# Can we find a computationally efficient *loop* algorithm?

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# Outline of Part I

1. **The Problem**
   - About new algorithms
   - Qualifiers
   - Buyer's Guide

# Outline of Part II

## 2 Bernstein - Sato - Tkachov

- BST polynomials
- Landau equations
- Summary of singular behavior
- BST in practice
- Contracting into one proof many different proofs
- Gram
- BST ready to use
- BST iteration never ends
- Tensor integrals
- Complex parameters
- Additional features
- Infrared
- Around threshold

# Outline of Part III

3. **New integral representations**
   - How to construct it

## Conclusions

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4 ⊕ ⊖

Part I

# Introduction

# the problem

There are $>$ 10 talks on evaluation of one-loop diagrams

**Question**

Can we find a computationally efficient *loop* algorithm to replace the brute-force methods?

list

**Comment**

Brute force is simple, but may demand very much patience (or faster hardware)

The sky, not the skull is the limit

# Rating new algorithms

Even disregarding the intrinsic necessity of some new algorithm, and even in case it has no intrinsic necessity at all, a probable decision about its truth is possible inductively by studying its *success*. Success here means fruitfulness in consequences, in particular in *verifiable* consequences, i.e., consequences demonstrable without the new algorithm, whose proofs with the help of the new algorithm, however, are considerably simpler and easier to discover, and make it possible to contract into one proof many different proofs.

# Definitions

**Theorem**

*Any* *algorithm* *aimed at reducing the analytical complexity of a (multi - loop) Feynman diagram is generally bound to*

- *replace the* *original integral* *with a sum of* *many simpler diagrams*,
- *introducing* *denominators* *that show zeros.*

**Definition**

An algorithm is *optimal* when

- there is a minimal number of terms,
- zeros of denominators correspond to solutions of Landau equations
- the nature of the singularities is not badly overestimated.

# Do not buy a product if

- results are not compact
- spurious singularities are induced
- around threshold (normal, pseudo, anomalous) behavior is not treated or not even understood
- extension to complex poles is not built-in

- results stop (well) below 7 legs
- tensor integrals are treated differently
- infrared and collinear behavior is not explicitly included
- terms $\mathcal{O}\left(\epsilon^k\right)$ are difficult to obtain

Part II

# Smoothness and numerical evaluation

# Prolegomena

## all one-loop

For all one-loop multi-leg diagrams we have developed computational techniques based on BST relations

## Example

We recall the definition of Bernstein - Sato polynomials: if $V(x)$ is a polynomial in several variables then

- there is a non-zero polynomial $b(\mu)$ and
- a differential operator $\mathcal{P}(\mu)$ with polynomial coefficients

such that

$$\mathcal{P}(\mu)\, V^{\mu+1}(x) = b(\mu)\, V^{\mu}(x).$$

# BST I

## Definition

The Bernstein-Sato polynomial is the monic polynomial of smallest degree amongst such $b(\mu)$. If $V(x)$ is a non-negative polynomial then $V^{\mu}(x)$, initially defined for $\mu$ with non-negative real part, can be analytically continued to a meromorphic distribution-valued function of $\mu$ by repeatedly using the functional equation

$$V^{\mu}(x) = \frac{1}{b(\mu)}\,\mathcal{P}(\mu)\,V^{\mu+1}(x)$$

# BST II

## Theorem

*For any finite set of polynomials $V_i(x)$, where $x = (x_1, \ldots, x_N)$ is a vector of Feynman parameters, there exists an identity of the following form (hereafter a BST identity):*

$$\mathcal{P}\,(x, \partial) \prod_i V_i^{\mu_i+1}(x) \;=\; B_V \prod_i V_i^{\mu_i}(x)$$

*where $\mathcal{P}$ is a polynomial of $x$ and $\partial_i = \partial/\partial x_i$; $B_V$ and all coefficients of $\mathcal{P}$ are polynomials of $\mu_i$ and of the coefficients of $V_i(x)$.*

