RESUMMATIONS IN QCD

- THE ANALYTIC VIEWPOINT -

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EG&R, DESY, 30/05/12

Outline

- Introducing resummations
- From factorization to resummation
- Recent developments: from theory ...
- ... to phenomenology
- Outlook

A FIRST LOOK

The virtues of large logs

- Multi-scale problems in renormalizable quantum field theories have perturbative corrections of the form $\alpha_s^n \log^k \left(Q_i^2/Q_j^2\right)$, which may spoil the reliability of the perturbative expansion. However, they carry important physical information!
  - Renormalization and factorization logs: $\alpha_s^n \log^n \left(Q^2/\mu^2\right)$
  - High-energy logs: $\alpha_s^n \log^{n-1} (s/t)$
  - Sudakov logs: $\alpha_s^n \log^{2n-1} (1-z)\ , \quad 1-z = W^2/Q^2,\ 1 - M^2/\hat{s}\ ,\ Q_\perp^2/Q^2,\ \ldots$

- Sudakov logs are universal: they originate from infrared and collinear singularities: they exponentiate and can be resummed

$$\underbrace{\frac{1}{\epsilon}}_{\text{virtual}} + \underbrace{(Q^2)^\epsilon \int_0^{m^2} \frac{dk^2}{(k^2)^{1+\epsilon}}}_{\text{real}} \qquad \Longrightarrow \qquad \ln(m^2/Q^2)$$

  - For inclusive observables: analytic resummation to high logarithmic accuracy.
  - For exclusive final states: parton shower event generators, (N)LL accuracy.

- Resummation probes the all-order structure of perturbation theory.
  - Power-suppressed corrections to QCD cross sections can be studied.
  - Links to the strong coupling regime can be established for SUSY gauge theories.

The perturbative exponent

A classic way to organize Sudakov logarithms is in terms of the Mellin (Laplace) transform of the momentum space cross section (Catani et al. 93),

$$
\begin{aligned}
d\sigma(\alpha_s, N) &= \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{k=0}^{2n} c_{nk} \log^k N + \mathcal{O}(1/N) \\
&= H(\alpha_s) \, \exp\left[\log N \, g_1(\alpha_s \log N) + g_2(\alpha_s \log N) + \alpha_s \, g_3(\alpha_s \log N) + \ldots \right] + \mathcal{O}(1/N)
\end{aligned}
$$

This displays the main features of Sudakov resummation

- Predictive: a k-loop calculation determines g_k and thus a whole tower of logarithms to all orders in perturbation theory.
- Effective:
  - the range of applicability of perturbation theory is extended (finite order: $\alpha_s \log^2 N$ small. NLL resummed: α_s small);
  - the renormalization scale dependence is naturally reduced.
- Theoretically interesting: resummation ambiguities related to the Landau pole give access to non-perturbative power-suppressed corrections.
- Well understood:
  - NLL Sudakov resummations exist for most inclusive observables at hadron colliders, NNLL and approximate N^3LL in simple cases.
  - Different `schools' (USA, Italian, SCET ...) compete, complacency is not an option, active and lively debate.

Color singlet hard scattering

A well-established formalism exists for distributions in processes that are electroweak at tree level (Gardi, Grunberg 07). For an observable r vanishing in the two-jet limit

$$\frac{d\sigma}{dr} = \delta(r)\left[1+\mathcal{O}(\alpha_s)\right] + C_R \frac{\alpha_s}{\pi}\left\{\left[-\frac{\log r}{r} + \frac{b_1 - d_1}{r}\right]_+ + \mathcal{O}(r^0)\right\} + \mathcal{O}(\alpha_s^2)$$

The Mellin (Laplace) transform, $\sigma(N) = \int_0^1 dr\,(1-r)^{N-1}\,\frac{d\sigma}{dr}$

exhibits log N singularities that can be organized in exponential form

$$\sigma\left(\alpha_s, N, Q^2\right) = H(\alpha_s)\,\mathcal{S}\left(\alpha_s, N, Q^2\right) + \mathcal{O}\left(1/N\right)$$

where the exponent of the \`Sudakov factor' is in turn a Mellin transform

$$\mathcal{S}\left(\alpha_s, N, Q^2\right) = \exp\left\{\int_0^1 \frac{dr}{r}\left[(1-r)^{N-1} - 1\right]\mathcal{E}\left(\alpha_s, r, Q^2\right)\right\}$$

