Constructing NLO SMEFT

a set of constructs, definitions, and propositions that present a systematic view of SMEFT

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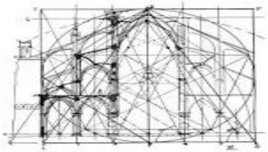

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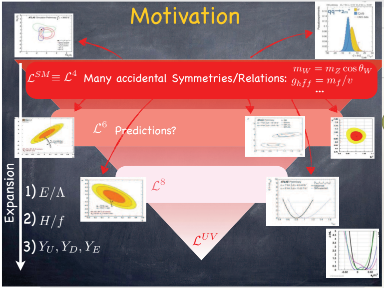
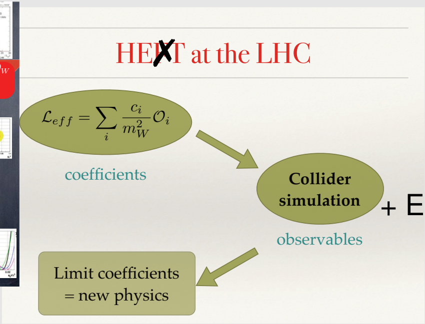
After the LHC Run 1, the SM has been completed, raising its status to that of a full theory. Despite its successes, this SM has shortcomings vis-à-vis cosmological observations. At the same time, while the LHC restarts for Run 2 at 13 *TeV*, there is presently a lack of direct evidence for new physics phenomena at the accelerator energy frontier. From this state of affairs arises the need for a consistent theoretical framework in which deviations from the SM predictions can be calculated. Such a framework should be applicable to comprehensively describe measurements in all sectors of particle physics: LHC Higgs measurements, past electroweak precision data, etc.

By simultaneously describing all existing measurements, this framework then becomes an intermediate step toward the next SM, hopefully revealing the underlying symmetries

SMEFT *is needed*





+ EWPD

It is manifestly of interest to formulate joint analysis where all of the data is fit simultaneously

The κ-framework: origin and problems

The original framework is defined in e-Print: arXiv:1209.0040
and has the following limitations:

- no κ touches kinematics. Therefore it works at the level of total cross-sections, not for differential distributions
- it is LO, partially accommodating factorizable QCD but not EW corrections
- it is not QFT-compatible (ad-hoc variation of the SM parameters, violates gauge symmetry and unitarity)

The role of SMEFT[1]

The role of SMEFT in paving the (as) Model Independent (as possible) road cannot be undermined.

Bringing SMEFT to NLO is the correct way for focusing in consistency of the approach where we can build POs that are QFT-compatible. Furthermore, NLO SMEFT means “calculate first, simplify later” and not “simplify first, calculate later”.

It is not justified to set individual Wilson coefficients to zero

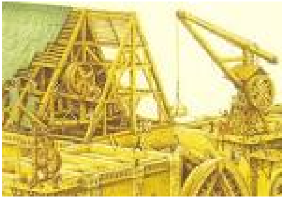
The precision of EWPD overcomes the loop suppression

No NLO SMEFT 

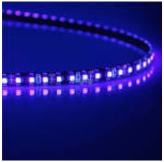
[1] arXiv:1505.02646, arXiv:1505.03706

Despite Wightman Axioms QFT is full of assumptions but, once you accept them, QFT is a non flexible working environment: you cannot work with the theory (pretending to get meaningful results) before constructing it

What can be said at all can be said clearly and whereof one cannot speak thereof one must be silent L. Wittgenstein



$\cdots$ constructing SMEFT



The UV connection

$$\mathscr{A} \;=\; \sum_{n=N}^{\infty} g^{n}\,\mathscr{A}_{n}^{(4)} + \sum_{n=N_6}^{\infty}\sum_{l=1}^{n}\sum_{k=1}^{\infty} g^{n}\,g^{l}{}_{4+2k}\,\mathscr{A}_{n\,l\,k}^{(4+2k)}$$

where g is the $SU(2)$ coupling constant and $g_{4+2k} = 1/(\sqrt{2}\,G_{\mathrm{F}}\,\Lambda^2)^k = g_6^k$, where G_{F} is the Fermi coupling constant and Λ is the scale around which new physics (NP) must be resolved. For each process N defines the $\mathrm{dim} = 4$ LO (e.g. $N = 1$ for $\mathrm{H} \to \mathrm{VV}$ etc. but $N = 3$ for $\mathrm{H} \to \gamma\gamma$). $N_6 = N$ for tree initiated processes and $N - 2$ for loop initiated ones. Here we consider single insertions of $\mathrm{dim} = 6$ operators, which defines NLO SMEFT.

Ex: HAA (tree) vertex generated by $\mathscr{O}^{(6)}_{\phi\mathrm{W}} = \left(\Phi^\dagger\,\Phi\right)\mathrm{F}^{a\,\mu\nu}\,\mathrm{F}^{a}_{\mu\nu}$, by $\mathscr{O}^{(8)}_{\phi\mathrm{W}} = \Phi^\dagger\,\mathrm{F}^{a\,\mu\nu}\,\mathrm{F}^{a}_{\mu\rho}\,\mathrm{D}^{\rho}\,\mathrm{D}_{\nu}\,\Phi$ etc.

