Higgs Effective Field Theory

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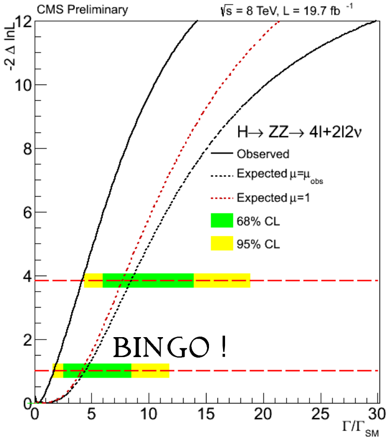
$d\sigma^{\text{off}} = \mu\, r\, d\sigma^{\text{peak}}$ $\quad r = \frac{\Gamma_H}{\Gamma_H^{SM}}$ ↬ assume $\mu = 1$ ⇝ measure r

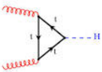
Combined limit

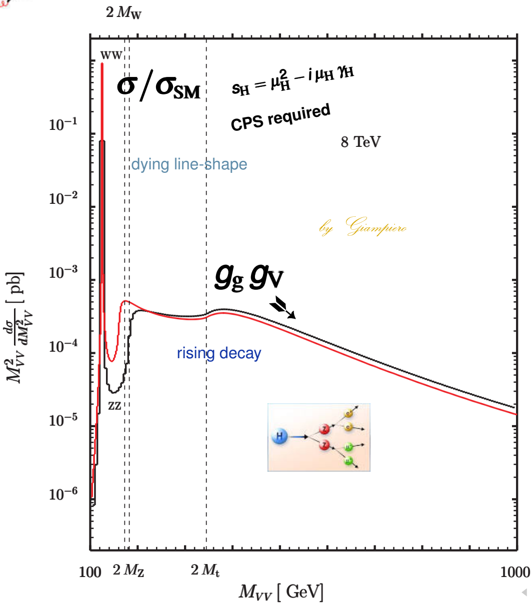
~ peak, exp resolution / SM width **2–3 *GeV*/4 *MeV***



- Combined observed (expected) values
  - $r = \Gamma/\Gamma_{SM} < 4.2$ (8.5) @ 95% CL

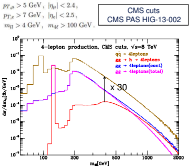
    (p-value = 0.02)
  - $r = \Gamma/\Gamma_{SM} = 0.3^{+1.5}_{-0.3}$
- equivalent to:
  - $\Gamma < 17.4$ (35.3) MeV @ 95% CL
  - $\Gamma = (1.4^{+6.1}_{-1.4})$ MeV

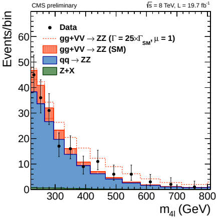




The big picture @ 8TeV

- Peak at Z mass due to singly resonant diagrams.
- Interference is an important effect.
- Destructive at large mass, as expected.
- With the standard model width, s_H, challenging to see enhancement/deficit due to Higgs channel.





dynamic QCD scales

OFF – SHELL I

We define an **off-shell production cross-section** (for all channels) as follows:

$$\sigma^{\text{prop}}_{ij\to\text{all}} = \frac{1}{\pi}\,\sigma_{ij\to\text{H}}\,\frac{s^2}{\left|s - s_{\text{H}}\right|^2}\,\frac{\Gamma^{\text{tot}}_{\text{H}}}{\sqrt{s}}$$

☞ When the cross-section $ij \to \mathrm{H}$ refers to an off-shell Higgs boson the choice of the QCD scales should be made according to the virtuality and not to a fixed value. Therefore, for the PDFs and $\sigma_{ij\to\mathrm{H+X}}$ one should select $\mu_F^2 = \mu_R^2 = z\,s/4$ ($z\,s$ being the invariant mass of the detectable final state).

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OFF – SHELL II

Let us consider the case of a *light Higgs boson; here, the common belief was that*

- ☛ the product of **on-shell production cross-section** (say in gluon-gluon fusion) and **branching ratios** reproduces the correct result to great accuracy. The expectation is based on the well-known result ($\Gamma_H \ll M_H$)

OFF

ON

$$\Delta_H = \frac{1}{\left(s - M_H^2\right)^2 + \Gamma_H^2 M_H^2} = \frac{\pi}{M_H \Gamma_H} \delta\left(s - M_H^2\right) + \mathrm{PV}\left[\frac{1}{\left(s - M_H^2\right)^2}\right]$$

where **PV** denotes the principal value (understood as a distribution). Furthermore s is the Higgs virtuality and M_H and Γ_H should be understood as $M_H = \mu_H$ and $\Gamma_H = \gamma_H$ and not as the corresponding on-shell values. In more simple terms,

- ☞ the first term puts you on-shell and the second one gives you the off-shell tail
- ☞ Δ_H is the Higgs propagator, there is no space for anything else in QFT (e.g. Breit-Wigner distributions).

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OFF – SHELL III

A short History of beyond ZWA (don't try fixing something that is already broken in the first place)

① There is an enhanced Higgs tail **Kauer - Passarino (arXiv:1206.4803)**: away from the narrow peak the propagator and the off-shell H width behave like

$$\Delta_{\rm H} \approx \frac{1}{\left(M^2_{\rm VV} - \mu^2_{\rm H}\right)^2}, \qquad \frac{\Gamma_{\rm H \to VV}\left(M_{\rm VV}\right)}{M_{\rm VV}} \sim G_{\rm F}\, M^2_{\rm VV}$$

② Introduce the notion of **∞-degenerate** solutions for the Higgs couplings to SM particles **Dixon - Li (arXiv:1305.3854), Caola - Melnikov(arXiv:1307.4935)**

③ Observe that the enhanced tail is obviously $\gamma_{\rm H}$ -independent and that this could be exploited to constrain the Higgs width model-independently

④ Use a matrix element method (MEM) to construct a kinematic discriminant to sharpen the constraint **Campbell, Ellis and Williams (arXiv:1311.3589)**

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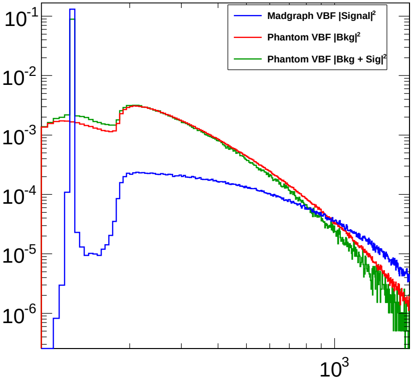
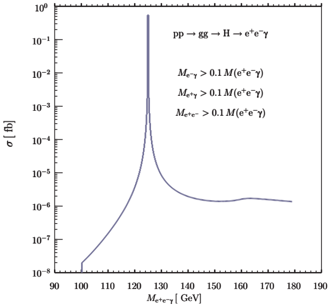
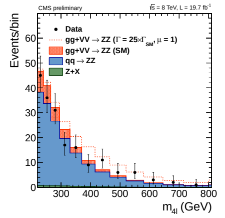
OFF – SHELL IV



Madgraph VBF |Signal|²
Phantom VBF |Bkg|²
Phantom VBF |Bkg + Sig|²
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^3



pp → gg → H → e⁺e⁻γ
M_{e⁻γ} > 0.1 M(e⁺e⁻γ)
M_{e⁺γ} > 0.1 M(e⁺e⁻γ)
M_{e⁺e⁻} > 0.1 M(e⁺e⁻γ)
σ [fb]
M_{e⁺e⁻γ} [GeV]



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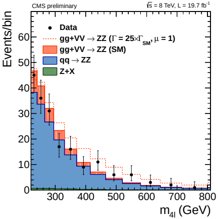
$$\sigma_{i\to H\to f} = (\sigma\cdot BR) = \frac{\sigma_i^{prod}\,\Gamma_f}{\gamma_H} \qquad \sigma_{i\to H\to f} \propto \frac{g_i^2 g_f^2}{\gamma_H} \qquad g_{i,f} = \xi\, g_{i,f}^{SM} \quad \gamma_H = \xi^4\, \gamma_H^{SM}$$

a consistent BSM interpretation?

On the whole, we have a constraint in the multidimensional κ-space

$$\kappa_g^2 = \kappa_g^2(\kappa_t,\kappa_b) \quad \kappa_H^2 = \kappa_H^2(\kappa_j, \forall j)$$

On-shell ∞-degeneracy
arXiv:1305.3854, 1307.4935, 1311.3589



The generalization is an ∞^2-degeneracy

$$g_i^2 g_f^2 = \gamma_H$$

$$g_i \Longleftrightarrow \kappa_j$$

Only on the assumption of degeneracy one can prove that off-shell effects measure γ_H

Simplified version

$$\frac{\Gamma_{gg}}{\Gamma_{gg}^{SM}(\mu_H)} = \frac{\kappa_t^2\cdot\Gamma_{gg}^{tt}(\mu_H)+\kappa_b^2\cdot\Gamma_{gg}^{bb}(\mu_H)+\kappa_t\kappa_b\cdot\Gamma_{gg}^{tb}(\mu_H)}{\Gamma_{gg}^{tt}(\mu_H)+\Gamma_{gg}^{bb}(\mu_H)+\Gamma_{gg}^{tb}(\mu_H)}$$

original κ-language

a combination of on-shell effects measuring

$g_i^2 g_f^2/\gamma_H$

and off-shell effects measuring

$g_i^2 g_f^2$

gives information on γ_H

without prejudices

The only limit to our realization of tomorrow will be our doubts of today

Memo:
Skip
meetings

Main Theorem:
HEFT is a realization of $\boldsymbol{\kappa}$-language

Definition:
$\boldsymbol{\kappa}$-language is BSM MI approach

Chapter IV
Renorm.
dim. reg. QFT
role of $\boldsymbol{\Lambda}$
top? - down

MHOU PO EFT

Strategy: How to interpret κ_X?