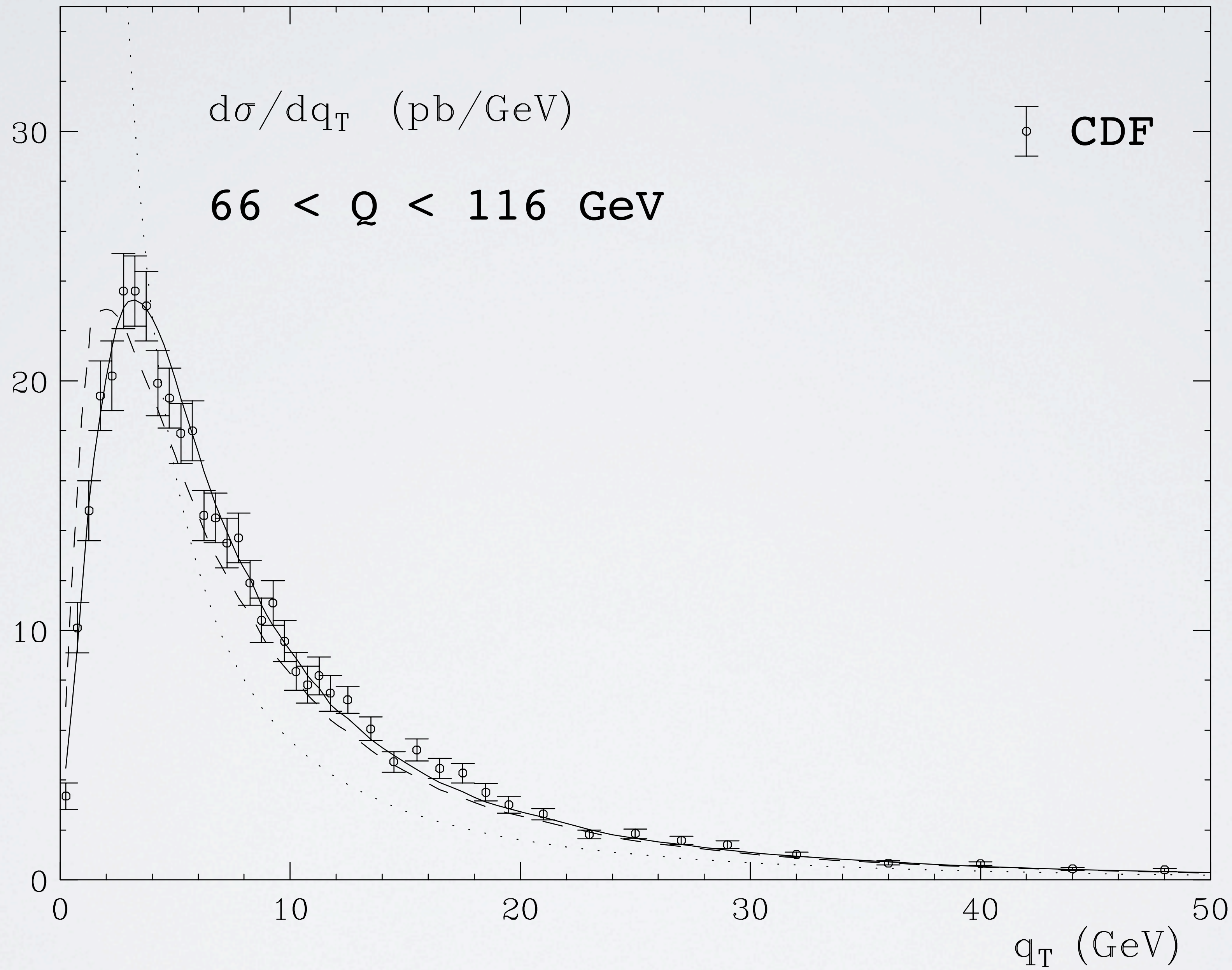
and the general form of the kernel is

$$\mathcal{E}\left(\alpha_s, r, Q^2\right) = \int_{r^2Q^2}^{rQ^2} \frac{d\xi^2}{\xi^2}\,A\left(\alpha_s(\xi^2)\right) + B\left(\alpha_s(rQ^2)\right) + D\left(\alpha_s(r^2Q^2)\right)$$

where A is the cusp anomalous dimension, and B and D have distinct physical characters.

Impact of resummation

Z-boson q_T spectrum at Tevatron (Kulesza et al. 03)



CDF data on Z production compared with QCD predictions at fixed order (dotted), with joint resummation (dashed), and with the inclusion of power corrections (solid).

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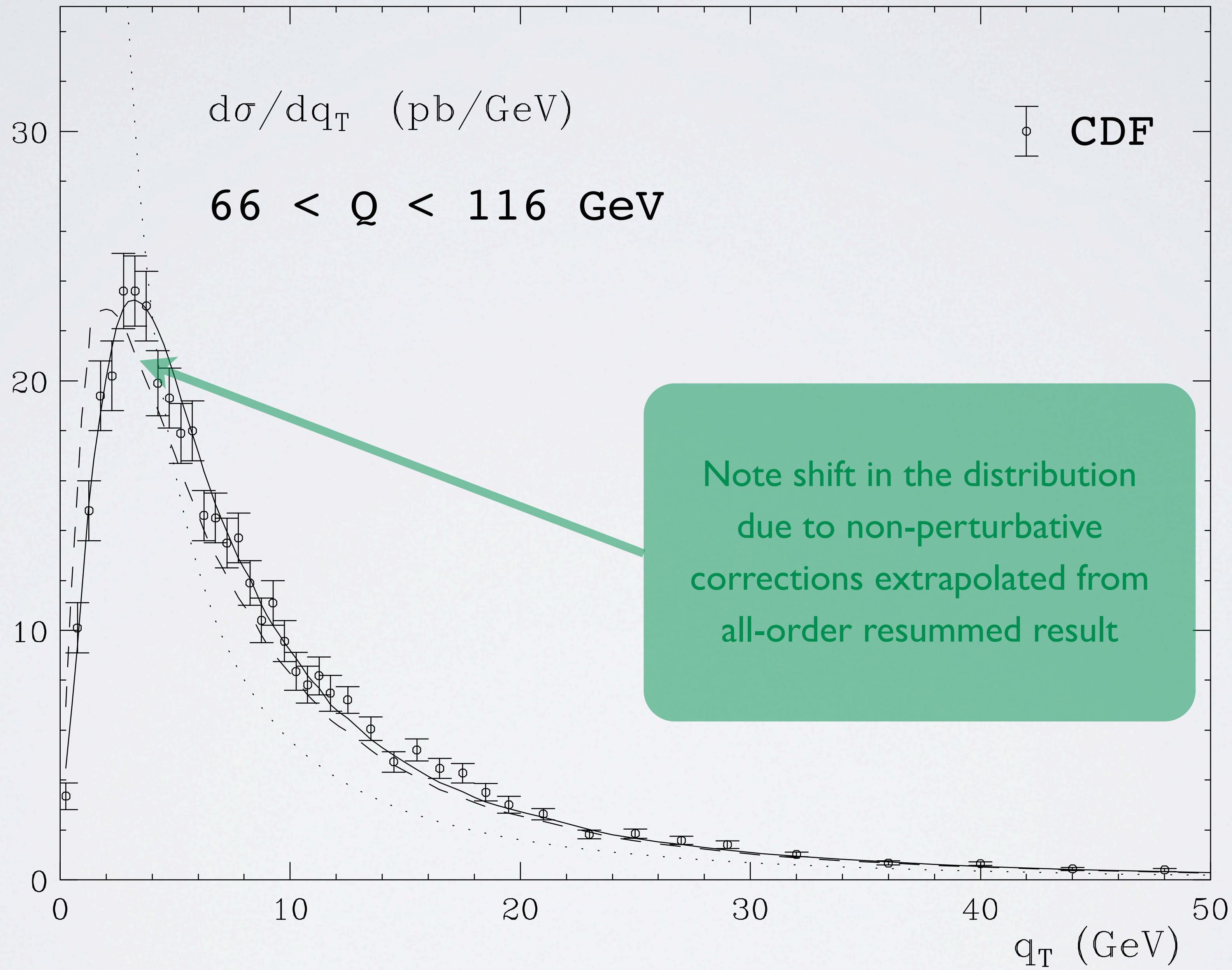