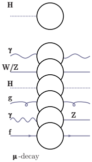
SMEFT ordertable for tree initiated $1 \to 2$ processes

$$
\begin{array}{llll}
g\,/\,\mathrm{dim} & \longrightarrow & & \\
\downarrow & g\,\mathscr{A}_1^{(4)} & +\,g\,g_6\,\mathscr{A}_{1,1,1}^{(6)} & +\,g\,g_8\,\mathscr{A}_{1,1,2}^{(8)} \\
 & g^3\,\mathscr{A}_3^{(4)} & +\,g^3\,g_6\,\mathscr{A}_{3,1,1}^{(6)} & +\,g^3\,g_6^2\,\mathscr{A}_{3,2,1}^{(6)} \\
 & \cdots\cdots & \cdots\cdots & \cdots\cdots
\end{array}
$$

- $g\,g_6\,\mathscr{A}_{1,1,1}^{(6)}$ LO SMEFT. There is also RG-improved LO (arXiv:1308.2627) and MHOU for LO SMEFT (arXiv:1508.05060)
- $g^3\,g_6\,\mathscr{A}_{3,1,1}^{(6)}$ (arXiv:1505.03706) NLO SMEFT
- $g\,g_8\,\mathscr{A}_{1,1,2}^{(8)}$ (arXiv:1510.00372), $g^3\,g_6^2\,\mathscr{A}_{3,2,1}^{(6)}$ MHOU for NLO SMEFT

N.B. g_8 denotes a single $\mathscr{O}^{(8)}$ insertion, g_6^2 denotes two, distinct, $\mathscr{O}^{(6)}$ insertions

Self-energies



$$
\begin{aligned}
\mathrm{S_{HH}} &= \frac{g^2}{16\,\pi^2}\,\Sigma_{\mathrm{HH}} = \frac{g^2}{16\,\pi^2}\,\left(\Sigma^{(4)}_{\mathrm{HH}} + g_6\,\Sigma^{(6)}_{\mathrm{HH}}\right) \\
S^{\mu\nu}_{\mathrm{AA}} &= \frac{g^2}{16\,\pi^2}\,\Sigma^{\mu\nu}_{\mathrm{AA}} \qquad \Sigma^{\mu\nu}_{\mathrm{AA}} = \Pi_{\mathrm{AA}}\,\mathrm{T}^{\mu\nu} \\
S^{\mu\nu}_{\mathrm{VV}} &= \frac{g^2}{16\,\pi^2}\,\Sigma^{\mu\nu}_{\mathrm{VV}} \qquad \Sigma^{\mu\nu}_{\mathrm{VV}} = \mathrm{D_{VV}}\,\delta^{\mu\nu} + \mathrm{P_{VV}}\,p^{\mu}\,p^{\nu} \\
\mathrm{D_{VV}} &= \mathrm{D}^{(4)}_{\mathrm{VV}} + g_6\,\mathrm{D}^{(6)}_{\mathrm{VV}} \qquad \mathrm{P_{VV}} = \mathrm{P}^{(4)}_{\mathrm{VV}} + g_6\,\mathrm{P}^{(6)}_{\mathrm{VV}} \\
S^{\mu\nu}_{\mathrm{ZA}} &= \frac{g^2}{16\,\pi^2}\,\Sigma^{\mu\nu}_{\mathrm{ZA}} + g_6\,\mathrm{T}^{\mu\nu}\,a_{\mathrm{AZ}} \qquad \Sigma^{\mu\nu}_{\mathrm{ZA}} = \Pi_{\mathrm{ZA}}\,\mathrm{T}^{\mu\nu} + \mathrm{P_{ZA}}\,p^{\mu}\,p^{\nu} \\
\mathrm{S_f} &= \frac{g^2}{16\,\pi^2}\,\left[\Delta_{\mathrm{f}} + \left(\mathrm{V_f} - \mathrm{A_f}\,\gamma^5\right)\,i\not{p}\right]
\end{aligned}
$$

Counterterms

$\Delta_{\mathrm{UV}} = \frac{2}{4-n} - \gamma - \ln\pi - \ln\frac{\mu_{\mathrm{R}}^2}{\mu^2}$
n is space-time dimension
loop measure $\mu^{4-n}\, d^n q$

μ_{R} ren. scale

$$Z_i \quad = \quad 1 + \frac{g^2}{16\pi^2}\left(dZ_i^{(4)} + g_6\, dZ_i^{(6)}\right)\Delta_{\mathrm{UV}}$$

With field/parameter counterterms we can make

$\mathrm{S_{HH}}, \Pi_{\mathrm{AA}}, \mathrm{D_{VV}}, \Pi_{\mathrm{ZA}}, \mathrm{V_f}, \mathrm{A_f}$ and the corresponding Dyson resummed propagators UV finite at $\mathscr{O}(g^2 g_6)$ (Q.E.D.)

which is enough when working under the assumption that gauge bosons couple to conserved currents

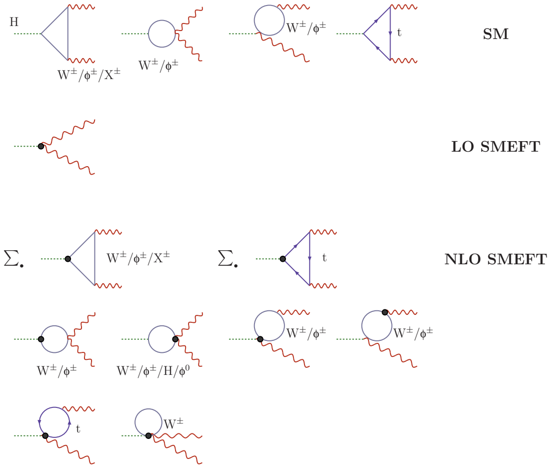
Mixing

Field/parameter counterterms are not enough to make UV finite the Green's functions with more than two legs. A mixing matrix among Wilson coefficients is needed:

$$a_i = \sum_j Z^{\mathrm{w}}_{ij}\, a^{\mathrm{ren}}_j \qquad Z^{\mathrm{w}}_{ij} = \delta_{ij} + \frac{g^2}{16\,\pi^2}\, dZ^{\mathrm{w}}_{ij}\, \Delta_{\mathrm{UV}}$$

$$|\, g^{\mathrm{N}} \mathscr{A}^{(4)}_{\mathrm{N}} + g^{\mathrm{K}} g_6\, \mathscr{A}^{(6)}_{\mathrm{K},1,1}\,|^2 \;\rightsquigarrow\; |\, g^{\mathrm{N}} \mathscr{A}^{(4)}_{\mathrm{N}}\,|^2 + 2\, g^{\mathrm{N}+\mathrm{K}} g_6\, \mathrm{Re}\left[\mathscr{A}^{(4)}_{\mathrm{N}}\right]^{\dagger} \mathscr{A}^{(6)}_{\mathrm{K},1,1}$$

