LoopVerein Summary

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Outline of Part I

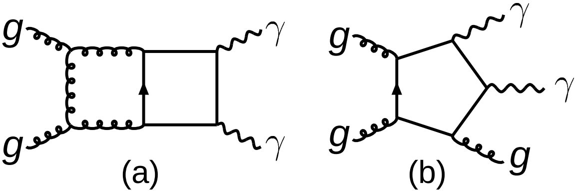
1. Goals for HO calculations
   - Challenges in NLO - NNLO
   - Complexity in N...NLO
   - Flowcharting NNLO
   - Computational challenge

Outline of Part II

2. **Presentations**
    - One loop, multi legs
    - Two loops, less legs
    - Mathematical structure of HO
    - Two loops, high energy
    - Two loops EW, QCD correction
    - EFT at NLO
    - Mixed EW + QCD
    - EW infrared evolution equations
    - Towards two loops in the MSSM
    - One loop Higgs decay
    - QCD corrections to squark loops

Part I

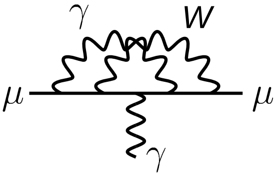
Goals and perspectives



(a)

(b)

or



NLO

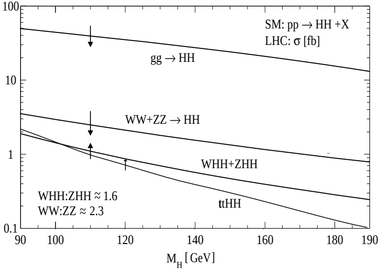
⊕

NNLO

in theory

in *practice*

⇓

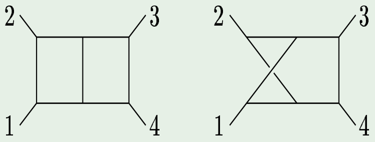


Challenge

Problem

HO perturbative QFT is a rather challenging field requiring:
clever ideas and algorithms

Example



Solutions

- importing new ideas from twistor developments into QCD

- new ideas from QCD into EW physics to confront the *practical difficulties* there,

- especially as concerns *massive Feynman diagrams*.

Complexity of an NLO(NNLO) process

(1, 2, 3)

1. *A variety of important processes will benefit from NLO(NNLO) computations*
2. *some in conjuction with resummation of large logs*
3. *Ideally, one would like a NLO(NNLO) program that mimic the experimental situation*

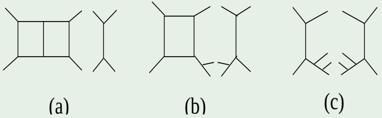
Complexity: $n!$ growth

Different amplitudes interfere

(a, b, c)

1. *virtual ⊗ tree*
2. *virtual ⊗ real*
3. *real ⊗ real*

Example



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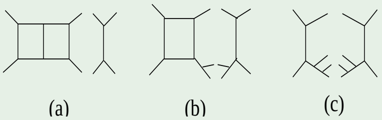
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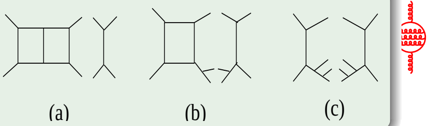

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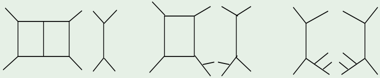

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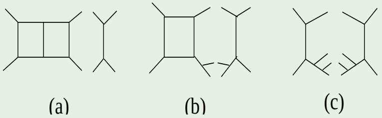

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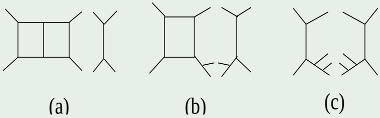
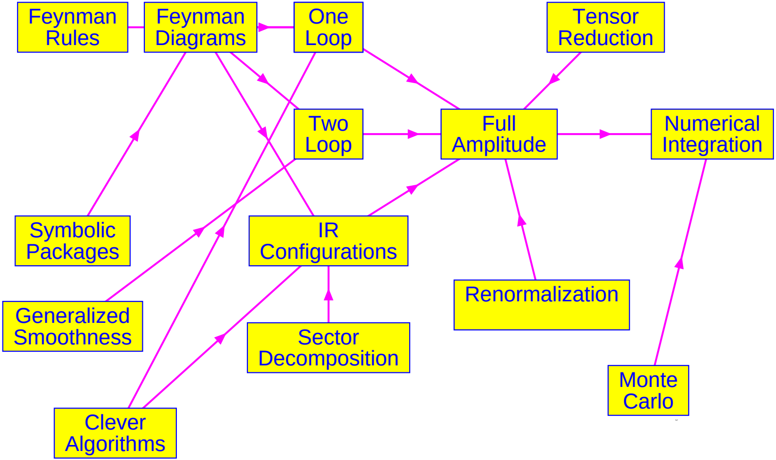


NNLO flowchart



Theorem

Denner - Dittmaier [2005]: you work in vain if you don't get to the end of the chain

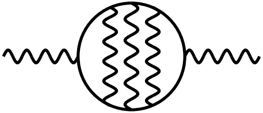
How to beat complexity?