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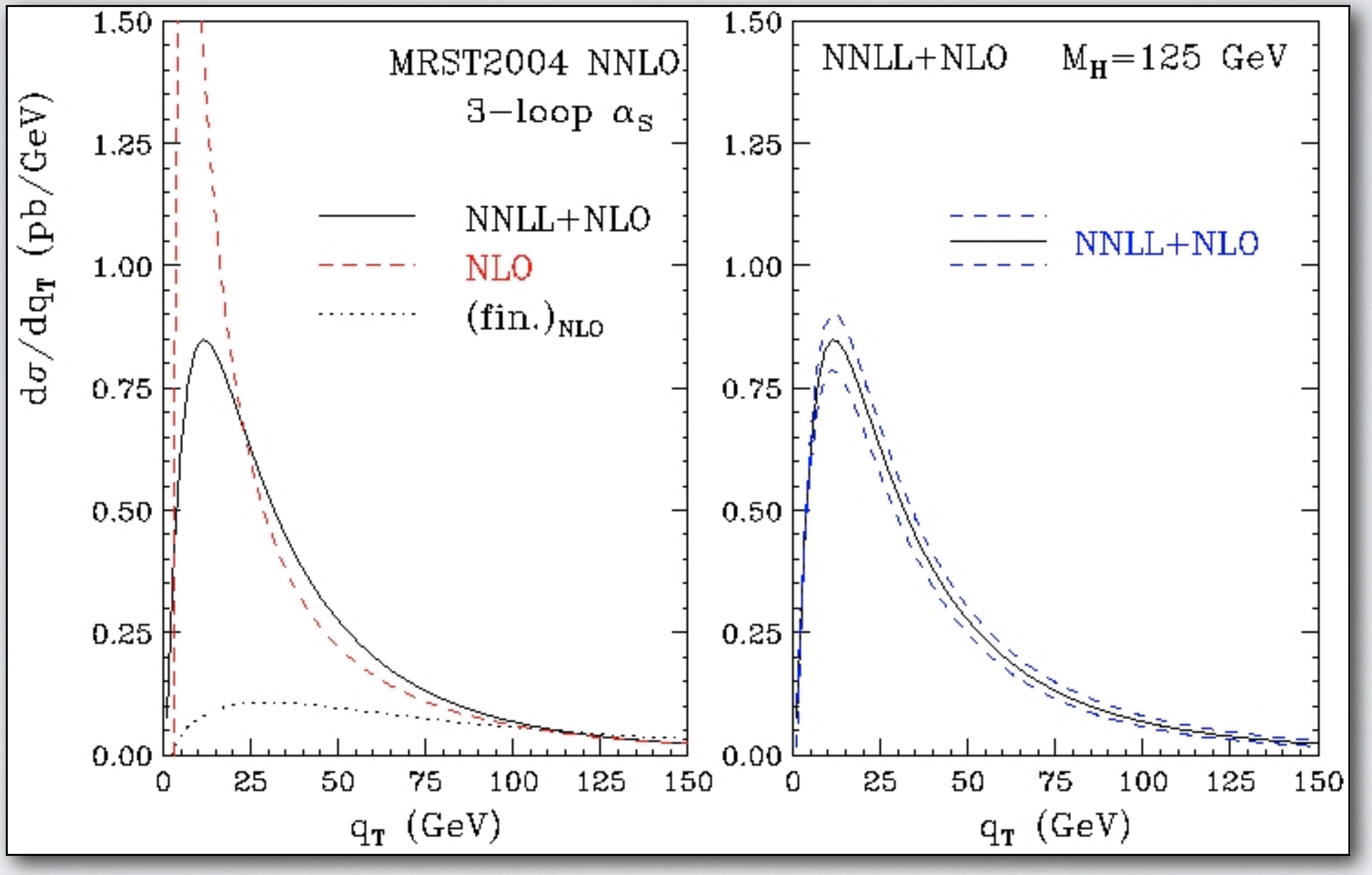
Predictions for the Higgs boson q_T spectrum at LHC (M. Grazzini, 05)



Predictions for the q_T spectrum of Higgs bosons produced via gluon fusion at the LHC, with and without resummation, and theoretical uncertainty band of the resummed prediction.