Remark negative bin entries judge the validity of the $\mathrm{dim} = 6$ "linear" approach (arXiv:1511.05170)



Diagrams contributing to the amplitude for $\mathbf{H} \to \boldsymbol{\gamma\gamma}$ in the $\mathbf{R}_\xi$ -gauge: SM (first row), LO SMEFT (second row), and NLO SMEFT. Black circles denote the insertion of one $\mathbf{dim} = \mathbf{6}$ operator. $\sum_\bullet$ implies summing over all insertions in the diagram (vertex by vertex). For triangles with internal charge flow $(\mathbf{t}, \mathbf{W}^{\pm}, \boldsymbol{\phi}^{\pm}, \mathbf{X}^{\pm})$ only the clockwise orientation is shown. Non-equivalent diagrams obtained by the exchange of the two photon lines are not shown. Higgs and photon wave-function factors are not included. The Fadeev-Popov ghost fields are denoted by $\mathbf{X}$.

❶ 

Define the following combinations of Wilson coefficients (where $s_\theta\,(c_\theta)$ denotes the sine(cosine) of the renormalized weak-mixing angle.

$$
\begin{aligned}
a_{\mathrm{ZZ}} &= s_\theta^2\, a_{\phi\mathrm{B}} + c_\theta^2\, a_{\phi\mathrm{W}} - s_\theta\, c_\theta\, a_{\phi\mathrm{WB}} \\
a_{\mathrm{AA}} &= c_\theta^2\, a_{\phi\mathrm{B}} + s_\theta^2\, a_{\phi\mathrm{W}} + s_\theta\, c_\theta\, a_{\phi\mathrm{WB}} \\
a_{\mathrm{AZ}} &= 2\, c_\theta\, s_\theta\, \left(a_{\phi\mathrm{W}} - a_{\phi\mathrm{B}}\right) + \left(2\, c_\theta^2 - 1\right)\, a_{\phi\mathrm{WB}}
\end{aligned}
$$

and compute the (on-shell) decay $\mathrm{H}(P) \to \mathrm{A}_\mu(p_1)\mathrm{A}_\nu(p_2)$ where the amplitude is

$$
\mathrm{A}^{\mu\nu}_{\mathrm{HAA}} = \mathscr{T}_{\mathrm{HAA}}\, T^{\mu\nu} \qquad M^2_{\mathrm{H}}\, T^{\mu\nu} = p_2^\mu\, p_1^\nu - p_1 \cdot p_2\, \delta^{\mu\nu}
$$

Remark The amplitude is made UV finite by mixing $\boldsymbol{a}_{\mathrm{AA}}$ with $\boldsymbol{a}_{\mathrm{AA}}, \boldsymbol{a}_{\mathrm{AZ}}, \boldsymbol{a}_{\mathrm{ZZ}}$ and $\boldsymbol{a}_{\mathrm{QW}}$ Q.E.D.

❷ 

Compute the (on-shell) decay $\mathrm{H}(P) \to \mathrm{A}_\mu(p_1)\mathrm{Z}_\nu(p_2)$. After adding 1PI and 1PR components we obtain

$$\mathrm{A}^{\mu\nu}_{\mathrm{HAZ}} = \mathscr{T}_{\mathrm{HAZ}}\, T^{\mu\nu} \qquad M^2_{\mathrm{H}}\, T^{\mu\nu} = p_2^\mu\, p_1^\nu - p_1 \cdot p_2\, \delta^{\mu\nu}$$

Remark The amplitude is made UV finite by mixing a_{AZ} with $a_{\mathrm{AA}}, a_{\mathrm{AZ}}, a_{\mathrm{ZZ}}$ and a_{QW} Q.E.D.

❸ 

Compute the (on-shell) decay $\mathrm{H}(P) \to \mathrm{Z}_\mu(p_1)\mathrm{Z}_\nu(p_2)$. The amplitude contains

- a $\mathscr{D}_{\mathrm{HZZ}}$ part proportional to $\delta^{\mu\nu}$ and
- a $\mathscr{P}_{\mathrm{HZZ}}$ part proportional to $p_2^\mu\, p_1^\nu$.

Remark Mixing of a_{ZZ} with other Wilson coefficients makes $\mathscr{P}_{\mathrm{HZZ}}$ UV finite, while the mixing of $a_{\phi\square}$ makes $\mathscr{D}_{\mathrm{HZZ}}$ UV finite Q.E.D.

❹ 

Compute the (on-shell) decay $\mathrm{H}(P) \to \mathrm{W}^-{}_{\mu}(p_1)\mathrm{W}^+{}_{\nu}(p_2)$. This process follows the same decomposition of $\mathrm{H} \to \mathrm{ZZ}$ and it is UV finite in the $\mathbf{dim} = 4$ part. However, for the $\mathbf{dim} = 6$ one, there are no Wilson coefficients left free in $\mathscr{P}_{\mathbf{HWW}}$ so that its UV finiteness follows from gauge cancellations

Proposition

this is the first part in proving closure of NLO SMEFT under renormalization Q.E.D.

Remark Mixing of $a_{\phi\mathrm{D}}$ makes $\mathscr{D}_{\mathbf{HWW}}$ UV finite Q.E.D.