① **measure κ**

$$\frac{\Gamma_{gg}}{\Gamma_{gg}^{SM}(m_H)} = \frac{\kappa_t^2 \cdot \Gamma_{gg}^{tt}(m_H) + \kappa_b^2 \cdot \Gamma_{gg}^{bb}(m_H) + \kappa_t \kappa_b \cdot \Gamma_{gg}^{tb}(m_H)}{\Gamma_{gg}^{tt}(m_H) + \Gamma_{gg}^{bb}(m_H) + \Gamma_{gg}^{tb}(m_H)}$$

② **find $\mathcal{O}_i \Leftrightarrow \kappa_x$** (epistemological stop, true ESM believers stop here)

$$\mathcal{L}_{ESM} = \mathcal{L}_{SM} + \sum_{n>4} \sum_{i=1}^{N_n} \frac{a_i^n}{\Lambda^{n-4}} \mathcal{O}_i^{(d=n)}$$

③ **find $\{\mathcal{L}_{BSM}\}$** that produces $\mathcal{O}_i$

Chapter II
Nature of $\mathcal{O}^{\mathbf{d}}$
Chapter III
Ontology of HEFT

is there a QFT behind degeneracy?

Corollary:
$\boldsymbol{\kappa}$-language requires insertion of $\mathcal{O}^{\mathbf{d}}$ operators in SM loops

annotated DIAGRAMMATICA

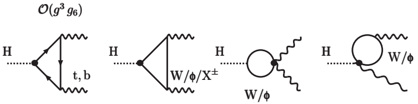


Figure 3: Example of one-loop SM diagrams with $\mathcal{O}$-insertions, contributing to the amplitude for $\mathrm{H} \to \gamma\gamma$

$\searrow$ κ_f , A_{NF} $\searrow$ κ_W , A_{NF}

$\upuparrows$

mix under ren. with $\mathcal{O}(g\, g_6)$ $\nearrow$ A_{NF}

$\downdownarrows$

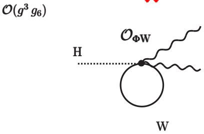
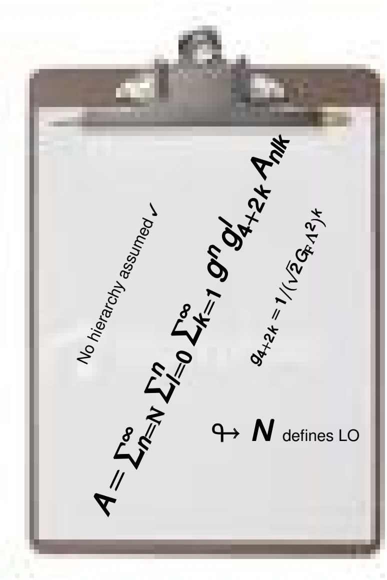


Figure 4: Example of one-loop $\mathcal{O}$-diagrams, contributing to the amplitude for $\mathrm{H} \to \gamma\gamma$

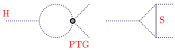


✍**PTG:** **T** - generated in at least one extension of SM

Note that for
$\Lambda \approx 5$ *TeV*
we have

$$1/(\sqrt{2} G_F \Lambda^2) \approx g^2/(4\pi),$$

i.e. ➽ the contributions of $d = 6$ operators are ≊ loop effects.
➽ ➽ For higher scales, loop contributions tend to be more important (≽)



PTG - operators versus LG - operators, cf. Einhorn, Wudka, ...

✍ It can be argued that (at LO) the basis operator should be chosen from among the **PTG operators**

✍ take $\mathcal{O}^{(6)}_{\rm LG}$, contract two lines, is ren of some $\mathcal{O}^{(4)}$

a SM vertex with $\mathcal{O}^{(6)}_{\rm PTG}$ required ... same order

$1/\Lambda$ expansion → power-counting ✓

LG → low-energy analytic structure ✗

PROPOSITION: There are two ways of formulating HEFT

a) mass-dependent scheme(s) or **Wilsonian** HEFT

b) mass-independent scheme(s) or **Continuum** HEFT (CHEFT)

- only **a)** is conceptually consistent with the image of an EFT as a low-energy approximation to a high-energy theory
- however, inclusion of NLO corrections is only meaningful in **b)** since we cannot regularize with a cutoff and NLO requires regularization
  - There is an additional problem, CHEFT requires evolving our theory to lower scales until we get below the "heavy-mass" scale where we use $\mathscr{L} = \mathscr{L}_{\mathbf{SM}} + d\mathscr{L}$, $d\mathscr{L}$ encoding matching corrections at the boundary. Therefore, CHEFT does not integrate out heavy degrees of freedom but removes them compensating for by an appropriate matching calculation

☞ *Not quite the same as it is usually discussed (no theory approaching the boundary from above ...)* cf. low-energy SM, weak effects on $g-2$ etc.

✍ $\dim\phi = d/2 - 1$

$\dim \mathcal{O}^{d} = N_{\phi} \dim\phi + N_{der}$

For $d \geq 3$ there is a finite number of relevant + marginal operators

For $d \geq 1$ there is a finite number of irrelevant operators

Sounds good for finite dependence on high-energy theory

✍ This assumes that high-energy theory is weakly coupled

✍ Dimensional arguments work for LO HEFT

✍ In NLO HEFT scaling may break down, implying appeal to a particular renormalization scheme

Ren. group should only be applied to EFTs that are nearly massless

Decoupling theorem fails for CHEFT, but, arguably this does not prevent them from supporting a well defined scheme, but decoupling must be inserted in the form of matching calculations (which we don't have ...)

✍ Match Feynman diagrams $\in$ HEFT with corresponding **1**(light)**PI** diagrams $\in$ high-energy theory (and discover that Taylor-expanding is not always a good idea)

Having said that ... no space left for annotations

Renormalization

FP-sector: handle with care

✓ Make finite all Green's functions

Schemes: remember $\beta_{\rm QED}$ in large m_e -limit

$$g = g_{\rm ren}\left[1+\frac{g^2_{\rm ren}}{16\,\pi^2}\left(dZ_g + g_6\,dZ_g^{(6)}\right)\frac{1}{\bar{\varepsilon}}\right]$$

✓ Don't forget background

$$M_{\rm W} = M^{\rm ren}_{\rm W}\left[1+\frac{1}{2}\,\frac{g^2_{\rm ren}}{16\,\pi^2}\left(dZ_{M_{\rm W}} + g_6\,dZ_{M_{\rm W}}^{(6)}\right)\frac{1}{\bar{\varepsilon}}\right]$$

etc.

Oops! ... $4f\,\mathcal{O}$ needed for $\mathbf{H}\to b\bar{b}$

Wilson coefficients $\rightarrow$ W_i

$\frac{1}{\bar{\varepsilon}} = \frac{2}{\varepsilon} - \gamma - \ln\pi + \ln\mu_R$

$\mathbf{H}\to\gamma\gamma$ not finite

$$W_i = \sum_j Z^{\rm wc}_{ij}\,W^{\rm ren}_j$$

$$Z^{\rm wc}_{ij} = \delta_{ij} + \frac{g^2_{\rm ren}}{16\,\pi^2}\,dZ^{\rm wc}_{ij}\,\frac{1}{\bar{\varepsilon}}$$

Einhorn, Wudka

$\mathcal{O}$ is PTG

$\mathcal{O}$ is LG

Grzadkowski, Iskrzynski, Misiak, Rosiek

Appendix C. Dimension-Six Basis Operators for the SM[22].

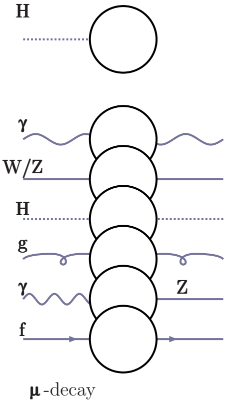
X^3 (LG)		φ^6 and $\varphi^4 D^2$ (PTG)		$\psi^2\varphi^3$ (PTG)	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$ (LG)		$\psi^2 X\varphi$ (LG)		$\psi^2\varphi^2 D$ (PTG)	
$Q_{\varphi G}$	$\varphi^\dagger \varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r) \tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger \varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r) \widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger \varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r) \tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r) \widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger \varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) \varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r) \tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger \tau^I \varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) \varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

Table C.1: Dimension-six operators other than the four-fermion ones.

[22] These tables are taken from [5], by permission of the authors.

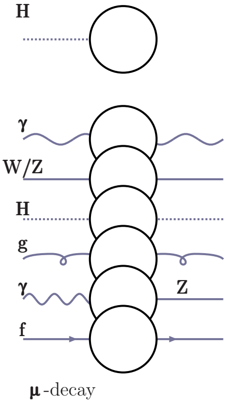
✍ Effective Lagrangians cannot be blithely used without acknowledging implications of their choice
ex: non gauge-invariant, intended to be used in U-gauge
ex: $\mathbf{H} \to \mathbf{WW^*}$ is virtual $\mathbf{W}$ + something else, depending on the operator basis

✓ Tadpoles $\mapsto$ β_H



✓ Tadpoles $\mapsto \beta_H$

✓ $\Phi = Z_\phi^{1/2} \Phi_R$ etc.

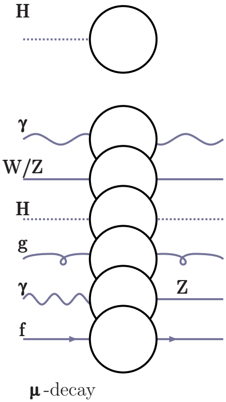


✓ Tadpoles $\mapsto$ $\beta_{\mathbf{H}}$

✓ $\Phi = Z_{\phi}^{1/2}\,\Phi_{\mathbf{R}}$ etc.

