Precision Higgs Physics
the Next Step

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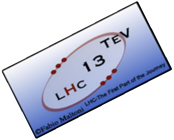
LHC – The first part of the journey, July 8−12, 2013, KITP Santa Barbara



Marcela, Fabio, Matthias and Lian-Tao

Outline: there are three vital steps or stages one must climb

- Theoretical precision: Missing Higher Orders (MHO)
- On – Off Shell: the Dalitz sector

- BSM: SM $\oplus$ $d = 6$ operators

Chiara Mariotti, Reisaburo Tanaka
Ansgar Denner and André David

Here we go

Prolegomena

From my Logbook:
now we must move on to the next step

melting BSM*-physics with high-precision* SM*-technology* The question has been repeated many times

But:

- **Answers** converging around *Not yet*
- *Meanwhile, it came dangerously close to realizing a nightmare, of Physics done by sub-sets of diagrams instead of cuts.*

WELL, SEVERAL YEARS AGO WE AVOIDED THAT FATE, MAY BE THE HISTORY WILL REPEAT ITSELF?

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Missing Higher Orders

What is THU?

The traditional way for estimating THEORETICAL UNCERTAINTIES associated to collider physics is based on the notion of **QCD** scale variation

We introduce the concept of

- **MHO**(**MHOU**), missing higher order (uncertainty), which has to do with the TRUNCATION ERROR IN THE PERTURBATIVE EXPANSION;

In the past 30 years the commonly accepted way for estimating MHOU has been based on scale variations.

Consider an observable $\boldsymbol{\sigma}(\boldsymbol{Q},\boldsymbol{\mu})$ where

- $\boldsymbol{Q}$ is the typical scale of the process and
- $\boldsymbol{\mu} \equiv \{\boldsymbol{\mu}_{\mathbf{R}},\boldsymbol{\mu}_{\mathbf{F}}\}$ are the renormalization and factorization scales. The conventional strategy defines

$$\sigma_{\xi}^{-} = \min\left\{\sigma\left(Q,\frac{\mu}{\xi}\right),\sigma\left(Q,\xi\,\mu\right)\right\},$$
$$\sigma_{\xi}^{+} = \max\left\{\sigma\left(Q,\frac{\mu}{\xi}\right),\sigma\left(Q,\xi\,\mu\right)\right\},$$

- selects a value for $\boldsymbol{\xi}$ (typically $\boldsymbol{\xi}=\mathbf{2}$) and predicts $\boldsymbol{\sigma}^{-} \leqq \boldsymbol{\sigma} \leqq \boldsymbol{\sigma}^{+}$

There is an open and debatable question on how to assign a probability distribution function (**pdf) to the MHOU**

- the generally accepted one is based on a Gaussian (or log-normal) distribution centered at $\sigma(Q,Q)$. What to use for the standard deviation, remains an open problem.
- Alternatively, it can be assumed that the pdf is a flat-box

Recently, there has been a proposal by **Cacciari and Houdeau**, based on a flat (**uninformative**) *Bayesian prior* for the MHOU.

More generally, dependence on scales is only part of the problem: indeed, the MHO problem is based on the following fact: given an observable $\mathscr{O}$, related to a perturbative series

$$\mathscr{O} \asymp \sum_{n=0}^{\infty} c_n g^n$$

➼ how should we interpret the relation?

- The perturbative expansion is unlikely to converge, **Simon,** 1972
- the asymptotic behavior of the coefficients is expected to be

$$c_n \quad \sim \quad K\, n^{\alpha}\, \frac{n!}{S^n}, \quad n \to \infty$$

Vainshtein 1994

The requirement of Eq.(1) ($\asymp$) is not a formal one, it has a physical content: it means that there is a smooth transition between the system with interaction and the system without it, **Fischer** 1995. Furthermore, Borel and Carleman proved that there are analytic functions corresponding to arbitrary asymptotic power series.

We (**A. David and I**) will not be able to answer general questions (e.g. to prove uniqueness) but concentrate on

predicting higher orders

using the well-known concept of “ series acceleration”, i.e. one of a collection of **sequence transformations (ST)** for improving the rate of convergence of a series.

- If the original series is divergent, the **ST** acts as an extrapolation method
- in the case of **infinite sums**, **ST**s have the effect that sums that formally diverge may return a result that can be interpreted as evaluation of the **analytic extension of the series for the sum**.

- the relation between **Borel summation** (usual method applied for summing divergent series) and these extrapolation methods is known
- Note that the definition of a sum of a **factorially divergent** series, including those with non-alternating coefficients, is always **equivalent** to Borel's definition, **Suslov** 2005

Example

$$S_\infty = \sum_{n=0}^{\infty} n!\, z^{n+1} = e^{-1/z}\,\mathrm{E}i\left(\frac{1}{z}\right)$$

where the **exponential integral** is a single-valued function in the plane cut along the negative real axis.
However, for $z > 0$ $\mathrm{E}i(z)$ can be computed to great accuracy using several Chebyshev expansions. Note that the r.h.s. is the **Borel sum of the series**.

Levin $\boldsymbol{\tau}$-transform, given the partial sum

$$S_n = \sum_{i=0}^{n} \gamma_i z^i, \quad \text{define the } \boldsymbol{\tau}\text{-transform as}$$

$$\tau_k = \frac{N_k}{D_k},$$

$$N_k = \sum_{i=1}^{k} W(k,i)\, S_i, \qquad D_k = \sum_{i=1}^{m} W(k,i),$$

$$W(k,i) = (-1)^i \begin{pmatrix} k \\ i \end{pmatrix} \frac{(i)_{k-1}}{\Delta S_{i-1}}$$

where $(z)_a = \Gamma(z+a)/\Gamma(z)$ is the Pochhammer symbol and Δ is the usual forward-difference operator, $\Delta S_n = S_{n+1} - S_n$.

Weniger δ-transform

$$\delta_k(\beta) = \frac{\sum_{i=0}^{k} W^{\delta}(k,i,\beta)\, S_i}{\sum_{i=0}^{k} W^{\delta}(k,i,\beta)}$$

$$W^{\delta}(k,i,\beta) = (-1)^i \binom{k}{i} \frac{(\beta+i)_{k-1}}{(\beta+k)_{k-1}} \frac{1}{\gamma_{i+1}\, z^{i+1}}$$

The whole strategy is based on the fact that one can predict the coefficients by

- constructing an approximant with the known terms of the series $(\gamma_0, \ldots, \gamma_n)$ and
- expanding the approximant in a Taylor series. The first n terms of this series will exactly agree with those of the original series and

the subsequent terms may be treated as the predicted coefficients. i.e. if $S_1, \ldots, S_k$ are known, one computes

$$\tau_k - S_k \;=\; \bar{\gamma}_{k+1}\, z^{k+1} + \mathcal{O}\left(z^{k+2}\right)$$

➽ $\bar{\gamma}_{k+1}$ is the prediction for γ_{k+1}

How to use it?

Consider a specific example, $\mathbf{gg \to H}$. Define

$$\sigma_{\rm gg}\left(\tau\,,M_{\rm H}^2\right) \;=\; \sigma^0_{\rm gg}\left(\tau\,,M_{\rm H}^2\right) K_{\rm gg}\left(\tau\,,M_{\rm H}^2\,,\alpha_{\rm s}\right)$$

where $\boldsymbol{\tau = M_{\rm H}^2/s}$ and $\boldsymbol{\sigma^0_{\rm gg}}$ is the LO cross section. The K-factor admits a formal power expansion in $\boldsymbol{\alpha_{\rm s}(\mu_{\rm R})}$

$$K_{\rm gg}\left(\tau\,,M_{\rm H}^2\,,\alpha_{\rm s}\right) \;=\; 1+\sum_{n=1}^{\infty}\alpha_{\rm s}^n(\mu_{\rm R})\,K^n_{\rm gg}$$

Known coefficients are **11.879** and **72.254**

In their recent work, Ball et al, (Ball:2013bra) computed (at $\sqrt{s} = 8$ *TeV*)

$$\alpha_s^3\left(\frac{M_H}{2}\right) K_{gg}^3\left(\mu = \frac{M_H}{2}\right) = 0.323 \pm 0.059$$

$$\alpha_s^3\left(M_H\right) K_{gg}^3\left(\mu = M_H\right) = 0.527 \pm 0.043$$

$$\alpha_s^3\left(2\,M_H\right) K_{gg}^3\left(\mu = 2\,M_H\right) = 0.729 \pm 0.032$$

Warming up with two coefficients ☛

$$\tau_2 - S_2 = \frac{\gamma_2^2}{\gamma_1} z^3 + \mathcal{O}\left(z^4\right)$$

- applied to the **ggF** series gives

$$\mathbf{346.42} \leqq \gamma_3\left(\mu = M_{\rm H}\right) \leqq \mathbf{407.48} \qquad (\text{Ball:2013bra})$$
$$\bar{\gamma}_3\left(\mu = M_{\rm H}\right) = \mathbf{439.48} \quad \Lleftarrow \quad \text{predicted}$$

✷ which has the correct sign and the right order of magnitude.

Introducing

$$S_{\mathrm{N},n} = \sum_{k=0}^{n} \gamma_k z^k + \sum_{k=n+1}^{\mathrm{N}} \bar{\gamma}_k z^k,$$

and $\boldsymbol{\delta_{\mathrm{N},n}}$ etc, constructed accordingly, our strategy for estimating **MHO and MHOU** can be summarized as follows:

- we select a scale, $\boldsymbol{\mu = M_{\mathrm{H}}}$ for gg-fusion
- ESTIMATE THE UNCERTAINTY DUE TO HIGHER ORDERS AT THAT SCALE, I.E. THE (SCALE VARIATION) UNCERTAINTY AT THE CHOSEN SCALE IS PART OF THE UNCERTAINTY DUE TO HIGHER ORDERS AND SHOULD NOT BE COUNTED TWICE

∴ we compare

$$\sigma_{gg}^{S,n} = \sigma_{gg}^{0}(\mu = M_H)\, S_{n,3}(\mu = M_H)$$
$$\sigma_{gg}^{\delta,n} = \sigma_{gg}^{0}(\mu = M_H)\, \delta_{n,3}(\mu = M_H)$$

Our conclusion is that, to a very good accuracy,

$$\sigma_{gg} \in \left[\sigma_{gg}^{S,3}, \sigma_{gg}^{\delta,5}\right]$$

with a flat interval of **16.37%**.

The uncertainty on the width, induced by the error on the coefficient $\gamma_3(\mu = M_H)$ brings it to 26.01%

✂ N^3LO & QCD scales var.