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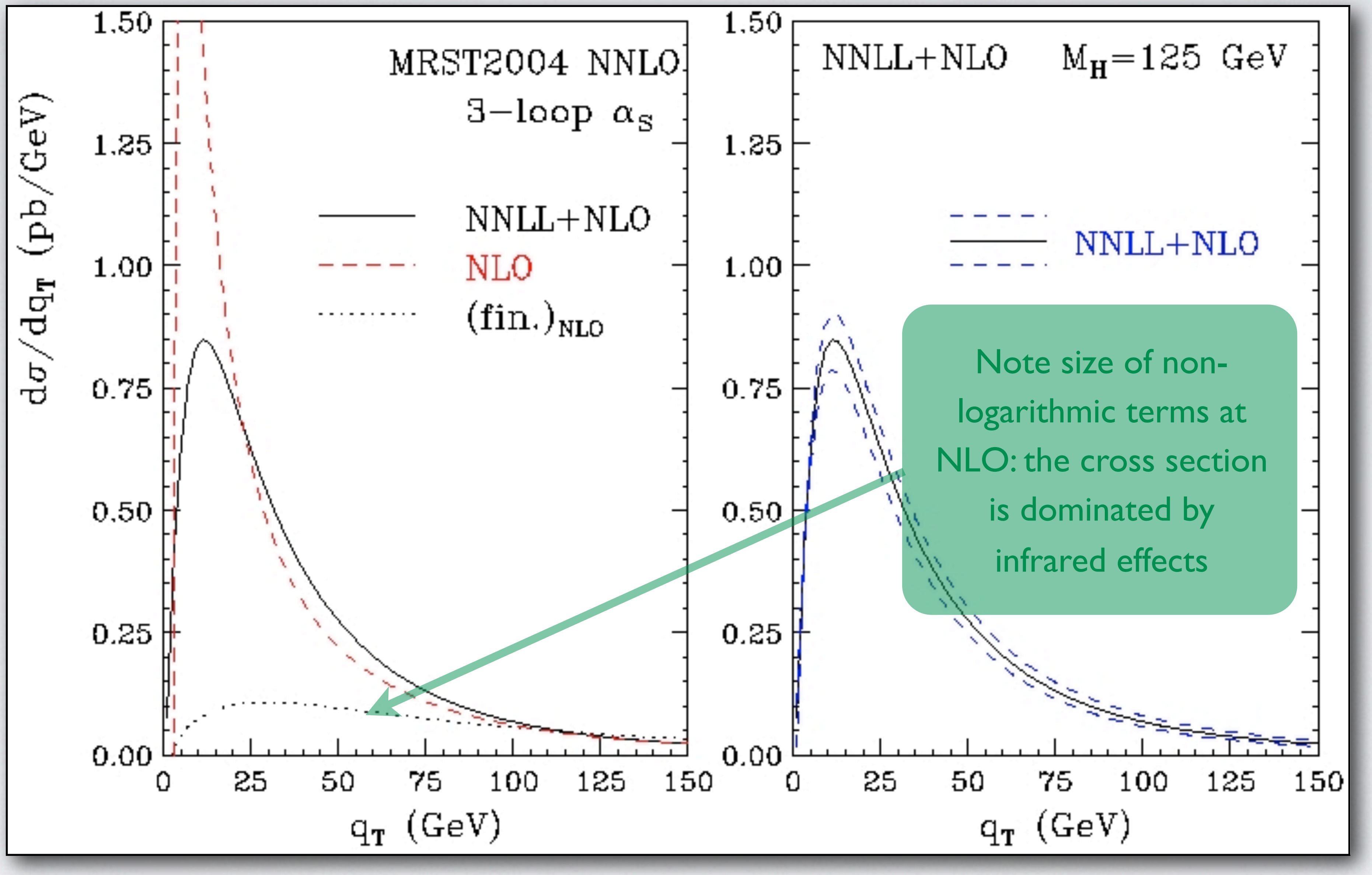

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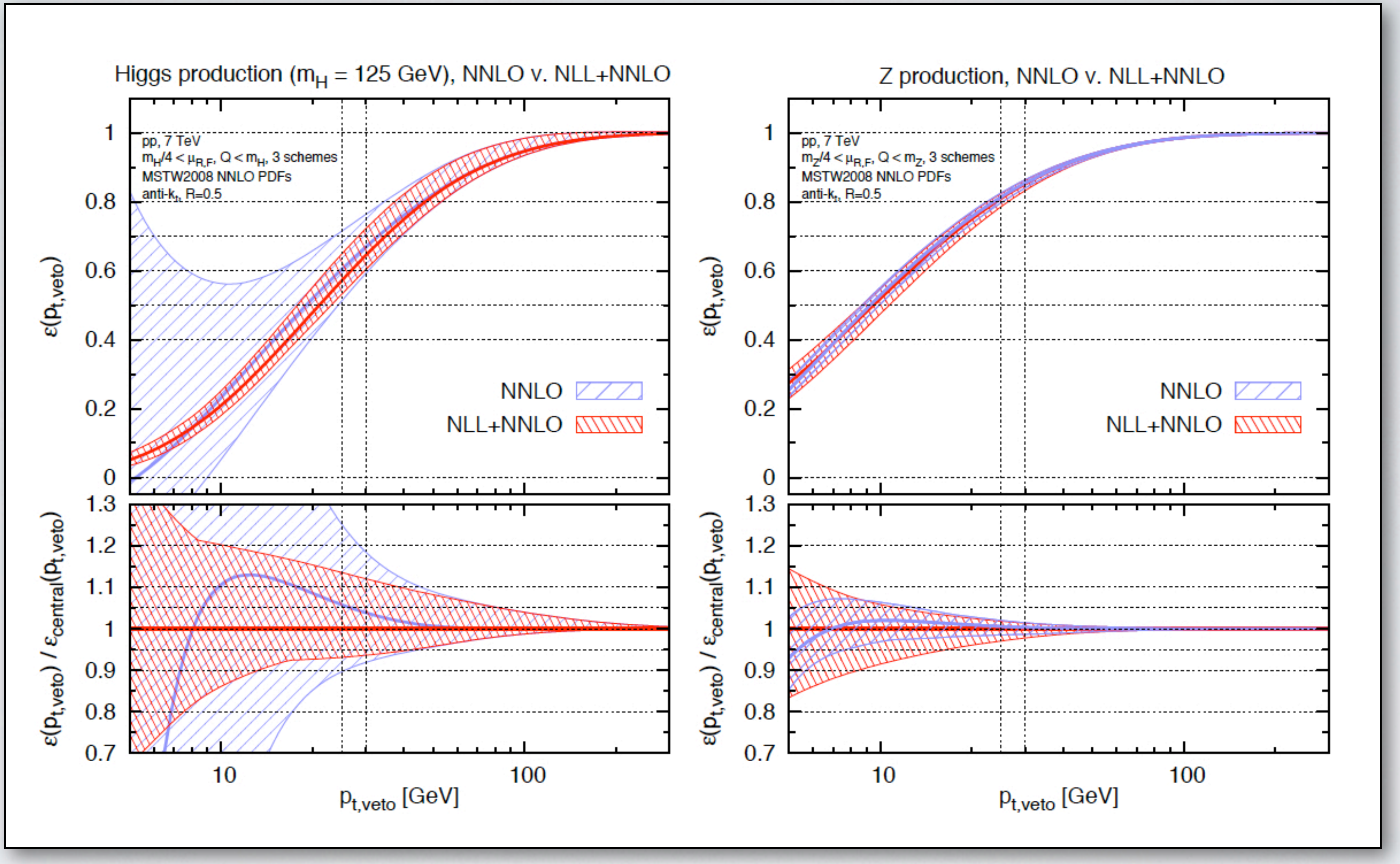
Complex observables