☛

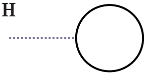
$$Z_{\phi} = 1 + \frac{g^2}{16\pi^2}\left(\delta Z_{\phi}^{(4)} + g_6\,\delta Z_{\phi}^{(6)}\right)$$

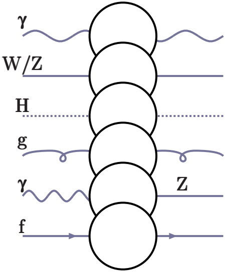


- ✓ Tadpoles $\mapsto$ β_{H}
- ✓ $\Phi = Z_{\phi}^{1/2}\,\Phi_{\mathrm{R}}$ etc.
- ☛

$$Z_{\phi} = 1 + \frac{g^2}{16\pi^2}\left(\delta Z_{\phi}^{(4)} + g_6\,\delta Z_{\phi}^{(6)}\right)$$

- ✓ Self-energies UV $\mathscr{O}^{(4)}, \mathscr{O}^{(6)}$ -finite





μ -decay

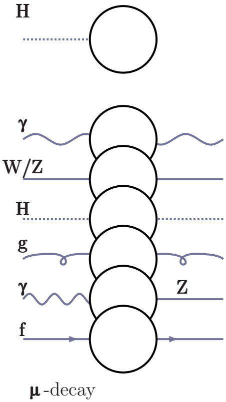
✓ Tadpoles ↦ β_{H}

✓ $\Phi = Z_\phi^{1/2}\,\Phi_{\mathrm{R}}$ etc.

☛ $$Z_\phi = 1 + \frac{g^2}{16\pi^2}\left(\delta Z_\phi^{(4)} + g_6\,\delta Z_\phi^{(6)}\right)$$

✓ Self-energies UV $\mathcal{O}^{(4)}, \mathcal{O}^{(6)}$ -finite

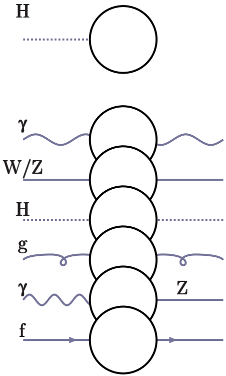
☛ μ-decay



- ✓ Tadpoles $\mapsto$ β_{H}
- ✓ $\Phi = Z_{\phi}^{1/2}\,\Phi_{\mathrm{R}}$ etc.
- ☛

$$Z_{\phi} = 1 + \frac{g^2}{16\pi^2}\left(\delta Z_{\phi}^{(4)} + g_6\,\delta Z_{\phi}^{(6)}\right)$$

- ✓ Self-energies UV $\mathscr{O}^{(4)}, \mathscr{O}^{(6)}$ -finite
- ☛ μ -decay
- ✓ $g \rightarrow g_{\mathrm{R}}$

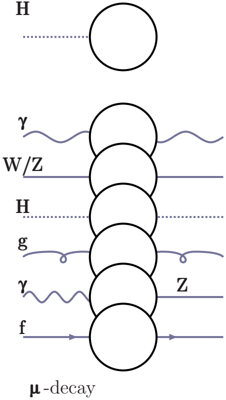


μ -decay

- ✓ Tadpoles $\mapsto$ β_{H}
- ✓ $\Phi = Z_{\phi}^{1/2}\,\Phi_{\mathrm{R}}$ etc.
- ☛

$$Z_{\phi} = 1 + \frac{g^2}{16\pi^2}\left(\delta Z_{\phi}^{(4)} + g_6\,\delta Z_{\phi}^{(6)}\right)$$

- ✓ Self-energies UV $\mathscr{O}^{(4)}, \mathscr{O}^{(6)}$ -finite
- ☛ μ -decay
- ✓ $g \to g_{\mathrm{R}}$
- ✓ Finite ren.



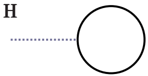
- ✓ Tadpoles $\mapsto \beta_{\mathrm{H}}$
- ✓ $\Phi = Z_{\phi}^{1/2}\,\Phi_{\mathrm{R}}$ etc.
- ☛

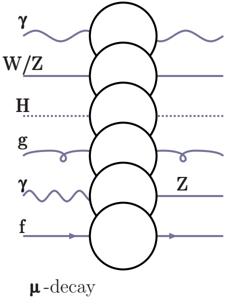
$$Z_{\phi} = 1 + \frac{g^2}{16\pi^2}\left(\delta Z_{\phi}^{(4)} + g_6\,\delta Z_{\phi}^{(6)}\right)$$

- ✓ Self-energies UV $\mathscr{O}^{(4)}, \mathscr{O}^{(6)}$ -finite
- ☛ μ -decay
- ✓ $g \rightarrow g_{\mathrm{R}}$
- ✓ Finite ren.
- ☛

$$M_{\mathrm{R}}^2 = M_{\mathrm{W}}^2\left[1 + \frac{g_{\mathrm{R}}^2}{16\pi^2}\left(\mathrm{Re}\,\Sigma_{\mathrm{WW}} - \delta Z_{\mathrm{M}}\right)\right]$$

- ✓ etc Propagators finite and μ_{R} -independent





EXAMPLE UV

H-propagator

$$\Delta_{\rm H}^{-1} = Z_{\rm H}\left(-s + Z_{m_{\rm H}}\,M_{\rm H}^2\right) - \frac{1}{(2\,\pi)^4\,i}\,\Sigma_{\rm HH}$$

$$Z_{\rm H} = 1 + \frac{g_{\rm R}^2}{16\,\pi^2}\left(\delta Z_{\rm H}^{(4)} + g_6\,\delta Z_{\rm H}^{(6)}\right)\frac{1}{\bar{\varepsilon}}$$

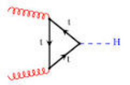
$$\delta Z_{\rm H}^{(4)} = 16\left[\frac{1}{288}\left(82 - \frac{16}{c_\theta^2} - 25\,\frac{s_\theta}{c_\theta} - 14\,s_\theta^2 - 14\,s_\theta\,c_\theta\right) - \frac{3}{32}\,\frac{m_{\rm b}^2 + m_{\rm t}^2}{M^2}\right]$$

$$\delta Z_{\rm H}^{(6)} = \frac{1}{6\,\sqrt{2}}\left[\frac{5}{c_\theta^2} + 12 - 18\,\frac{m_{\rm b}^2 + m_{\rm t}^2}{M^2} - 21\,\frac{m_{\rm H}^2}{M^2}\right]a_{\phi\Box} + \text{etc}$$

EXAMPLE finite ren.

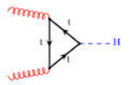
$$m_{\rm H}^2 = M_{\rm H}^2 \left[1 + \frac{g_{\rm R}^2}{16\,\pi^2} \left({\rm d}M_{\rm H}^{(4)} + g_6\,{\rm d}M_{\rm H}^{(6)} \right) \right]$$

$$\begin{aligned}
\frac{M_H^2}{16}\,{\rm d}M_{\rm H}^{(4)} &= \frac{1}{16}\,M_{\rm W}^2 \left(\frac{1}{c_\theta^4} + 2 \right) \\
&- \frac{3}{32}\,\frac{M_{\rm t}^2}{M_{\rm W}^2} \left(M_{\rm H}^2 - 4\,M_{\rm t}^2 \right) B_0 \left(-M_{\rm H}^2\,;M_{\rm t}\,,M_{\rm t} \right) \\
&- \frac{3}{32}\,\frac{M_{\rm b}^2}{M_{\rm W}^2} \left(M_{\rm H}^2 - 4\,M_{\rm b}^2 \right) B_0 \left(-M_{\rm H}^2\,;M_{\rm b}\,,M_{\rm b} \right) \\
&- \frac{9}{128}\,\frac{M_{\rm H}^4}{M_{\rm W}^2}\,B_0 \left(-M_{\rm H}^2\,;M_{\rm H}\,,M_{\rm H} \right) \\
&- \frac{1}{64} \left(\frac{M_{\rm H}^4}{M_{\rm W}^2} - 4\,M_{\rm H}^2 - 12\,M_{\rm W}^2 \right) B_0 \left(-M_{\rm H}^2\,;M_{\rm W}\,,M_{\rm W} \right) \\
&- \frac{1}{128} \left(\frac{M_{\rm H}^4}{M_{\rm W}^2} - 4\,\frac{M_{\rm H}^2}{c_\theta^2} + 12\,\frac{M_{\rm W}^2}{c_\theta^4} \right) B_0 \left(-M_{\rm H}^2\,;M_{\rm Z}\,,M_{\rm Z} \right) \Big]
\end{aligned}$$



✓ requires $Z_{\mathrm{H}}, Z_{\mathrm{g}}, Z_{g}, Z_{g_{\mathrm{s}}}$

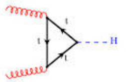
v_{H} = Higgs virtuality



✓ requires $Z_{\mathrm{H}}, Z_{\mathrm{g}}, Z_{g}, Z_{g_{\mathrm{S}}}$

✓ It is $\mathscr{O}^{(4)}$ -finite but not $\mathscr{O}^{(6)}$ -finite

$v_{\mathrm{H}} =$ Higgs virtuality



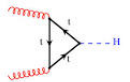
- ✓ requires $Z_{\mathrm{H}}, Z_{\mathrm{g}}, Z_{g}, Z_{g_{\mathrm{S}}}$
- ✓ It is $\mathscr{O}^{(4)}$ -finite but not $\mathscr{O}^{(6)}$ -finite
- ✓ involves $a_{\phi\mathrm{D}}$, $a_{\phi\Box}$, $a_{\mathrm{t}\phi}$, $a_{\mathrm{b}\phi}$, $a_{\phi\mathrm{W}}$, $a_{\phi\mathrm{g}}$, a_{tg}, a_{bg},

$v_{\mathrm{H}} =$ Higgs virtuality

$$a_{\mathrm{tg}} = W_1 \quad a_{\mathrm{bg}} = W_2 \quad a_{\phi\mathrm{g}} = W_3$$

$$a_{\mathrm{b}\phi} + \frac{1}{4} a_{\phi\mathrm{D}} - a_{\Phi\mathrm{W}} - a_{\phi\Box} = W_4$$

$$a_{\mathrm{t}\phi} - \frac{1}{4} a_{\phi\mathrm{D}} + a_{\Phi\mathrm{W}} + a_{\phi\Box} = W_5$$