$\sigma_{gg} \in [18.90, 21.93]\ pb$

completion & MHO ★★★

$\sigma_{gg} \in [20.13, 23.42]\ pb$

NNLO ↠ +17% ↠ N^3LO → ≈ +7% ↠ completion

The advantages of the method are that

- the result does not depend on the choice of the parameter expansion (it is based on “ PARTIAL SUMS)” ✓
- it takes into account the nature of the coefficients, i.e. that the **known terms** of the perturbative expansion in gg-fusion are **positive** ✓

BU1

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▸ BU1

pdf

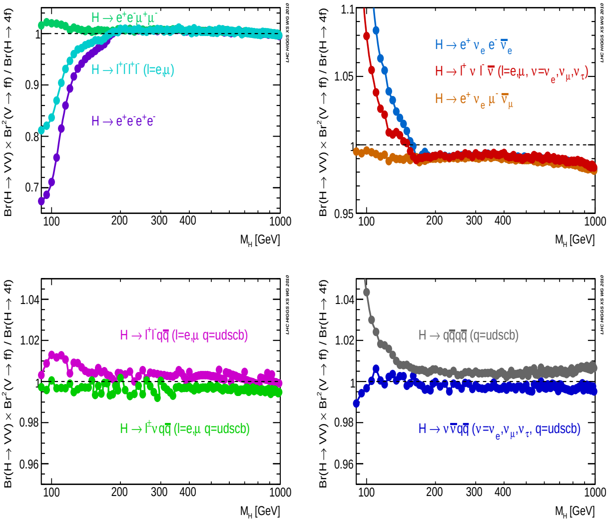
The corresponding pdf could be derived by following the work of Cacciari and Houdeau giving

$$P_{\mathrm{CH}}(\sigma) = N_\sigma^{-1} \begin{cases} \left(\frac{\Delta\sigma}{\sigma_+ - \sigma}\right)^5 & \text{if} \quad \sigma \leqq \sigma_- \\ 1 & \text{if} \quad \sigma_- \leqq \sigma \leqq \sigma_+ \\ \left(\frac{\Delta\sigma}{\sigma - \sigma_-}\right)^5 & \text{if} \quad \sigma > \sigma_+ \end{cases}$$

$$\sigma_- = \sigma_{\mathrm{gg}}^{\mathrm{S},3} \qquad \sigma_+ = \sigma_{\mathrm{gg}}^{\delta,4}$$

$$\Delta\sigma = \sigma_+ - \sigma_- \qquad N_\sigma = \frac{3}{2}\Delta\sigma$$

Pseudo Observables

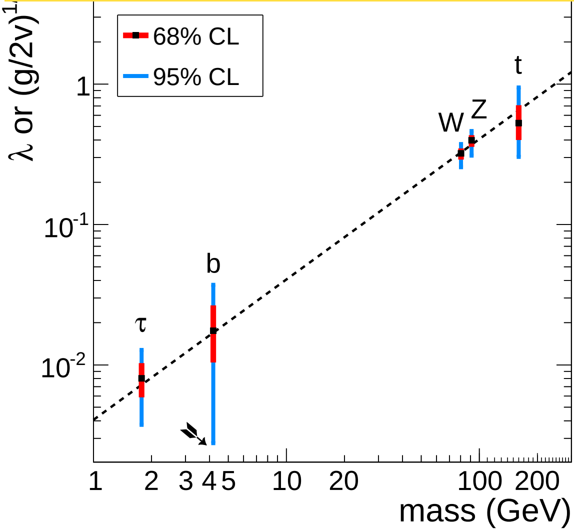

H → e+e-μ+μ-
H → l+l-l+l- (l=e,μ)
H → e+e-e+e-
H → e+ νe e- ν̄e
H → l+ ν l- ν̄ (l=e,μ, ν=νe,νμ,ντ)
H → e+ νe μ- ν̄μ
H → l+l-qq̄ (l=e,μ q=udscb)
H → l+νqq̄ (l=e,μ q=udscb)
H → qq̄qq̄ (q=udscb)
H → νν̄qq̄ (ν=νe,νμ,ντ, q=udscb)
Br(H → VV) × Br²(V → ff) / Br(H → 4f)
MH [GeV]
LHC HIGGS XS WG 2010

These plots are one of the best examples that

$$\mathrm{BR}\,(\mathrm{H} \to \mathrm{VV}) \quad \otimes \quad \mathrm{BR}^2\,(V \to \bar{\mathrm{f}}\mathrm{f})$$
$$\neq$$
$$\mathrm{BR}\,(\mathrm{H} \to 4\,\mathrm{f})$$

Trivial but true, $\nrightarrow$ $\mathbf{H} \to \mathbf{VV}$ is not a physical OBSERVABLE, eventually it can be *defined* as “ PSEUDO-OBSERVABLE”

censored



The previous plot (**couplings** ⇄ **masses**) is another example that

POs can be defined (couplings) **Iff** the rules of the game are respected

☛

- MODEL-INDEPENDENT couplings are *extracted* in some *effective* way that includes **QCD** but not **NLO EW**
- If one wants to obtain the **SM** (the straight line) ↬ use RUNNING MASSES $m_f(M_H)$

Prototyping

Theorem

$$\nexists \quad \mathrm{H} \to \mathrm{Z} + \gamma,\ \mathrm{H} \to \mathrm{VV} \quad \textit{etc.}$$

do not exist/make sense since $\Downarrow$

$$✂\mathbf{V} \quad \notin \quad |\,\mathbf{in/out}> \quad \textit{bases of the Hilbert space}$$

High Precision Road

Dalitz Decay?

$M_{\rm H} = 125.5\ GeV$ $\qquad$ $\mathrm{BR}\left(\mathrm{H} \to \mathrm{e}^{+}\mathrm{e}^{-}\right) = 5.1 \times 10^{-9}$

while a *naive* estimate gives

$$\mathrm{BR}\left(\mathrm{H} \to \mathrm{Z}\gamma\right) \mathrm{BR}\left(\mathrm{Z} \to \mathrm{e}^{+}\mathrm{e}^{-}\right) = 5.31 \times 10^{-5}$$

4 **orders of magnitude larger**

How much is the corresponding PO extracted from full Dalitz Decay?

We could expect $\mathbf{\Gamma}\left(\mathbf{H} \to \mathbf{e}^{+}\mathbf{e}^{-}\boldsymbol{\gamma}\right) = \mathbf{5.7\%}\,\mathbf{\Gamma}\left(\mathbf{H} \to \boldsymbol{\gamma\gamma}\right)$ but photon isolation must be discussed.

Categories

Terminology:

The name **Dalitz Decay** *must be reserved for the full process*

$$\mathbf{H \to \bar{f}f\gamma}$$

Subcategories:

$$\left\{ \begin{array}{l} H \to Z^{*} \left(\to \bar{f}f\right) + \gamma \\ H \to \gamma^{*} \left(\to \bar{f}f\right) + \gamma \\ H \to Z_{c} \left(\to \bar{f}f\right) + \gamma \end{array} \right.$$

✂ **unphysical**[1]

✂ **unphysical**

PO[2]

---

[1] Z^{*} is the off-shell Z

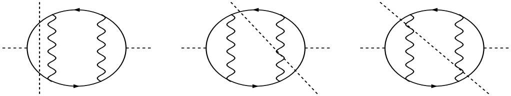
[2] Z_{c} is the Z at its complex pole

Understanding the problem

$$\mathbf{H} \to \bar{\mathrm{f}}\mathrm{f} \quad \text{or} \quad \mathbf{H} \to \bar{\mathrm{f}}\mathrm{f} + n\gamma\,?$$

Go to two-loop, the process is considerably more complex than, say, $\mathbf{H} \to \gamma\gamma$ *because of the role played by* **QED** *and* **QCD** *corrections*.

The ingredients needed are better understood in terms of cuts of the three-loop $\mathbf{H}$ self-energy ☛



Moral: *Unless you* **Isolate photons**

you don't know which process you are talking about

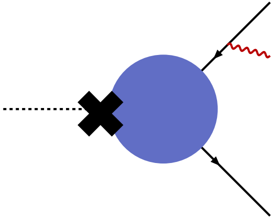
$\mathbf{H} \to \bar{\mathbf{f}}\mathbf{f}$ NNLO or $\mathbf{H} \to \bar{\mathbf{f}}\mathbf{f}\boldsymbol{\gamma}$ NLO

The complete **S**-matrix element will read as follows:

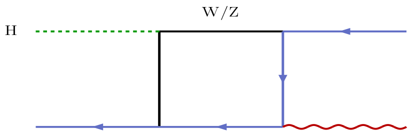
$$
\begin{aligned}
S &= \left|A^{(0)}\left(\mathrm{H}\to\bar{\mathrm{f}}\mathrm{f}\right)\right|^2 \\
&+ 2\,\mathrm{Re}\left[\mathrm{A}^{(0)}\left(\mathrm{H}\to\bar{\mathrm{f}}\mathrm{f}\right)\right]^{\dagger}\mathrm{A}^{(1)}\left(\mathrm{H}\to\bar{\mathrm{f}}\mathrm{f}\right) \\
&+ \left|A^{(0)}\left(\mathrm{H}\to\bar{\mathrm{f}}\mathrm{f}\gamma\right)\right|^2 ✗ \\
&+ 2\,\mathrm{Re}\left[\mathrm{A}^{(0)}\left(\mathrm{H}\to\bar{\mathrm{f}}\mathrm{f}\right)\right]^{\dagger}\mathrm{A}^{(2)}\left(\mathrm{H}\to\bar{\mathrm{f}}\mathrm{f}\right) \\
&+ 2\,\mathrm{Re}\left[\mathrm{A}^{(0)}\left(\mathrm{H}\to\bar{\mathrm{f}}\mathrm{f}\gamma\right)\right]^{\dagger}\mathrm{A}^{(1)}\left(\mathrm{H}\to\bar{\mathrm{f}}\mathrm{f}\gamma\right) ✗ \\
&+ \left|A^{(0)}\left(\mathrm{H}\to\bar{\mathrm{f}}\mathrm{f}\gamma\gamma\right)\right|^2.
\end{aligned}
$$

Don't get trapped by your intuition, the **IR/collinear** stuff will not survive in the limit $m_f \to 0$

There are **genuinely non-QED(QCD)** terms surviving the **zero-Yukawa** limit (a result known since the '80s)



Dalitz box



- **Collinear/Virtual** cancel in the total ✗
- **Gram and Cayley** do not generate real singularities ✗
- *Plenty* of hard stuff around ✌

Only the total *Dalitz Decay* has a meaning and *can be differentiated through cuts*

- The most important is the definition of *visible photon* to distinguish between $\bar{f}f$ and $\bar{f}f\gamma$
- Next cuts are on $M(\bar{f}f)$ to *isolate* pseudo-observables
- One has to distinguish:
  - $H \to \bar{f}f+$ **soft(collinear)** photon(s) which is part of the real corrections to be added to the virtual ones in order to obtain $H \to \bar{f}f$ at (N)NLO
  - a **visible** photon and a soft $\bar{f}f$-pair where you probe the Coulomb pole and get large (logarithmic) corrections that must be exponentiated.

Unphysical $\mathbf{H \to Z\gamma \to \bar{f}f\gamma}$ and $\mathbf{H \to \gamma\gamma \to \bar{f}f\gamma}$

- ✂ None of these contributions exists by itself, each of them is **NOT even gauge invariant.** One can put cuts and
  - with a small window around the $\mathbf{Z}$-peak the pseudo-observable $\mathbf{H \to Z_c\gamma}$ can be enhanced, but there is a contamination due to many non-resonant backgrounds ✓
  - Beware of generic statements *box contamination in* $\mathbf{H \to Z\gamma}$ *is known to be small* and of *ad-hoc* definition of gauge-invariant **splittings** ✓
  - at small di-lepton invariant masses $\boldsymbol{\gamma^*}$ dominates ✓

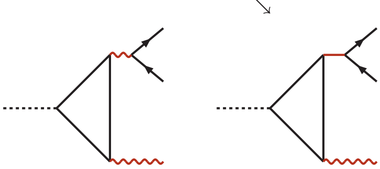
Partial Summary

- $\mathbf{H \to \bar{f}f}$ is well defined and $\mathbf{H \to \bar{f}f + \gamma}$ ($\boldsymbol{\gamma}$ **soft+collinear**) is part of the corresponding NLO corrections

- $\mathbf{H \to Z\gamma}$ is not well defined being a gauge-variant part of $\mathbf{H \to \bar{f}f + \gamma}$ ($\boldsymbol{\gamma}$ **visible**) and can be *extracted* (☛ in a PO sense) by *cutting the di-lepton invariant mass*.

the best that we can hope to achieve is simply to misunderstand at a deeper level

Intuition



$\not\in$ Facts of life with non-resonant

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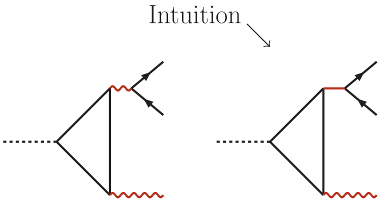

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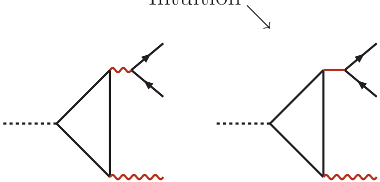


