## Higgs Pseudo - Observables

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But I talk, so it's my responsibility
All that theories can tell us is how the world could be (van Fraassen 1991)


## Outlines



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## $(1,2)$

(1) From signal and background at LHC to the definition of Pseudo - Obervablespseudo-observability fits nicely with structural realism, evenin the absence of further metaphysical explication (Higgs)

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(1) From signal and background at LHC to the definition of Pseudo - Obervables
2) and Feynman diagrams on the second Riemann sheet,
what else, but the inevitable!
The suggestion that particles might be seen as aspects of pseudo-observability fits nicely with structural realism, even in the absence of further metaphysical explication (Higgs)

## $\pi \rho 0 \lambda 0 \gamma O S$

## Before the entry of the chorus

- All the questions in this talk are not really urgent, as nothing will be really measurable by LHC before 2013, 2014 or beyond
- but they will be if you consider as relevant to have results published in such a way that theorists can later enter their general model parameters, calculate resulting POs and see how well data constrains this model.
- even if it will take 10 years to reach enough $\mathrm{fb}^{-1}$;
- even if it is not LEP anymore
$\mapsto$ study tools that Exps can use in future analysis to extract Higgs POs


## Oldies but Goldies

## Experimenters

(should) extract (unfold ?) so-called realistic observables from raw data, e.g. $M(\gamma \gamma)$ in $\sigma(p p \rightarrow \gamma \gamma+X)$ and need

- to present results in a form that can be useful for comparing them with theoretical predictions, i.e. the results should be transformed into POs


## Theorists

(should) compute POs

- using the best available technology and satisfying a list of demands from the self-consistency of the underlying theory


## The search for a mechanism explaining EWSB

has been a major goal for many years, in particular the search for a SM Higgs boson. As a result of this an intense effort in the theoretical community has been made to produce the most accurate NLO and NNLO predictions

## However, there is a point

that has been ignored: the Higgs boson is an unstable particle and should be removed from the |in/out > bases in the $\mathbf{H}$ space, without destroying unitarity of the theory. Therefore, concepts as the

- production or partial decay widths of an unstable particle should be replaced by a conventionalized definition which respects first principles of QFT


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## Prolegomena II

## Example

Combine $g g \rightarrow H$ with $H \rightarrow \gamma \gamma$. The full process is

$$
\begin{aligned}
p p & \rightarrow \gamma \gamma+X \\
& =[\text { Signal }] \mathrm{pp} \rightarrow \operatorname{gg}(\rightarrow \mathrm{H} \rightarrow \gamma \gamma)+\mathrm{X}
\end{aligned}
$$

and by a non-resonant background. The question is:

- how to extract from the data, without ambiguities, a PO H partial decay width into $\gamma \gamma$ which does not violate first principles?
Once again,
- Higgs boson $\notin \mid$ in $>$;
- $<\gamma \gamma$ out $\mid H$ in $>$ not definable in QFT.


## Background: comment

Perhaps we have been too busy with polynomial 20 gluons,

## but

The $\bar{q} q \rightarrow \gamma \gamma$ background was computed with NLO XX. However,

- XX is not an event Monte Carlos suitable for the detector simulation.

Hence LO YY is used to produce events and then $\sigma$, the $\mathbf{p}_{\mathrm{T}}$ and the $\mathbf{M}_{\gamma \gamma}$ distributions are reweighted to XX.

- Theory issues exist independently of those experimental detector-related aspects and must be tackled anyway


## The mother of all POs

## resummed propagators

Skeleton expansion of the self-energy $S=16 \pi^{4} i \Sigma$ with propagators that are resummed up to $\mathcal{O}(n)$

$$
\begin{aligned}
\Delta_{i}^{(0)}(s) & =\frac{1}{s-m_{i}^{2}} \\
\Delta_{i}^{(n)}(s) & =-\Delta_{i}^{(0)}(s)\left[1+\Delta_{i}^{(0)}(s) \Sigma_{i i}^{(n)}\left(s, \Delta_{i}^{(n-1)}(s)\right)\right]^{-1},
\end{aligned}
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\end{aligned}
$$

## a dressed propagator is the formal limit

$$
\begin{aligned}
& \bar{\Delta}_{i}(s)=\lim _{n \rightarrow \infty} \Delta_{i}^{(n)}(s) \\
& \bar{\Delta}_{i}(s)=-\Delta_{i}^{(0)}(s)\left[1+\Delta_{i}^{(0)}(s) \Sigma_{i i}\left(s, \bar{\Delta}_{i}(s)\right)\right]^{-1}
\end{aligned}
$$