✓ requires $Z_{\mathrm{H}}, Z_{\mathrm{g}}, Z_{g}, Z_{g_{\mathrm{S}}}$

✓ It is $\mathcal{O}^{(4)}$ -finite but not $\mathcal{O}^{(6)}$ -finite

✓ involves $a_{\phi\mathrm{D}}$, $a_{\phi\Box}$, $a_{\mathrm{t}\phi}$, $a_{\mathrm{b}\phi}$, $a_{\phi\mathrm{W}}$, $a_{\phi\mathrm{g}}$, a_{tg}, a_{bg},

$v_{\mathrm{H}} =$ Higgs virtuality

$$a_{\mathrm{tg}} = W_1 \quad a_{\mathrm{bg}} = W_2 \quad a_{\phi\mathrm{g}} = W_3$$

$$a_{\mathrm{b}\phi} + \frac{1}{4}\, a_{\phi\mathrm{D}} - a_{\Phi\mathrm{W}} - a_{\phi\Box} = W_4$$

$$a_{\mathrm{t}\phi} - \frac{1}{4}\, a_{\phi\mathrm{D}} + a_{\Phi\mathrm{W}} + a_{\phi\Box} = W_5$$

✓ requires *extra* renormalization

$$W_i = \sum_j Z^{\mathrm{mix}}_{ij}\, W^{\mathrm{R}}_j\,(\mu_{\mathrm{R}})$$

$$Z^{\mathrm{mix}}_{ij} = \delta_{ij} + \frac{g g_{\mathrm{S}}}{16\,\pi^2}\,\delta Z^{\mathrm{mix}}_{ij}\,\frac{1}{\varepsilon}$$

$$\delta Z^{\mathrm{mix}}_{31(2)} = -\frac{1}{2\sqrt{2}}\,\frac{M_{\mathrm{t(b)}}}{M_{\mathrm{W}}}$$

✓ Define building blocks

$$\frac{8\,\pi^2}{i\,g_S^2}\,\frac{M_W}{M_q^2}\,\mathrm{A}_q^{LO} \;=\; 2 - \left(4\,M_q^2 - v_H\right)\,C_0\left(-v_H, 0, 0\,;\, M_q, M_q, M_q\right)$$

$$\frac{32\,\pi^2}{i\,g_S^2}\,\frac{M_W^2}{M_q}\,\mathrm{A}_q^{nf} \;=\; 8\,M_q^4\,C_0\left(-v_H, 0, 0\,;\, M_q, M_q, M_q\right) + v_H\left[1 - B_0\left(-v_H\,;\, M_q, M_q\right)\right] - 4\,M_q^2$$

✓ Define (process dependent) κ-factors

$$\kappa_{\mathrm{b}} = 1 + g_6 \left[\frac{1}{2} \frac{M_{\mathrm{b}}}{M_{\mathrm{W}}} W_2^{\mathrm{R}} - \frac{1}{\sqrt{2}} W_4^{\mathrm{R}} \right]$$

$$\kappa_{\mathrm{t}} = 1 + g_6 \left[\frac{1}{2} \frac{M_{\mathrm{t}}}{M_{\mathrm{W}}} W_1^{\mathrm{R}} - \frac{1}{\sqrt{2}} W_5^{\mathrm{R}} \right]$$

✓ Define (process dependent) κ-factors

$$\kappa_{\rm b} = 1 + g_6 \left[\frac{1}{2} \frac{M_{\rm b}}{M_{\rm W}} W_2^{\rm R} - \frac{1}{\sqrt{2}} W_4^{\rm R} \right]$$

$$\kappa_{\rm t} = 1 + g_6 \left[\frac{1}{2} \frac{M_{\rm t}}{M_{\rm W}} W_1^{\rm R} - \frac{1}{\sqrt{2}} W_5^{\rm R} \right]$$

✓ Obtain the **4+6** amplitude

$$\mathrm{A}^{(4+6)} = g \sum_{\rm q=b,t} \kappa_{\rm q}\, \mathrm{A}_{\rm q}^{\rm LO} + i \frac{g_6\, g_{\rm S}}{\sqrt{2}} \frac{M_{\rm H}^2}{M_{\rm W}} W_3^{\rm R} + g_6\, g \left[W_1^{\rm R}\, \mathrm{A}_{\rm t}^{\rm nf} + W_2^{\rm R}\, \mathrm{A}_{\rm b}^{\rm nf} \right]$$

✓ Define (process dependent) κ-factors

$$\kappa_b = 1 + g_6 \left[\frac{1}{2}\frac{M_b}{M_W} W_2^R - \frac{1}{\sqrt{2}} W_4^R\right]$$

$$\kappa_t = 1 + g_6 \left[\frac{1}{2}\frac{M_t}{M_W} W_1^R - \frac{1}{\sqrt{2}} W_5^R\right]$$

✓ Obtain the **4+6** amplitude

$$A^{(4+6)} = g \sum_{q=b,t} \kappa_q A_q^{LO} + i \frac{g_6 g_S}{\sqrt{2}} \frac{M_H^2}{M_W} W_3^R + g_6 g \left[W_1^R A_t^{nf} + W_2^R A_b^{nf}\right]$$

✓ Derive true relation

$$A^{(4+6)}(gg \to H) = g_g(v_H) A^{(4)}(gg \to H)$$

✓ Define (process dependent) κ-factors

$$\kappa_b = 1 + g_6 \left[\frac{1}{2} \frac{M_b}{M_W} W_2^R - \frac{1}{\sqrt{2}} W_4^R \right]$$

$$\kappa_t = 1 + g_6 \left[\frac{1}{2} \frac{M_t}{M_W} W_1^R - \frac{1}{\sqrt{2}} W_5^R \right]$$

✓ Obtain the **4+6** amplitude

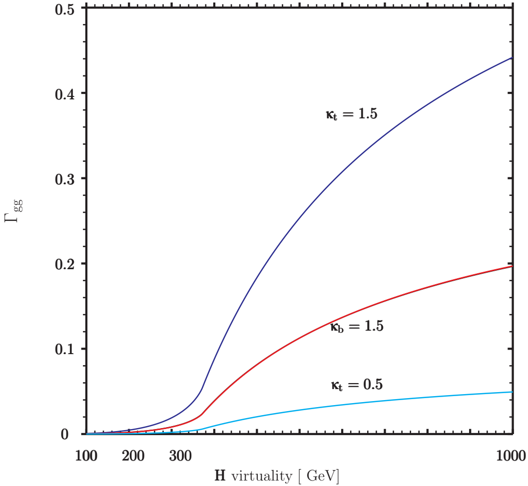
$$A^{(4+6)} = g \sum_{q=b,t} \kappa_q A_q^{LO} + i \frac{g_6 g_S}{\sqrt{2}} \frac{M_H^2}{M_W} W_3^R + g_6 g \left[W_1^R A_t^{nf} + W_2^R A_b^{nf} \right]$$

✓ Derive true relation

$$A^{(4+6)}(gg \to H) = g_g(v_H) A^{(4)}(gg \to H)$$

✓ Effective (running) scaling (g_i) is not a κ (constant) parameter (unless $\mathcal{O}^{(6)} = 0$ and $\kappa_b = \kappa_t$)

☛ Non-factorizable not included



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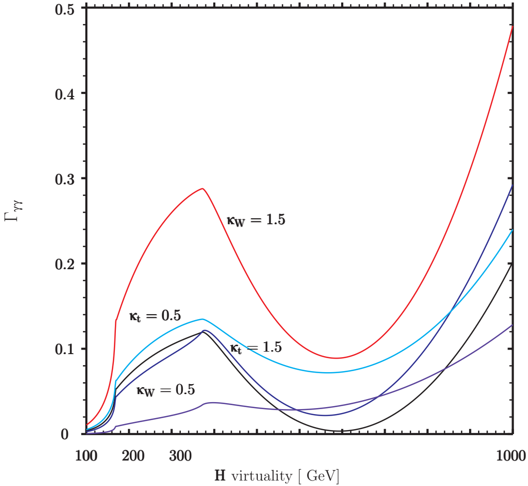

✓ SCALE dependence (no subtraction point)

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- ✓ Consider $\mathbf{H} \to \boldsymbol{\gamma\gamma}$

$$
\begin{aligned}
Z_{ij}^{\mathrm{mix}} &= \delta_{ij} + \frac{g_{\mathrm{R}}^2}{16\,\pi^2}\left[\delta Z_{ij}^{\mathrm{mix}}\,\frac{1}{\bar{\varepsilon}} + \Delta_{ij}\,\ln\frac{M_{\mathrm{H}}^2}{\mu_{\mathrm{R}}^2}\right] \\
W_1 &= a_{\gamma\gamma} = s_\theta\, c_\theta\; a_{\Phi\mathrm{WB}} + c_\theta^2\; a_{\phi\mathrm{B}} + s_\theta^2\; a_{\phi\mathrm{W}} \\
M_{\mathrm{W}}^2\,\Delta_{11} &= \frac{1}{4}\left[8\,s_\theta^2\,\left(2\,s_\theta^2 - c_\theta^2\right)\,M_{\mathrm{W}}^2 + \left(4\,s_\theta^2\,c_\theta^2 - 5\right)\,M_{\mathrm{H}}^2\right]
\end{aligned}
$$

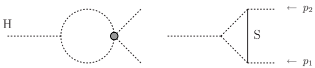
✓ SCALE dependence (no subtraction point)

✓ Consider $\mathbf{H} \to \gamma\gamma$

$$
\begin{aligned}
Z_{ij}^{\text{mix}} &= \delta_{ij} + \frac{g_{\rm R}^2}{16\,\pi^2}\left[\delta Z_{ij}^{\text{mix}}\,\frac{1}{\bar{\varepsilon}} + \Delta_{ij}\,\ln\frac{M_{\rm H}^2}{\mu_{\rm R}^2}\right] \\
W_1 &= a_{\gamma\gamma} = s_\theta\,c_\theta\; a_{\Phi\rm WB} + c_\theta^2\; a_{\phi\rm B} + s_\theta^2\; a_{\phi\rm W} \\
M_{\rm W}^2\,\Delta_{11} &= \frac{1}{4}\left[8\,s_\theta^2\left(2\,s_\theta^2 - c_\theta^2\right)M_{\rm W}^2 + \left(4\,s_\theta^2\,c_\theta^2 - 5\right)M_{\rm H}^2\right]
\end{aligned}
$$