$\notin$ Facts of life with non-resonant

Results: leptons

$$m\left(\bar{\mathrm{f}}\mathrm{f}\right) > 0.1\,M_{\mathrm{H}} \qquad m\left(\mathrm{f}\gamma\right) > 0.1\,M_{\mathrm{H}} \qquad m\left(\bar{\mathrm{f}}\gamma\right) > 0.1\,M_{\mathrm{H}}$$

$$\Gamma_{\mathrm{NLO}} = 0.233\,keV \quad \oplus \quad \begin{cases} \Gamma_{\mathrm{LO}} = 0.29 \times 10^{-6}\,keV & \mathrm{e} \\ \Gamma_{\mathrm{LO}} = 0.012\,keV & \mu \\ \Gamma_{\mathrm{LO}} = 3.504\,keV & \tau \end{cases}$$

☛ LO and NLO **do not interfere** (as long as masses are neglected in NLO), they belong to different helicity sets. Cuts à la **Dicus and Repko**

Results: quarks

$m\left(\bar{\mathrm{f}}\mathrm{f}\right) > 0.1\, M_{\mathrm{H}} \qquad m\left(\mathrm{f}\gamma\right) > 0.1\, M_{\mathrm{H}} \qquad m\left(\bar{\mathrm{f}}\gamma\right) > 0.1\, M_{\mathrm{H}}$

$$\begin{cases} \Gamma_{\mathrm{LO}} = 0.013\; keV & \Gamma_{\mathrm{NLO}} = 0.874\; keV \quad \mathrm{d} \\ \Gamma_{\mathrm{LO}} = 8.139\; keV & \Gamma_{\mathrm{NLO}} = 0.866\; keV \quad \mathrm{b} \end{cases}$$

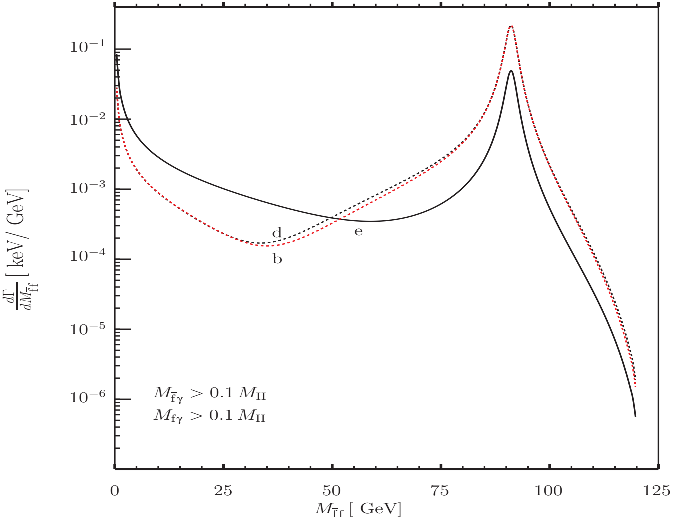
☛ Note the effect of m_t

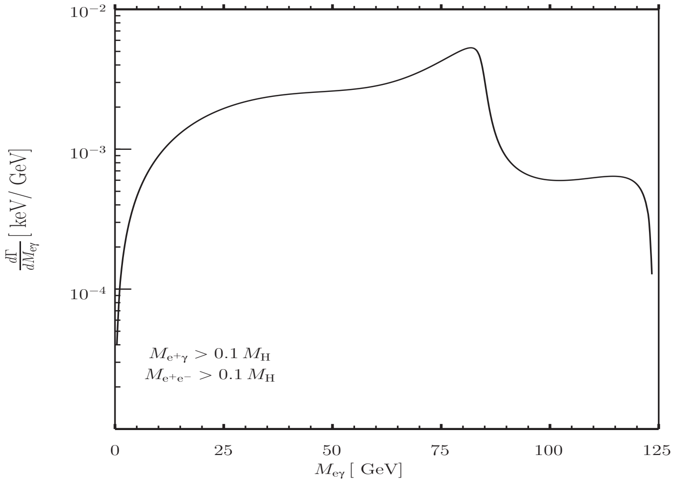
Cutting

$$m\,(\mathrm{f}\gamma) > 0.1\,M_{\mathrm{H}} \qquad m\,(\bar{\mathrm{f}}\gamma) > 0.1\,M_{\mathrm{H}}$$

	$\Gamma_{\mathrm{NLO}}[\,keV]$ $m\,(\bar{\mathrm{f}}\mathrm{f}) > 0.1\,M_{\mathrm{H}}$	$m\,(\bar{\mathrm{f}}\mathrm{f}) > 0.6\,M_{\mathrm{H}}$
l	0.233	0.188
d	0.874	0.835
b	0.866	0.831

	$\Gamma_{\mathrm{LO}}[\,keV]$ $m\,(\bar{\mathrm{f}}\mathrm{f}) > 0.1\,M_{\mathrm{H}}$	$m\,(\bar{\mathrm{f}}\mathrm{f}) > 0.6\,M_{\mathrm{H}}$
μ	0.012	0.010
d	0.013	0.011
b	8.139	6.745


10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$\frac{d\Gamma}{dM_{\bar{f}f}}$ [keV/ GeV]
d
e
b
$M_{\bar{f}\gamma} > 0.1\ M_H$
$M_{f\gamma} > 0.1\ M_H$
0
25
50
75
100
125
$M_{\bar{f}f}$ [GeV]


10^{-2}
10^{-3}
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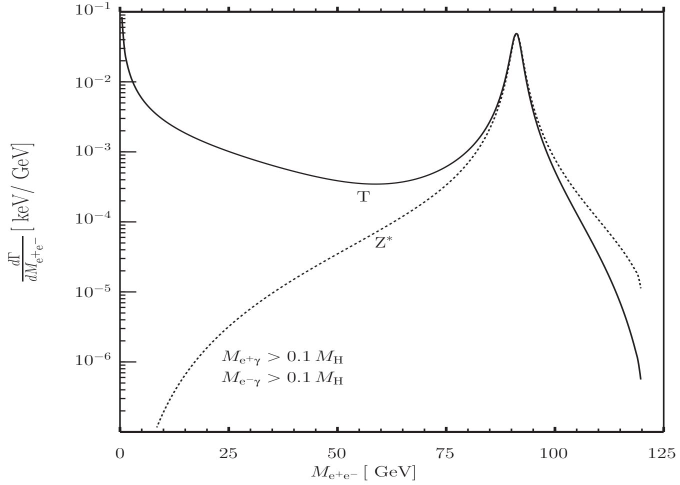
$\frac{d\Gamma}{dM_{e\gamma}}$ [keV/ GeV]
$M_{e^+\gamma} > 0.1\,M_H$
$M_{e^+e^-} > 0.1\,M_H$
0
25
50
75
100
125
$M_{e\gamma}$ [GeV]


10^{-1}
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10^{-6}
$\frac{d\Gamma}{dM_{e^+e^-}}$ [keV/ GeV]
T
Z^*
$M_{e^+\gamma} > 0.1\, M_H$
$M_{e^-\gamma} > 0.1\, M_H$
0
25
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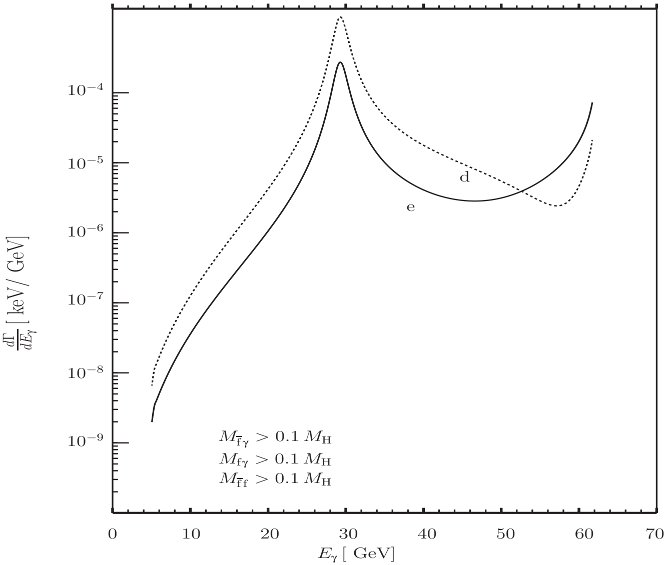
75
100
125
$M_{e^+e^-}$ [GeV]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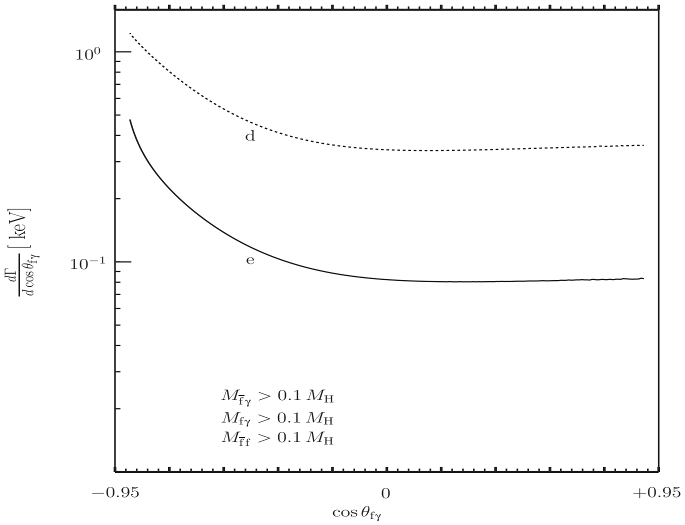
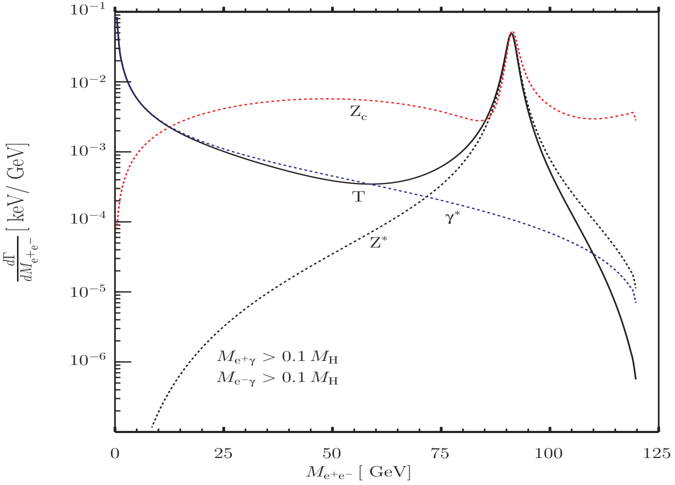
10^{-9}
$\frac{d\Gamma}{dE_\gamma}$ [keV/ GeV]
d
e
$M_{\overline{f}\gamma} > 0.1\, M_H$
$M_{f\gamma} > 0.1\, M_H$
$M_{\overline{f}f} > 0.1\, M_H$
0
10
20
30
40
50
60
70
E_γ [GeV]



10^0
10^{-1}
d
e
$\frac{d\Gamma}{d\cos\theta_{f\gamma}}$ [keV]
$M_{\bar{f}\gamma} > 0.1\ M_H$
$M_{f\gamma} > 0.1\ M_H$
$M_{\bar{f}f} > 0.1\ M_H$
-0.95
0
$+0.95$
$\cos\theta_{f\gamma}$

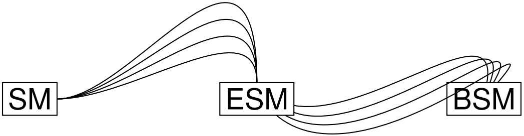

10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$\frac{d\Gamma}{dM_{e^+e^-}}$ [keV/ GeV]
Z_c
T
γ^*
Z^*
$M_{e^+\gamma} > 0.1\, M_H$
$M_{e^-\gamma} > 0.1\, M_H$
0
25
50
75
100
125
$M_{e^+e^-}$ [GeV]

Observable	Pseudo-Observable
$\mathrm{H} \to \gamma\gamma$	
$\mathrm{H} \to \bar{\mathrm{f}}\mathrm{f}\gamma$	☞ $\mathrm{H} \to \mathrm{Z}\gamma$
$\mathrm{H} \to \bar{\mathrm{f}}\mathrm{f}$	
$\mathrm{H} \to \bar{\mathrm{f}}\mathrm{f}\bar{\mathrm{f}}'\mathrm{f}'$	☞ $\mathrm{H} \to \mathrm{VV}, \mathrm{Z}\gamma$

One needs to define when it is $\mathbf{4f}$ final state and when it is PAIR CORRECTION to $\mathbf{2f}$ final state (as it was done at LEP2)

Effective Field Theory

Lets' consider the following path



$$\mathscr{L}_{\rm ESM} = \mathscr{L}_{\rm SM} + \sum_{n>4} \sum_{i=1}^{N_n} \frac{a_i^n}{\Lambda^{n-4}} \mathscr{O}_i^{(d=n)}$$

$$\exists(\exists\,!) \quad \mathscr{L}_{\rm UCSM} \rightarrowtail \mathscr{O}_i\,?$$

UV completion of the SM (UCSM) or ESM?