❺ 

Compute the (on-shell) decay $\mathbf{H}(P) \to \mathbf{b}(p_1)\overline{\mathbf{b}}(p_2)$.

Remark

- It is $\mathbf{dim} = 4$ UV finite and
- mixing of $a_{\mathbf{d}\phi}$ makes it UV finite also at $\mathbf{dim} = 6$ Q.E.D.

❻

Compute the (on-shell) decay $\mathrm{Z}(P) \to \mathrm{f}(p_1)\bar{\mathrm{f}}(p_2)$. It is $\mathbf{dim} = \mathbf{4}$ UV finite and we introduce

$$
\begin{aligned}
a_{\mathrm{l\,W}} &= s_\theta\, a_{\mathrm{l\,WB}} + c_\theta\, a_{\mathrm{l\,BW}} & a_{\mathrm{l\,B}} &= s_\theta\, a_{\mathrm{l\,BW}} - c_\theta\, a_{\mathrm{l\,WB}} \\
a_{\mathrm{d\,W}} &= s_\theta\, a_{\mathrm{d\,WB}} + c_\theta\, a_{\mathrm{d\,BW}} & a_{\mathrm{d\,B}} &= s_\theta\, a_{\mathrm{d\,BW}} - c_\theta\, a_{\mathrm{d\,WB}} \\
a_{\mathrm{u\,W}} &= s_\theta\, a_{\mathrm{u\,WB}} + c_\theta\, a_{\mathrm{u\,BW}} & a_{\mathrm{u\,B}} &= c_\theta\, a_{\mathrm{u\,WB}} - s_\theta\, a_{\mathrm{u\,BW}}
\end{aligned}
$$

$$
\begin{aligned}
a^{(3)}_{\phi\mathrm{l}} - a^{(1)}_{\phi\mathrm{l}} &= \frac{1}{2}\,(a_{\phi\mathrm{l\,V}} + a_{\phi\mathrm{l\,A}}) & a_{\phi\mathrm{l}} &= \frac{1}{2}\,(a_{\phi\mathrm{l\,A}} - a_{\phi\mathrm{l\,V}}) \\
a_{\phi\mathrm{u\,V}} &= a^{(3)}_{\phi\mathrm{q}} + a_{\phi\mathrm{u}} + a^{(1)}_{\phi\mathrm{q}} & a_{\phi\mathrm{u\,A}} &= a^{(3)}_{\phi\mathrm{q}} - a_{\phi\mathrm{u}} + a^{(1)}_{\phi\mathrm{q}} \\
a_{\phi\mathrm{d\,V}} &= a^{(3)}_{\phi\mathrm{q}} - a_{\phi\mathrm{d}} - a^{(1)}_{\phi\mathrm{q}} & a_{\phi\mathrm{d\,A}} &= a^{(3)}_{\phi\mathrm{q}} + a_{\phi\mathrm{d}} - a^{(1)}_{\phi\mathrm{q}}
\end{aligned}
$$

and obtain that (Q.E.D.)

$\mathbf{Z} \to \bar{\mathbf{l}}\mathbf{l}$ requires mixing of $\boldsymbol{a}_{\mathbf{lBW}}, \boldsymbol{a}_{\boldsymbol{\phi}\mathbf{l\,A}}$ and $\boldsymbol{a}_{\boldsymbol{\phi}\mathbf{l\,V}}$ with other coefficients,

$\mathbf{Z} \to \bar{\mathbf{u}}\mathbf{u}$ requires mixing of $\boldsymbol{a}_{\mathbf{uBW}}, \boldsymbol{a}_{\boldsymbol{\phi}\mathbf{u\,A}}$ and $\boldsymbol{a}_{\boldsymbol{\phi}\mathbf{u\,V}}$ with other coefficients,

$\mathbf{Z} \to \bar{\mathbf{d}}\mathbf{d}$ requires mixing of $\boldsymbol{a}_{\mathbf{dBW}}, \boldsymbol{a}_{\boldsymbol{\phi}\mathbf{d\,A}}$ and $\boldsymbol{a}_{\boldsymbol{\phi}\mathbf{d\,V}}$ with other coefficients,

$\mathbf{Z} \to \bar{\boldsymbol{\nu}}\boldsymbol{\nu}$ requires mixing of $\boldsymbol{a}_{\boldsymbol{\phi\nu}} = \mathbf{2}\,(\boldsymbol{a}^{(\mathbf{1})}_{\boldsymbol{\phi}\mathbf{l}} + \boldsymbol{a}^{(\mathbf{3})}_{\boldsymbol{\phi}\mathbf{l}})$ with other coefficients.

❼ 

At this point we are left with the universality of the electric charge. In QED there is a Ward identity telling us that e is renormalized in terms of vacuum polarization and Ward-Slavnov-Taylor identities allow us to generalize the argument to the full SM.

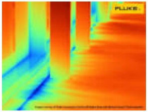
We can give a quantitative meaning to the the previous statement by saying that the contribution from vertices (at zero momentum transfer) exactly cancel those from (fermion) wave function renormalization factors. Therefore,

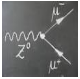
Compute the vertex $\mathrm{A\bar{f}f}$ (at $q^2 = 0$) and the f wave function factor in SMEFT, proving that the WST identity can be extended to $\mathrm{dim} = 6$; this is non trivial since there are no free Wilson coefficients in these terms (after the previous steps); (non-trivial) finiteness of $\mathrm{e^+e^- \to \bar{f}f}$ follows.

Proposition

This is the second part in proving closure of NLO SMEFT under renormalization Q.E.D.

