = \frac{1}{g^2_{\rm YM}} \int d^4x \, \mathrm{Tr} \Big\{ (\partial_\mu A_\nu - \partial_\nu A_\mu) \partial^\mu A^\nu + 2\mathrm{i}\, \partial_\mu A_\nu \big[A^\mu, A^\nu\big]$$
$$- \, 2\bar\Lambda_{\dot\alpha} \bar{\not{D}}^{\dot\alpha\beta} \Lambda_\beta + \mathrm{i}\big(\partial^\mu A^\nu - \partial^\nu A^\mu\big) \Lambda\cdot\Lambda \, C_{\mu\nu}$$
$$+ \, H_c H^c + H_c \, \bar\eta^c_{\mu\nu} \Big(\big[A^\mu, A^\nu\big] + \frac{1}{2} \Lambda\cdot\Lambda \, C^{\mu\nu} \Big) \Big\} \quad .$$

The deformed gauge theory action

- Integrating on the auxiliary field H_c, we get

$$\tilde{S} = \frac{1}{g^2_{\rm YM}} \int d^4x \; {\rm Tr}\Big\{ \frac{1}{2} F^2_{\mu\nu} - 2\bar{\Lambda}_{\dot{\alpha}} \bar{D\!\!\!/}^{\dot{\alpha}\beta} \Lambda_\beta + {\rm i}\, F^{\mu\nu}\, \Lambda\cdot\Lambda\, C_{\mu\nu} - \frac{1}{4}\Big(\Lambda\cdot\Lambda\, C_{\mu\nu}\Big)^2 \Big\}$$

$$= \frac{1}{g^2_{\rm YM}} \int d^4x \; {\rm Tr}\Big\{ \Big(F^{(-)}_{\mu\nu} + \frac{\rm i}{2}\Lambda\cdot\Lambda\, C_{\mu\nu}\Big)^2 + \frac{1}{2} F_{\mu\nu}\widetilde{F}^{\mu\nu} - 2\bar{\Lambda}_{\dot{\alpha}} \bar{D\!\!\!/}^{\dot{\alpha}\beta} \Lambda_\beta \Big\} \,,$$

i.e., exactly the action of Seiberg's $\mathcal{N} = 1/2$ gauge theory.

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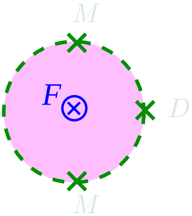
$\rightsquigarrow$ How is the instantonic sector affected?

The graviphoton in D(-1) disks

- Inserting $V_{\mathcal{F}}$ in a disk with all boundary on D(-1)'s is perfectely analogous to the D3 case (but we have non momenta).

- The only possible diagram is



$$\langle\!\langle\, V_M V_M V_D V_{\mathcal{F}} \,\rangle\!\rangle$$

$$= \frac{\pi^2}{2} 2\pi\alpha')^{\frac{1}{2}} \mathrm{tr}\Big(M'\cdot M' D_c\Big) \bar{\eta}^c_{\mu\nu} \mathcal{F}_{\dot\alpha\dot\beta} \left(\bar{\sigma}_{\nu\mu}\right)^{\dot\alpha\dot\beta}$$

$$= -\frac{1}{2}\, \mathrm{tr}\Big(M'\cdot M' D_c\Big) C^c ,$$

where

$$C^c = \frac{1}{4} \bar{\eta}^c_{\mu\nu}\, C^{\mu\nu} .$$

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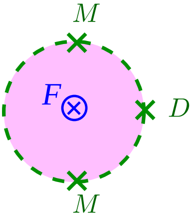


- The only possible diagram is

$$
\begin{aligned}
&\langle\!\langle\, V_M V_M V_D V_{\mathcal{F}} \,\rangle\!\rangle \\
&= \frac{\pi^2}{2} 2\pi\alpha')^{\frac{1}{2}} \mathrm{tr}\Big(M'\cdot M' D_c\Big) \bar{\eta}^c_{\mu\nu} \mathcal{F}_{\dot\alpha\dot\beta} \,(\bar\sigma_{\nu\mu})^{\dot\alpha\dot\beta} \\
&= -\frac{1}{2}\,\mathrm{tr}\Big(M'\cdot M' D_c\Big) C^c \;,
\end{aligned}
$$