## Complex pole

## The Higgs boson complex pole

$\mathbf{S}_{\mathrm{H}}$ is the solution of the equation

$$
\mathbf{s}_{\mathbf{H}}-M_{H}^{2}+\Sigma_{H H}\left(\mathbf{s}_{\mathbf{H}}, M_{H}^{2}, \xi\right)=0,
$$

where $M_{H}^{2}$ is the renormalized mass; all local CTs have been introduced to make the off-shell $\Sigma$ UV finite.

## NI 1

$$
\frac{\partial}{\partial \xi} s_{H}=0
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## NI 1

## NI 2

$$
\frac{\partial}{\partial \xi} s_{H}=0
$$

$$
\frac{\partial}{\partial \xi} \Sigma_{H H}\left(s_{H}, M_{H}^{2}, \xi\right)=0,
$$

## Corollary

## From NIs it follows:

$$
\frac{\partial}{\partial \xi} \Sigma_{H H}^{(1)}\left(s_{H}, s_{H}, \xi\right)=0,
$$

- at one-loop, the Higgs complex pole is gauge parameter independent if the self-energy is computed at $\mathbf{M}_{\mathrm{H}}^{2}=\mathbf{s}_{\mathrm{H}}$,
- the basis of the so-called complex-mass scheme
- higher than one-loop ... no time ...


## S - matrix

## At the parton level

the $S$-matrix for the process $\mathbf{i} \rightarrow \mathbf{f}$ can be written as

$$
S_{f i}=V_{i}(s) \Delta_{H}(s) V_{f}(s)+B_{i f}(s),
$$

- $V_{i}$ is the production vertex $i \rightarrow H$
- $V_{f}$ is the decay vertex $H \rightarrow f$
- $\Delta_{H}$ the $\mathbf{H}$ re-summed propagator
- $B_{\text {if }}$ is the non-resonant background


## Modification of the LSZ reduction

## Step 1

$$
<f \text { out }|H><H| i \text { in }>+\sum_{n \neq H}<f \text { out }|n><n| i \text { in }>
$$

$\{n\} \oplus H$ is a complete set of states

## Step 2

$$
\begin{aligned}
\Pi_{H H}(s) & =\frac{\Sigma_{H H}(s)-\Sigma_{H H}\left(s_{H}\right)}{s-s_{H}} \\
\Delta_{H H}(s) & =\left(s-s_{H}\right)^{-1}\left[1+\Pi_{H H}(s)\right]^{-1} \\
Z_{H} & =1+\Pi_{H H} .
\end{aligned}
$$

## Modification of the LSZ reduction

## Step 3

$$
S_{f i}=\left[Z_{H}^{-1 / 2}(s) V_{i}(s)\right] \frac{1}{s-s_{H}}\left[Z_{H}^{-1 / 2}(s) V_{f}(s)\right]+B_{i f}(s)
$$

## Step 4

$$
\begin{aligned}
S\left(H_{c} \rightarrow f\right) & =Z_{H}^{-1 / 2}\left(s_{H}\right) V_{f}\left(s_{H}\right) \\
S_{f i} & =\frac{S\left(i \rightarrow H_{C}\right) S\left(H_{C} \rightarrow f\right)}{s-s_{H}}+\text { non resonant terms. }
\end{aligned}
$$

## Main result

## Example: $g g \rightarrow \gamma \gamma$

$$
\begin{aligned}
\frac{1}{s} \int d \Phi_{f}\left(P_{H},\left\{p_{f}\right\}\right) & \\
& \times\left|\frac{S_{i}\left(s_{H}\right) S_{f}\left(s_{H}\right)}{s-s_{H}}\right|^{2}=\frac{\mu_{H}^{5}}{s\left|s-s_{H}\right|^{2}} \\
\mu_{H} \Gamma\left(H_{C} \rightarrow f\right) & =\frac{(2 \pi)^{4}}{2} \int d \Phi_{f}\left(P_{H},\left\{p_{f}\right\}\right) \\
& \times \sum_{\text {spins }}\left|S\left(H_{C} \rightarrow f\right)\right|^{2}
\end{aligned}
$$