The IR connection (e.g. $Z \to \bar{l}\, l$)





$$= \rho_Z^f \gamma^\mu \left[\left(I_f^{(3)} + i\, a_L \right) \gamma_+ - 2\, Q_f\, \kappa_Z^f \sin^2\theta + i\, a_Q \right]$$

$$\mathscr{A}_\mu^{\text{tree}} \quad = \quad g\, \mathscr{A}_{1\,\mu}^{(4)} + g\, g_6\, \mathscr{A}_{1\,\mu}^{(6)}$$

$$\mathscr{A}_{1\,\mu}^{(4)} = \frac{1}{4\, c_\theta}\, \gamma_\mu \left(v_L + \gamma^5 \right) \qquad\qquad \mathscr{A}_{1\,\mu}^{(6)} = \frac{1}{4}\, \gamma_\mu \left(V_l + A_l\, \gamma^5 \right)$$

$$\begin{aligned} V_l \quad &= \quad \frac{s_\theta^2}{c_\theta} \left(4\, s_\theta^2 - 7 \right) a_{AA} + c_\theta \left(1 + 4\, s_\theta^2 \right) a_{ZZ} + s_\theta \left(4\, s_\theta^2 - 3 \right) a_{AZ} \\ &+ \quad \frac{1}{4\, c_\theta} \left(7 - s_\theta^2 \right) a_{\phi D} + \frac{2}{c_\theta}\, a_{\phi 1 V} \\ A_l \quad &= \quad \frac{s_\theta^2}{c_\theta}\, a_{AA} + c_\theta\, a_{ZZ} + s_\theta\, a_{AZ} - \frac{1}{4\, c_\theta}\, a_{\phi D} + \frac{2}{c_\theta}\, a_{\phi L A} \end{aligned}$$

After UV renormalization, i.e. after counterterms and mixing have been introduced, we perform analytic continuation in n (space-time dimension), $n = 4 + \varepsilon$ with ε positive.

$$\mathscr{A}^{\mathrm{tree}\,,\,1\mathrm{L}} \quad = \quad \overline{u}_1\, \mathscr{A}_\mu^{\mathrm{tree}\,,\,1\mathrm{L}}\, v_2\, e^\mu\,(\lambda\,,\,P)$$

$$\Gamma\left(\mathrm{Z}\to\bar{\mathrm{l}}+\mathrm{l}\right)\,|_{\mathrm{div}} \quad = \quad \frac{2}{3}\,\frac{1}{(2\,\pi)^2}\,\sum_{\mathrm{spin}}\int d\Phi_{1\to 2}\;\mathrm{Re}\left[\mathscr{A}^{\mathrm{tree}}\right]^{\dagger}\mathscr{A}^{1\mathrm{L}}\,|_{\mathrm{div}}$$

$\boldsymbol{(\varepsilon\,,\,m_{\mathrm{f}})}$ -scheme for (IR , collinear) singularities

$$\frac{1}{\hat{\varepsilon}} \quad = \quad \frac{2}{\varepsilon}+\overline{\gamma}-\ln\frac{M_{\mathrm{W}}^2}{\mu^2} \qquad \mathrm{L}_{\mathrm{cw}}=\ln\frac{m_{\mathrm{l}}^2}{M_{\mathrm{W}}^2} \qquad \mathrm{L}_{\mathrm{cz}}=\ln\frac{m_{\mathrm{l}}^2}{M_{\mathrm{Z}}^2}$$

$$\overline{\gamma} \quad = \quad \gamma+\ln\pi \qquad \mathrm{L}=\ln\frac{M_{\mathrm{Z}}^2}{M_{\mathrm{W}}^2}$$

IR /collinear divergent factor

$$\mathscr{F}^{\mathrm{virt}} = -2\left(\frac{1}{\hat{\varepsilon}}+\overline{\gamma}\right)(1+\mathrm{L}_{\mathrm{cZ}})-\mathrm{L}_{\mathrm{cZ}}^2-4\,\mathrm{L}_{\mathrm{cZ}}\,\mathrm{L}+3\,\mathrm{L}_{\mathrm{cZ}}-4\,\mathrm{L}$$
$$-\;2\ln\frac{M_{\mathrm{W}}^2}{\mu^2}(1+\mathrm{L}_{\mathrm{cZ}})+2-8\,\zeta(2)$$

Sub-amplitudes

$$\Gamma_0^{(4)} = \frac{1}{2}\left(1-4\,s_\theta^2+8\,s_\theta^4\right)\frac{1}{c_\theta^2}=\frac{1}{4}\left(1+v_{\mathrm{l}}^2\right)\frac{1}{c_\theta^2}$$

$$\Gamma_{0\,\mathrm{A}}^{(4)} = 2\left(1-4\,s_\theta^2\right)\frac{s_\theta}{c_\theta}=2\,v_{\mathrm{l}}\,\frac{s_\theta}{c_\theta}$$

$$\Gamma_0^{(6)} = -\left(3-16\,s_\theta^2+8\,s_\theta^4\right)\frac{s_\theta^2}{c_\theta^2}\,a_{\mathrm{AA}}+\left(1-8\,s_\theta^4\right)a_{\mathrm{ZZ}}-\left(1-8\,s_\theta^2+8\,s_\theta^4\right)\frac{s_\theta}{c_\theta}\,a_{\mathrm{AZ}}$$
$$+\;\frac{1}{4}\left(3-16\,s_\theta^2+8\,s_\theta^4\right)\frac{1}{c_\theta^2}\,a_{\phi\,\mathrm{D}}+\frac{1}{c_\theta^2}\,a_{\phi\,1\,\mathrm{A}}+\left(1-4\,s_\theta^2\right)\frac{1}{c_\theta^2}\,a_{\phi\,1\,\mathrm{V}}$$

Proposition

The infrared/collinear part of the one-loop virtual corrections shows double factorization.