Bottom-up or top-down approach to **ESM**?

- How many facts the theory explains: it is a draw
- Having the fewer auxiliary hypothesis: **SM** $\twoheadrightarrow$ **UCSM** superior
- Analogy: **SM** should be augmented by all possible terms consistent with symmetries $\twoheadrightarrow$ **ESM**

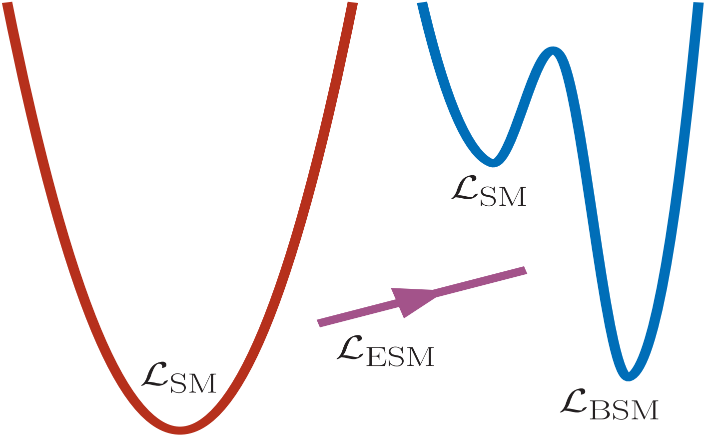
The regulative ideal of an ultimate theory remains a powerful aesthetic ingredient

How to interpret κ_X?

$$\frac{\Gamma_{gg}}{\Gamma_{gg}^{SM}(m_H)} = \frac{\kappa_t^2 \cdot \Gamma_{gg}^{tt}(m_H) + \kappa_b^2 \cdot \Gamma_{gg}^{bb}(m_H) + \kappa_t \kappa_b \cdot \Gamma_{gg}^{tb}(m_H)}{\Gamma_{gg}^{tt}(m_H) + \Gamma_{gg}^{bb}(m_H) + \Gamma_{gg}^{tb}(m_H)}$$

Space of Lagrangians (arXiv:1202.3144, arXiv:1202.3415, arXiv:1202.3697)

Wilson coefficients in $\mathscr{L}_{\rm ESM}$ are assumed to be small enough that they can be treated at leading order.



Strategy

① **measure κ**

$$\frac{\Gamma_{gg}}{\Gamma^{SM}_{gg}(m_H)} = \frac{\kappa_t^2 \cdot \Gamma^{tt}_{gg}(m_H) + \kappa_b^2 \cdot \Gamma^{bb}_{gg}(m_H) + \kappa_t \kappa_b \cdot \Gamma^{tb}_{gg}(m_H)}{\Gamma^{tt}_{gg}(m_H) + \Gamma^{bb}_{gg}(m_H) + \Gamma^{tb}_{gg}(m_H)}$$

② **find $\mathcal{O}_i \Leftrightarrow \kappa_x$** (epistemological stop, true ESM believers stop here)

$$\mathcal{L}_{ESM} = \mathcal{L}_{SM} + \sum_{n>4} \sum_{i=1}^{N_n} \frac{a_i^n}{\Lambda^{n-4}} \mathcal{O}_i^{(d=n)}$$

③ **find $\{\mathcal{L}_{BSM}\}$** that produces $\mathcal{O}_i$

κ_X **cannot** be arbitrary shifts of the **SM** diagrams

$$\frac{\Gamma_{gg}}{\Gamma_{gg}^{SM}(m_H)} = \frac{\kappa_t^2 \cdot \Gamma_{gg}^{tt}(m_H) + \kappa_b^2 \cdot \Gamma_{gg}^{bb}(m_H) + \kappa_t \kappa_b \cdot \Gamma_{gg}^{tb}(m_H)}{\Gamma_{gg}^{tt}(m_H) + \Gamma_{gg}^{bb}(m_H) + \Gamma_{gg}^{tb}(m_H)}$$

↬ they require an **underlying** (at least **effective**) theory

We define an effective Lagrangian based on

a **linear** representation of the EW gauge symmetry with a Higgs-doublet field, restricting ourselves to **dimension**-**6** operators relevant for Higgs physics **Buchmuller:1985jz, Grzadkowski:2010es**.

☛ **Disclaimer: it is impossible to quote all who have contributed. For what is relevant here:**

- **Yellow Report HXSWG vol. 3: A. David, A. Denner, M. Dührssen, M. Grazzini, C. Grojean, K. Prokofiev, G. Weiglein, M. Zanetti, S. Dittmaier, G. Passarino and M. Spira**
- **Contino:2013kra**
- **Corbett:2013hia**
- **Elias-Miro:2013gya**

Lagrangian

$$\mathscr{L}_{\rm eff} = \mathscr{L}_{\rm SM}^{(4)} + \frac{1}{\Lambda^2}\sum_k \alpha_k \mathscr{O}_k,$$

$$\begin{aligned}\mathscr{L}_{\rm SM}^{(4)} = & -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}\mathrm{W}^I_{\mu\nu}\mathrm{W}^{I\mu\nu} - \frac{1}{4}\mathrm{B}_{\mu\nu}\mathrm{B}^{\mu\nu} \\ & + (D_\mu\Phi)^\dagger(D^\mu\Phi) + m^2\Phi^\dagger\Phi - \frac{1}{2}\lambda(\Phi^\dagger\Phi)^2 \\ & + i\bar{\mathrm{l}}\not{D}\mathrm{l} + i\bar{\mathrm{e}}\not{D}\mathrm{e} + i\bar{\mathrm{q}}\not{D}\mathrm{q} + i\bar{\mathrm{u}}\not{D}\mathrm{u} + i\bar{\mathrm{d}}\not{D}\mathrm{d} \\ & - (\bar{\mathrm{l}}\Gamma_{\mathrm{e}}\mathrm{e}\Phi + \bar{\mathrm{q}}\Gamma_{\mathrm{u}}\mathrm{u}\widetilde{\Phi} + \bar{\mathrm{d}}\Gamma_{\mathrm{d}}\mathrm{d}\Phi + \mathrm{h.c.}),\end{aligned}$$

Operators

Φ^6 and $\Phi^4 D^2$	$\psi^2\Phi^3$	X^3
$\mathcal{O}_{\Phi} = (\Phi^\dagger\Phi)^3$	$\mathcal{O}_{e\Phi} = (\Phi^\dagger\Phi)(\bar{l}\,\Gamma_e e\Phi)$	$\mathcal{O}_G = f^{ABC} G^{A\nu}_{\mu} G^{B\rho}_{\nu} G^{C\mu}_{\rho}$
$\mathcal{O}_{\Phi\Box} = (\Phi^\dagger\Phi)\Box(\Phi^\dagger\Phi)$	$\mathcal{O}_{u\Phi} = (\Phi^\dagger\Phi)(\bar{q}\,\Gamma_u u\tilde{\Phi})$	$\mathcal{O}_{\widetilde{G}} = f^{ABC} \widetilde{G}^{A\nu}_{\mu} G^{B\rho}_{\nu} G^{C\mu}_{\rho}$
$\mathcal{O}_{\Phi D} = (\Phi^\dagger D^\mu\Phi)^*(\Phi^\dagger D_\mu\Phi)$	$\mathcal{O}_{d\Phi} = (\Phi^\dagger\Phi)(\bar{q}\,\Gamma_d d\Phi)$	$\mathcal{O}_W = \varepsilon^{IJK} W^{I\nu}_{\mu} W^{J\rho}_{\nu} W^{K\mu}_{\rho}$
		$\mathcal{O}_{\widetilde{W}} = \varepsilon^{IJK} \widetilde{W}^{I\nu}_{\mu} W^{J\rho}_{\nu} W^{K\mu}_{\rho}$

$X^2\Phi^2$	$\psi^2 X\Phi$	$\psi^2\Phi^2 D$
$\mathcal{O}_{\Phi G} = (\Phi^\dagger\Phi) G^A_{\mu\nu} G^{A\mu\nu}$	$\mathcal{O}_{uG} = (\bar{q}\sigma^{\mu\nu}\frac{\lambda^A}{2}\Gamma_u u\tilde{\Phi}) G^A_{\mu\nu}$	$\mathcal{O}^{(1)}_{\Phi l} = (\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{l}\gamma^\mu l)$
$\mathcal{O}_{\Phi\widetilde{G}} = (\Phi^\dagger\Phi)\widetilde{G}^A_{\mu\nu} G^{A\mu\nu}$	$\mathcal{O}_{dG} = (\bar{q}\sigma^{\mu\nu}\frac{\lambda^A}{2}\Gamma_d d\Phi) G^A_{\mu\nu}$	$\mathcal{O}^{(3)}_{\Phi l} = (\Phi^\dagger i\overleftrightarrow{D}^I_\mu\Phi)(\bar{l}\gamma^\mu\tau^I l)$
$\mathcal{O}_{\Phi W} = (\Phi^\dagger\Phi) W^I_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}_{eW} = (\bar{l}\sigma^{\mu\nu}\Gamma_e e\tau^I\Phi) W^I_{\mu\nu}$	$\mathcal{O}_{\Phi e} = (\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{e}\gamma^\mu e)$
$\mathcal{O}_{\Phi\widetilde{W}} = (\Phi^\dagger\Phi)\widetilde{W}^I_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}_{uW} = (\bar{q}\sigma^{\mu\nu}\Gamma_u u\tau^I\tilde{\Phi}) W^I_{\mu\nu}$	$\mathcal{O}^{(1)}_{\Phi q} = (\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{q}\gamma^\mu q)$
$\mathcal{O}_{\Phi B} = (\Phi^\dagger\Phi) B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dW} = (\bar{q}\sigma^{\mu\nu}\Gamma_d d\tau^I\Phi) W^I_{\mu\nu}$	$\mathcal{O}^{(3)}_{\Phi q} = (\Phi^\dagger i\overleftrightarrow{D}^I_\mu\Phi)(\bar{q}\gamma^\mu\tau^I q)$
$\mathcal{O}_{\Phi\widetilde{B}} = (\Phi^\dagger\Phi)\widetilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{eB} = (\bar{l}\sigma^{\mu\nu}\Gamma_e e\Phi) B_{\mu\nu}$	$\mathcal{O}_{\Phi u} = (\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{u}\gamma^\mu u)$
$\mathcal{O}_{\Phi WB} = (\Phi^\dagger\tau^I\Phi) W^I_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{uB} = (\bar{q}\sigma^{\mu\nu}\Gamma_u u\tilde{\Phi}) B_{\mu\nu}$	$\mathcal{O}_{\Phi d} = (\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{d}\gamma^\mu d)$
$\mathcal{O}_{\Phi\widetilde{W}B} = (\Phi^\dagger\tau^I\Phi)\widetilde{W}^I_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dB} = (\bar{q}\sigma^{\mu\nu}\Gamma_d d\Phi) B_{\mu\nu}$	$\mathcal{O}_{\Phi ud} = i(\tilde{\Phi}^\dagger D_\mu\Phi)(\bar{u}\gamma^\mu\Gamma_{ud} d)$

IN A COMPLETE ANALYSIS ALL 59 INDEPENDENT OPERATORS OF Grzadkowski:2010es), INCLUDING 25 FOUR-FERMION OPERATORS, HAVE TO BE CONSIDERED IN ADDITION TO THE SELECTED 34 OPERATORS

In **weakly interacting** theories the dimension-6 operators involving field strengths can only result from loops, while the others also result from tree diagrams (Arzt:1994gp). The operators involving dual field strengths tensors or complex Wilson coefficients violate CP.