## real life: $\gamma \gamma$



## real life: 4 leptons



## Strategy

## We have four parameters, all PO

$$
s_{H}=\mu_{H}^{2}-i \mu_{H} \gamma_{H} . \quad \sigma\left(\mu_{H}\right)_{i j \rightarrow H}, \quad \Gamma\left(\mu_{H}\right)_{H \rightarrow x X},
$$

- to use in a fit to the (box-detector) experimental distribution (of course, after folding with PDFs);
- these quantities are universal, uniquely defined, and in one-to-one correspondence with corrected experimental data;
- after that one could start comparing the results of the fit with a XM calculation.
$\leadsto$ Proposed initial step: unfold RO into something with idealised cuts and then use the PO approach to fit that.


## PO, once again

## Why PO language?

- POs are the the 'moneta franca' of LHC, creating an awful lot of wealth . . . . . . . There are reasons why the chain

MCT $\rightarrow$ detector simulation-selection cuts
$\rightarrow$ realistic distributions $\rightarrow$ unfolded distributions
should be replaced by
MCT $\rightarrow$ box acceptance
$\rightarrow$ (unfolded particle-level) distributions
Since detector-experimental issues are a moving target only PO are the Frankish language to understand LHC.

## Why?

PO

- POs transform the universal intuition of a QFT-non-existing quantity into an archetype,
- PO $\equiv$ the archetypal model after which theoretical calculations are patterned without worries on generating and detector-simulating events for signal and background


## Glossary

## Example

- RD = real data
- RO = from real data $\rightarrow$ distributions with cuts $\equiv \mathbf{R O}$
- diphoton pairs $(E, p) \rightarrow M(\gamma \gamma)$;
- PO = transform the universal intuition of a QFT-non-existing quantity into an archetype, e.g. $\sigma(g g \rightarrow H), \Gamma(H \rightarrow \gamma \gamma)$,
- $\mathrm{RO}_{\text {th }}\left(m_{H}, \Gamma(H \rightarrow \gamma \gamma), \ldots\right)$ fitted to $\mathrm{RO}_{\text {exp }}($ e.g. $\mathrm{RO}=M(\gamma \gamma))$ defines and extracts $m_{H}$ etc.


## Steps

- go via idealised (model-independent?) RO distributions and from there then going to the POs.
- Step 0) Use a (new) MCT - the PO code - to fit ROs
- Step 1) Understand differences with a standard event generator plus detector simulation plus calibrating the method/event generator used (which differ from the PO-code in its theoretical content)
- Step $\geq 2$ ) Let's see ......


## Lep example of RO



## Plan

## Example

$$
\begin{array}{ccc}
\mathrm{RO}_{\text {exp }} & \overrightarrow{\text { fit }} & \mathrm{RO}_{\text {th }}\left(\mathrm{PO}_{1} \ldots \mathrm{PO}_{n}\right) \\
& & \downarrow \\
\mathrm{PO}_{1}^{\mathrm{XM}} \ldots \mathrm{PO}_{n}^{\mathrm{XM}} & \text { comparison } & \mathrm{PO}_{1} \ldots \mathrm{PO}_{n} \\
& & \left(\mathrm{PO}_{1} \otimes \mathrm{PO}_{2} \ldots \mathrm{PO}_{n}\right)
\end{array}
$$

- $\mathrm{XM}=$ any Model


## Once again

on-shell $\rightarrow|\mathbf{H}>\rightarrow|$ FS $>\quad \leftarrow \quad$ does not exist
$\downarrow$
well defined
$\mathrm{RO}_{\mathrm{FS}}^{\mathrm{th}}\left(m_{H} \Gamma\left(H_{c} \rightarrow F S\right), \ldots\right)$
$\mathrm{RO}_{\mathrm{FS}}^{\exp } \quad \rightarrow \quad m_{H}, \Gamma\left(H_{C} \rightarrow F S\right) \leftarrow$ extracted

## Generalization



Figure: Gauge-invariant breakdown of the triply-resonant $g g \rightarrow 4 \mathrm{f}$ signal into $g g \rightarrow H$ production, $H \rightarrow W^{+} W^{-}$decay and subsequent $W \rightarrow \bar{f} f$ decays.