# BST III

## Quadratic forms

If the polynomial $V$ is of second degree we have a master formula: write the polynomial as $V(x) = x^t\, H\, x + 2\, K^t\, x + L$, where $x^t = (x_1, ..., x_n)$, $H$ is an $n \times n$ matrix, $K$ is an $n$ vector. The solution to the problem of determining the polynomial $\mathcal{P}$ is as follows:

$$
\begin{aligned}
\mathcal{P} &= 1 - \frac{(x - X_v)^t\, \partial_x}{2\,(\mu + 1)}, \qquad B_v = L - K^t\, H^{-1}\, K\,, \\
X_v &= -H^{-1}\, K\,.
\end{aligned}
$$

# BST IV

## Application

$$
\begin{aligned}
V^{\mu}(x) &= \frac{1}{B_v}\left[1-\frac{(x-X_v)^t\,\partial_x}{2\,(\mu+1)}\right] V^{\mu+1}(x),\\
V^{-1}(x) &= \frac{1}{B_v}\left[1-\frac{1}{2}\,(x-X_v)^t\,\partial_x\,\ln V(x)\right].
\end{aligned}
$$

# B of BST and Landau singularities

## One-loop $N$-point scalar function

$$G_N = \frac{\mu^\epsilon}{i\,\pi^2} \int d^n q \prod_{i=1}^{N} \left[ (q + k_{i-1})^2 + m_i^2 - i\,0 \right]^{-1},$$

$$k_0 = 0, \quad k_i = p_1 + \ldots + p_i$$

## Landau equations

$$\forall\, i \qquad \left( (q + k_i)^2 + m_i^2 \right)\, \alpha_i = 0,$$

$$\sum_{i=1}^{N} (q + k_i)^\mu\, \alpha_i = 0$$

# Equivalence I

## Step I

$$G_N = \left(\frac{\mu^2}{\pi}\right)^{\epsilon} \Gamma\left(N - 2 + \frac{\epsilon}{2}\right) \times \prod_{i=1}^{N-1} \int_0^{x_{i-1}} dx_i \, V^{2-N-\epsilon/2}$$

## Step II

$$V = x^t H x + 2 K^t x + L = y^t M y, \quad y^t = (x^t, 1),$$

$$M = \begin{pmatrix} H & K \\ K^t & L \end{pmatrix}$$

# Equivalence II

**Theorem**

*One can show that*

$$B_N = \frac{\det(M)}{\det(H)},$$

- *The necessary condition for the leading Landau singularity of $G_N$ is $\det(M) = 0$.*

# More on LE

- It is easily seen that $B_N = 0$ induces a pinch on the integration contour at $(X_1 \ldots X_{N-1})$.
- At $B_N = 0$ we encounter a singularity if

$$0 < X_{N-1} < \ldots < X_1 < 1$$

otherwise the diagram is regular.

# To summarize:

- at the leading Landau singularity of $G_N$, the so-called anomalous threshold (AT), we have $B_N = 0$;

- conversely, $B_N = 0$ is the condition to have a proper solution for the system of Landau equations corresponding to $G_N$.

**Comment**

Note that AT is not directly related through unitarity to physical processes (cut diagrams).

**Question**

For a large class of applications the relevant question is whether or not AT $\in \mathcal{R} \equiv$ physical region.

# AT I

## Example

The solution to this problem, for arbitrary $N$ is technically complicated; here we illustrate a simple case, a scalar one-loop vertex with the following configuration (e.g. $H \to WW$),

$$m_i \;=\; m, p^2_{2,3} = -M^2 \leq 0, p^2_1 = -r\,.$$

For real vectors it follows that $\mathcal{R}$ is defined by

$$r \;\geq\; 4\,M^2(s\text{-channel}), \quad r \leq 0(t\text{-channel}).$$

The diagram has an AT at

$$r \;=\; r_{AT} = 4\,M^2\left(1 - \frac{M^2}{4\,m^2}\right), \quad \text{iff} \quad M^2 > 2\,m^2.$$

# AT II

The kind of singularity depends on the value of $M^2$:

## Classification

- if $0 < M^2 < 2\,m^2$ there is no AT,
- if $2\,m^2 < M^2 < 4\,m^2$ there is an unphysical AT ($0 < r_{AT} < 4\,m^2$) and, finally,
- if $M^2 > 4\,m^2$ there is a physical AT at $r_{AT} < 0$.