✓ etc

What do we lose without matching?



toy model: S dark Higgs field

$$\mathscr{L} = \mathscr{L}_{\rm SM} - \frac{1}{2}\,\partial_\mu S\,\partial_\mu S - \frac{1}{2}\,M_{\rm S}^2\,S^2 + \mu_{\rm S}\,\Phi^\dagger\,\Phi\,S$$

$$I^{\rm DR}_{\rm eff} = \frac{3}{4}\,g\,\frac{M_{\rm H}^2}{M_{\rm W}\Lambda^2}\left[\left(\frac{1}{2}\,s - 3\,M_{\rm H}^2\right)\left(\frac{1}{\bar{\varepsilon}} - \ln\frac{-s-i0}{\mu_{\rm R}^2}\right) + \quad \text{finite part}\right]$$

$$I_{\rm full} = -\frac{3}{2}\,g\,\frac{M_{\rm H}^2\,\mu_{\rm S}^2}{M_{\rm W}\,M_{\rm S}^2}\left[1 - \frac{1}{4}\,\frac{s}{M_{\rm S}^2} - \left(1 - \frac{1}{2}\,\frac{s}{M_{\rm S}^2}\right)\ln\frac{-s-i0}{M_{\rm S}^2} + \mathscr{O}\left(\frac{s^2}{M_{\rm S}^4}\right)\right]$$

full starts at $\mathscr{O}(\mu_{\rm S}^2/M_{\rm S}^2)$
eff starts at $\mathscr{O}(s/\Lambda^2)$
large mass expansion of **full** follows from Mellin-Barnes expansion and not from Taylor expansion

✓ Background? Consider $\bar{\mathbf{u}}\mathbf{u} \to \mathbf{ZZ}$

✓ Background? Consider $\bar{u}u \to ZZ$

✓ The following Wilson coefficients appear:

$$
\begin{aligned}
W_1 &= a_{\gamma\gamma} = s_\theta c_\theta\, a_{\Phi WB} + c_\theta^2\, a_{\phi B} + s_\theta^2\, a_{\phi W} \\
W_2 &= a_{ZZ} = -s_\theta c_\theta\, a_{\Phi WB} + s_\theta^2\, a_{\phi B} + c_\theta^2\, a_{\phi W} \\
W_3 &= a_{\gamma Z} = 2\, s_\theta\, c_\theta \left(a_{\phi W} - a_{\phi B} \right) + \left(c_\theta^2 - s_\theta^2 \right) a_{\Phi WB} \\
W_4 &= a_{\phi D} \\
W_5 &= a_{\phi q}^{(3)} + a_{\phi q}^{(1)} - a_{\phi u} \\
W_6 &= a_{\phi q}^{(3)} + a_{\phi q}^{(1)} + a_{\phi u}
\end{aligned}
$$

✓ Background? Consider $\bar{\mathbf{u}}\mathbf{u} \rightarrow \mathbf{ZZ}$

✓ The following Wilson coefficients appear:

$$
\begin{aligned}
W_1 &= a_{\gamma\gamma} = s_\theta c_\theta\, a_{\Phi \mathrm{WB}} + c_\theta^2\, a_{\phi \mathrm{B}} + s_\theta^2\, a_{\phi \mathrm{W}} \\
W_2 &= a_{\mathrm{ZZ}} = -s_\theta c_\theta\, a_{\Phi \mathrm{WB}} + s_\theta^2\, a_{\phi \mathrm{B}} + c_\theta^2\, a_{\phi \mathrm{W}} \\
W_3 &= a_{\gamma \mathrm{Z}} = 2\, s_\theta\, c_\theta \left(a_{\phi \mathrm{W}} - a_{\phi \mathrm{B}} \right) + \left(c_\theta^2 - s_\theta^2 \right) a_{\Phi \mathrm{WB}} \\
W_4 &= a_{\phi \mathrm{D}} \\
W_5 &= a_{\phi \mathrm{q}}^{(3)} + a_{\phi \mathrm{q}}^{(1)} - a_{\phi \mathrm{u}} \\
W_6 &= a_{\phi \mathrm{q}}^{(3)} + a_{\phi \mathrm{q}}^{(1)} + a_{\phi \mathrm{u}}
\end{aligned}
$$

✓ Define

$$
\mathrm{A}^{\mathrm{LO}} = \frac{M_{\mathrm{Z}}^4}{t^2} + \frac{M_{\mathrm{Z}}^4}{u^2} - \frac{t}{u} - \frac{u}{t} - 4\, \frac{M_{\mathrm{Z}}^2 s}{tu}
$$

✓ Obtain the result ($\bar{\mathbf{u}}\mathbf{u} \to \mathbf{ZZ}$)

$$\sum_{\text{spin}} \left| \mathrm{A}^{(4+6)} \right|^2 = g^4 \, \mathrm{A}^{\mathrm{LO}} \left[\mathrm{F}^{\mathrm{LO}}(s_\theta) + \frac{g_6}{\sqrt{2}} \sum_{i=1}^{6} \mathrm{F}^{i}(s_\theta) \, W_i \right]$$

✓ Obtain the result ($\bar{\mathbf{u}}\mathbf{u} \to \mathbf{ZZ}$)

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✓ Background changes!

✓ Obtain the result ($\bar{\mathbf{u}}\mathbf{u} \to \mathbf{ZZ}$)

$$\sum_{\text{spin}} \left| \mathrm{A}^{(4+6)} \right|^2 = g^4 \, \mathrm{A}^{\mathrm{LO}} \left[\mathrm{F}^{\mathrm{LO}}(s_\theta) + \frac{g_6}{\sqrt{2}} \sum_{i=1}^{6} \mathrm{F}^i(s_\theta) \, W_i \right]$$

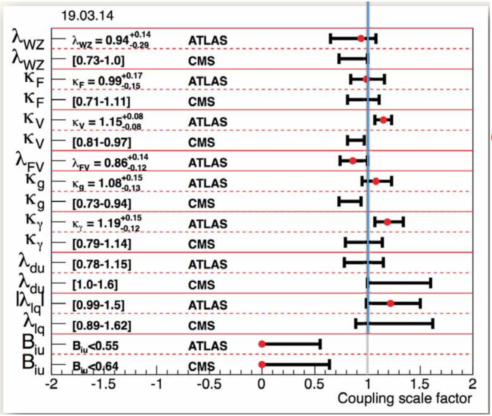
✓ Background changes!

✓ Note that

$$\begin{aligned} \mathrm{F}^{\mathrm{LO}} &\approx -0.57 \quad \mathrm{F}^1 \approx +2.18 \quad \mathrm{F}^2 \approx -3.31 \\ \mathrm{F}^3 &\approx +4.07 \quad \mathrm{F}^4 \approx -2.46 \quad \mathrm{F}^4 \approx -2.46 \quad \mathrm{F}^6 \approx -5.81 \end{aligned}$$

[from arXiv:1403.7191]

FUTURE (Moriod EW **2014**)



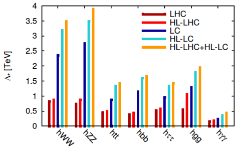


FIG. 2: Effective new physics scales Λ_* extracted from the Higgs coupling measurements collected in Table I. T for the loop-induced couplings to gluons and photons contain only the contribution of the contact terms, as t of the loop terms are already disentangled at the level of the input values Δ. (The ordering of the columns fr right corresponds to the legend from up to down.)

34

$$\mathscr{L} = \mathscr{L}_4 + \sum_{n>4} \sum_{i=1}^{N_n} \frac{a_i^n}{\Lambda^{n-4}} \mathcal{O}_i^{(d=n)}$$

TH is improving with NLO κ-language

NLO κ-language is NOT a simple scaling

Thanks for your attention

Backup Slides

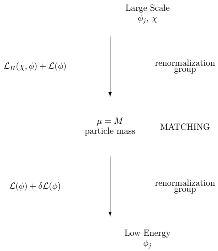


Figure 4: The general form of a matching calculation.

terms.

In this region, the physics is described by a set fields, χ, describing the heaviest particles, of mass M, and a set of light particle fields, ϕ, describing all the lighter particles. The Lagrangian has the form

$$\mathcal{L}_H(\chi, \phi) + \mathcal{L}(\phi)\,, \tag{3.15}$$

where $\mathcal{L}(\phi)$ contains all the terms that depend only on the light fields, and $\mathcal{L}_H(\chi, \phi)$ is everything else. You then evolve the theory down to lower scales. As long as no particle masses are encountered, this evolution is described by the renormalization group. However, when μ goes below the mass, M, of the heavy particles, you should change the effective theory to a new theory without the heavy particles. In the process, the parameters of the theory change, and new, nonrenormalizable interactions may be introduced. Thus the Lagrangian of the effective theory below M has the form

$$\mathcal{L}(\phi) + \delta\mathcal{L}(\phi)\,, \tag{3.16}$$

Increasing COMPLEXITY

✓ $\mathbf{H \to \gamma\gamma}$

① **3** LO amplitudes $\mathbf{A_t^{LO}, A_b^{LO}, A_W^{LO}}$, **3** $\boldsymbol{\kappa}$-factors

② **6** Wilson coefficients & non-factorizable amplitudes

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✓ $\mathbf{H \to ZZ}$

① **1** LO amplitude

② **6** NLO amplitudes, **6** $\boldsymbol{\kappa}$-factors

$$\delta^{\mu\nu} \sum_{i=\mathrm{t,b,B}} \mathrm{A}_{i,\mathrm{D}}^{\mathrm{NLO}} + p_2^{\mu} p_1^{\nu} \sum_{i=\mathrm{t,b,B}} \mathrm{A}_{i,\mathrm{P}}^{\mathrm{NLO}}$$

② **16** Wilson coefficients & non-factorizable amplitudes

Increasing COMPLEXITY

✓ $\mathbf{H \to \gamma\gamma}$

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✓ $\mathbf{H \to ZZ}$

① **1** LO amplitude

② **6** NLO amplitudes, **6** $\boldsymbol{\kappa}$-factors

$$\delta^{\mu\nu} \sum_{i=\mathrm{t,b,B}} \mathrm{A}_{i,\mathrm{D}}^{\mathrm{NLO}} + p_2^{\mu} p_1^{\nu} \sum_{i=\mathrm{t,b,B}} \mathrm{A}_{i,\mathrm{P}}^{\mathrm{NLO}}$$

② **16** Wilson coefficients & non-factorizable amplitudes

✓ etc.