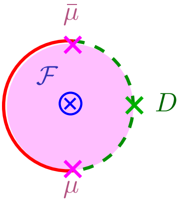
where

$$
C^c = \frac{1}{4}\bar\eta^c_{\mu\nu}\, C^{\mu\nu} \;.
$$

The graviphoton in mixed disks

- We can also insert $V_{\mathcal{F}}$ in a disk with mixed b.c.'s.
  - There is a possible diagram

$$\langle\!\langle\, V_{\bar{\mu}} V_{\mu} V_D V_{\mathcal{F}} \,\rangle\!\rangle$$



The graviphoton in mixed disks

- We can also insert $V_{\mathcal{F}}$ in a disk with mixed b.c.'s.
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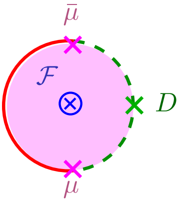

- We have different b.c.s on the two parts of the boundary, but the spin fields in the RR vertex $V_{\mathcal{F}}$ have the same identification on both:

$$S^{\dot{\alpha}} S^{(+)}(z) = \tilde{S}^{\dot{\alpha}} \tilde{S}^{(+)}(\bar{z})\Big|_{z=\bar{z}} .$$

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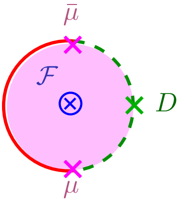

- This is because we chose D(-1)'s to represent instantons with self-dual f.s. and $\mathcal{F}_{\mu\nu}$ to be anti-self-dual.

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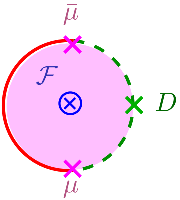

- The $\mu, \bar\mu$ vertices contain bosonic twist fields with correlator

$$\Delta(y_1)\,\bar\Delta(y_2) \sim (y_1 - y_2)^{-\frac{1}{2}}\ .$$

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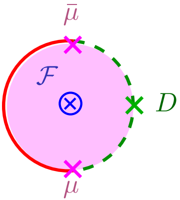

- Taking into account all correlators, the $\mathrm{SL}(2,\mathbb{R})$ gauge fixing, the integrations and the normalizations, we find the result

$$-\frac{\pi^2}{2}(2\pi\alpha')^{\frac{1}{2}}\mathrm{tr}\Big(\bar\mu_u \mu^u D_c\Big)\bar\eta^c_{\mu\nu}\mathcal{F}_{\dot\alpha\dot\beta}\,(\bar\sigma^{\nu\mu})^{\dot\alpha\dot\beta}$$
$$=\frac{1}{2}\,\mathrm{tr}\Big(\bar\mu_u \mu^u D_c\Big)C^c\ .$$

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Effects of the graviphoton on the moduli measure

- No other disk diagrams contribute in our $\alpha' \to 0$ limit.
- The two terms above are linear in the auxiliary field D_c ⇝ deform the bosonic ADHM constraints to

$$W^c + \mathrm{i}\bar{\eta}^c_{\mu\nu}\left[a'^{\mu}, a'^{\nu}\right] + \boxed{\tfrac{\mathrm{i}}{2}\left(M'\cdot M' + \mu^u\bar{\mu}_u\right)C^c} = \mathbf{0}\ .$$

- This is the only effect of the chosen anti-self-dual graviphoton bckg.
- Had we chosen a self-dual graviphoton, we would have no effect.

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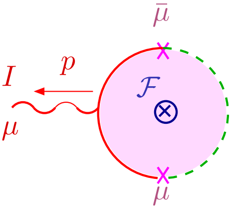
The emitted gauge field in presence of $C_{\mu\nu}$



- In the graviphoton background, we have the extra emission diagram

$$\langle\!\langle\, V_{\bar\mu}\, \mathcal{V}_{A^I_\mu}(-p)\, V_\mu V_{\mathcal{F}} \,\rangle\!\rangle$$
$$= 2\pi^2\,(2\pi\alpha')^{\frac{1}{2}}\,(T^I)^v{}_u\, p^\nu\, (\bar\sigma_{\nu\mu})^{\dot\alpha\dot\beta}\, \mathcal{F}_{\dot\alpha\dot\beta}\, \mu^u\, \bar\mu_v\, \mathrm{e}^{-\mathrm{i}p\cdot x_0}$$
$$= \frac{1}{2}\,(T^I)^v{}_u\, p^\nu\, \bar\eta^c_{\nu\mu}\, \mu^u\, \bar\mu_v\, C^c\, \mathrm{e}^{-\mathrm{i}p\cdot x_0} \quad ,$$

- No other diagrams with only two moduli contribute to the emission of a gauge field.