# BST for one-loop I

## Definition

$$G_N^n = \frac{\mu^{4-n}}{i\,\pi^2} \int d^n q \prod_{i=1}^{N} \left[(q + k_{i-1})^2 + m_i^2 - i\,0\right]^{-1}$$

$$= \left(\frac{\mu^2}{\pi}\right)^{4-n} \Gamma\left(N - \frac{n}{2}\right) \prod_{i=1}^{N-1} \int_0^{x_{i-1}} dx_i \; V^{n/2-N}$$

# BST for one-loop II

## Recursion, $N < 5$

After BST $\oplus$ integration-by-parts

$$2\, B_N\, G_N^n = (N - n + 1)\, \frac{\mu^4}{\pi^2}\, G_N^{n+2} + \sum_{l=0}^{N-1} (X_l - X_{l+1})\, G_{N-1}^n[l]$$

$$X_0 = 1, \quad X_{N-1} = 0$$

$[l]$ removes $\left[(q + k_{l-1})^2 + m_l^2\right]^{-1}$ in $G_N^n$

# BST for one-loop III

## $N = 5, 6,\ n = 4$

Delete the term $\propto G_N^{6-\epsilon}$

- $N = 5$, due to $N - 5 + \epsilon$
- $N = 6$, vanishing of $\det(H)$

## $N \geq 7$

- modify the algorithm

# BST for one-loop IV

## Definitions

- Let $N = 6 + d$, $\{k\} = \{k_1 \ldots k_d\}$ (arbitrarily chosen); let $H_{\{k\}}$ be the $5 \times 5$ matrix $H \to H_{\{k\}}$ by dropping the $d$ rows and columns $k_1 \ldots k_d$;
- let $M_{\{k\}}$ be the $6 \times 6$ matrix $M \to M_{\{k\}}$ obtained accordingly

## Define

$$X_I^{\{k\}} = \det_{I,6} M_{\{k\}} \quad B_N^{\{k\}} = \det M_{\{k\}}$$

where $\det_{i,j} M$ is the co-determinant of the element $M_{ij}$

# BST for one-loop V

**$N \geq 7,\ n = 4$**

$$G_N^n = -\frac{1}{2\,B_N^{\{k\}}}\,\Gamma\,(N-3)\sum_{l=0}^{6}\left(X_l^{\{k\}} - X_{l+1}^{\{k\}}\right) \times \prod_{i=1}^{N-2}\int_0^{x_{i-1}} dx_i\, V^{3-N}[l]$$

# BST as new algorithm

## intrinsic necessity?

Decomposition $N \to N-1$ is known

## usefulness

- is based on having a simple prescription for computing the coefficients of the decomposition
- whose meaning is easy to understand

## Connection

with leading and sub-leading Landau singularities

## Easy

to iterate, until the exponent of each polynomial reaches $-\epsilon/2$

# Public Vices and Private Virtues of Gram

## Problem

- $\text{Gram}^{-k}$ in standard reduction

## Vices

- for legs $=5$ Gram $=0$ close to the physical boundary

## Virtues

- any test is of the form

$$S = \frac{SE}{G} = 0$$

- standard reduction is unbeatable

## Comment

- for BST zero Gram is a virtue ($B_N \propto G_N^{-1}$)

# BST and smoothness

## Where to stop recursion?

- Smoothness for our integrands requires that the kernel and its first $d$ derivatives be continuous functions
- $d$ should be as large as possible
- however, in most of the cases we will be satisfied with absolute convergence, e.g. logarithmic singularities of the kernel
- this is particularly true around the zeros of $B_N$

