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	Hig	gs Pseudo	- Observal	bles	
		<u>.</u>			
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Loops and Legs in Quantum Field Theory, 25 - 30 April 2010, Wörlitz, Germany



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Based on work done in collaboration with Christian Sturm and Sandro Uccirati

Thanks: A. Denner, S. Dittmaier, M. Dührssen, M. Grünewald, S. Heinemeyer, C. Mariotti, R. Tanaka

But I talk, so it's my responsibility

All that theories can tell us is how the world could be (van Fraassen 1991)





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Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
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		of Pseud	do - Obervables			

and Feynman diagrams on the second Riemann sheet,

what else, but the inevitable! The suggestion that particles might be seen as aspects of pseudo-observability fits nicely with structural realism, even in the absence of further metaphysical explication (Higgs)

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Outlines Introduction Complex poles Extracting POs	Complexity	Numerica
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Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
$\pi \rho \mathbf{o} \lambda \mathbf{o} \gamma$	os				

Before the entry of the chorus

- All the questions in this talk are not really urgent, as nothing will be really measurable by LHC before 2013, 2014 or beyond
- **but they will be** if you consider as relevant to have results published in such a way that theorists can later enter their general model parameters, calculate resulting **POs** and see how well data constrains this model.
 - even if it will take 10 years to reach enough fb^{-1} ;
 - even if it is not LEP anymore
- → study tools that Exps can use in future analysis to extract Higgs POs



Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Oldie	s but Gold	lies			

Experimenters

(should) extract (**unfold ?**) so-called *realistic observables* from raw data, e.g. $M(\gamma\gamma)$ in $\sigma(pp \rightarrow \gamma\gamma + X)$ and need

 to present results in a form that can be useful for comparing them with theoretical predictions, i.e. the results should be transformed into POs

Theorists

(should) compute POs

 using the best available technology and satisfying a list of demands from the self-consistency of the underlying theory

has been a major goal for many years, in particular the search for a **SM Higgs boson**. As a result of this an intense effort in the theoretical community has been made to produce the most accurate NLO and NNLO predictions

However, there is a point

that has been ignored: the Higgs boson is an unstable particle and should be removed from the |in/out > bases in the H - space, without destroying **unitarity** of the theory. Therefore, concepts as the

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Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Prole	domena II				

Example

Combine $gg \rightarrow H$ with $H \rightarrow \gamma \gamma$. The full process is

$$pp \rightarrow \gamma\gamma + X$$

= $\left[\text{Signal} \right] pp \rightarrow gg(\rightarrow H \rightarrow \gamma\gamma) + X_{p}$

and by a non-resonant background. The question is:

 how to extract from the data, without ambiguities, a PO H partial decay width into γγ which does not violate first principles?

Once again,

- Higgs boson $\notin | in >$;
- $< \gamma \gamma$ out |H in > not definable in QFT.



Perhaps we have been too busy with polynomial 20 gluons,

but

The $\overline{q}q \rightarrow \gamma\gamma$ background was computed with NLO XX. However,

• xx is not an event Monte Carlos suitable for the detector simulation.

Hence LO YY is used to produce events and then σ , the \mathbf{p}_{τ} and the $\mathbf{M}_{\gamma\gamma}$ distributions are reweighted to xx.

 Theory issues exist independently of those experimental detector-related aspects and must be tackled anyway



The mother of all POs

resummed propagators

Skeleton expansion of the self-energy $S = 16 \pi^4 i \Sigma$ with propagators that are resummed up to O(n)

$$\begin{aligned} \Delta_i^{(0)}(s) &= \frac{1}{s - m_i^2}, \\ \Delta_i^{(n)}(s) &= -\Delta_i^{(0)}(s) \left[1 + \Delta_i^{(0)}(s) \Sigma_{ii}^{(n)}\left(s, \Delta_i^{(n-1)}(s)\right) \right]^{-1}, \end{aligned}$$

a dressed propagator is the formal limit

$$\begin{split} \overline{\Delta}_i(s) &= \lim_{n \to \infty} \Delta_i^{(n)}(s), \\ \overline{\Delta}_i(s) &= -\Delta_i^{(0)}(s) \left[1 + \Delta_i^{(0)}(s) \Sigma_{ii}\left(s, \overline{\Delta}_i(s)\right) \right]^{-1}, \end{split}$$

The mother of all POs

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Skeleton expansion of the self-energy $S = 16 \pi^4 i \Sigma$ with propagators that are resummed up to $\mathcal{O}(n)$

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Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Comp	olex pole				

The Higgs boson complex pole

 $\boldsymbol{s}_{\scriptscriptstyle H}$ is the solution of the equation

$${f s}_{f H} - M_{\!H}^2 + \Sigma_{{\scriptscriptstyle H}{\scriptscriptstyle H}}({f s}_{f H}, M_{\!H}^2, \xi) ~=~ 0,$$

where M_{H}^{2} is the renormalized mass; all **local** CTs have been introduced to make the off-shell Σ UV finite.