Problems

- Memory
- Performance
- Software

Levels

- Analytic (approximations?): symbolic programs
- Numerical: stability & cancellations



3-loop 4-graviton $\approx 10^{21}$ terms / diag

Example

- parallelization
- automatization
- standardization

Part II

LoopVerein Sections

(D. Eiras, W. Bernreuther, J. Bluemlein, P. Ruiz-Femenia, S. Dittmaier, G. Weiglein)

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2. *Heavy quark vertex functions at order α_s^2: some applications*
3. *HO harmonic polylogarithms and sums*
4. *RG analysis in NRQCD for squark pair production propagator*
5. *Techniques for one-loop tensor integrals in many-particle processes*
6. *Electroweak Precision Observables in the MSSM: New results on two-loop Yukawa corrections*

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Denner et al. - Complete $\mathcal{O}(\alpha)$, $2 \to 4$

- relative corrections for $\nu_\tau \tau^+ \mu^- \bar{\nu}_\mu$ in the threshold region ↘

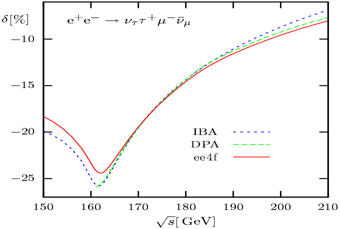
Message is

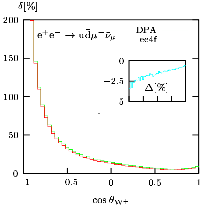
$|$ 4f - DPA $| \approx 2\%$
$\sqrt{s} < 170\,\mathrm{GeV}$

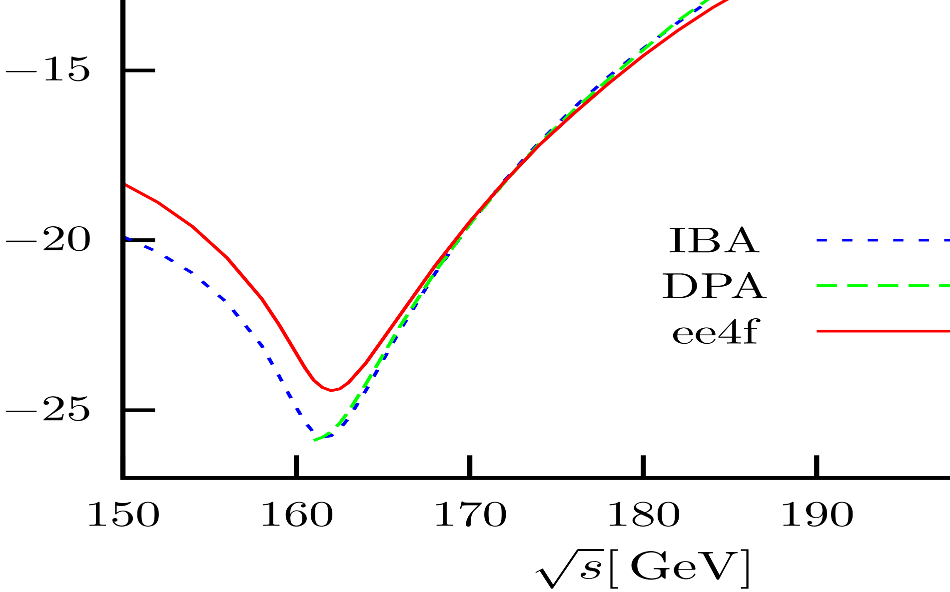
⋆ Beyond DPA

Example

Note angular dependence of full correction - DPA ↓






−15
−20
−25
150
160
170
180
190
$\sqrt{s}$[GeV]
IBA
DPA
ee4f

Dittmaier et al. - 2 → 4 NLO technically feasible

Techniques at work

They have been applied to a realistic 2 → 4 process

⋆ special award

(1- and 2- point, 3- and 4- point, 5- and 6- point, Furthemore,)

1. *integrals → stable direct calculation*
2. *integrals → PVR ⊕ expansion in determinants ⊕ analytical special cases (note PVR → BFR for those who dislike me)*
3. *integrals → stable reduction without Gram determinants*
4. *dim. reg. for IR possible, complex masses available*

Dittmaier et al. - $2 \to 4$ NLO technically feasible

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Hollik et al. - Complete $\mathcal{O}\left(\alpha^2\right)$, $\sin^2\theta_{\rm eff}$