# Useful results

## Introduce

$$\int dS_n(\{x\}) \;=\; \prod_{i=1}^{n} \int_0^{x_{i-1}} dx_i \,,$$

## Results I

$$\begin{aligned} C_0 \;&\equiv\; G_3^4 = \frac{1}{B_3}\Big[\int dS_2(\{x\}) \ln V(x_1,x_2) \\ &- \frac{1}{2}\sum_{i=0}^{2} (X_i - X_{i+1}) \int dS_1(x) \ln V[i](x) + \frac{1}{2}\Big] \end{aligned}$$

# Useful results I

$$
\begin{aligned}
D_0 \;&\equiv\; G_4^4 = -\frac{3}{4\,B_4^2}\Big[\int dS_3(\{x\})\,\ln V(x_1,x_2,x_3) \\
&-\; \frac{1}{3}\sum_{i=0}^{3}(X_i - X_{i+1})\int dS_2(\{x\})\,\ln V[i](x) + \frac{1}{9}\Big] \\
&+\; \frac{1}{2\,B_4}\sum_{i=0}^{3}(X_i - X_{i+1})\,C_0[i+1], \\
E_0 \equiv G_5^4 \;&=\; \frac{1}{4\,B_5}\sum_{i=0}^{4}(X_i - X_{i+1})\,D_0[i+1], \\
F_0 \equiv G_6^4 \;&=\; \frac{1}{6\,B_6}\sum_{i=0}^{5}(X_i - X_{i+1})\,E_0[i+1]
\end{aligned}
$$

# Useful results II

## Example

If absolute convergence is not enough for your integrator BST can do! All integrals are of the form

$$\int dS_k(\{x\}) \ln V(\{x\})$$

## $V \ln V$

$$2\,B_1 \int_0^1 dx\ \ln V(x) = \int_0^1 dx\ V(x)\left[1 - 3\ \ln V(x)\right]$$

$$-\quad X\ V(0)\left[1 - \ln V(0)\right] - (1 - X)\ V(1)\left[1 - \ln V(1)\right]$$

$$2\,B_2 \int dS_2(\{x\})\ \ln V(x_1, x_2) = -2 \int dS_2(\{x\})\, V(x_1, x_2)\left[1 - 2\ \ln V(x_1, x_2)\right]$$

$$+\quad \int_0^1 dx \sum_{i=0}^{2} (X_i - X_{i+1})\ V(\widehat{i\ i+1})\left[1 - \ln V(\widehat{i\ i+1})\right]$$

etc etc

# Same algorithm at work

## Easy to prove

tensor integrals $\rightarrow$ BST $\rightarrow$ smooth integrands

- Sorry, no space left . . . , just one example

## $N = 6,\, k \leq 5 \rightarrow N > 6,\, k \leq 2N - 7$

$$
\begin{aligned}
2\, B_6\, F^{\mu_1 \cdots \mu_k} \quad &= \quad \sum_{i=0}^{5} \Bigg[ \left( X_i - X_{i+1} \right) E^{\mu_1 \cdots \mu_k}[i] \\
&+ \quad \frac{1}{2} \sum_{j=1}^{5} \left( H^{-1}_{ij} - H^{-1}_{i+1\,j} \right) E^{\{\mu_1 \cdots \mu_{k-1}}[i]\, p_j^{\mu_k\}} \Big|_{n=6} \Bigg]
\end{aligned}
$$

# Complex poles

- If CP are present, internal $m_i^2 = \mu_i^2 - i\,\mu_i\,\gamma_i$ or external $M_i^2 = \Lambda_i^2 - i\,\Lambda_i\,\Gamma_i$, one should remember that they are lying on the second Riemann sheet; let

$$\begin{aligned} \zeta &= V(m_1^2 \dots m_i^2\,,\, M_1^2 \dots M_j^2\,;\, x_1 \dots x_n) - i\,0, \\ z &= V(\mu_1^2 \dots \mu_i^2\,,\, \Lambda_1^2 \dots \Lambda_j^2\,;\, x_1 \dots x_n) - i\,0 \end{aligned}$$