Jet veto efficiency in Higgs and Z production (Banfi et al., 03/12)



Comparison of NNLO fixed order results and matched resummed NLL-NNLO results for Higgs production with a jet veto (left) and Z production with a jet veto (right). For inclusion of NNLL, see also Becher and Neubert, 05/12.

Note the sharp reduction of the theoretical uncertaitnty upon resummation

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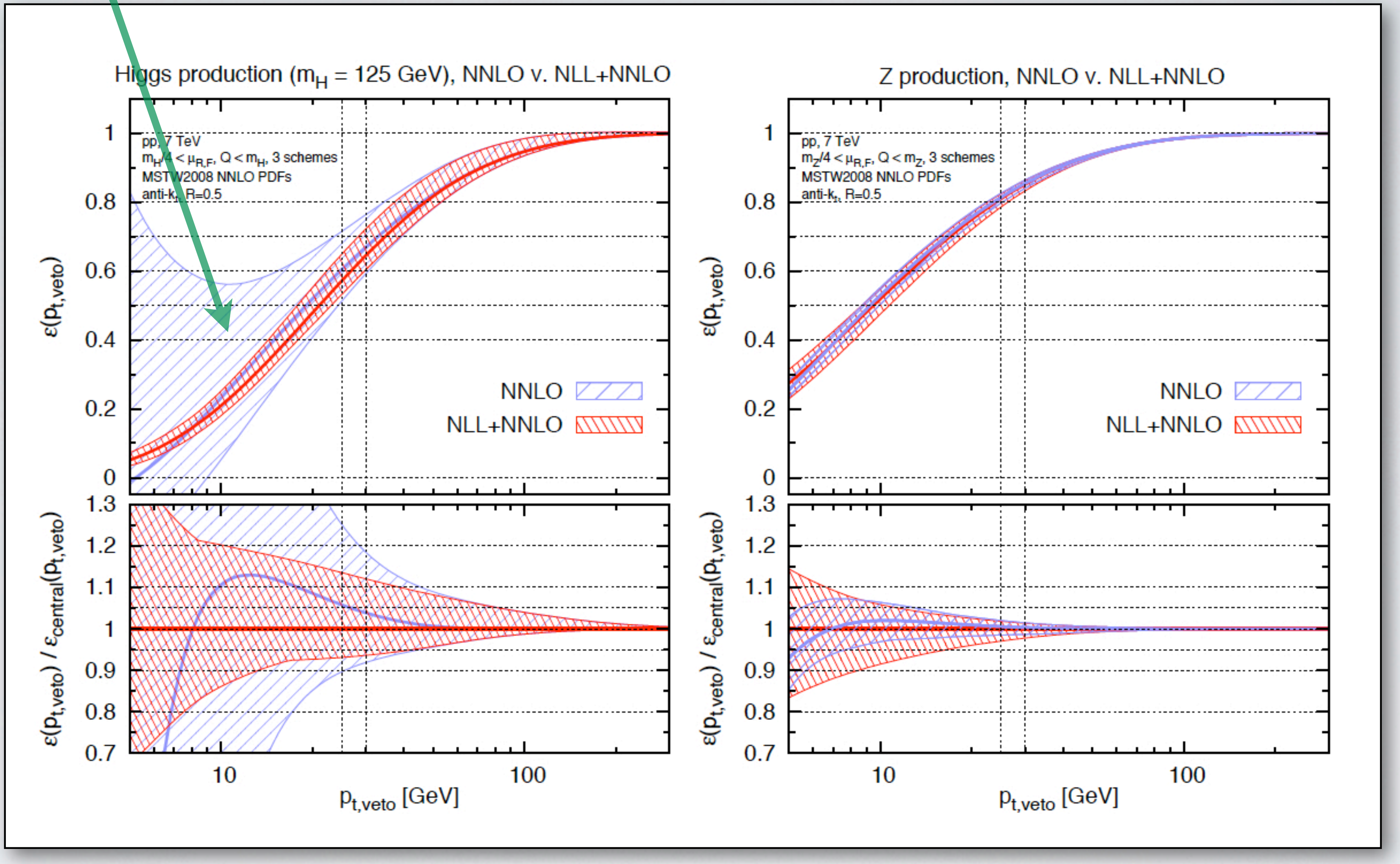

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Note the sharp reduction of the theoretical uncertaitnty upon resummation ...

... which does not always take place!