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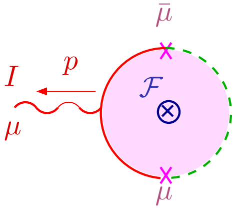

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The classical solution

- Taking into account also "undeformed" emission diagram discussed before, the emission amplitude is

$$A^I_\mu(p) = \mathrm{i}\,(T^I)^v{}_u\, p^\nu\, \bar\eta^c_{\nu\mu} \Big[(T^c)^u{}_v + (S^c)^u{}_v \Big]\, \mathrm{e}^{-\mathrm{i}p\cdot x_0} \quad ,$$

where $(T^I)^v{}_u$ are the $\mathrm{U}(N)$ generators and

$$(T^c)^u{}_v = w^u{}_{\dot\alpha}\, (\tau^c)^{\dot\alpha}{}_{\dot\beta}\, \bar w^{\dot\beta}_v \quad , \quad (S^c)^u{}_v = -\frac{\mathrm{i}}{2}\, \mu^u \bar\mu_v\, C^c \ .$$

- From this we obtain the profile of the classical solution

$$A^I_\mu(x) = \int \frac{d^4p}{(2\pi)^2}\, A^I_\mu(p)\, \frac{1}{p^2}\, \mathrm{e}^{\mathrm{i}p\cdot x}$$

$$= 2\, (T^I)^v{}_u \Big[(T^c)^u{}_v + (S^c)^u{}_v \Big]\, \bar\eta^c_{\mu\nu} \frac{(x-x_0)^\nu}{(x-x_0)^4}$$

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- The above solution represents the leading term at long distance of the deformed instanton solution in the singular gauge.
- However, above appeared the unconstrained moduli $\mu, \bar{\mu}, w, \bar{w}$.
  - We need to enforce the deformed ADHM contraints, for $k = 1$:

$$W^c + \boxed{\tfrac{\mathrm{i}}{2}\Big(M'\cdot M' + \mu^u \bar{\mu}_u\Big) C^c} = 0\,,$$

$$w^u{}_{\dot\alpha}\,\bar{\mu}_u + \mu^u \bar{w}_{\dot\alpha u} = 0\,.$$

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$$\begin{aligned} W^c + \boxed{\tfrac{\mathrm{i}}{2}\Big(M'\cdot M' + \mu^u\bar{\mu}_u\Big)C^c} &= \mathbf{0}\,, \\ w^u{}_{\dot\alpha}, \bar{\mu}_u + \mu^u\bar{w}_{\dot\alpha u} &= \mathbf{0}\,. \end{aligned}$$

The classical solution in the true moduli space

- Using the ADHM constraints, the solution can be written as

$$A^I_\mu(x) = 2\Big(\mathcal{M}^{cb}\,\mathrm{Tr}\big(T^I t^b\big) + W^c\,\mathrm{Tr}\big(T^I t^0\big) + \mathrm{Tr}\big(T^I S^c\big)\Big) \times \bar{\eta}^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^4}\,.$$

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- On the bosonic ADHM constraints,

$$W^c = -\frac{\mathrm{i}}{2}\Big(M'\cdot M' + \mu^u\bar{\mu}_u\Big)C^c \equiv \hat{W}^c.$$

Without the RR deformation, W^c would vanish.

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- The matrix $\mathcal{M}$ is $\mathcal{M}^{ab} = W^0\sqrt{W_0^2 - |\vec{W}|^2}\,\big(\mathcal{R}^{-\frac{1}{2}}\big)^{ab}$, with $\big(\mathcal{R}\big)^{ab} = W_0^2\,\delta^{ab} - W^a W^b$, where

$$W^0 = w^u{}_{\dot\alpha}\,\bar{w}^{\dot\alpha}{}_u\,.$$

At $C_c = 0$, $W_0^2 = 2\rho^2$, where $\rho =$ size of the instanton.