- We must replace

$$\ln\zeta \quad\to\quad \ln\zeta + 2\,i\,\pi\,\theta(-\mathrm{Re}\,z)\,\mathrm{sign}(\mathrm{Im}\,z)$$

# Non deliverable in this talk

**IR**

classification of infrared divergent one-loop virtual configurations

BST & IR, extraction of IR pole and IR-finite part

**Real**

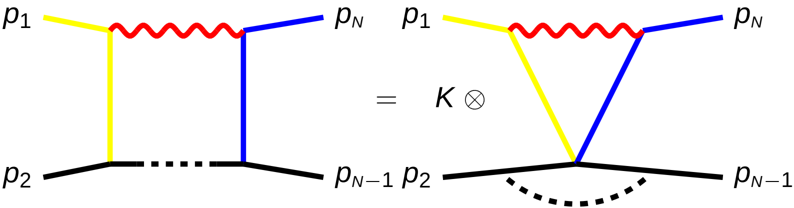
inclusion of real IR divergent diagrams in the BST scheme

**Collinear**

collinear divergent one-loop configurations (à la Sudakov), e.g.

$$C_0(s,0,0\,;\,m,m,m) \sim = \frac{1}{2}\ln^2\left(-\frac{m^2}{s}\right),$$

# However




- BST computable finite reminder

# $B_N \approx 0, B_{N-1} \approx 0$ etc

## Drawback

BST violates one of the requirements: if $B_N \approx 0$ then the nature of the singularity

$$C_0 \sim \rho_3 \ln B_3, \; D_0 \sim \rho_4 B_4^{-1/2}, \quad \text{etc}$$

is overestimated

## Solution

- if the (reduced) diagram is regular at $(X_1, \ldots, X_{N-1})$ Taylor expand
- otherwise use Mellin - Barnes to get as many terms as possible
- or

Part III

# The Uccirati Variant

# Beyond Nielsen - Goncharov

**New**

FD $\equiv$ integral representations

**Theorem**

$$\int dC_k(\{x\})\frac{1}{A}\ln\left(1+\frac{A}{B}\right) \quad or \quad \int dC_k(\{x\})\frac{1}{A}\,\mathrm{Li}_n\left(\frac{A}{B}\right)$$

where $A, B$ are multivariate polynomials in the Feynman parameters. One-(Two-) loop diagrams are always reducible to combinations of integrals of this type where the usual monomials that appear in the integral representation of Nielsen - Goncharov generalized polylogarithms are replaced by multivariate polynomials of arbitrary degree.

# Example

## General $C_0$: definitions

$$
\begin{aligned}
C_0 &= \int dS_2\, V^{-1-\epsilon/2}(x_1, x_2), \\
V(x_1, x_2) &= x^t H\, x + 2\, K^t\, x + L = Q(x_1, x_2) + B, \\
H_{ij} &= -p_i \cdot p_j, \quad L = m_1^2, \\
K_1 &= \frac{1}{2}(p_1 \cdot p_1 + m_2^2 - m_1^2), \\
K_2 &= \frac{1}{2}(P \cdot P - p_1 \cdot p_1 + m_3^2 - m_2^2),
\end{aligned}
$$