$$\Gamma\left(\mathrm{Z} \to \bar{\mathrm{l}} + \mathrm{l}\right)\big|_{\mathrm{div}} \quad = \quad -\frac{g^4}{384\,\pi^3}\, M_{\mathrm{Z}}\, s_\theta^2\, \mathscr{F}^{\,\mathrm{virt}} \left[\Gamma_0^{(4)}\,\left(1 + g_6\,\Delta\Gamma\right) + g_6\,\Gamma_0^{(6)}\right]$$

$$\Delta\Gamma \quad = \quad 2\left(2 - s_\theta^2\right) a_{\mathrm{AA}} + 2\, s_\theta^2\, a_{\mathrm{ZZ}} + 2\,\frac{c_\theta^3}{s_\theta}\, a_{\mathrm{AZ}} - \frac{1}{2}\,\frac{1}{s_\theta^2\, c_\theta^2}\, a_{\phi\,\mathrm{D}}$$

Next we compute $\mathrm{Z}(P) \to \mathrm{l}(p_1) + \bar{\mathrm{l}}(p_2) + \gamma(k)$, obtaining

$$\Gamma\left(\mathrm{Z} \to \bar{\mathrm{l}} + \mathrm{l} + \gamma\right) = \frac{1}{3}\,\frac{1}{(2\,\pi)^5}\sum_{\text{spin}}\int d\Phi_{1\to 3}\,|\,\mathscr{A}^{\text{real}}\,|^2$$

$$\mathscr{A}^{\text{real}} = \overline{u}_1\,\mathscr{A}^{\text{real}}_{\mu\nu}\,v_2\,e^{\mu}(\lambda\,,\,P)\,e^{\nu}(\sigma\,,\,k)$$

We split the total into

- "approximated", $n \neq 4$, approximated phase-space, reproducing the exact structure of singularities
- "remainder", $n = 4$, finite

After expanding in $\varepsilon = n - 4$ we obtain an overall infrared/collinear (real) factor

$$\begin{aligned}\mathscr{F}^{\mathrm{real}} &= -2\left(\frac{1}{\hat{\varepsilon}}+\overline{\gamma}\right)\left(1+\mathrm{L_{cz}}\right)-\mathrm{L_{cz}^2}-2\,\mathrm{L_{cz}}\,\mathrm{L}+3\,\mathrm{L_{cz}}-2\,\mathrm{L}\\ &- 2\ln\frac{M_Z^2}{\mu^2}\left(1+\mathrm{L_{cz}}\right)+1-4\,\zeta(2)\end{aligned}$$

and a partial width integrated over the whole photon phase space

$$\Gamma^{\mathrm{app}}\left(\mathrm{Z}\to\bar{\mathrm{l}}+\mathrm{l}+(\gamma)\right) = \frac{g^4}{384\,\pi^3}\,M_Z\,s_\theta^2\,\mathscr{F}^{\mathrm{real}}\left[\Gamma_0^{(4)}\left(1+g_6\,\Delta\Gamma\right)+g_6\,\Gamma_0^{(6)}\right]$$

Proposition

The infrared/collinear part of the real corrections shows double factorization. The total = virtual + real is IR /collinear finite at $\mathscr{O}(g^4\,g_6)$ (Q.E.D.).

Assembling everything gives

$$
\begin{aligned}
\Gamma^{\mathrm{l}}_{\mathrm{QED}} &= \frac{3}{4}\,\Gamma^{\mathrm{l}}_{0}\,\frac{\alpha}{\pi}\,\left(1 + g_6\,\Delta^{(6)}_{\mathrm{QED}}\right) \qquad \Gamma^{\mathrm{l}}_{0} = \frac{G_{\mathrm{F}}\,M^3_{\mathrm{Z}}}{24\,\sqrt{2}\,\pi}\,\left(v^2_{\mathrm{l}} + 1\right) \\
\Delta^{(6)}_{\mathrm{QED}} &= 2\,\left(2 - s^2_{\theta}\right)\,a_{\mathrm{AA}} + 2\,s^2_{\theta}\,a_{\mathrm{ZZ}} + 2\,\left(\frac{c^3_{\theta}}{s_{\theta}} + \frac{512}{26}\,\frac{v_{\mathrm{L}}}{v^2_{\mathrm{L}} + 1}\right)\,a_{\mathrm{AZ}} \\
&- \frac{1}{2}\,\frac{c^2_{\theta}}{s^2_{\theta}}\,a_{\phi\,\mathrm{D}} + \frac{1}{v^2_{\mathrm{L}} + 1}\,\delta^{(6)}_{\mathrm{QED}} \\
\delta^{(6)}_{\mathrm{QED}} &= \left(1 - 6\,v_{\mathrm{l}} - v^2_{\mathrm{l}}\right)\,\frac{1}{c^2_{\theta}}\,\left(s_{\theta}\,a_{\mathrm{AA}} - \frac{1}{4}\,a_{\phi\,\mathrm{D}}\right) \\
&+ \left(1 + 2\,v_{\mathrm{l}} - v^2_{\mathrm{l}}\right)\,\left(a_{\mathrm{ZZ}} + \frac{s_{\theta}}{c_{\theta}}\,a_{\mathrm{AZ}}\right) \\
&+ \frac{2}{c^2_{\theta}}\,\left(a_{\phi\,\mathrm{l}\,\mathrm{A}} + v_{\mathrm{l}}\,a_{\phi\,\mathrm{l}\,\mathrm{V}}\right)
\end{aligned}
$$

NLO SMEFT for Higgs and EW precision data

No NP yet?