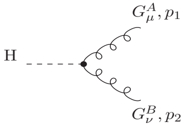
The parameters of the SM Lagrangian g, g', λ, m^2, and Γ_f keep their meaning in the presence o dimension-6 operators.

10.4.2 Higgs vertices

Here we list the most important Feynman rules for vertices involving exactly one physical Higgs boson These are given in terms of the above-defined physical fields and parameters. In the coefficients o dimension-6 couplings we replaced v^2 by the Fermi constant via $v^2 = 1/(\sqrt{2}G_F)$.

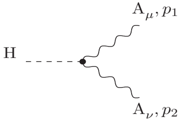
The triple vertices involving one Higgs boson read:

Hgg coupling:



$$= \mathrm{i}\frac{2g}{M_\mathrm{W}}\frac{1}{\sqrt{2}G_F\Lambda^2}\left[\alpha_{GG}(p_{2\mu}p_{1\nu} - p_1p_2g_{\mu\nu}) + \alpha_{G\widetilde{G}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right]\delta^{AB}, \quad (155$$

HAA coupling:



$$= \mathrm{i}\frac{2g}{M_\mathrm{W}}\frac{1}{\sqrt{2}G_F\Lambda^2}\left[\alpha_{\mathrm{AA}}(p_{2\mu}p_{1\nu} - p_1p_2g_{\mu\nu}) + \alpha_{\mathrm{A}\widetilde{\mathrm{A}}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right], \quad (156$$

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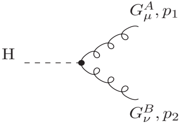

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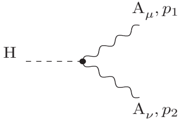

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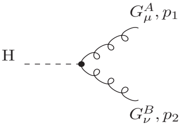

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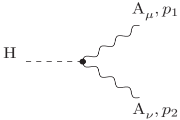

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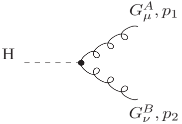

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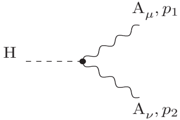

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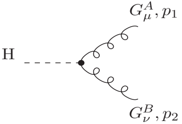

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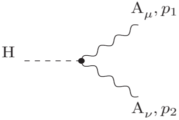

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Since κ_t, κ_b etc. can be made different **ONLY** by inserting $\mathcal{O}$ operators in **SM** vertices

Vademecum (**NLO + EFT**) trainee

- the **EFT** part has to be implemented into existing (**EW + QCD**) codes: formulation in arbitrary gauge (not U-gauge restricted) is needed
- Renormalization for the full **SM + EFT** Lagrangian is needed

If one restricts the analysis to the calculation of **on-shell matrix** elements then additional operators are eliminated by the **Equations-Of-Motion** (EOM).

given a theory with a Lagrangian $\mathscr{L}[\phi]$ consider an effective Lagrangian $\mathscr{L}_{\text{eff}} = \mathscr{L} + g\,\mathscr{O} + g'\,\mathscr{O}''$ where

$$\mathscr{O} - \mathscr{O}'' \quad = \quad F[\phi]\,\delta\mathscr{L}/\delta\phi$$

and F is some local functional of ϕ. The effect of $\mathscr{O}'$ on $\mathscr{L}_{\text{eff}} = \mathscr{L} + g\,\mathscr{O}$ is

to shift $g \to g + g'$ and to replace $\phi \to \phi + g'\,F$

Caveat

Only $\boldsymbol{S}$-matrix elements will be the same for equivalent operators but not the Green's functions $\therefore$

- since we are working with **unstable particles**,
- since we are inserting operators **inside loops**,
- since we want to use (off-shell) $\boldsymbol{S}, \boldsymbol{T}$ and $\boldsymbol{U}$ parameters to **constrain** the Wilson coefficients,

↬ the use of EOM should be taken with extreme caution

➼ **(Wudka:1994ny) even if the S-matrix elements cannot distinguish between two equivalent operators $\mathcal{O}$ and $\mathcal{O}'$, there is a large quantitative difference whether the underlying theory can generate $\mathcal{O}'$ or not. It is equally reasonable not to eliminate redundant operators and, eventually, exploit redundancy to check S-matrix elements.**

T, L operators

The $d = 6$ operators are supposed to arise from a local Lagrangian, containing heavy degrees of freedom, ONCE THE LATTER ARE INTEGRATED OUT (the correspondence Lagrangians $\longrightarrow$ effective operators is not bijective) These operators are of two different origins:

- ***T*-operators** are those that arise from the tree-level exchange of some heavy degree of freedom
- ***L*-operators** are those that arise from loops of heavy degrees of freedom.

The L-operators are usually not included in the analysis. **See recent results in Einhorn:2013kja**

UV

Insertion of $d = 6$ operators in loops

We have to deal with

- renormalization of composite operators
- absorbing UV divergences to all orders and of maintaining the independence of arbitrary UV scale cutoff, problems that require the introduction of all possible terms allowed by the symmetries Georgi:1994qn,Kaplan:1995uv (EFT renormalization à la BPHZ?)
- Special care should be devoted in avoiding double-counting when we consider insertion of T-operators in loops and L-operators as well.

Caveat

Note that for

$\Lambda \approx 5\ TeV$

we have

$$1/(\sqrt{2}G_F\Lambda^2) \approx g^2/(4\pi)$$,

i.e. ⤐ the contributions of $d = 6$ operators are ≊ loop effects.

⤐ ⤐ For higher scales, loop contributions tend to be more important (⋡)

UV

UV Characteristic

- Operators normally alter the UV power-counting of a SM diagram
- but THERE ARE OPERATORS THAT DO NOT CHANGE THE UV POWER-COUNTING: we say that a set of SM diagrams is UV-scalable w.r.t. a combination of $d = 6$ operators if
  - their sum is UV finite
  - all diagrams in the set are scaled by the same combination of $d = 6$ operators.
- these diagrams are UV admissible

Example: SM loops dressed only with UV-admissible operators

$\mathrm{H} \to \gamma\gamma$

For $\mathrm{H} \to \gamma\gamma$ the SM amplitude reads

$$\mathscr{M}_{\mathrm{SM}} = F_{\mathrm{SM}} \left(\delta^{\mu\nu} + 2\, \frac{p_1^{\nu} p_2^{\mu}}{\overline{\mathrm{M}}_{\mathrm{H}}^{2}} \right) e_{\mu}\,(p_1)\, e_{\nu}\,(p_2)$$

$$F_{\mathrm{SM}} = -g\,\overline{\mathrm{M}}\, F_{\mathrm{SM}}^{\mathrm{W}} - \frac{1}{2}\, g\, \frac{M_{\mathrm{t}}^2}{\overline{\mathrm{M}}}\, F_{\mathrm{SM}}^{\mathrm{t}} - \frac{1}{2}\, g\, \frac{M_{\mathrm{b}}^2}{\overline{\mathrm{M}}}\, F_{\mathrm{SM}}^{\mathrm{b}}.$$

$$F_{\mathrm{SM}}^{\mathrm{W}} = 6 + \frac{\overline{\mathrm{M}}_{\mathrm{H}}^{2}}{\overline{\mathrm{M}}^{2}} + 6\left(\overline{\mathrm{M}}_{\mathrm{H}}^{2} - 2\,\overline{\mathrm{M}}^{2}\right) C_0\left(-\overline{\mathrm{M}}_{\mathrm{H}}^{2}, 0, 0; \overline{\mathrm{M}}, \overline{\mathrm{M}}, \overline{\mathrm{M}}\right),$$

$$F_{\mathrm{SM}}^{\mathrm{t}} = -8 - 4\left(\overline{\mathrm{M}}_{\mathrm{H}}^{2} - 4\, M_{\mathrm{t}}^2\right) C_0\left(-\overline{\mathrm{M}}_{\mathrm{H}}^{2}, 0, 0; M_{\mathrm{t}}, M_{\mathrm{t}}, M_{\mathrm{t}}\right),$$

We only need a subset of operators $\curvearrowright$

$$
\begin{aligned}
\widetilde{\mathscr{L}} &= A^1_{\mathrm{V}} \left(\Phi^\dagger \Phi - v^2\right) F^a_{\mu\nu} F^a_{\mu\nu} + A^2_{\mathrm{V}} \left(\Phi^\dagger \Phi - v^2\right) F^0_{\mu\nu} F^0_{\mu\nu} \\
&+ A^3_{\mathrm{V}} \Phi^\dagger \tau_a \Phi F^a_{\mu\nu} F^0_{\mu\nu} + \frac{1}{2} A_{\partial\Phi}\, \partial_\mu \left(\Phi^\dagger \Phi\right) \partial_\mu \left(\Phi^\dagger \Phi\right) \\
&+ A^1_{\Phi} \left(\Phi^\dagger \Phi\right) \left(D_\mu \Phi\right)^\dagger D_\mu \Phi + A^3_{\Phi} \left(\Phi^\dagger D_\mu \Phi\right) \left[\left(D_\mu \Phi\right)^\dagger \Phi\right] \\
&+ \frac{1}{4\sqrt{2}} \frac{M_{\mathrm{t}}}{\overline{\mathrm{M}}} A^1_{\mathrm{f}} \left(\Phi^\dagger \Phi - v^2\right) \overline{\psi}_{\mathrm{L}} \Phi\, \mathrm{t}_{\mathrm{R}} \\
&+ \frac{1}{4\sqrt{2}} \frac{M_{\mathrm{b}}}{\overline{\mathrm{M}}} A^2_{\mathrm{f}} \left(\Phi^\dagger \Phi - v^2\right) \overline{\psi}_{\mathrm{L}} \Phi^c\, \mathrm{b}_{\mathrm{R}} + \text{h. c.}
\end{aligned}
$$

$$
A^0_{\Phi} = A^1_{\Phi} + 2\, \frac{A^3_{\Phi}}{\hat{s}^2_\theta} + 4\, A_{\partial\Phi}.
$$

✌

$$\mathscr{M}_{\mathrm{H}\to\gamma\gamma} = \left(4\sqrt{2}\,G_{\mathrm{F}}\right)^{1/2}\left\{-\frac{\alpha}{\pi}\left[C_{\mathrm{W}}^{\gamma\gamma}\,F_{\mathrm{SM}}^{\mathrm{W}} + 3\sum_{\mathrm{q}} Q_{\mathrm{q}}^{2}\,C_{\mathrm{q}}^{\gamma\gamma}\,F_{\mathrm{SM}}^{\mathrm{q}}\right] + F_{\mathrm{AC}}\right\}$$

$$F_{\mathrm{AC}} = \frac{g_6}{\sqrt{2}}\,\overline{\mathrm{M}}_{\mathrm{H}}^{2}\left(\hat{s}_{\theta}^{2}\,A_{\mathrm{V}}^{1} + \hat{c}_{\theta}^{2}\,A_{\mathrm{V}}^{2} + \hat{c}_{\theta}\,\hat{s}_{\theta}\,A_{\mathrm{V}}^{3}\right).$$

$$g_6 = \frac{1}{G_{\mathrm{F}}\,\Lambda^{2}} = 0.085736\left(\frac{TeV}{\Lambda}\right)^{2}$$