Complex observables

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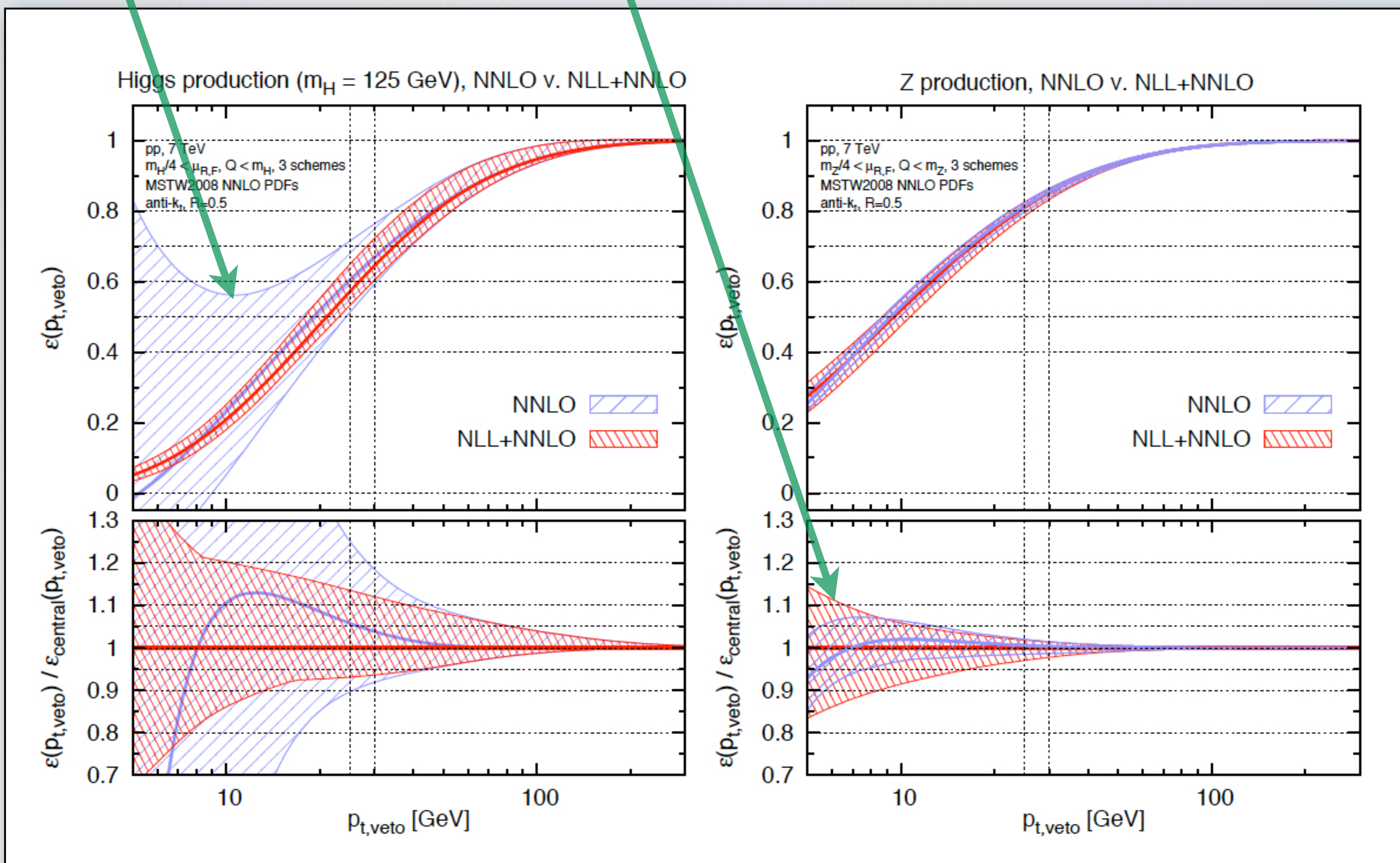

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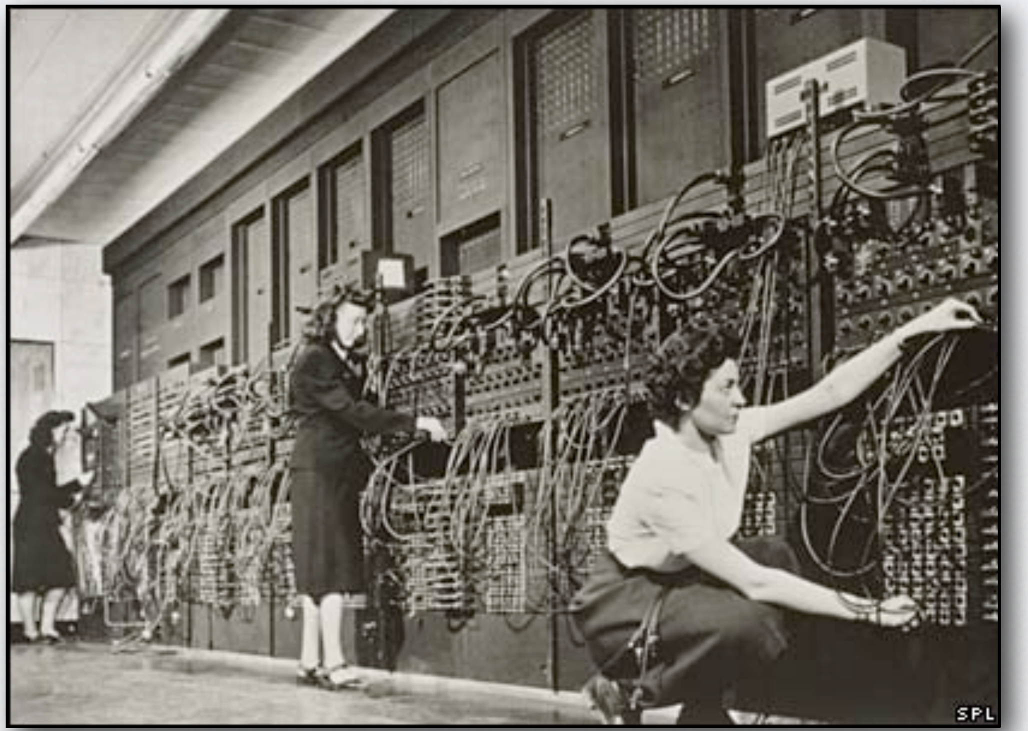
A SECOND LOOK



Factorization

All factorizations separating dynamics at different energy scales lead to resummation of logarithms of the ratio of scales.

Renormalization is a textbook example.

- Renormalization factorizes cutoff dependence.

$$G_0^{(n)}\left(p_i, \Lambda, g_0\right) = \prod_{i=1}^{n} Z_i^{1/2}\left(\Lambda/\mu, g(\mu)\right)\ G_R^{(n)}\left(p_i, \mu, g(\mu)\right)$$

- Factorization requires the introduction of an arbitrarily chosen scale μ.