- strong compensation of M_H dependence between $\Delta\kappa$ and M_W

results for $\Delta\kappa$ $\left([\Delta\kappa] = 10^{-4}\right)$

$\sin^2\theta_{\rm eff} =: (1 - M_W^2/M_Z^2)\,(1 + \Delta\kappa)$

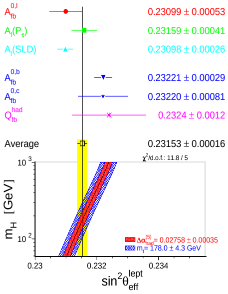
$M_H\,[GeV]$	$\mathcal{O}\,(\alpha)$	$\mathcal{O}\left(\alpha^2\right)$	prev. calc.
100	438.937	-0.633(1)	-0.63
200	419.599	-2.161(1)	-2.16
600	379.560	-5.008(1)	-5.01
1000	358.619	-4.733(1)	-4.73

$M_H\,[GeV]$	2 ferm. loops	1 ferm. loop
100	13.758	-14.391(1)
200	13.758	-15.919(1)
600	13.758	-18.766(1)
1000	13.758	-18.491(1)

$M_W = 80.426$ GeV, $M_Z = 91.1876$ GeV,
$\Gamma_Z = 2.4952$ GeV, $m_t = 178.0$ GeV,
$\Delta\alpha\left(M_Z^2\right) = 0.05907$, $\alpha_s\left(M_Z^2\right) = 0.117$,
$G_F = 1.16637 \times 10^{-5}$.

Towards full $\mathcal{O}\left(\alpha^2\right)$ for $\Delta\kappa$

$\sin^2\theta_{\rm eff}$ is a precision observable of central importance for test of the SM



Blümlein - Nielsen and harmonic polylogarithms

Observables in QED + EW in the $s \to \infty$ and heavy mass expansions often give Nielsen or harmonic polylogarithms

Weight	Number of					
	Sums	a-basic sums	Sums $\neg\{-1\}$	a-basic sums	Sums $i > 0$	a-basic sums
1	2	2	1	1	1	1
2	6	3	3	2	2	1
3	18	8	7	4	4	2
4	54	18	17	7	8	3
5	162	48	41	16	16	6
6	486	116	99	30	32	9
7	1458	312	239	68	64	18

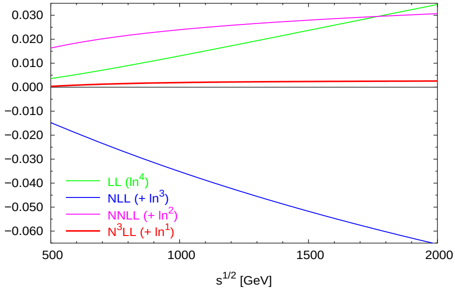
Transforming single–scale expressions into Mellin–space leads to significant simplifications, $\Longleftarrow$ level of complexity for harmonic sums

Basic functions

- 5 $\to$ 2 - loop DIS
- 8 $\to$ 3 - loop anom. dimensions

Jantzen - EW in the high energy limit

Evaluation of FD ($s \to \infty$) by expansion by regions and Mellin-Barnes representation



Example

two-loop EW corrections to, e.g. $e^+e^- \to q\bar{q} \equiv$ form factor of the vector current in massive $SU(2)$

Conclusion

- individual ln represent corrections of several % each
- the sum is quite small

Bernreuther - quark vertex functions at order α_s^2

Goal

Differential description of $e^+e^- \to \gamma, Z \to Q\bar{Q}X$ to $\mathcal{O}\left(\alpha_s^2\right)$ for $Q = b, t$, in the continuum

Example

heavy quark anomalous magnetic moments a_Q^γ:

- $a_t^\gamma(1loop) = 1.53 \times 10^{-2}$,
- $a_t^\gamma(1+2loop) = 2.00 \times 10^{-2}$
- $a_b^\gamma(1loop) = -8.4 \times 10^{-3}$,
- $a_b^\gamma(1+2loop) = -1.50 \times 10^{-2}$

previously known:

$d\sigma(4jet, LO)$, $d\sigma(3jet, NLO)$

Results

- $e^+e^- \to \gamma, Z \to Q\bar{Q}$ at $\mathcal{O}\left(\alpha_s^2\right)$
- Analytical results for $VQ\bar{Q}$ $(V = \gamma, Z)$ vector(axial) FF for arbitrary q^2, including threshold and asymptotic expansions