✎ ***g*** **finite renormalization**

$$g_{\rm exp}^2 = G^2 \left[1 + 2\,\frac{G^2}{16\,\pi^2} \left(\mathrm{d}G^{(4)} + g_6\,\mathrm{d}G^{(6)} \right) \right] \qquad G^2 = 4\,\sqrt{2}\,G_{\rm F}\,M_{\rm W}^2$$

✓ $\mathbf{d}\boldsymbol{G}^{\mathbf{(4,6)}}$ from $\boldsymbol{\mu}$-decay

✎ g finite renormalization

$$g^2_{\rm exp} = G^2 \left[1 + 2\,\frac{G^2}{16\,\pi^2}\,\left(\mathrm{d}G^{(4)} + g_6\,\mathrm{d}G^{(6)}\right)\right] \qquad G^2 = 4\,\sqrt{2}\,G_{\rm F}\,M^2_{\rm W}$$

- ✓ $\mathbf{d}G^{(4,6)}$ from μ -decay
- ✓ Involving $\Sigma_{\rm WW}(0)$ (easy)

✎ *g* finite renormalization

$$g^2_{\text{exp}} = G^2 \left[1 + 2\,\frac{G^2}{16\,\pi^2} \left(\mathrm{d}G^{(4)} + g_6\,\mathrm{d}G^{(6)} \right) \right] \qquad G^2 = 4\,\sqrt{2}\,G_{\mathrm{F}}\,M^2_{\mathrm{W}}$$

✓ $\mathbf{d}G^{(4,6)}$ from μ -decay

✓ Involving $\Sigma_{\mathbf{WW}}(\mathbf{0})$ (easy)

✗ and vertices & boxes (not easy with $\mathscr{O}^{(6)}$ -insertions)

H wave function renormalization $\mathbf{1} - \frac{1}{2}\,\frac{g^2_{\rm exp}}{16\pi^2}\,\delta\mathscr{Z}_{\mathrm{H}}$

$$
\begin{aligned}
\delta\mathscr{Z}^{(4)}_{\mathrm{H}} \;=\;& \frac{3}{2}\,\frac{M^2_{\mathrm{t}}}{M^2_{\mathrm{W}}}\,B^f_0\left(-M^2_{\mathrm{H}}\,;M_{\mathrm{t}},M_{\mathrm{t}}\right) + \frac{3}{2}\,\frac{M^2_{\mathrm{b}}}{M^2_{\mathrm{W}}}\,B^f_0\left(-M^2_{\mathrm{H}}\,;M_{\mathrm{b}},M_{\mathrm{b}}\right) \\
-\;& B^f_0\left(-M^2_{\mathrm{H}}\,;M_{\mathrm{W}},M_{\mathrm{W}}\right) - 1/2\,\frac{1}{c^2_\theta}\,B^f_0\left(-M^2_{\mathrm{H}}\,;M_{\mathrm{Z}},M_{\mathrm{Z}}\right) \\
+\;& \frac{3}{2}\left(M^2_{\mathrm{H}} - 4\,M^2_{\mathrm{t}}\right)\frac{M^2_{\mathrm{t}}}{M^2_{\mathrm{W}}}\,B^p_0\left(-M^2_{\mathrm{H}}\,;M_{\mathrm{t}},M_{\mathrm{t}}\right) + \frac{3}{2}\left(M^2_{\mathrm{H}} - 4\,M^2_{\mathrm{b}}\right)\frac{M^2_{\mathrm{b}}}{M^2_{\mathrm{W}}}\,B^p_0\left(-M^2_{\mathrm{H}}\,;M_{\mathrm{b}},M_{\mathrm{b}}\right) \\
+\;& \frac{1}{4}\left(\frac{M^4_{\mathrm{H}}}{M^2_{\mathrm{W}}} - 4\,M^2_{\mathrm{H}} + 12\,M^2_{\mathrm{W}}\right)B^p_0\left(-M^2_{\mathrm{H}}\,;M_{\mathrm{W}},M_{\mathrm{W}}\right) + \frac{1}{8}\left(\frac{M^4_{\mathrm{H}}}{M^2_{\mathrm{W}}} - 4\,\frac{M^2_{\mathrm{H}}}{c^2_\theta} + 12\,\frac{M^2_{\mathrm{Z}}}{c^2_\theta}\right)B^p_0\left(-M^2_{\mathrm{H}}\,;M_{\mathrm{Z}},M_{\mathrm{Z}}\right) \\
+\;& \frac{9}{8}\,\frac{M^4_{\mathrm{H}}}{M^2_{\mathrm{W}}}\,B^p_0\left(-M^2_{\mathrm{H}}\,;M_{\mathrm{H}},M_{\mathrm{H}}\right)
\end{aligned}
$$

etc.

Fine points on PTG versus LG $\mathscr{O}^{(\mathbf{6})}$ operators

✓ Proposition: if we assume that the high-energy theory is

① weakly-coupled and

② renormalizable

Fine points on PTG versus LG $\mathcal{O}^{(6)}$ operators

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✓ it follows that the PTG/LG classification of arXiv:1307.0478 (used here) is correct.

Fine points on PTG versus LG $\mathcal{O}^{(6)}$ operators

✓ Proposition: if we assume that the high-energy theory is

① weakly-coupled and

② renormalizable

✓ it follows that the PTG/LG classification of arXiv:1307.0478 (used here) is correct.

✓ If we do not assume the above but work always in some EFT context (i.e.. also the next high-energy theory is EFT, possibly involving some strongly interacting theory) then classification changes, see Eqs. (A1-A2) of arXiv:1305.0017v2

STU: (combination of) Wilson coefficients

$$W_1 = a_{\gamma\gamma} = s_\theta c_\theta\, a_{\Phi WB} + c_\theta^2\, a_{\phi B} + s_\theta^2\, a_{\phi W}$$

$$W_2 = a_{ZZ} = -s_\theta c_\theta\, a_{\Phi WB} + s_\theta^2\, a_{\phi B} + c_\theta^2\, a_{\phi W}$$

$$W_3 = a_{\gamma Z} = 2\, s_\theta\, c_\theta \left(a_{\phi W} - a_{\phi B} \right) + \left(c_\theta^2 - s_\theta^2 \right) a_{\Phi WB}$$

$W_4 = a_{\phi D}$ $\qquad$ $W_5 = a_{\phi \Box}$

$W_6 = a_{bWB}$ $\qquad$ $W_7 = a_{bBW}$

$W_8 = a_{tWB}$ $\qquad$ $W_9 = a_{tBW}$

$W_{10} = a_{b\phi}$ $\qquad$ $W_{11} = a_{t\phi}$

$$a_{qW} = s_\theta\, a_{qWB} + c_\theta\, a_{qBW} \qquad a_{qB} = s_\theta\, a_{qBW} - c_\theta\, a_{qWB}$$

$$W_{12} = a_{\phi bA} \qquad W_{13} = a_{\phi bV}$$
$$W_{14} = a_{\phi tA} \qquad W_{15} = a_{\phi tV}$$

$$a_{\phi bV} = a_{\phi q}^{(3)} - a_{\phi b} - a_{\phi q}^{(1)} \qquad a_{\phi bA} = a_{\phi q}^{(3)} + a_{\phi b} - a_{\phi q}^{(1)}$$
$$a_{\phi tV} = a_{\phi q}^{(3)} - a_{\phi t} - a_{\phi q}^{(1)} \qquad a_{\phi tA} = a_{\phi q}^{(3)} + a_{\phi t} - a_{\phi q}^{(1)}$$

STU: building blocks $\gamma - \gamma$

$$
\begin{aligned}
\Sigma_{\gamma\gamma}(s) &= \Pi_{\gamma\gamma}(s)\, s \\
\Pi_{\gamma\gamma}(s) &= \frac{g^2 s_\theta^2}{16\,\pi^2}\,\Pi^{(4)}_{\gamma\gamma}(s) + \frac{g^2 g_6}{16\sqrt{2}\,\pi^2}\sum_{i=1}^{11}\Pi^{(6)}_{\gamma\gamma\,i}(s)\, W_i
\end{aligned}
$$

$$
\Pi^{(4)}_{\gamma\gamma}(0) = 3\, a_0^f\left(M_{\mathrm{W}}\right) + \frac{1}{9}\left[1 - 4\, a_0^f\left(M_{\mathrm{b}}\right) - 16\, a_0^f\left(M_{\mathrm{t}}\right)\right]
$$