A study of $\mathcal{SM}$-deviations: here the reference process is $\mathbf{gg \to H}$

✓ $\boldsymbol{\kappa}$-approach: write the amplitude as

$$\mathbf{A^{gg}} = \sum_{\mathbf{q=t,b}} \boldsymbol{\kappa}^{\mathbf{gg}}_{\mathbf{q}} \mathscr{A}^{\mathbf{gg}}_{\mathbf{q}} + \boldsymbol{\kappa}^{\mathbf{gg}}_{c}$$

$\mathscr{A}^{\mathbf{gg}}_{\mathbf{t}}$ being the SM t-loop etc. The contact term (which is the LO SMEFT) is given by $\boldsymbol{\kappa}^{\mathbf{gg}}_{c}$. Furthermore

$$\boldsymbol{\kappa}^{\mathbf{gg}}_{\mathbf{q}} = 1 + \Delta \boldsymbol{\kappa}^{\mathbf{gg}}_{\mathbf{q}}$$

Compute

$$\mathbf{R} = \sigma\left(\kappa_{\mathbf{q}}^{\mathbf{gg}}, \kappa_{\mathbf{c}}^{\mathbf{gg}}\right)/\sigma_{\mathbf{SM}} - 1 \quad [\%]$$

❶ In LO SMEFT κ_c is non-zero and $\kappa_q = 1$. [2] You measure a deviation and you get a value for κ_c

❷ However, at NLO $\Delta\kappa_q$ is non zero and you get a degeneracy

❸ The interpretation in terms of κ_c^{LO} or in terms of $\{\kappa_c^{\mathrm{NLO}}, \Delta\kappa_q^{\mathrm{NLO}}\}$ could be rather different.

[2] Certainly true in the linear realization

Going interpretational

$$\mathrm{A}^{\mathrm{gg}}_{\mathrm{SMEFT}} = \frac{g\,g_S^2}{\pi^2}\sum_{\mathrm{q=t,b}} \kappa^{\mathrm{gg}}_{\mathrm{q}}\,\mathscr{A}^{\mathrm{gg}}_{\mathrm{q}} + 2\,g_S\,g_6\,\frac{s}{M_W^2}\,a_{\phi\mathrm{g}} + \frac{g\,g_S^2\,g_6}{\pi^2}\sum_{\mathrm{q=t,b}} \mathscr{A}^{\mathrm{NF;gg}}_{\mathrm{q}}\,a_{\mathrm{qg}}$$

Remark use arXiv:1505.03706, adopt Warsaw basis (arXiv:1008.4884), eventually work in the Einhorn-Wudka PTG scenario (arXiv:1307.0478)

1. LO SMEFT: $\kappa_{\mathrm{q}} = 1$ and $a_{\phi\mathrm{g}}$ is scaled by $1/16\,\pi^2$ being LG (blue color)
2. NLO PTG-SMEFT: $\kappa_{\mathrm{q}} \neq 1$ but only PTG operators inserted in loops (non-factorizable terms absent), $a_{\phi\mathrm{g}}$ scaled as above
3. NLO full-SMEFT: $\kappa_{\mathrm{q}} \neq 1$ LG/PTG operators inserted in loops (non-factorizable terms present), LG coefficients scaled as above

At NLO, $\Delta\kappa = g_6\,\rho$

Warsaw basis

$$
\begin{aligned}
g_6^{-1} &= \sqrt{2}\, G_F \Lambda^2 \\
4\pi\alpha_s &= g_S^2 \\
\rho_t^{gg} &= a_{\phi W} + a_{t\phi} + 2\, a_{\phi\Box} - \frac{1}{2}\, a_{\phi D} \\
\rho_b^{gg} &= a_{\phi W} - a_{b\phi} + 2\, a_{\phi\Box} - \frac{1}{2}\, a_{\phi D}
\end{aligned}
$$

Relaxing the PTG assumption introduces non-factorizable sub-amplitudes proportional to a_{tg}, a_{bg} with a mixing among $\{a_{\phi g}, a_{tg}, a_{bg}\}$. Meanwhile, renormalization has made one-loop SMEFT finite, e.g. in the G_F-scheme, with a residual μ_R-dependence.

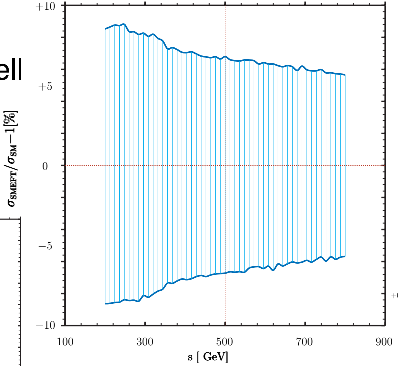
What are POs? Experimenters collapse some “primordial quantities” (say number of observed events in some pre-defined set-up) into some “secondary quantities” which we feel closer to the theoretical description of the phenomena.

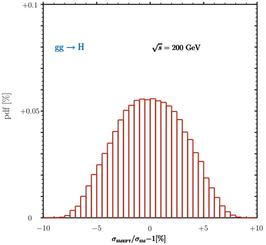
Residues of resonant poles, κ-parameters and Wilson coefficients are different layers of POs support $|a_i| \in [-1, +1]$