✌ the scaling factors are given by

$$C^{\gamma\gamma}_{\mathrm{W}} = \frac{1}{4}\,\overline{\mathrm{M}}^{\,2}\Bigl\{1 + \frac{g_6}{4\sqrt{2}}\Bigl[8\,A^3_{\mathrm{V}}\,\hat{c}_\theta\,\Bigl(\hat{s}_\theta + \frac{1}{\hat{s}_\theta}\Bigr) + A^0_{\Phi}\Bigr]\Bigr\}$$

$$C^{\gamma\gamma}_{\mathrm{t}} = \frac{1}{8}\,M^2_{\mathrm{t}}\Bigl\{1 + \frac{g_6}{4\sqrt{2}}\Bigl[8\,A^3_{\mathrm{V}}\,\hat{c}_\theta\,\Bigl(\hat{s}_\theta + \frac{1}{\hat{s}_\theta}\Bigr) + A^0_{\Phi} - A^1_{\mathrm{f}}\Bigr]\Bigr\}$$

$$C^{\gamma\gamma}_{\mathrm{b}} = \frac{1}{8}\,M^2_{\mathrm{b}}\Bigl\{1 + \frac{g_6}{4\sqrt{2}}\Bigl[8\,A^3_{\mathrm{V}}\,\hat{c}_\theta\,\Bigl(\hat{s}_\theta + \frac{1}{\hat{s}_\theta}\Bigr) + A^0_{\Phi} - A^2_{\mathrm{f}}\Bigr]\Bigr\}$$

The amplitude is the sum of

- the $\mathbf{W}, \mathbf{t}$ and $\mathbf{b}$ SM components, each **scaled** by some combination of Wilson coefficients, and of
- a **contact** term

The latter is $\mathcal{O}(g_6)$ while the rest of the corrections is $\mathcal{O}\left(\frac{\alpha}{\pi} g_6\right)$.
However, one should remember that

- $\mathcal{O}^i_{\mathbf{V}}$ are operators of L-type, i.e. they arise from loop correction in the complete theory

$\therefore$, the corresponding coefficients are expected to be very small although this is only an argument about naturalness without a specific quantitative counterpart (apart from a $1/(16\pi^2)$ factor from loop integration)

Glimpsing at the headlines of the **complete** calculation for
$\mathbf{H \to \gamma\gamma}$

☛

- **SM** loops, dressed with admissible operators
- **New 33** loop-diagrams
- **Counter-terms**

Amplitude in *internal* notations ☛

```
g HAA= -int(q)*Qs(-1,[q]^2+mt^2)*Qs(-1,[q+p1]^2+mt^2)*Qs(-1,[q+p1+p2]^2+mt^2)*3*trace*(
  ( -1/2*g*mt/M + L^-2 *( 4*r2^-1*M^2*af1 - 2*M*aV1*mt - 1/2*a3K*M*g*mt + 2*adK*M*g*mt))*
  (-i_*(gd(s,q)+gd(s,p1)+gd(s,p2))+mt)*
  VAtt(nu,p2)*(-i_*(gd(s,q)+gd(s,p1))+mt)*
  VAtt(mu,p1)*(-i_*gd(s,q)+mt)+
  ( -1/2*g*mt/M + L^-2 *( 4*r2^-1*M^2*af1 - 2*M*aV1*mt - 1/2*a3K*M*g*mt + 2*adK*M*g*mt))*
  ( i_*gd(s,q)+mt)*
  VAtt(mu,p1)*( i_*(gd(s,q)+gd(s,p1))+mt)*
  VAtt(nu,p2)*( i_*(gd(s,q)+gd(s,p1)+gd(s,p2))+mt))-
  int(q)*Qs(-1,[q]^2+mb^2)*Qs(-1,[q+p1]^2+mb^2)*Qs(-1,[q+p1+p2]^2+mb^2)*trace*(
  ( -1/2*g*mb/M + L^-2 *( - 4*r2^-1*M^2*af2 - 2*M*aV1*mb - 1/2*a3K*M*g*mb + 2*adK*M*g*mb))*
  (-i_*(gd(s,q)+gd(s,p1)+gd(s,p2))+mb)*
  VAbb(nu,p2)*(-i_*(gd(s,q)+gd(s,p1))+mb)*
  VAbb(mu,p1)*(-i_*gd(s,q)+mb)+
  ( -1/2*g*mb/M + L^-2 *( - 4*r2^-1*M^2*af2 - 2*M*aV1*mb - 1/2*a3K*M*g*mb + 2*adK*M*g*mb))*
  ( i_*gd(s,q)+mb)*
  VAbb(mu,p1)*( i_*(gd(s,q)+gd(s,p1))+mb)*
  VAbb(nu,p2)*( i_*(gd(s,q)+gd(s,p1)+gd(s,p2))+mb))+
  + i_*L^-2 *(
     + 8*M*(sth^2*aV1 + cth^2*aV2 + sth*cth*aV3)*(p1(nu)*p2(mu) - d_(mu,nu)*p1.p2))+
  int(q)*Qs(-1,[q]^2+M^2)*Qs(-1,[q+p1]^2+M^2)*Qs(-1,[q+p1+p2]^2+M^2)*(
  dia1*VHWW(al,be,-q,q+p1+p2)*VAWmWp(nu,be,si,p2,-q-p1-p2,q+p1)*VAWmWp(mu,si,al,p1,-q-p1,q)+
  dia2*VHWW(be,al,q+p1+p2,-q)*VAWmWp(mu,al,si,p1,q,-q-p1)*VAWmWp(nu,si,be,p2,q+p1,-q-p1-p2)+
  dia3*VHPmWp(al,-p1-p2,-q)*VAWmWp(nu,al,be,p2,-q-p1-p2,q+p1)*VAPpWm(mu,be,p1,-q-p1)+

  .................

  dia30*VAAWP(mu,nu,al,p1,p2)*VHPpWm(al,-p1-p2,-q))+
  int(q)*Qs(-1,[q]^2+M0^2)*Qs(-1,[q+p1+p2]^2+M0^2)*(
  dia31*VHP0P0(-p1-p2,-q,q+p1+p2)*VAAP0P0(mu,nu,p1,p2))+
  int(q)*Qs(-1,[q]^2+mh^2)*Qs(-1,[q+p1+p2]^2+mh^2)*(
  dia32*VHHH(-p1-p2,q+p1,-q)*VAAHH(mu,nu,p1,p2))+
  int(q)*Qs(-1,[q]^2+M^2)*(dia33*VHAAWW(mu,nu,si,si));
```

```
id VHPmPp(p1?,p2?,p3?)=
  - 1/2*M^-1*mh^2*g
  + L^-2 * (
  - 2*M*mh^2*aV1 - 2*p2.p3*a1K*M*g + 1/2*(mh^2 + 2*p1.p1)*a3K*M*g - 2*(mh^2 + 2*p1.p1)*adK*M*g
*
id VHPmWp(be?,p1?,p2?)=
  - 1/2*(p1(be) - p2(be))*i_*g
  + L^-2 * (
  - 2*p2(be)*i_*a1K*M^2*g - 2*(p1(be) - p2(be))*i_*M^2*aV1
  - 1/2*(p1(be) - p2(be))*i_*a3K*M^2*g + 2*(p1(be) - p2(be))*i_*adK*M^2*g);
*
id VHPpWm(be?,p1?,p2?)=
  - 1/2*(p1(be) - p2(be))*i_*g
  + L^-2 * (
  - 2*p2(be)*i_*a1K*M^2*g - 2*(p1(be) - p2(be))*i_*M^2*aV1
  - 1/2*(p1(be) - p2(be))*i_*a3K*M^2*g + 2*(p1(be) - p2(be))*i_*adK*M^2*g);
*
id VHWW(al?,be?,p2?,p3?)=
  - d_(al,be)*M*g
  + L^-2 * (
  - 4*d_(al,be)*M^3*aV1 - d_(al,be)*a3K*M^3*g + 2*d_(al,be)*a1K*M^3*g
  + 4*d_(al,be)*adK*M^3*g + 8*(p2(be)*p3(al) - d_(al,be)*p2.p3)*M*aV1);
*
id VHZZ(al?,be?,p2?,p3?)=
  - d_(al,be)*M*cth^-2*g
  + L^-2 * (
  - 4*d_(al,be)*M^3*aV1*cth^-2 + d_(al,be)*a3K*M^3*cth^-2*g
  + 2*d_(al,be)*a1K*M^3*cth^-2*g + 4*d_(al,be)*adK*M^3*cth^-2*g
  - 8*(p2(be)*p3(al) - d_(al,be)*p2.p3)*M*aV3*cth*sth
  + 8*(p2(be)*p3(al) - d_(al,be)*p2.p3)*M*aV2*sth^2
  + 8*(p2(be)*p3(al) - d_(al,be)*p2.p3)*M*aV1*cth^2);
*
```

$\mathcal{O}(g^3)$

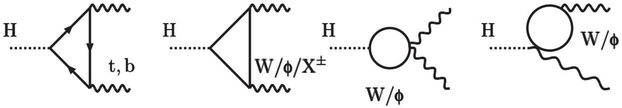
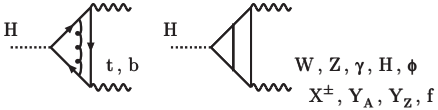


Figure 1: The three families of diagrams contributing to the amplitude for H $\rightarrow \gamma\gamma$; W/ϕ denotes a W-line or a ϕ-line. $X^{\pm}$ denotes a FP-ghost line

$\mathcal{O}(g^5)$



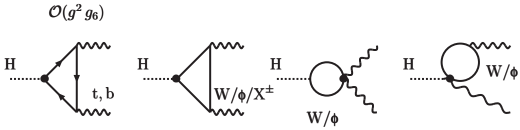


Figure 3: Example of one-loop SM diagrams with $\mathcal{O}$-insertions, contributing to the amplitude for $\mathrm{H} \to \gamma\gamma$

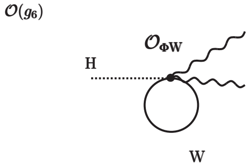


Figure 4: Example of one-loop $\mathcal{O}$-diagrams, contributing to the amplitude for $\mathrm{H} \to \gamma\gamma$

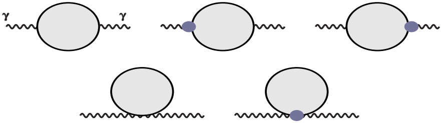


Figure 5: The photon self-energy with inclusion of $\mathcal{O}$-operators into SM one-loop diagrams. The last diagram contains vertices, like $\mathrm{AAHH}, \mathrm{AA}\phi^0\phi^0$, that do not belong to the SM part.

Renormalization

$$g = g_{\rm ren}\left[1 + \frac{g^2_{\rm ren}}{16\,\pi^2}\left(dZ_g + g_6\,dZ^{(6)}_g\right)\frac{1}{\varepsilon}\right]$$

$$M_{\rm W} = M^{\rm ren}_{\rm W}\left[1 + \frac{1}{2}\,\frac{g^2_{\rm ren}}{16\,\pi^2}\left(dZ_{M_{\rm W}} + g_6\,dZ^{(6)}_{M_{\rm W}}\right)\frac{1}{\varepsilon}\right]$$

etc.

Wilson coefficients $\rightarrow$ W_i

$$W_i = \sum_j Z^{\rm wc}_{ij}\,W^{\rm ren}_j$$

$$Z^{\rm wc}_{ij} = \delta_{ij} + \left(g^2_{\rm ren}\,dZ^{\rm wc}_{ij} + dZ^{{\rm wc},6}_{ij}\right)\frac{1}{\varepsilon}$$

Conclusions?

✎ Data-driven Theory? Or

If you’re looking for your lost keys, failing to find them in the kitchen is not evidence against their being somewhere else in the house

Conclusions?