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Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Comp	olex pole				

The Higgs boson complex pole

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where M_{H}^{2} is the renormalized mass; all **local** CTs have been introduced to make the off-shell Σ UV finite.

NI 1
$$\frac{\partial}{\partial \xi} s_{\mu} = 0$$

NI 2
$$\frac{\partial}{\partial \xi} \Sigma_{HH}(s_H, M_H^2, \xi) = 0,$$

Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Corolla	ry				

From NIs it follows:

$$rac{\partial}{\partial\xi} \Sigma^{(1)}_{\scriptscriptstyle HH}(s_{\scriptscriptstyle H},s_{\scriptscriptstyle H},\xi) ~=~ 0,$$

- at one-loop, the Higgs complex pole is **gauge parameter** independent if the self-energy is computed at $M_{H}^{2} = s_{H}$,
- the basis of the so-called complex-mass scheme
- higher than one-loop ... no time ...

Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
S - mat	rix				

At the parton level

the S-matrix for the process $\boldsymbol{i} \rightarrow \boldsymbol{f}$ can be written as

$$S_{fi} = V_i(s) \Delta_{H}(s) V_f(s) + B_{if}(s),$$

- V_i is the production vertex i → H
- V_f is the decay vertex
 H → f

- Δ_H the H re-summed propagator
- *B_{if}* is the **non-resonant background**



Modification of the LSZ reduction

Step 1

$$< f \text{ out } | H > < H | i \text{ in } > + \sum_{n \neq H} < f \text{ out } | n > < n | i \text{ in } >$$

$\{n\} \oplus H$ is a complete set of states

Step 2

$$\begin{aligned} \Pi_{HH}(s) &= \frac{\Sigma_{HH}(s) - \Sigma_{HH}(s_{H})}{s - s_{H}}, \\ \Delta_{HH}(s) &= (s - s_{H})^{-1} \left[1 + \Pi_{HH}(s) \right]^{-1}, \\ Z_{H} &= 1 + \Pi_{HH}. \end{aligned}$$

Modification of the LSZ reduction

Step 3

$$S_{fi} = \left[Z_{H}^{-1/2}(s) V_{i}(s)\right] \frac{1}{s-s_{H}} \left[Z_{H}^{-1/2}(s) V_{f}(s)\right] + B_{if}(s)$$

Step 4

$$\begin{array}{lll} S\left(H_{c} \rightarrow f\right) &=& Z_{H}^{-1/2}(s_{H}) \ V_{f}(s_{H}), \\ \\ S_{fi} &=& \displaystyle \frac{S\left(i \rightarrow H_{c}\right) \ S\left(H_{c} \rightarrow f\right)}{s-s_{H}} + \text{non resonant terms.} \end{array}$$

Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Main I	result				

Example: $gg \rightarrow \gamma \gamma$

$$\frac{1}{s} \int d\Phi_f \left(P_H, \{ p_f \} \right) \qquad \left| \frac{S_i(s_H) S_f(s_H)}{s - s_H} \right|^2 = \frac{\mu_H^5}{s | s - s_H |^2} \\ \times \sigma(\mu_H)_{gg \to H} \otimes \Gamma(\mu_H)_{H \to \gamma\gamma}.$$

$$egin{aligned} \mu_{H}\, \Gamma\left(\mathcal{H}_{m{c}}
ightarrow f
ight) &=& rac{(2\,\pi)^{4}}{2}\,\int\, d\Phi_{f}\left(\mathcal{P}_{H},\, \{m{p}_{f}\}
ight) \ & imes &\sum_{ ext{spins}} \left|\mathcal{S}\left(\mathcal{H}_{m{c}}
ightarrow f
ight)
ight|^{2}, \end{aligned}$$

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Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
real lif	e: 4 lent	ons			



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Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Strate	gy				

We have four parameters, all PO

$$\mathbf{s}_{H} = \mu_{H}^{2} - i \,\mu_{H} \,\gamma_{H}. \quad \sigma(\mu_{H})_{ij \to H}, \quad \Gamma(\mu_{H})_{H \to XX},$$

- to use in a fit to the (box-detector) experimental distribution (of course, after folding with PDFs);
- these quantities are universal, uniquely defined, and in one-to-one correspondence with corrected experimental data;
- after that one could start comparing the results of the fit with a XM calculation.
- Proposed initial step: unfold RO into something with idealised cuts and then use the PO approach to fit that.



Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
PO. c	once again				

Why PO language?