- Results must be independent of the arbitrary choice of μ.

$$\frac{dG_0^{(n)}}{d\mu} = 0 \quad \to \quad \frac{d\log G_R^{(n)}}{d\log\mu} = -\sum_{i=1}^{n} \gamma_i\left(g(\mu)\right) \ .$$

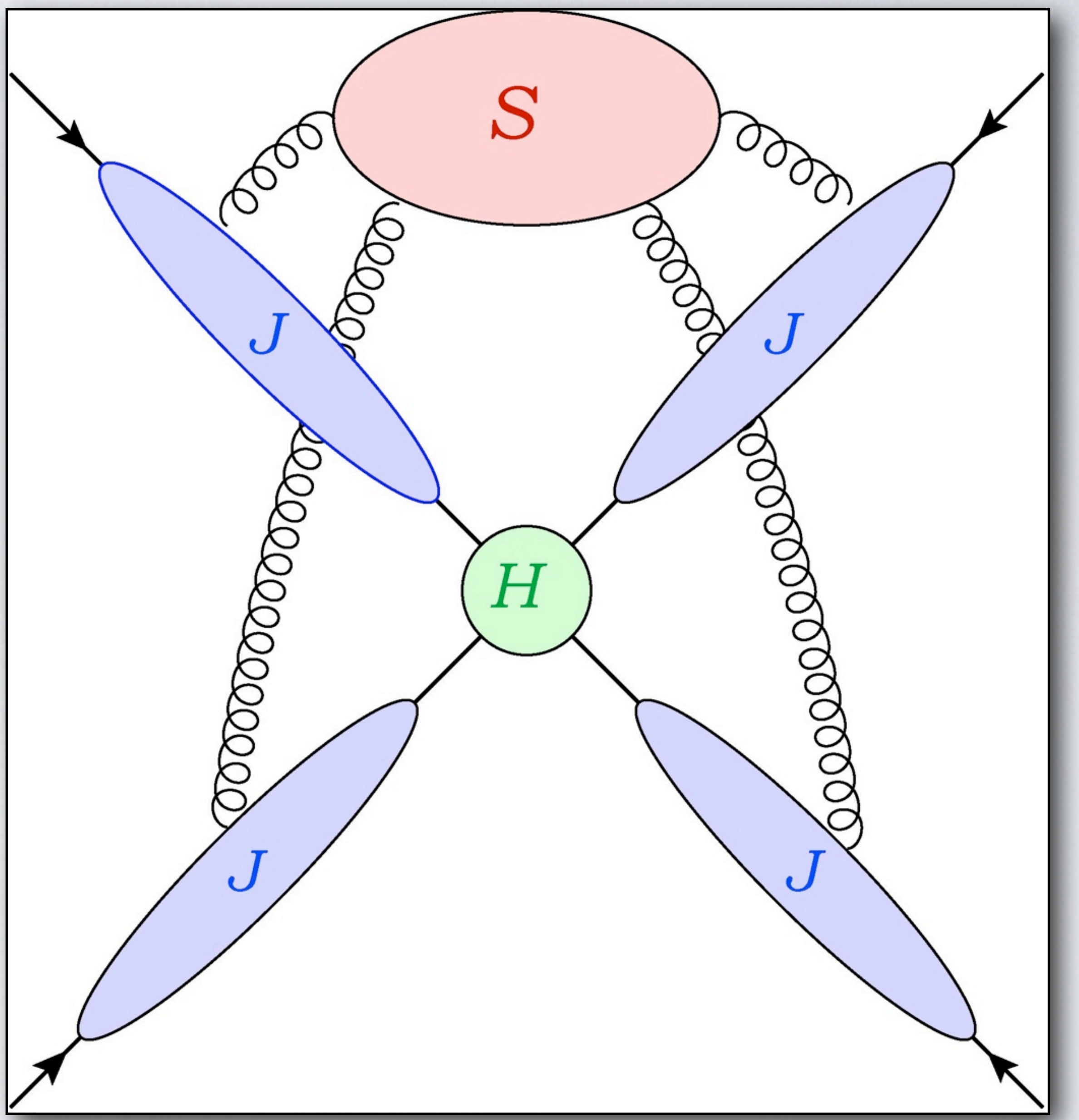
- The simple functional dependence of the factors is dictated by separation of variables.

- Proving factorization is the difficult step: it requires all-order diagrammatic analyses. Evolution equations follow automatically.

- Solving RG evolution resums logarithms of Q^2/μ^2 into $\alpha_s(\mu^2)$.

Soft-collinear factorization

- Sudakov logarithms are remainders of infrared and collinear divergences.
- Divergences arise in scattering amplitudes from leading regions in loop momentum space.
- Power-counting arguments show that soft gluons decouple from the hard subgraph.
- Ward identities decouple soft gluons from jets and restrict color transfer to the hard part.
- Jet functions J represent color singlet evolution of external hard partons.
- The soft function S is a matrix mixing the available color representations.
- In the planar limit soft exchanges are confined to wedges: S is proportional to the identity.
- Beyond the planar limit S is determined by an anomalous dimension matrix Γ_S.
- The matrix Γ_S correlates color exchange with kinematic dependence.