$$
\begin{aligned}
\Pi^{(6)}_{\gamma\gamma\,1}(0) &= -\left(1-8\,s_\theta^2+2\,s_\theta^4\right)\,a_0^f\left(M_{\rm W}\right) \\
&- \frac{1}{2}\,\frac{M_{\rm H}^2}{M_{\rm W}^2}\,a_0^f\left(M_{\rm H}\right)-\frac{1}{2}\,\frac{1}{c_\theta^2}\,a_0^f\left(M_{\rm Z}\right) \\
&- \frac{4}{9}\,s_\theta^2\left[16\left(1-\frac{1}{2}\,s_\theta^2\right)\,a_0^f\left(M_{\rm t}\right)\right. \\
&+ \left.4\left(1-\frac{1}{2}\,s_\theta^2\right)\,a_0^f\left(M_{\rm b}\right)+17\left(1-\frac{35}{34}\,s_\theta^2\right)\right] \\
\Pi^{(6)}_{\gamma\gamma\,2}(0) &= s_\theta\,c_\theta\left\{\frac{2}{9}\left[35+16\,a_0^f\left(M_{\rm t}\right)+4\,a_0^f\left(M_{\rm b}\right)\right]-2\,a_0^f\left(M_{\rm W}\right)\right\} \\
\Pi^{(6)}_{\gamma\gamma\,3}(0) &= s_\theta\,c_\theta\left\{4\left(1-\frac{35}{18}\,c_\theta^2\right)+4\left(1-\frac{1}{2}\,s_\theta^2\right)\,a_0^f\left(M_{\rm W}\right)\right. \\
&- \left.\frac{8}{9}\,c_\theta^2\left[4\,a_0^f\left(M_{\rm t}\right)+a_0^f\left(M_{\rm b}\right)\right]\right\} \\
\Pi^{(6)}_{\gamma\gamma\,4}(0) &= c_\theta^2\left\{-\frac{3}{2}\,a_0^f\left(M_{\rm W}\right)+\frac{1}{18}\left[16\,a_0^f\left(M_{\rm t}\right)+4\,a_0^f\left(M_{\rm b}\right)-1\right]\right\} \\
\Pi^{(6)}_{\gamma\gamma\,6}(0) &= -2\,\frac{M_{\rm b}^2}{M_{\rm W}^2}\,s_\theta\left[a_0^f\left(M_{\rm b}\right)+1\right] \\
\Pi^{(6)}_{\gamma\gamma\,8}(0) &= -4\left(c_\theta^2-s_\theta^2\right)\,s_\theta\,\frac{M_{\rm b}^2}{M_{\rm W}^2}\left[a_0^f\left(M_{\rm t}\right)+1\right] \\
\Pi^{(6)}_{\gamma\gamma\,9}(0) &= 8\,s_\theta^2\,c_\theta\,\frac{M_{\rm t}^2}{M_{\rm W}^2}\left[a_0^f\left(M_{\rm t}\right)+1\right]
\end{aligned}
$$

$\mathcal{STU}$: building blocks $\mathbf{Z}$–$\boldsymbol{\gamma}$

$$
\begin{aligned}
\Sigma_{\mathrm{Z}\gamma}(s) &= \Pi_{\mathrm{Z}\gamma}(s)\, s \\
\Pi_{\mathrm{Z}\gamma}(s) &= \frac{g^2}{16\,\pi^2}\,\frac{s_\theta}{c_\theta}\,\Pi^{(4)}_{\mathrm{Z}\gamma}(s) + \frac{g^2 g_6}{16\,\sqrt{2}\,\pi^2}\,\sum_{i=1}^{15}\Pi^{(6)}_{\mathrm{Z}\gamma\, i}(s)\, W_i - \frac{g_6}{\sqrt{2}}\, W_3
\end{aligned}
$$

$$
\begin{aligned}
\Pi^{(4)}_{\mathrm{Z}\gamma}(0) &= \frac{1}{6}\left(19 - 18\, s^2_\theta\right) a^f_0\left(M_{\mathrm{W}}\right) - \frac{2}{9}\left(3 - 8\, s^2_\theta\right) a^f_0\left(M_{\mathrm{t}}\right) \\
&- \frac{1}{9}\left(3 - 4\, s^2_\theta\right) a^f_0\left(M^{\mathrm{b}}\right) + \frac{1}{18}\left(21 - 2\, s^2_\theta\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pi^{(6)}_{Z\gamma\,1}(0) &= \frac{s_\theta}{c_\theta}\Big[\frac{1}{3}\left(1+6\,c_\theta^4\right)a_0^f\left(M_W\right)+\frac{4}{9}\left(5-8\,c_\theta^4\right)a_0^f\left(M_t\right) \\
&+ \frac{2}{9}\left(1-4\,c_\theta^4\right)a_0^f\left(M_b\right)-\frac{1}{9}\left(33-122\,s_\theta^2+70\,s_\theta^4\right)\Big] \\
\Pi^{(6)}_{Z\gamma\,2}(0) &= s_\theta\,c_\theta\Big[+2\left(3-c_\theta^2\right)a_0^f\left(M_W\right)-\frac{32}{9}\,s_\theta^2\,a_0^f\left(M_t\right) \\
&- \frac{8}{9}\,s_\theta^2\,a_0^f\left(M_b\right)-\frac{2}{9}\left(8-35\,c_\theta^2\right)\Big] \\
\Pi^{(6)}_{Z\gamma\,3}(0) &= -\frac{1}{18}\left(33-174\,s_\theta^2+140\,s_\theta^4\right)+\frac{1}{3}\left(2-9\,s_\theta^2+6\,s_\theta^4\right)a_0^f\left(M_W\right) \\
&- \frac{1}{4}\,\frac{M_H^2}{M_W^2}\,a_0^f\left(M_H\right)-\frac{1}{4}\,\frac{1}{c_\theta^2}\,a_0^f\left(M_Z\right)-\frac{2}{9}\left(3-24\,s_\theta^2+16\,s_\theta^4\right)a_0^f\left(M_t\right)-\frac{1}{9}\left(3-12\,s_\theta^2+8\,s_\theta^4\right)a_0^f\left(M_b\right) \\
\Pi^{(6)}_{Z\gamma\,4}(0) &= \frac{1}{s_\theta\,c_\theta}\Big[-\frac{1}{24}\left(19-56\,s_\theta^2+36\,s_\theta^4\right)a_0^f\left(M_W\right)+\frac{1}{18}\left(3-24\,s_\theta^2+16\,s_\theta^4\right)a_0^f\left(M_t\right) \\
&+ \frac{1}{36}\left(3-12\,s_\theta^2+8\,s_\theta^4\right)a_0^f\left(M_b\right)-\frac{1}{72}\left(21+4\,s_\theta^4\right)\Big] \\
\Pi^{(6)}_{Z\gamma\,6}(0) &= \frac{1}{4\,c_\theta}\,\frac{M_b^2}{M_W^2}\left(1-4\,c_\theta^2\right)\left[a_0^f\left(M_b\right)-1\right] \\
\Pi^{(6)}_{Z\gamma\,7}(0) &= -\frac{M_b^2}{M_W^2}\,s_\theta^2\left[a_0^f\left(M_b\right)+1\right] \\
\Pi^{(6)}_{Z\gamma\,8}(0) &= -\frac{1}{4\,c_\theta}\,\frac{M_t^2}{M_W^2}\left(5-34\,c_\theta^2+32\,c_\theta^4\right)\left[a_0^f\left(M_t\right)-1\right] \\
\Pi^{(6)}_{Z\gamma\,9}(0) &= \frac{1}{2}\,s_\theta\,\frac{M_t^2}{M_W^2}\left(7-16\,s_\theta^2\right)\left[a_0^f\left(M_t\right)+1\right]
\end{aligned}
$$

$$\Pi^{(6)}_{Z\gamma\,13}(0) \quad = \quad -\frac{2}{3}\,\frac{s_\theta}{c_\theta}\,\frac{M_b^2}{M_W^2}\left[a_0^f\left(M_b\right)+1\right]$$

$$\Pi^{(6)}_{Z\gamma\,15}(0) \quad = \quad -\frac{4}{3}\,\frac{s_\theta}{c_\theta}\,\frac{M_t^2}{M_W^2}\left[a_0^f\left(M_t\right)+1\right]$$