## General setup

$\phi \sigma^{2}$ theory with $M_{\phi}>2 m_{\sigma}$; the $\phi$ propagator is

$$
\Delta=\left[s-M_{\phi}^{2}+\Sigma_{\phi \phi}(s)\right]^{-1},
$$

The inverse function, $\Delta^{-1}(s)$

- is analytic in the entire $s$-plane except for a cut

$$
\left[4 m_{\sigma}^{2} \rightarrow \infty\right] ;
$$

- is defined above the cut, $\Delta^{-1}(s+i 0)$ and the analytical continuation downwards is to the 2nd Riemann sheet

$$
\Delta_{2}^{-1}(s-i 0)=\Delta^{-1}(s+i 0)=\Delta^{-1}(s-i 0)+2 i \pi \rho(s),
$$

$2 i \pi \rho(s)$ is the discontinuity across the cut.

## The logarithm

## We need a few definitions which will help

 the understanding of the procedure for the analytical continuation of functions defined through a parametric integral representation
## Logarithm

- Step $1 \ln ^{(k)} z=\ln ^{(0)} z+2 i \pi k, k=0, \pm 1, \ldots$ where $\operatorname{In}^{(0)} z$ denotes the principal branch $(-\pi<\arg (z) \leq+\pi)$.
- Step 2 Let $z_{ \pm}=z_{0} \pm i 0$ and $z=z_{R}+i z_{l}$, define

$$
\ln ^{ \pm}\left(z ; z_{ \pm}\right)=\left\{\begin{array}{l}
\ln z \pm 2 i \pi \theta\left(-z_{0}\right) \theta\left(\mp z_{l}\right) \\
\ln z \pm 2 i \pi \theta\left(-z_{R}\right) \theta\left(\mp z_{l}\right)
\end{array}\right.
$$

## The logarithm II

## first definition of the $\mathrm{In}^{ \pm}$-functions

is most natural in defining analytical continuation of Feynman integrals with a smooth limit into the theory of stable particles; the reason is simple,

- in case some of the particles are taken to be unstable we have to perform analytical continuation only when the corresponding Feynman diagram, in the limit of all (internal) stable particles, develops an imaginary part (e.g. above some normal threshold);
- However, in all cases where the analytical expression for the diagram is known, one can easily see that the result does not change when replacing $z_{0}$ with $z_{R}$, the second variant.


## The di-logarithm

## Example

$$
\begin{aligned}
\mathrm{Li}_{2}^{(0,0)}(z) \quad & 0<\arg (z-1)<2 \pi, \\
\mathrm{Li}_{2}^{(n, m)}(z)= & \mathrm{Li}_{2}^{(0,0)}(z)+2 n \pi i \ln { }^{(0)} z+4 m \pi^{2}
\end{aligned}
$$

## Question: given

$$
\begin{aligned}
\operatorname{Li}_{2}\left(M^{2}+i 0\right) & =-\int_{0}^{1} \frac{d x}{x} \ln \left(1-M^{2} x-i 0\right), \\
\operatorname{Im} \mathrm{Li}_{2}\left(M^{2}+i 0\right) & =\pi \ln M^{2} \theta\left(M^{2}-1\right),
\end{aligned}
$$

how do we understandbf analytical continuation in terms of an integral representation?

## di-logarithm II

## Let us consider the analytical continuation

- from $z^{+}=M^{2}+i 0$ to $z=M^{2}-i M \Gamma$ and define

$$
\begin{aligned}
I & =-\int_{0}^{1} \frac{d x}{x} \ln ^{-}\left(1-z x ; 1-z^{+} x\right) \\
\chi(x) & =1-z x=1-\left(M^{2}-i M \Gamma\right) x
\end{aligned}
$$

- If $M^{2}>1 \chi$ crosses the positive imaginary axis

$$
I=\mathrm{Li}_{2}^{(0,0)}(z)+2 i \pi \ln M^{2}
$$

which is not the expected result

## di-logarithm III

## The mismatch can be understood by observing that

- $\mathrm{In}^{-} \chi$ has a cut $[0,+i \infty]$ and, in the process of continuation, with $x \in[0,1]$, we have been crossing the cut.
- The solution consists in deforming the integration contour, therefore defining a new integral,

$$
I_{c}=\int_{c} \frac{d x}{x} \ln ^{-}\left(1-z x ; 1-z^{+} x\right)
$$

where $C=C_{0}+C^{\prime}$

- $C_{0}=\left\{0 \leq x \leq 1 / M^{2}-\epsilon \oplus 1 / M^{2}+\epsilon \leq x \leq 1\right.$,
- $C^{\prime}(u):\left\{x=u+i \frac{1-M^{2} u}{M \Gamma}\right\}, \frac{1}{M^{2}+\Gamma^{2}} \leq u \leq \frac{1}{M^{2}}$