- Higgs-landscape: asking the right questions takes as much skill as giving the right answers
- *A conclusion is the place where you got tired of thinking* (Arthur Bloch)
- *I am turned into a sort of machine for observing facts and grinding out conclusions* (Charles Darwin)
- *El sueño de la razõn produce monstruos* (Francisco Goya)

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Thanks for your attention

Backup

assumptions/inferences

- Given the (few) known coefficients in the perturbative expansion we estimate the next (few) coefficients and the corresponding partial sums by means of sequence transformations. This is the first step towards “ reconstructing ” the physical observable.
- The sequence transformations have been tested on a number of test sequences.
- A function can be uniquely determined by its asymptotic expansion if certain conditions are satisfied (Sokal).
- Borel procedure is a summation method which, under the above conditions, determines uniquely the sum of the series. It should be taken into account that there is a large class of series that have Borel sums (analytic in the cut-plane) and there is evidence that Levin-Weniger transforms produce approximations to these Borel sums. This is one of the arguments of plausibility supporting our results.
- The QCD scale variation uncertainty decreases when we include new (estimated) partial sums.
- All known and predicted coefficients are positive and all transforms predict convergence within a narrow interval.
- Missing a formal proof of uniqueness, we assume uninformative prior between the last known partial sum and the (largest) predicted partial sum.

Return

Structure of the calculation

- Process: $\mathrm{H} \to \bar{\mathrm{f}}\mathrm{f}\gamma$, $\mathrm{f} = \mathrm{l}, \mathrm{q}$,
  including b with non-zero m_{t}
- Setup: $m_{\mathrm{f}} = 0$ at NLO. Calculation based on helicity amplitudes
  LO and NLO do not interfere (with $m_{\mathrm{f}} = 0$)

Cuts available in the H rest-frame

Please complain but it took years to interface *POWHEG* and *Prophecy4f* ......

$\mathrm{gg} \to \bar{\mathrm{f}}\mathrm{f}\gamma$? Can be done, *But* ......

HTO-DALITZ Features

- Internal cross-check, loops are evaluated both *analytically* and *numerically* (using BST-algorithm)
- The code makes extensive use of *In-House* abbreviation algorithms (if $a+b$ appears twice or more it receives an abbreviation and it is pre-computed only once).
- All functions are **collinear-free**
- High performances thanks to gcc-4.8.0
- *Open MPI* version under construction, *GPU* version in a preliminar phase
- Returns the full result and also the unphysical components

Man at work

- Extensions: as it was done during Lep times, there are diagrams where both the Z and the γ propagators should be Dyson-improved, i.e.

$$\alpha_{\mathrm{QED}}(0) \to \alpha_{\mathrm{QED}}(\mathrm{virtuality}) \qquad \rho_{\mathrm{f}} - \text{parameter included}$$

- However, the interested sub-sets are not gauge invariant, $\therefore$ appropriate subtractions must be performed (at virtuality $= 0$, s_{Z}, the latter being the Z complex-pole).

Misunderstandings

- use $M\left(\bar{\mathrm{f}}\mathrm{f}\gamma\right)$ and require $\mid M - M_{\mathrm{Z}} \mid \leqq n\Gamma_{\mathrm{Z}}$. *This is not the photon we are discussing*
  Photons are collinear to leptons only if emitted by leptons but those are Yukawa-suppressed.
  In any case $M\left(\bar{\mathrm{f}}\mathrm{f}\gamma\right) = M_{\mathrm{H}}$ or it is *Not* Dalitz decay
- Requiring a cut on the opening angle between leptons and the photon to define *isolated photons* is highly recommended, *But* at the moment we are still in the Higgs rest-frame (*Miracles take a bit longer*)

Return

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Return

with the physical mass parameters

$$\begin{aligned} M_\mathrm{W}^2 &= \frac{1}{4}g^2v^2\left[1+2\frac{v^2}{\Lambda^2}\alpha_{\Phi\mathrm{W}}\right], \\ M_\mathrm{Z}^2 &= \frac{1}{4}(g^2+g'^2)v^2\left[1+\frac{v^2}{2\Lambda^2}\left(4\alpha_{\mathrm{ZZ}}+\alpha_{\Phi D}\right)\right], \\ M_\mathrm{H}^2 &= \lambda v^2\left[1+\frac{v^2}{2\Lambda^2}\left(4\alpha_{\Phi\Box}-\frac{6}{\lambda}\alpha_\Phi-\alpha_{\Phi D}\right)\right], \\ m_\mathrm{f} &= \frac{1}{\sqrt{2}}U^{\hat{\mathrm{f}}}\Gamma_\mathrm{f}U^{\mathrm{f},\dagger}v\left[1-\frac{1}{2}\frac{v^2}{\Lambda^2}\alpha_{\mathrm{f}\phi}\right]. \end{aligned} \tag{151}$$

In (150) we have used the usual 't Hooft–Feynman gauge-fixing term

$$\mathcal{L}_\mathrm{fix} = -C_+C_- - \frac{1}{2}(C_\mathrm{Z})^2 - \frac{1}{2}(C_\mathrm{A})^2 - \frac{1}{2}C_G^AC_G^A \tag{152}$$

with

$$C_G^A = \partial_\mu G^{A\mu}, \qquad C_\mathrm{A} = \partial_\mu \mathrm{A}^\mu, \qquad C_\mathrm{Z} = \partial_\mu \mathrm{Z}^\mu + M_\mathrm{Z}\phi^0, \qquad C_\pm = \partial_\mu \mathrm{W}^{\pm\mu} \pm \mathrm{i}M_\mathrm{W}\phi^\pm \tag{153}$$

in terms of the physical fields and parameters, which gives rise to the same propagators as in the SM.

In the following, the abbreviations c_w and s_w are defined via the physical masses

$$c_\mathrm{w} = \frac{M_\mathrm{W}}{M_\mathrm{Z}}, \qquad s_\mathrm{w} = \sqrt{1-c_\mathrm{w}^2}. \tag{154}$$

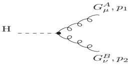
The parameters of the SM Lagrangian g, g', λ, m^2, and Γ_f keep their meaning in the presence of dimension-6 operators.

10.4.2 Higgs vertices

Here we list the most important Feynman rules for vertices involving exactly one physical Higgs boson. These are given in terms of the above-defined physical fields and parameters. In the coefficients of dimension-6 couplings we replaced v^2 by the Fermi constant via $v^2 = 1/(\sqrt{2}G_F)$.

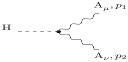
The triple vertices involving one Higgs boson read:

Hgg coupling:



$$= \mathrm{i}\frac{2g}{M_\mathrm{W}}\frac{1}{\sqrt{2}G_F\Lambda^2}\left[\alpha_{GG}(p_{2\mu}p_{1\nu}-p_1p_2g_{\mu\nu})+\alpha_{G\tilde{G}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right]\delta^{AB}, \tag{155}$$

HAA coupling:



$$= \mathrm{i}\frac{2g}{M_\mathrm{W}}\frac{1}{\sqrt{2}G_F\Lambda^2}\left[\alpha_{\mathrm{AA}}(p_{2\mu}p_{1\nu}-p_1p_2g_{\mu\nu})+\alpha_{\mathrm{A}\tilde{\mathrm{A}}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right], \tag{156}$$

with the physical mass parameters

$$\begin{aligned}
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M_{\mathrm{Z}}^2 &= \frac{1}{4}(g^2 + g'^2) v^2 \left[1 + \frac{v^2}{2\Lambda^2}\left(4\alpha_{\mathrm{ZZ}} + \alpha_{\Phi D}\right)\right], \\
M_{\mathrm{H}}^2 &= \lambda v^2 \left[1 + \frac{v^2}{2\Lambda^2}\left(4\alpha_{\Phi\Box} - \frac{6}{\lambda}\alpha_{\Phi} - \alpha_{\Phi D}\right)\right], \\
m_{\mathrm{f}} &= \frac{1}{\sqrt{2}} U^{\mathrm{f}} \Gamma_{\mathrm{f}} U^{\mathrm{f},\dagger} v \left[1 - \frac{1}{2}\frac{v^2}{\Lambda^2}\alpha_{\mathrm{f}\phi}\right].
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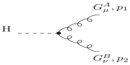
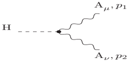


$$= \mathrm{i}\frac{2g}{M_{\mathrm{W}}}\frac{1}{\sqrt{2}G_F\Lambda^2}\left[\alpha_{GG}(p_{2\mu}p_{1\nu} - p_1 p_2 g_{\mu\nu}) + \alpha_{G\tilde{G}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right]\delta^{AB}, \tag{155}$$

HAA coupling:



$$= \mathrm{i}\frac{2g}{M_{\mathrm{W}}}\frac{1}{\sqrt{2}G_F\Lambda^2}\left[\alpha_{\mathrm{AA}}(p_{2\mu}p_{1\nu} - p_1 p_2 g_{\mu\nu}) + \alpha_{\mathrm{A}\tilde{\mathrm{A}}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right], \tag{156}$$

with the physical mass parameters

$$\begin{aligned} M_{\rm W}^2 &= \frac{1}{4} g^2 v^2 \left[1 + 2\frac{v^2}{\Lambda^2}\alpha_{\Phi \rm W}\right], \\ M_{\rm Z}^2 &= \frac{1}{4}(g^2+g'^2) v^2 \left[1 + \frac{v^2}{2\Lambda^2}\left(4\alpha_{\rm ZZ} + \alpha_{\Phi D}\right)\right], \\ M_{\rm H}^2 &= \lambda v^2 \left[1 + \frac{v^2}{2\Lambda^2}\left(4\alpha_{\Phi\Box} - \frac{6}{\lambda}\alpha_{\Phi} - \alpha_{\Phi D}\right)\right], \\ m_{\rm f} &= \frac{1}{\sqrt{2}} U^{\rm f}\Gamma_{\rm f} U^{{\rm f},\dagger} v \left[1 - \frac{1}{2}\frac{v^2}{\Lambda^2}\alpha_{{\rm f}\phi}\right]. \end{aligned} \tag{151}$$

In (150) we have used the usual 't Hooft–Feynman gauge-fixing term

$$\mathcal{L}_{\rm fix} = -C_+C_- - \frac{1}{2}(C_{\rm Z})^2 - \frac{1}{2}(C_{\rm A})^2 - \frac{1}{2}C_G^A C_G^A \tag{152}$$

with

$$C_G^A = \partial_\mu G^{A\mu}, \qquad C_{\rm A} = \partial_\mu {\rm A}^\mu, \qquad C_{\rm Z} = \partial_\mu {\rm Z}^\mu + M_{\rm Z}\phi^0, \qquad C_\pm = \partial_\mu {\rm W}^{\pm\mu} \pm {\rm i} M_{\rm W}\phi^\pm \tag{153}$$

in terms of the physical fields and parameters, which gives rise to the same propagators as in the SM.

In the following, the abbreviations $c_{\rm w}$ and $s_{\rm w}$ are defined via the physical masses

$$c_{\rm w} = \frac{M_{\rm W}}{M_{\rm Z}}, \qquad s_{\rm w} = \sqrt{1 - c_{\rm w}^2}. \tag{154}$$

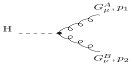
The parameters of the SM Lagrangian g, g', λ, m^2, and $\Gamma_{\rm f}$ keep their meaning in the presence of dimension-6 operators.

10.4.2 Higgs vertices

Here we list the most important Feynman rules for vertices involving exactly one physical Higgs boson. These are given in terms of the above-defined physical fields and parameters. In the coefficients of dimension-6 couplings we replaced v^2 by the Fermi constant via $v^2 = 1/(\sqrt{2}G_F)$.