–

$\mathcal{STU}$: building blocks $\mathbf{Z}$–$\mathbf{Z}$

$$
\begin{aligned}
\Sigma_{\mathrm{ZZ}}(s) &= \mathrm{S}_{\mathrm{ZZ}} + \Pi_{\mathrm{ZZ}}\, s + \mathscr{O}(s^2) \\
\mathrm{S}_{\mathrm{ZZ}} &= \frac{g^2}{16\,\pi^2\,c_\theta^2}\,\mathrm{S}^{(4)}_{\mathrm{ZZ}} + \frac{g^2\,g_6}{16\,\sqrt{2}\,\pi^2}\sum_{i=1}^{15}\mathrm{S}^{(6)}_{\mathrm{ZZ}\,i}\,W_i \\
\Pi_{\mathrm{ZZ}} &= \frac{g^2}{16\,\pi^2\,c_\theta^2}\,\Pi^{(4)}_{\mathrm{ZZ}} + \frac{g^2\,g_6}{16\,\sqrt{2}\,\pi^2}\sum_{i=1}^{15}\Pi^{(6)}_{\mathrm{ZZ}\,i}\,W_i
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{S}^{(4)}_{\mathrm{ZZ}} \quad = \quad & \left(M_{\mathrm{Z}}^2 - \frac{1}{3}\, M_{\mathrm{H}}^2 + \frac{1}{12}\, \frac{M_{\mathrm{H}}^4}{M_{\mathrm{Z}}^2}\right) B_0^f\left(-M_{\mathrm{Z}}^2\, ; M_{\mathrm{H}}, M_{\mathrm{Z}}\right) \\
+ \quad & \frac{1}{18}\left[\left(7 - 16\, c_\theta^2 - 64\, c_\theta^2\, s_\theta^2\right) M_{\mathrm{t}}^2 + \left(17 - 8\, c_\theta^2 - 32\, c_\theta^2\, s_\theta^2\right) M_{\mathrm{Z}}^2\right] B_0^f\left(-M_{\mathrm{Z}}^2\, ; M_{\mathrm{t}}, M_{\mathrm{t}}\right) \\
+ \quad & \frac{1}{18}\left[\left(5 + 4\, c_\theta^2 - 8\, c_\theta^2\, s_\theta^2\right) M_{\mathrm{Z}}^2 - \left(17 - 8\, c_\theta^2 + 16\, c_\theta^2\, s_\theta^2\right) M_{\mathrm{b}}^2\right] B_0^f\left(-M_{\mathrm{Z}}^2\, ; M_{\mathrm{b}}, M_{\mathrm{b}}\right) \\
+ \quad & \frac{1}{12}\left[\left(1 - 20\, c_\theta^2 + 36\, c_\theta^2\, s_\theta^2\right) M_{\mathrm{Z}}^2 - 16\left(5 - 3\, s_\theta^2\right) M_{\mathrm{Z}}^2\, c_\theta^6\right] B_0^f\left(-M_{\mathrm{Z}}^2\, ; M_{\mathrm{W}}, M_{\mathrm{W}}\right) \\
+ \quad & \frac{1}{12}\left(M_{\mathrm{Z}}^4 - 2\, M_{\mathrm{H}}^2\, M_{\mathrm{Z}}^2 + M_{\mathrm{H}}^4\right) B_0^p\left(0\, ; M_{\mathrm{H}}, M_{\mathrm{Z}}\right) + \frac{2}{3}\left(M_{\mathrm{Z}}^2 + \frac{M_{\mathrm{Z}}^4}{M_{\mathrm{H}}^2 - M_{\mathrm{Z}}^2} - \frac{3}{8}\, M_{\mathrm{H}}^2 + \frac{1}{8}\, \frac{M_{\mathrm{H}}^4}{M_{\mathrm{Z}}^2}\right) a_0^f\left(M_{\mathrm{H}}\right) \\
+ \quad & \frac{1}{4}\left(M_{\mathrm{Z}}^2 - \frac{8}{3}\, \frac{M_{\mathrm{Z}}^4}{M_{\mathrm{H}}^2 - M_{\mathrm{Z}}^2} - \frac{1}{3}\, M_{\mathrm{H}}^2\right) a_0^f\left(M_{\mathrm{Z}}\right) - \frac{4}{27}\left(2 + c_\theta^2 - 5\, c_\theta^2\, s_\theta^2\right) M_{\mathrm{Z}}^2 \\
\Pi^{(4)}_{\mathrm{ZZ}} \quad = \quad & \frac{5}{6}\left(M_{\mathrm{Z}}^2 - \frac{1}{5}\, M_{\mathrm{H}}^2\right) B_0^p\left(0\, ; M_{\mathrm{H}}, MZ\right) + \frac{1}{18}\left(7 - 16\, c_\theta^2 - 64\, c_\theta^2\, s_\theta^2\right) M_{\mathrm{t}}^2\, B_0^p\left(0\, ; M_{\mathrm{t}}, M_{\mathrm{t}}\right) \\
- \quad & \frac{1}{18}\left(17 - 8\, c_\theta^2 + 16\, c_\theta^2\, s_\theta^2\right) M_{\mathrm{b}}^2\, B_0^p\left(0\, ; M_{\mathrm{b}}, M_{\mathrm{b}}\right) + \frac{1}{3}\left[5\, M_{\mathrm{Z}}^2\, c_\theta^2 - 4\left(5 - 3\, s_\theta^2\right) M_{\mathrm{Z}}^2\, c_\theta^6\right] B_0^p\left(0\, ; M_{\mathrm{W}}, M_{\mathrm{W}}\right) \\
- \quad & \frac{1}{24}\left(M_{\mathrm{Z}}^4 - 2\, M_{\mathrm{H}}^2\, M_{\mathrm{Z}}^2 + M_{\mathrm{H}}^4\right) B_0^s\left(0\, ; M_{\mathrm{H}}, MZ\right) + \frac{1}{12}\left(1 + \frac{M_{\mathrm{Z}}^2}{M_{\mathrm{H}}^2 - M_{\mathrm{Z}}^2}\right) a_0^f\left(M_{\mathrm{H}}\right) \\
- \quad & \frac{1}{12}\, \frac{M_{\mathrm{Z}}^2}{M_{\mathrm{H}}^2 - M_{\mathrm{Z}}^2}\, a_0^f\left(M_{\mathrm{Z}}\right) + \frac{4}{27}\left(2 + c_\theta^2 - 5\, c_\theta^2\, s_\theta^2\right)
\end{aligned}
$$


KEEP
CALM
TO
BE
CONTINUED

The life and death of $\boldsymbol{\mu_{\mathrm{R}}}$

✓ $\boldsymbol{\gamma}$ bare propagator

$$
\begin{aligned}
\Delta_{\gamma}^{-1} &= -s - \frac{g^2}{16\,\pi^2}\,\Sigma_{\gamma\gamma}(s) \\
\Sigma_{\gamma\gamma}(s) &= \left(\mathrm{D}^{(4)} + g_6\,\mathrm{D}^{(6)}\right)\frac{1}{\bar{\varepsilon}} + \sum_{x\in\mathscr{X}}\left(\mathrm{L}_i^{(4)} + g_6\,\mathrm{L}_i^{(6)}\right)\ln\frac{x}{\mu_{\mathrm{R}}^2} + \Sigma_{\gamma\gamma}^{\mathrm{rest}} \\
\{\mathscr{X}\} &= \{s\,,\,m^2\,,\,m_0^2\,,\,m_{\mathrm{H}}^2\,,\,m_{\mathrm{t}}^2\,,\,m_{\mathrm{b}}^2\}
\end{aligned}
$$

The life and death of μ_{R}

✓ **γ** bare propagator

$$
\begin{aligned}
\Delta_{\gamma}^{-1} &= -s - \frac{g^2}{16\,\pi^2}\,\Sigma_{\gamma\gamma}(s) \\
\Sigma_{\gamma\gamma}(s) &= \Big(\mathrm{D}^{(4)} + g_6\,\mathrm{D}^{(6)}\Big)\,\frac{1}{\bar{\varepsilon}} + \sum_{x \in \mathscr{X}} \Big(\mathrm{L}_i^{(4)} + g_6\,\mathrm{L}_i^{(6)}\Big)\,\ln\frac{x}{\mu_{\mathrm{R}}^2} + \Sigma_{\gamma\gamma}^{\mathrm{rest}} \\
\{\mathscr{X}\} &= \{s\,,\,m^2\,,\,m_0^2\,,\,m_{\mathrm{H}}^2\,,\,m_{\mathrm{t}}^2\,,\,m_{\mathrm{b}}^2\}
\end{aligned}
$$

✓ **γ** renormalized propagator

$$
\begin{aligned}
\Delta_{\gamma}^{-1}\Big|_{\mathrm{ren}} &= -Z_{\gamma}\,s - \frac{g^2}{16\,\pi^2}\,\Sigma_{\gamma\gamma}(s) \\
&= -s - \frac{g^2}{16\,\pi^2}\,\Sigma_{\gamma\gamma}^{\mathrm{ren}}(s)
\end{aligned}
$$

The life and death of $\boldsymbol{\mu_{\mathrm{R}}}$

$$\Sigma_{\gamma\gamma}^{\text{ren}}(s) = \sum_{x\in\mathscr{X}} \left(\mathrm{L}_i^{(4)} + g_6\,\mathrm{L}_i^{(6)}\right) \ln\frac{x}{\mu_{\mathrm{R}}^2} + \Sigma_{\gamma\gamma}^{\text{rest}}$$

✓ finite renormalization

$$\Sigma_{\gamma\gamma}^{\text{ren}}(s) = \Pi_{\gamma\gamma}^{\text{ren}}(s)\,s$$

$$\frac{\partial}{\partial\mu_{\mathrm{R}}}\left[\Pi_{\gamma\gamma}^{\text{ren}}(s) - \Pi_{\gamma\gamma}^{\text{ren}}(0)\right] = 0$$

The life and death of μ_{R}

$$\Sigma^{\text{ren}}_{\gamma\gamma}(s) \;=\; \sum_{x\in\mathscr{X}} \left(\mathrm{L}_i^{(4)} + g_6\,\mathrm{L}_i^{(6)}\right) \ln\frac{x}{\mu_{\mathrm{R}}^2} + \Sigma^{\text{rest}}_{\gamma\gamma}$$

✓ finite renormalization

$$\Sigma^{\text{ren}}_{\gamma\gamma}(s) \;=\; \Pi^{\text{ren}}_{\gamma\gamma}(s)\, s$$

$$\frac{\partial}{\partial\mu_{\mathrm{R}}}\left[\Pi^{\text{ren}}_{\gamma\gamma}(s) - \Pi^{\text{ren}}_{\gamma\gamma}(0)\right] \;=\; 0$$

✓ including $\mathscr{O}^{(6)}$ contribution. There is no μ_{R} problem when a subtraction point is available.

$\mathscr{O}^{(6)} \to \mathscr{O}^{(4)} \to$ field(parameter) redefinition

$$
\begin{aligned}
\mathscr{L} &= -\partial_\mu \mathrm{K}^\dagger \partial^\mu \mathrm{K} - \mu^2 \mathrm{K}^\dagger \mathrm{K} \\
&- \frac{1}{2}\lambda \left(\mathrm{K}^\dagger \mathrm{K}\right)^2 - \frac{1}{2} M_0^2 \phi_0^2 - M^2 \phi^+ \phi^- + g^2 \frac{a_\phi}{\Lambda^2} \left(\mathrm{K}^\dagger \mathrm{K}\right)^3 \\
&- g \frac{a_{\phi\Box}}{\Lambda^2} \mathrm{K}^\dagger \mathrm{K} \Box \mathrm{K}^\dagger \mathrm{K} - g \frac{a_{\phi\mathrm{D}}}{\Lambda^2} \left|\mathrm{K}^\dagger \partial^\mu \mathrm{K}\right|^2
\end{aligned}
$$

$$
\sqrt{2}\,\mathrm{K}_1 = \mathrm{H} + 2\frac{M}{g} + i\,\phi_0 \qquad\qquad \mathrm{K}_2 = i\,\phi^-
$$

Requires

$$\mu^2 = \beta_{\rm H} - 2\,\frac{\lambda}{g^2}\,M^2 \qquad\qquad \lambda = \frac{1}{4}\,g^2\,\frac{M^2_{\rm H}}{M^2}$$

$$\mathrm{H} \quad\rightarrow\quad \left[1 - \left(a_{\phi\mathrm{D}} - 4\,a_{\phi\Box}\right)\,\frac{M^2_{\rm H}}{g^2\Lambda^2}\right]\mathrm{H}$$

$$M_{\rm H} \quad\rightarrow\quad \left[1 + \left(a_{\phi\mathrm{D}} - 4\,a_{\phi\Box} + 24\,a_{\phi}\right)\,\frac{M^2_{\rm H}}{g^2\Lambda^2}\right]M_{\rm H}$$

etc. with non-trivial effects on the $\boldsymbol{S}$-matrix