- POs are the the 'moneta franca' of LHC, creating an awful lot of wealth There are reasons why the chain
 - $\textbf{MCT} \ \ \rightarrow \ \ detector \ simulation-selection \ cuts$
 - $\rightarrow \ \ \text{realistic distributions} \rightarrow \ \text{unfolded distributions}$

should be replaced by

- MCT \rightarrow box acceptance
 - \rightarrow (unfolded particle-level) distributions

Since detector-experimental issues are a moving target only PO are the *Frankish language* to understand LHC.



Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Why?					

PO

 POs transform the universal intuition of a QFT-non-existing quantity into an archetype,

 PO = the archetypal model after which theoretical calculations are patterned without worries on generating and detector-simulating events for signal and background

Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Glossa	ary				

Example

- RD = real data
- **RO** = from *real data* \rightarrow distributions with cuts \equiv **RO**
 - diphoton pairs $(E, p) \rightarrow M(\gamma \gamma)$;
- **PO** = transform the *universal intuition* of a *QFT-non-existing* quantity into an *archetype*, e.g. $\sigma(gg \rightarrow H), \Gamma(H \rightarrow \gamma\gamma),$
 - $\text{RO}_{\text{th}}(m_H, \Gamma(H \to \gamma \gamma), ...)$ fitted to RO_{exp} (e.g. $\text{RO} = M(\gamma \gamma)$) defines and extracts m_H etc.

Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Steps					

- go via idealised (model-independent?) **RO** distributions and from there then going to the **POs**.
 - Step 0) Use a (new) MCT the PO code to fit ROs
 - Step 1) Understand differences with a standard event generator plus detector simulation plus calibrating the method/event generator used (which differ from the PO-code in its theoretical content)

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● Step ≥ 2) Let's see

Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Lep e	xample of	fRO			



Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Plan					







Outline	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Gei	neralization				



3

Figure: Gauge-invariant breakdown of the triply-resonant $gg \rightarrow 4f$ signal into $gg \rightarrow H$ production, $H \rightarrow W^+W^-$ decay and subsequent $W \rightarrow \bar{f}f$ decays.

 $\phi \sigma^2$ theory with $M_{\phi} > 2 m_{\sigma}$; the ϕ propagator is

$$\Delta = \left[s - M_{\phi}^2 + \Sigma_{\phi\phi}(s) \right]^{-1},$$

The inverse function, $\Delta^{-1}(s)$

- is analytic in the entire *s*-plane except for a cut $[4 m_{\sigma}^2 \rightarrow \infty];$
- is defined above the cut, Δ⁻¹(s + i0) and the analytical continuation downwards is to the 2nd Riemann sheet

$$\Delta_2^{-1}(s-i0) = \Delta^{-1}(s+i0) = \Delta^{-1}(s-i0) + 2i\pi\rho(s),$$

 $2 i \pi \rho(s)$ is the discontinuity across the cut.



Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
The lo	ogarithm				

We need a few definitions which will help

the understanding of the procedure for the analytical continuation of functions defined through a **parametric integral representation**

Logarithm

• Step 1
$$\ln^{(k)} z = \ln^{(0)} z + 2i\pi k$$
, $k = 0, \pm 1, \ldots$ where $\ln^{(0)} z$ denotes the principal branch $(-\pi < \arg(z) \le +\pi)$

• Step 2 Let $z_{\pm} = z_0 \pm i 0$ and $z = z_R + i z_I$, define

$$\ln^{\pm} (z; z_{\pm}) = \begin{cases} \ln z \pm 2 i \pi \theta (-z_0) \theta (\mp z_l) \\ \ln z \pm 2 i \pi \theta (-z_R) \theta (\mp z_l) \end{cases}$$

first definition of the In[±] -functions

is most natural in defining analytical continuation of Feynman integrals with a smooth limit into the theory of stable particles; the reason is simple,

- in case some of the particles are taken to be unstable we have to perform analytical continuation only when the corresponding Feynman diagram, in the limit of all (internal) stable particles, develops an imaginary part (e.g. above some normal threshold);
- However, in all cases where the analytical expression for the diagram is known, one can easily see that the result does not change when replacing z_0 with z_R , the second variant.

Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
The d	i-logarith	m			

Example

$$\begin{array}{lll} {\rm Li}_2^{(0,0)}(z) & 0 < \arg(z-1) < 2\,\pi, \\ {\rm Li}_2^{(n,m)}(z) & = & {\rm Li}_2^{(0,0)}(z) + 2\,n\,\pi\,i\,\ln^{(0)}z + 4\,m\,\pi^2 \end{array}$$

Question: given

$$\text{Li}_{2}(M^{2} + i 0) = -\int_{0}^{1} \frac{dx}{x} \ln (1 - M^{2} x - i 0),$$

$$\text{Im} \operatorname{Li}_{2} (M^{2} + i 0) = \pi \ln M^{2} \theta (M^{2} - 1),$$

how do we understandbf analytical continuation in terms of an integral representation?