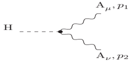
The triple vertices involving one Higgs boson read:

Hgg coupling:



$$= {\rm i}\frac{2g}{M_{\rm W}}\frac{1}{\sqrt{2}G_F\Lambda^2}\left[\alpha_{GG}(p_{2\mu}p_{1\nu} - p_1p_2 g_{\mu\nu}) + \alpha_{G\tilde{G}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right]\delta^{AB}, \tag{155}$$

HAA coupling:



$$= {\rm i}\frac{2g}{M_{\rm W}}\frac{1}{\sqrt{2}G_F\Lambda^2}\left[\alpha_{\rm AA}(p_{2\mu}p_{1\nu} - p_1p_2 g_{\mu\nu}) + \alpha_{{\rm A}\tilde{\rm A}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right], \tag{156}$$

with the physical mass parameters

$$
\begin{aligned}
M_{\mathrm{W}}^2 &= \frac{1}{4}g^2v^2\left[1+2\frac{v^2}{\Lambda^2}\alpha_{\Phi\mathrm{W}}\right],\\
M_{\mathrm{Z}}^2 &= \frac{1}{4}(g^2+g'^2)v^2\left[1+\frac{v^2}{2\Lambda^2}\left(4\alpha_{\mathrm{ZZ}}+\alpha_{\Phi D}\right)\right],\\
M_{\mathrm{H}}^2 &= \lambda v^2\left[1+\frac{v^2}{2\Lambda^2}\left(4\alpha_{\Phi\Box}-\frac{6}{\lambda}\alpha_{\Phi}-\alpha_{\Phi D}\right)\right],\\
m_{\mathrm{f}} &= \frac{1}{\sqrt{2}}U^{\hat{\mathrm{f}}}\Gamma_{\mathrm{f}}U^{\mathrm{f},\dagger}v\left[1-\frac{1}{2}\frac{v^2}{\Lambda^2}\alpha_{\mathrm{f}\phi}\right].
\end{aligned}
\tag{151}
$$

In (150) we have used the usual 't Hooft–Feynman gauge-fixing term

$$\mathcal{L}_{\mathrm{fix}} = -C_+C_- - \frac{1}{2}(C_{\mathrm{Z}})^2 - \frac{1}{2}(C_{\mathrm{A}})^2 - \frac{1}{2}C_G^AC_G^A \tag{152}$$

with

$$C_G^A = \partial_\mu G^{A\mu},\qquad C_{\mathrm{A}} = \partial_\mu \mathrm{A}^\mu,\qquad C_{\mathrm{Z}} = \partial_\mu \mathrm{Z}^\mu + M_{\mathrm{Z}}\phi^0,\qquad C_\pm = \partial_\mu \mathrm{W}^{\pm\mu} \pm \mathrm{i}M_{\mathrm{W}}\phi^\pm \tag{153}$$

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In the following, the abbreviations c_{w} and s_{w} are defined via the physical masses

$$c_{\mathrm{w}} = \frac{M_{\mathrm{W}}}{M_{\mathrm{Z}}},\qquad s_{\mathrm{w}} = \sqrt{1-c_{\mathrm{w}}^2}. \tag{154}$$

The parameters of the SM Lagrangian g, g', λ, m^2, and Γ_{f} keep their meaning in the presence of dimension-6 operators.

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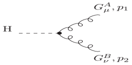
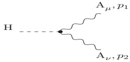
Hgg coupling:



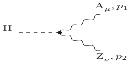
$$= \mathrm{i}\frac{2g}{M_{\mathrm{W}}}\frac{1}{\sqrt{2}G_F\Lambda^2}\left[\alpha_{GG}(p_{2\mu}p_{1\nu}-p_1p_2g_{\mu\nu})+\alpha_{G\tilde{G}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right]\delta^{AB}, \tag{155}$$

HAA coupling:



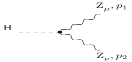
$$= \mathrm{i}\frac{2g}{M_{\mathrm{W}}}\frac{1}{\sqrt{2}G_F\Lambda^2}\left[\alpha_{\mathrm{AA}}(p_{2\mu}p_{1\nu}-p_1p_2g_{\mu\nu})+\alpha_{\mathrm{A}\tilde{\mathrm{A}}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right], \tag{156}$$

HAZ coupling:



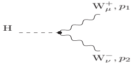
$$= \mathrm{i}\frac{g}{M_W}\frac{1}{\sqrt{2}G_F\Lambda^2}\left[\alpha_{AZ}(p_{2\mu}p_{1\nu} - p_1 p_2 g_{\mu\nu}) + \alpha_{A\tilde{Z}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right], \quad (157)$$

HZZ coupling:



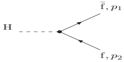
$$\begin{aligned} &= \mathrm{i}g\frac{M_Z}{c_w}g_{\mu\nu}\left[1 + \frac{1}{\sqrt{2}G_F\Lambda^2}\left(\alpha_{\Phi W} + \alpha_{\Phi\Box} + \frac{1}{4}\alpha_{\Phi D}\right)\right] \\ &\quad + \mathrm{i}\frac{2g}{M_W}\frac{1}{\sqrt{2}G_F\Lambda^2}\left[\alpha_{ZZ}(p_{2\mu}p_{1\nu} - p_1 p_2 g_{\mu\nu}) + \alpha_{Z\tilde{Z}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right], \end{aligned} \quad (158)$$

HWW coupling:



$$\begin{aligned} &= \mathrm{i}gM_W g_{\mu\nu}\left[1 + \frac{1}{\sqrt{2}G_F\Lambda^2}\left(\alpha_{\Phi W} + \alpha_{\Phi\Box} - \frac{1}{4}\alpha_{\Phi D}\right)\right] \\ &\quad + \mathrm{i}\frac{2g}{M_W}\frac{1}{\sqrt{2}G_F\Lambda^2}\left[\alpha_{\Phi W}(p_{2\mu}p_{1\nu} - p_1 p_2 g_{\mu\nu}) + \alpha_{\Phi\tilde{W}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right], \end{aligned} \quad (159)$$

Hff coupling:

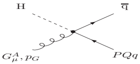


$$= -\mathrm{i}\frac{g}{2}\frac{m_f}{M_W}\left[1 + \frac{1}{\sqrt{2}G_F\Lambda^2}\left(\alpha_{\Phi W} + \alpha_{\Phi\Box} - \frac{1}{4}\alpha_{\Phi D} - \alpha_{f\phi}\right)\right], \quad (160)$$

where $f = e, u, d$.

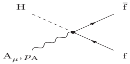
The quadruple vertices involving one Higgs field, one gauge boson and a fermion–antifermion pair are given by ($q = u, d$, $f = u, d, \nu_l, e$, and $\hat{f} = q$ for $f = u, d$ and $\hat{f} = l$ for $f = e$):

$Hgq\overline{q}$ coupling:



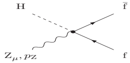
$$= \mathrm{i}g\frac{m_q}{M_W}\mathrm{i}p_{G\nu}\sigma^{\mu\nu}\frac{\lambda^A}{2}\left[\frac{1+\gamma_5}{2}\alpha_{qG} + \frac{1-\gamma_5}{2}\alpha^*_{qG}\right], \quad (161)$$

$\mathrm{HAf\bar{f}}$ coupling:



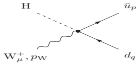
$$= \mathrm{i}g\frac{m_f}{M_\mathrm{W}}\mathrm{i}p_{A\nu}\sigma^{\mu\nu}\left[\frac{1+\gamma_5}{2}(\alpha_{\mathrm{fB}}c_\mathrm{w} + 2I_3^f\alpha_{\mathrm{fW}}s_\mathrm{w}) + \frac{1-\gamma_5}{2}(\alpha^*_{\mathrm{fB}}c_\mathrm{w} + 2I_3^f\alpha^*_{\mathrm{fW}}s_\mathrm{w})\right], \tag{162}$$

$\mathrm{HZf\bar{f}}$ coupling:



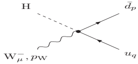
$$= \mathrm{i}g\frac{m_f}{M_\mathrm{W}}\mathrm{i}p_{\mathrm{Z}\nu}\sigma^{\mu\nu}\left[\frac{1+\gamma_5}{2}(2I_3^\mathrm{f}\alpha_{\mathrm{fW}}c_\mathrm{w} - \alpha_{\mathrm{fB}}s_\mathrm{w}) + \frac{1-\gamma_5}{2}(2I_3^\mathrm{f}\alpha^*_{\mathrm{fW}}c_\mathrm{w} - \alpha^*_{\mathrm{fB}}s_\mathrm{w})\right] + \mathrm{i}2M_\mathrm{Z}\gamma^\mu\left[\frac{1-\gamma_5}{2}\left(\alpha^{(1)}_{\Phi\hat{\mathrm{f}}} - 2I_3^\mathrm{f}\alpha^{(3)}_{\Phi\hat{\mathrm{f}}}\right) + \frac{1+\gamma_5}{2}\alpha_{\Phi\mathrm{f}}\right], \tag{163}$$

$\mathrm{HW^+d\overline{u}}$ coupling:



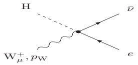
$$= \mathrm{i}g\frac{\sqrt{2}}{M_\mathrm{W}}\mathrm{i}p_{\mathrm{W}\nu}\sigma^{\mu\nu}V_{pq}\left[\frac{1+\gamma_5}{2}m_\mathrm{d}\alpha_{\mathrm{dW}} + \frac{1-\gamma_5}{2}m_\mathrm{u}\alpha^*_{\mathrm{uW}}\right] - \mathrm{i}\sqrt{2}M_\mathrm{W}\gamma^\mu\left[\frac{1-\gamma_5}{2}\,2\alpha^{(3)}_{\Phi\mathrm{q}}V_{pq} + \frac{1+\gamma_5}{2}(\Gamma_{\mathrm{ud}})_{pq}\alpha_{\Phi\mathrm{ud}}\right], \tag{164}$$

$\mathrm{HW^-u\overline{d}}$ coupling:



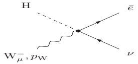
$$= \mathrm{i}g\frac{\sqrt{2}}{M_\mathrm{W}}\mathrm{i}p_{\mathrm{W}\nu}\sigma^{\mu\nu}V^\dagger_{pq}\left[\frac{1+\gamma_5}{2}m_\mathrm{u}\alpha_{\mathrm{uW}} + \frac{1-\gamma_5}{2}m_\mathrm{d}\alpha^*_{\mathrm{dW}}\right] - \mathrm{i}\sqrt{2}M_\mathrm{W}\gamma^\mu\left[\frac{1-\gamma_5}{2}\,2\alpha^{(3)}_{\Phi\mathrm{q}}V^\dagger_{pq} + \frac{1+\gamma_5}{2}(\Gamma^\dagger_{\mathrm{ud}})_{pq}\alpha^*_{\Phi\mathrm{ud}}\right], \tag{165}$$

$\mathrm{HW^+e\overline{\nu}_e}$ coupling:



$$= \mathrm{i}g\frac{\sqrt{2}}{M_\mathrm{W}}\mathrm{i}p_{\mathrm{W}\nu}\sigma^{\mu\nu}\frac{1+\gamma_5}{2}m_\mathrm{e}\alpha_{\mathrm{eW}} - \mathrm{i}\sqrt{2}M_\mathrm{W}\gamma^\mu\frac{1-\gamma_5}{2}\,2\alpha^{(3)}_{\Phi\mathrm{l}}, \tag{166}$$

$\mathrm{HW^-\nu_e e^+}$ coupling:



$$= \mathrm{i}g\frac{\sqrt{2}}{M_\mathrm{W}}\mathrm{i}p_{\mathrm{W}\nu}\sigma^{\mu\nu}\frac{1-\gamma_5}{2}m_\mathrm{e}\alpha^*_{\mathrm{eW}} - \mathrm{i}\sqrt{2}M_\mathrm{W}\gamma^\mu\frac{1-\gamma_5}{2}\,2\alpha^{(3)}_{\Phi l}. \tag{167}$$

Decoupling and $SU(2)_C$

- Heavy degrees of freedom $\hookrightarrow \mathbf{H} \to \gamma\gamma$: to be fully general one has to consider effects due to heavy fermions $\in R_f$ and heavy scalars $\in R_s$ of $SU(3)$. Colored scalars disappear from the low energy physics as their mass increases . However, the same is not true for fermions.

- Renormalization: whenever $\rho_{LO} \neq 1$, quadratic power-like contribution to $\Delta\rho$ are absorbed by renormalization of the new parameters of the model $\rightsquigarrow \rho$ is not a measure of the custodial symmetry breaking.
  Alternatively one could examine models containing $SU(2)_L \otimes SU(2)_R$ multiplets.

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