# General $C_0$: result

$$C_0 = \frac{1}{2}\sum_{i=0}^{2}(X_i - X_{i+1}) \times \int_0^1 \frac{dx}{Q(\widehat{i\; i+1})} \ln\left(1 + \frac{Q(\widehat{i\; i+1})}{B}\right)$$

$$Q(\widehat{0\;1}) = Q(1,x),\; Q(\widehat{1\;2}) = Q(x,x),\; Q(\widehat{2\;3}) = Q(x,0)$$

$$X^t = -K^t H^{-1},\; X_0 = 1,\; X_3 = 0$$

# Basics

## Define

$$\mathcal{L}_n(z) = z^n L_n(z) = z^n \int dC_n \left(\prod_{i=1}^{n} y_i\right)^{n-1} \left[1 + \prod_{j=1}^{n} y_j\, z\right]^{-n}$$

$$= \left(\frac{z}{n}\right)^n {}_{n+1}F_n\left((n)_{n+1}\,;\,(n+1)_n\,;\,-z\right),$$

$$\mathcal{L}_1(z) = -S_{0,1}(-z),$$

$$\mathcal{L}_2(z) = S_{0,1}(-z) - S_{1,1}(-z),$$

$$\mathcal{L}_3(z) = -\frac{1}{2}S_{0,1}(-z) + \frac{3}{2}S_{1,1}(-z) - S_{2,1}(-z),$$

# Problem

- For any quadratic form in $n$-variables

$$V(x) \quad = \quad (x - X)^t \; H \; (x - X) + B = Q(x) + B,$$

- we want to compute

$$I(n,\mu) \quad = \quad \int dC_n \, V^{-\mu} = \int dC_n \left[ Q(x) + B \right]^{-\mu}.$$

## Definition

- Consider the operator

$$\mathcal{P} \quad = \quad (x - X)^t \; \partial, \qquad \text{satisfying} \quad \mathcal{P}\, Q = 2\, Q$$

# Solution

## Introduce

$$J(\beta,\mu) \;=\; \int_0^1 dy\, y^{\beta-1}\, W^{-\mu}(y), \qquad W(y) = Q(x)\, y + B.$$

## Use

$$\left(\frac{1}{2}\mathcal{P} - y\,\partial_y\right) W^{-\mu} = 0 \quad\rightarrow\quad V^{-\mu} = \left(\beta + \frac{1}{2}\mathcal{P}\right) J(\beta,\mu),$$

$$I(n,\mu) \;=\; \int dC_n \left(\beta + \frac{1}{2}\mathcal{P}\right) J(\beta,\mu),$$

# Further definitions

## Define

$$
\begin{aligned}
f([x]) &= f(x_1\,,\cdots,x_n),\\
f({}_i[x]) &= f(x_1\,,\cdots,x_i=0\,,x_n),\\
f([x]_i) &= f(x_1\,,\cdots,x_i=1\,,x_n),\\
\int dC_n &= \int_0^1\prod_{i=1}^{n}dx_i\,, \qquad \int dC_{n,j}=\int_0^1\prod_{i=1,i\neq j}^{n}dx_i\,.
\end{aligned}
$$

# Results I

## Example

- For $\mu = 1$ it is convenient to choose $\beta = 1$, to obtain

$$
\begin{aligned}
I(n,1) &= \left(\frac{n}{2} - 1\right) \int dC_n \, L_1([x]) \\
&- \frac{1}{2} \sum_{i=1}^{n} \int dC_{n,i} \left\{ X_i \, L_1({}_i\,[x]) - (1 - X_i) \, L_1([x]_i) \right\}
\end{aligned}
$$

# Results II

## Example

- For $\mu = 2$ it is more convenient to write

$$V^{-2} = \left(2 + \frac{1}{2}\mathcal{P}\right)^2 J(2,2) = \left(2 + \frac{1}{2}\mathcal{P}\right)^2 L_2.$$

- integration-by-parts follows
- additional work (along the same lines) is needed to deal with surface terms ...