Let us consider the analytical continuation

• from $z^+ = M^2 + i 0$ to $z = M^2 - i M \Gamma$ and define

$$I = -\int_0^1 \frac{dx}{x} \ln^- (1 - zx; 1 - z^+ x),$$

$$\chi(x) = 1 - zx = 1 - (M^2 - iM\Gamma)x$$

• If $M^2 > 1 \chi$ crosses the positive imaginary axis

$$I = \operatorname{Li}_2^{(0,0)}(z) + 2 \, i \, \pi \, \ln M^2,$$

which is not the expected result

The mismatch can be understood by observing that

- In⁻ χ has a cut [0, +i∞] and, in the process of continuation, with x ∈ [0, 1], we have been crossing the cut.
- The solution consists in **deforming the integration contour**, therefore defining a new integral,

$$I_c = \int_c \frac{dx}{x} \ln^-(1-zx; 1-z^+x),$$

where $C = C_0 + C'$

- $C_0 = \{0 \le x \le 1/M^2 \epsilon \oplus 1/M^2 + \epsilon \le x \le 1,$
- C'(u) : $\{x = u + i \frac{1 M^2 u}{M\Gamma}\}$, $\frac{1}{M^2 + \Gamma^2} \le u \le \frac{1}{M^2}$

di-logarithm IV

Result:

- The integral over C' is downwards on the first quadrant an upwards on the second (along the cut of ln^-);
- Integration of \ln^- over C' gives $-2i\pi(\ln M^2 \ln z)$. showing that

$$Li_{2}^{(1,0)}(z) = I_{c},$$

 the correct analytical continuation. Therefore we can extend our integral, by modifying the contour of integration, to reproduce the right analytical continuation

$$\operatorname{Li}_{n} \stackrel{\operatorname{Analyt.Cont.}}{\longmapsto} \operatorname{Li}_{n}^{-}, \qquad \operatorname{Li}_{n+1}^{-}(z) \neq \int_{0}^{z} \frac{dx}{x} \operatorname{Li}_{n}^{-}(x),$$

Deformation: example



Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Com	plexificatio	on: examp	le		



natural choice: real kin \rightarrow complex inv

$$PO = \sigma \left(gg \to Hj\right) \left(s_{H}, s, t\right)$$
$$t = -\frac{s}{2} \left[1 - \frac{s_{H}}{s} - \left(1 - 4\frac{s_{H}}{s}\right)^{1/2} \cos \theta\right]$$
$$u = -\frac{s}{2} \left[1 - \frac{s_{H}}{s} + \left(1 - 4\frac{s_{H}}{s}\right)^{1/2} \cos \theta\right]$$

Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
Schem	nes				

Schemes

- **RMRP** the usual **on-shell** scheme where all masses and all Mandelstam invariants are real;
- **CMRP** the **complex mass** scheme with complex internal *W* and *Z* poles (extendable to top complex pole) but with real, external, on-shell Higgs, W, Z, etc. legs and with the standard LSZ wave-function renormalization;

CMCP the (complete) **complex mass** scheme with complex, external, Higgs (W, Z, etc.) where the LSZ procedure is carried out at the **Higgs complex pole** (on the second Riemann sheet).

No theoretical uncertainty; only the CMCP scheme is fully consistent.







and $\sqrt{s} = 14$ TeV (black)





Figure: $\sigma(pp \rightarrow H)$ at $\sqrt{s} = 3$ TeV for CMRP (red) and CMCP (blue). Dashed lines give the scale uncertainty



Figure: Weak one-loop radiative corrections to $H \rightarrow \overline{b}b$; RMRP (red), CMRP (black dotted) and CMCP scheme (black)





Figure: $\Gamma(H \rightarrow \gamma \gamma)$; RMRP (dotted line), CMRP (dashed line) and the CMCP (solid line)

Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
$\epsilon\pi\iota\lambda o\gamma c$	os				

We get a consistent PO definition of mass, width, couplings, meaning we can write σ (pp \rightarrow H) \otimes BR(H \rightarrow X) as product of **POS** This is needed if we want published results in such a way that theorists can later enter their general model parameters, calculate resulting **POS** and see how well data constrains this model

Happening at Higgs Cross Section Working Group https://twiki.cem.ch/twiki/bin/view/LHCPhysics



Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
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We get a consistent PO definition of mass, width, couplings, meaning we can write $\sigma (pp \rightarrow H) \otimes BR(H \rightarrow X)$ as product of **POs**

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Outlines	Introduction	Complex poles	Extracting POs	Complexity	Numerica
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2	This is n way mode see h	eeded if we wan that theorists ca el parameters, co pow well data co	t published resul n later enter thei alculate resulting nstrains this moo	ts in such a ir general g POs and lel	

Happening at Higgs Cross Section Working Group https://twiki.cern.ch/twiki/bin/view/LHCPhysics