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- The $N \times N$ matrices t^a and t^0, depending on the moduli $w, \bar{w}$, generate a $\mathrm{u}(2)$ subalgebra
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An explicit case of the solution

- We can write the above general expression choosing a particular solution to the ADHM constraints, to make contact with the literature [Grassi et al, 2003, Britto et al, 2003].
- Decomposing $u=(\dot\alpha, i)$ with $\dot\alpha = 1,2$ and $i=3,\ldots,N$, the bosonic ADHM constraints are solved by

$$\begin{cases} w^{\dot\beta}{}_{\dot\alpha} = \rho\,\delta^{\dot\beta}{}_{\dot\alpha} + \frac{1}{4\rho}\,\hat{W}_c\,(\tau^c)^{\dot\beta}{}_{\dot\alpha}\ , \\ w^{i}{}_{\dot\alpha} = 0\ . \end{cases}$$

- Having fixed $w, \bar{w}$, the fermionic constraints are solved by

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Moreover, up to a $\mathrm{U}(N-2)$ rotation, we can choose a single ... μ^{3} being $\neq 0$.

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An explicit case of the solution

- The instanton gauge field $(A_\mu)^u{}_v$ reduces then to

$$(A_\mu)^{\dot\alpha}{}_{\dot\beta} = \Big\{ \rho^2 (\tau_c)^{\dot\alpha}{}_{\dot\beta} - \frac{\mathrm{i}}{4}\Big(M'\cdot M' + \mu^3\bar\mu_3\Big) C_c\, \delta^{\dot\alpha}{}_{\dot\beta} + \frac{1}{32\rho^2}\Big(|\vec{C}|^2 (\tau_c)^{\dot\alpha}{}_{\dot\beta} - 2 C_c C^b (\tau_b)^{\dot\alpha}{}_{\dot\beta}\Big) M'\cdot M'\, \mu^3\bar\mu_3 \Big\}\, \bar\eta^c_{\mu\nu}\, \frac{(x-x_0)^\nu}{(x-x_0)^4}$$

and

$$(A_\mu)^3{}_3 = -\frac{\mathrm{i}}{2}\,\mu^3\bar\mu_3\, C_c\, \bar\eta^c_{\mu\nu}\, \frac{(x-x_0)^\nu}{(x-x_0)^4} \quad .$$

This agrees with [Britto et al, 2003].

Additional remarks

- The gaugino emission is *not* modified *at the leading order* by the RR background.
- Subleading terms in the long-distance expansion of the solution arise from emission diagrams with more moduli insertions.
- At the field theory level, they correspond to having more source terms.
- This, is exactly the field-theoretical procedure utilized in [Grassi et al, 2003, Britto et al, 2003] to determine the (deformed) super-instanton profile,

Conclusions and perspectives

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Conclusions

- The instantonic sectors of (supersymmetric) YM theories is *really* described by D3/D(-1) systems.
- Disks (partly) attached to the D(-1)'s account, in the $\alpha' \to 0$ field theory limit for
  - the ADHM construction of instanton moduli space;
  - the classical profile of the instanton solution: the mixed disks are the source for it;
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- The open string realization of gauge theories is a very powerful tool, also in discussing possible deformations (induced by closed string backgrounds).
- In particular, the deformation of $\mathcal{N} = 1$ gauge theory to $\mathcal{N} = 1/2$ gauge theory is exactly described in the open string set-up by the inclusion of a particular Ramond-Ramond background.
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Perspectives

- Deformations of $\mathcal{N} = 2$ theories:
  - deformations of $\mathcal{N} = 2$ superspace by RR backgrounds (work in progress);
  - stringy interpretation of the deformations leading to the localization á la Nekrasov of the integrals on instanton moduli space (under investigation, in collab. also with Tor Vergata).
- Derivation of the effects of constant Ramond-Ramond field strengths (gauge theory action, instantons, etc) using Berkovits' formalism instead of RNS (work in progress).
- Derivation of the instantonic sector of non-commutative gauge theory from the string realization with constant $B_{\mu\nu}$ background.

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