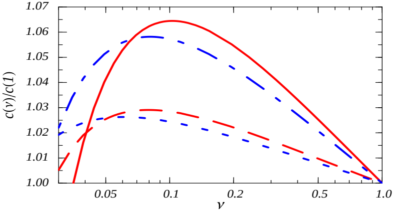
Ruiz-Femenia - RG analysis in vNRQCD

Goal

- EFT for a pair of non-relativistic squarks
- RG improved bound state energies, cross-sections at threshold

Results

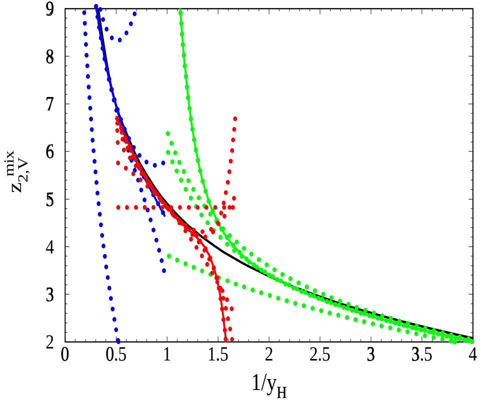
NLO-log running of the Wilson coeff. of the currents that create a pair of heavy squarks in an S- or P-wave close to threshold (velocity-NRQCD)



Example

- Wilson coeff. resum perturbative logs
- Coulomb Green function resums $(\alpha_s/v)^n$

Eiras - $\mathcal{O}(\alpha\,\alpha_s)$ on-shell fermion propagator



Exact vs. expansion

- Black exact result
- Blue large m_t
- Red $m_t \approx m_h$
- Green small m_t
- Dotted, lower order

P. Ciafaloni - Electroweak evolution equations

Goal

Evolution equations in the electroweak sector of the Standard Model analogous of DGLAP equations in QCD

Problem

Bloch-Nordsieck violation

Result

Infrared evolution equations in spontaneously broken theories

Example

Phenomenological relevance of EW evolution equations, resummation (at 1 TeV)

- $\left(\frac{\alpha_W}{\pi} \ln^2 \frac{s}{M^2}\right)^n$ 8 %
- $\left(\frac{\alpha_W}{\pi} \ln \frac{s}{M^2}\right)^n$ 14 %

Weiglein et al. - Two-loop Yukawa corrections

Goal

Two-loop Yukawa corrections for $\Delta\rho$ in the MSSM

Problem

$\mathcal{O}\left(\alpha_t^2\right), \mathcal{O}\left(\alpha_t\,\alpha_b\right), \mathcal{O}\left(\alpha_b^2\right)$ from

- SM fermions
- sfermions
- Higgs bosons
- higgsinos

Result

Corrections up to

- $\Delta M_W = +8\,\text{MeV}$
- $\Delta \sin^2\theta_{\text{eff}} = -4 \times 10^{-5}$

can be as large as SM quark loops and SUSY $\mathcal{O}\left(\alpha\alpha_s\right)$

Example

Remaining uncertanties

- $\delta M_W = (5-9)\,\text{MeV}$
- $\delta \sin^2\theta_{\text{eff}} = (5-7) \times 10^{-5}$

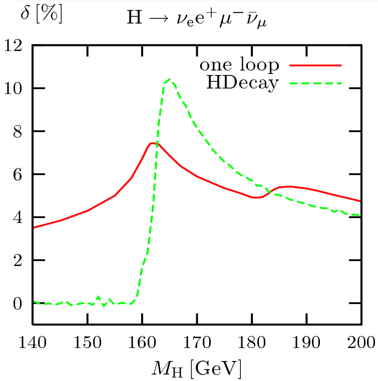
Weber - $H \rightarrow 4$ fermions

Goal

One-loop EW corrections to $H \rightarrow 4$ fermions

Result

- they amount to about $2-8\%$



Example

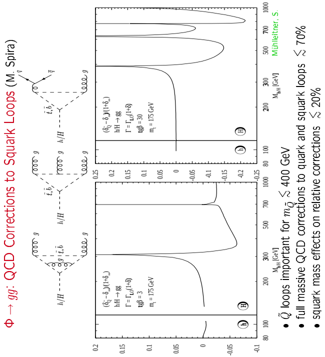
$H \rightarrow \nu_e e^+ \mu \bar{\nu}_\mu$ including a comparison with the HDecay program.

Spira - $\Phi \to gg$

Goal

Fully massive SUSY - QCD corrections

- Higgs searches at LHC + ILC belong to major endeavours



$\Phi \to gg$: QCD Corrections to Squark Loops (M. Spira)

- $\tilde Q$ loops important for $m_{\tilde Q} \lesssim 400$ GeV
- full massive QCD corrections to quark and squark loops $\lesssim 70\%$
- squark mass effects on relative corrections $\lesssim 20\%$

Example

- Fully massive QCD corrections to quark & squark loops up to 70 %
- squark mass effects on relative corrections up to 20 %