Part IV

# Conclusions

# Conclusions

**( 1 , 2, 3, 4, 5 )**

1. *High-Precision one-loop multi-leg calculations are doable; do it, do not introduce yet another algorithm!*
2. *(at least at the* parton *level)*
3. *It is a problem of* assembling, a huge assembling, *cumbersome and not so challenging,*
4. *at least no conceptual challenge,*
5. *unless* unstable particles *are present (but this would require another talk . . . )*

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# Have a look

back


- Numerical evaluation of loop integrals. Charalampos Anastasiou (Zurich, ETH) , Alejandro Daleo (Zurich U.) . ZU-TH-22-05, Nov 2005. 38pp. e-Print Archive: hep-ph/0511176
- A Numerical algorithm for efficient computations of one-gluon loop Feynman diagrams in QCD for a large number of external gluons. A.S. Kapoyannis, A.I. Karanikas, C.N. Ktorides (Athens U.) . Jul 2005. 26pp. Published in Comput.Phys.Commun.174:631-642,2006 e-Print Archive: hep-th/0507128
- Numerical contour integration for loop integrals. Y. Kurihara, T. Kaneko (KEK, Tsukuba) . KEK-CP-159, Mar 2005. 17pp. Published in Comput.Phys.Commun.174:530-539,2006 e-Print Archive: hep-ph/0503003
- General subtraction method for numerical calculation of one-loop QCD matrix elements. Z. Nagy (Zurich U.) , D.E. Soper (Oregon U., Dept. Math.) . 2004. 16pp. Prepared for Meeting of the European Network 'Physics at Colliders', Montpellier, France, 26-27 Sep 2004. Published in Acta Phys.Polon.B35:2557-2572,2004
- Precise predictions for Higgs production at e+ e- colliders and numerical calculation of one-loop integrals. M.M. Weber (Turin U.) . Oct 2004. 15pp. Talk given at Meeting of the European Network 'Physics at Colliders', Montpellier, France, 26-27 Sep 2004. Published in Acta Phys.Polon.B35:2655-2669,2004 e-Print Archive: hep-ph/0410166
- Numerical evaluation of one-loop diagrams near exceptional momentum configurations. W. Giele (Fermilab) , E.W.N. Glover (Durham U.) , G. Zanderighi (Fermilab) . FERMILAB-CONF-04-103-T, DCPT-04-72, IPPP-04-36, Jul 2004. 5pp. Talk given at 7th DESY Workshop on Elementary Particle Theory: Loops and Legs in Quantum Field Theory, Zinnowitz, Germany, 25-30 Apr 2004. Published in Nucl.Phys.Proc.Suppl.135:275-279,2004 Also in *Zinnowitz 2004, Loops and legs in quantum field theory* 275-279 e-Print Archive: hep-ph/0407016
- Formulae for a numerical computation of one-loop tensor integrals. Roberto Pittau (Granada U. CAFPE, Granada) . UG-FT-164-04, CAFPE-34-04, Jun 2004. 5pp. Talk given at International Conference on Linear Colliders (LCWS 04), Paris, France, 19-24 Apr 2004. e-Print Archive: hep-ph/0406105
- Loop integration results using numerical extrapolation for a non-scalar integral. E. de Doncker (Western Michigan U.) , Y. Shimizu, J. Fujimoto, F. Yuasa (KEK, Tsukuba) , K. Kaugars, L. Cucos, J. Van Voorst (Western Michigan U.) . KEK-CP-149, May 2004. 4pp. Published in Nucl.Instrum.Meth.A534:269-273,2004 e-Print Archive: hep-ph/0405098
- Recursive numerical calculus of one-loop tensor integrals. F. del Aguila, R. Pittau (CAFPE, Granada) . UG-FT-162-04, CAFPE-32-04, Apr 2004. 23pp. Erratum added online, feb/4/2005. Published in JHEP 0407:017,2004 e-Print Archive: hep-ph/0404120
- A Numerical algorithm for computing the complete set of one gluon loop diagrams in QCD on the basis of a single master expression. A.S. Kapoyannis, A.I. Karanikas, C.N. Ktorides (Athens U.) . UA-NPPS-1-04, Mar 2004. 17pp. e-Print Archive: hep-th/0403148
- General subtraction method for numerical calculation of one loop QCD matrix elements. Zoltan Nagy, Davison E. Soper (Oregon U.) . Aug 2003. 38pp. Published