## di-logarithm IV

## Result:

- The integral over $C^{\prime}$ is downwards on the first quadrant an upwards on the second (along the cut of $\mathrm{In}^{-}$);
- Integration of $\ln ^{-}$over $C^{\prime}$ gives $-2 i \pi\left(\ln M^{2}-\ln z\right)$, showing that

$$
\mathrm{Li}_{2}^{(1,0)}(z)=I_{c},
$$

- the correct analytical continuation. Therefore we can extend our integral, by modifying the contour of integration, to reproduce the right analytical continuation

$$
\mathrm{Li}_{n} \stackrel{\text { Analyt.Cont. }}{\longmapsto} \mathrm{Li}_{n}^{-}, \quad \mathrm{Li}_{n+1}^{-}(z) \neq \int_{0}^{z} \frac{d x}{x} \mathrm{Li}_{n}^{-}(x),
$$

## Deformation: example



## Complexification: example


natural choice: real kin $\rightarrow$ complex inv

$$
\begin{aligned}
\mathrm{PO} & =\sigma(g g \rightarrow H j)\left(s_{H}, s, t\right) \\
t & =-\frac{s}{2}\left[1-\frac{s_{H}}{s}-\left(1-4 \frac{s_{H}}{s}\right)^{1 / 2} \cos \theta\right] \\
u & =-\frac{s}{2}\left[1-\frac{s_{H}}{s}+\left(1-4 \frac{s_{H}}{s}\right)^{1 / 2} \cos \theta\right]
\end{aligned}
$$

## Schemes

## Schemes

RMRP the usual on-shell scheme where all masses and all Mandelstam invariants are real;
CMRP the complex mass scheme with complex internal $W$ and $Z$ poles (extendable to top complex pole) but with real, external, on-shell Higgs, $W, Z$, etc. legs and with the standard LSZ wave-function renormalization;
CMCP the (complete) complex mass scheme with complex, external, Higgs ( $W, Z$, etc.) where the LSZ procedure is carried out at the Higgs complex pole (on the second Riemann sheet).

No theoretical uncertainty; only the CMCP scheme is fully consistent.


Figure: $H \rightarrow \gamma \gamma$ (blue), $H \rightarrow g g$ (red)


Figure: $\Gamma(H \rightarrow g g)$ CMRP (dashed), CMCP (dotted)


Figure: $\sigma_{\text {cмср }} / \sigma_{\text {CMRP }}(p p \rightarrow H) \sqrt{s}=3 \mathrm{TeV}$ (red), $\sqrt{s}=10 \mathrm{TeV}$ (blue) and $\sqrt{s}=14 \mathrm{TeV}$ (black)


Figure: $\sigma(p p \rightarrow H)$ at $\sqrt{s}=3$ TeV for CMRP (red) and CMCP (blue).
Dashed lines give the scale uncertainty


Figure: Weak one-loop radiative corrections to $H \rightarrow \bar{b} b$; RMRP (red), CMRP (black dotted) and CMCP scheme (black)


Figure: $\Gamma(H \rightarrow \gamma \gamma)$; RMRP (dotted line), CMRP (dashed line) and the CMCP (solid line)

## $\epsilon \pi \iota \lambda O \gamma O S$

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## $(1,2$.

(1) We get a consistent PO definition of mass, width, couplings, meaning we can write $\sigma(p p \rightarrow H) \otimes B R(H \rightarrow X)$ as product of POs

This is needed if we want published results in such a way that theorists can later enter their general model parameters, calculate resulting POs and see how well data constrains this model

## Happening at Higgs Cross Section Working Group https://twiki.cern.ch/twiki/bin/view/LHCPhysics

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## (1, 2,)

(1) We get a consistent PO definition of mass, width, couplings, meaning we can write $\sigma(p p \rightarrow H) \otimes B R(H \rightarrow X)$ as product of $\mathbf{P O s}$
(2) This is needed if we want published results in such a way that theorists can later enter their general model parameters, calculate resulting POs and see how well data constrains this model

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