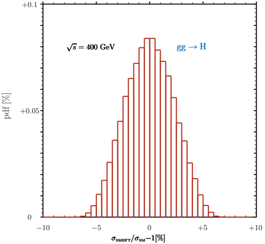
$gg \rightarrow H$ off-shell

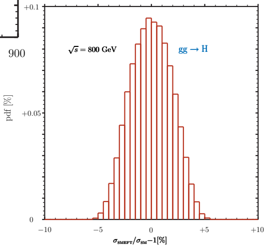
unif$(-1, 1)$

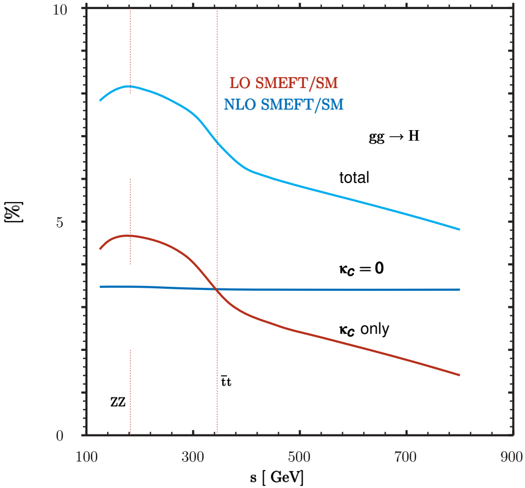
$\Lambda = 3$ *TeV*











Another reason to go NLO

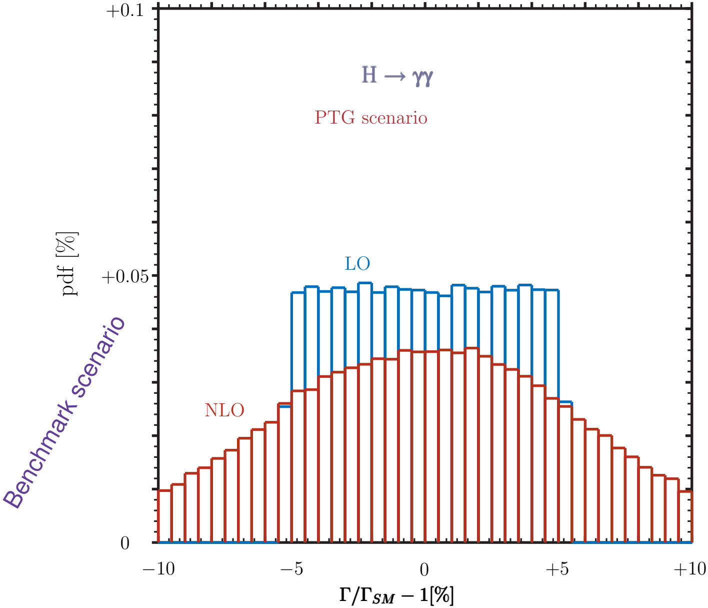
The contact term is real … $\kappa_c^{gg} \in \mathbb{R}$

$$\frac{g\, g_S^2\, g_6}{\pi^2} \sum_{q=t,b} \left[\Delta\kappa_q^{gg}\, \mathscr{A}_q^{gg} + \mathscr{A}_q^{NF;gg}\, a_{qg}\right] \in \mathbb{C}$$

$$2\, g_S\, g_6\, \frac{s}{M_W^2}\, a_{\phi g} \in \mathbb{R}$$

$$a_i = 1, \forall i$$

$$\Lambda = 3\ TeV$$

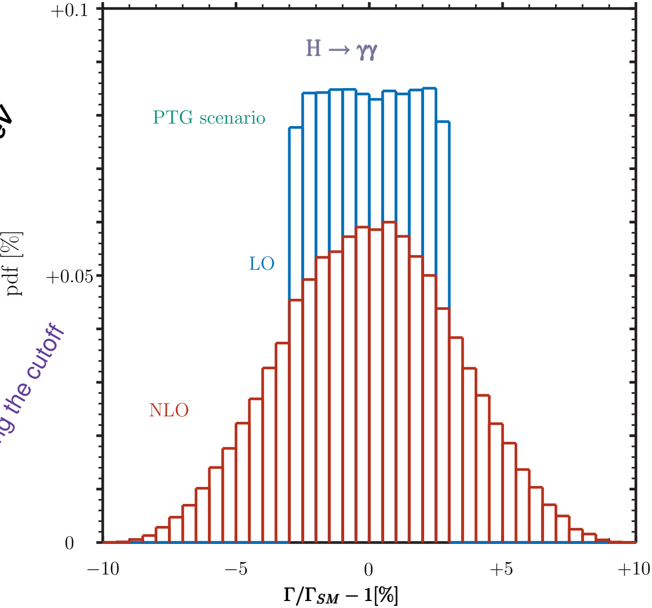


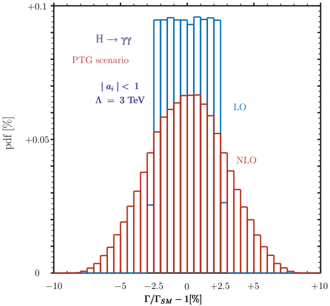
H → γγ
PTG scenario
LO
NLO
Benchmark scenario
pdf [%]
+0.1
+0.05
0
−10
−5
0
+5
+10
Γ/Γ_SM − 1[%]

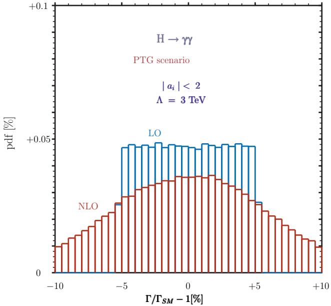
𝒢

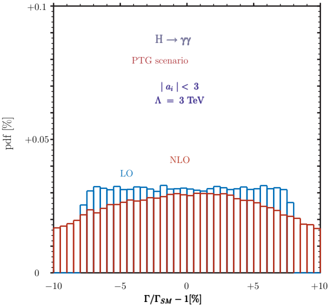
$\Lambda = 4$ TeV

Changing the cutoff









Changing the interval

It is an error to believe that rigour is the enemy of simplicity. On the contrary we find it confirmed by numerous examples that the rigorous method is at the same time the simpler and the more easily comprehended. The very effort for rigor forces us to find out simpler methods of proof D. Hilbert

To conclude, the journey to the next SM may require crossing narrow straits of precision physics. If that is what nature has in store for us, we must equip ourselves with both a range of concrete BSM models as well as a general SMEFT. Both will be indispensable tools in navigating an ocean of future experimental results.

It is possible that at some very large energy scale, all nonrenormalizable interactions disappear. This seems unlikely, given the difficulty with gravity. It is possible that the rules change drastically, it may even be possible that there is no end, simply more and more scales. H. Georgi

Thank you for your attention