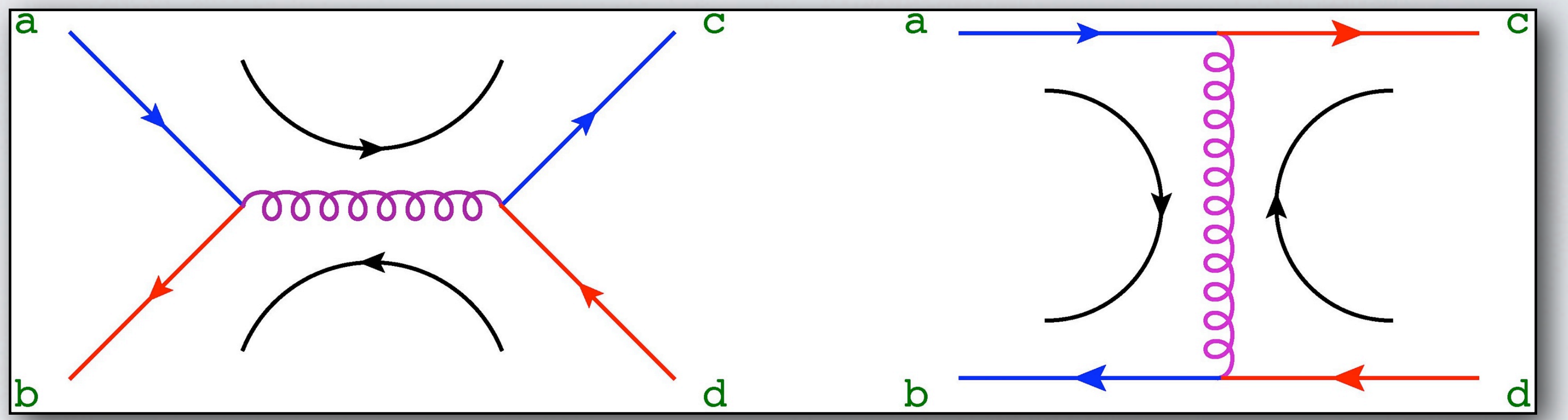


Leading integration regions in loop momentum space for soft-collinear factorization

Color flow

In order to understand the matrix structure of the soft function it is sufficient to consider the simple case of quark-antiquark scattering.

At tree level



Tree-level diagrams and color flows for quark-antiquark scattering

For this process only two color structures are possible. A basis in the space of available color tensors is

$$c^{(1)}_{abcd} = \delta_{ab}\delta_{cd}\,, \qquad c^{(2)}_{abcd} = \delta_{ac}\delta_{bd}$$

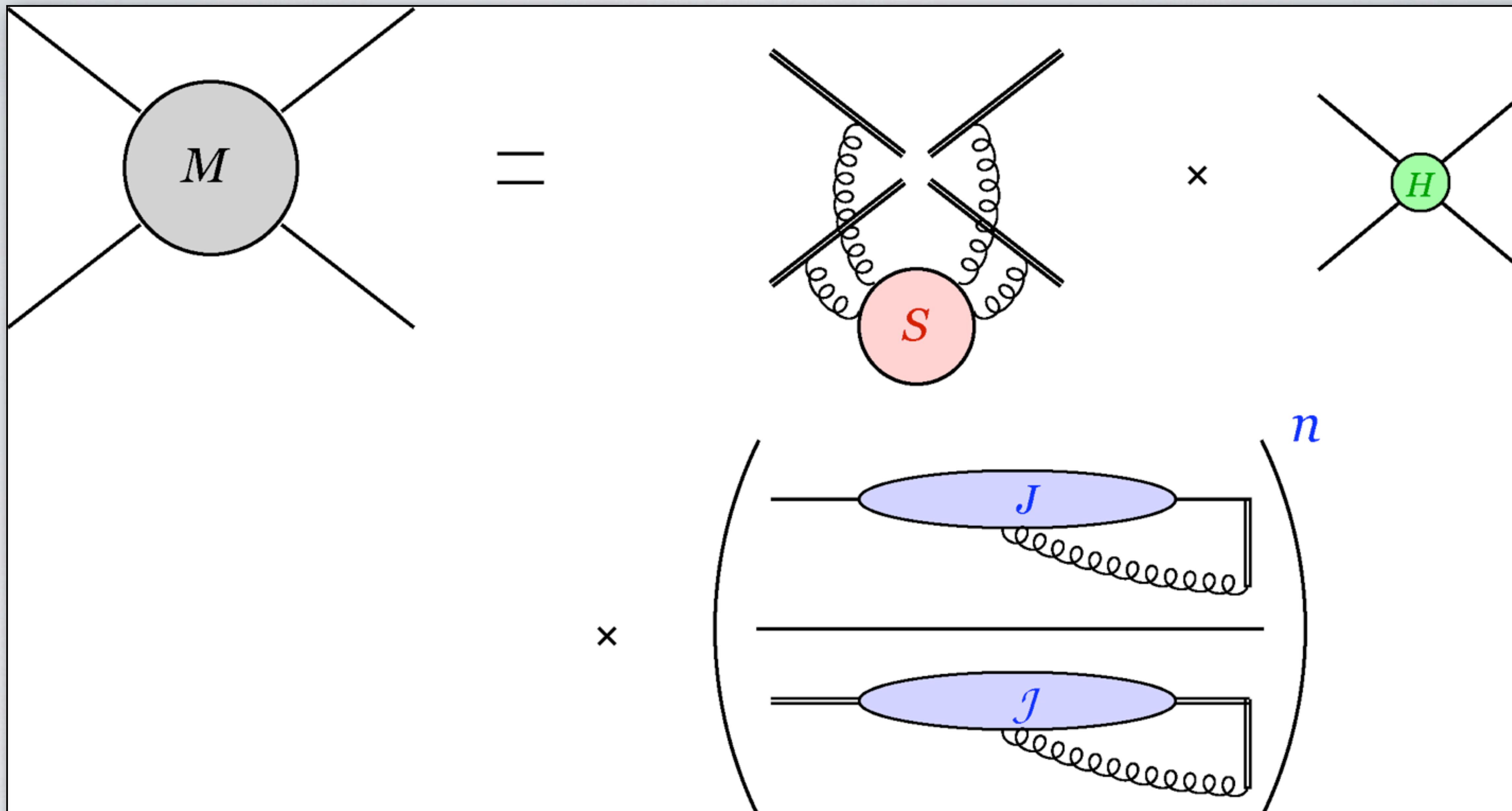
The matrix element is a vector in this space, and the Born cross section is

$$\mathcal{M}_{abcd} = \mathcal{M}_1\, c^{(1)}_{abcd} + \mathcal{M}_2\, c^{(2)}_{abcd} \longrightarrow \sum_{\text{color}} |\mathcal{M}|^2 = \sum_{J,L} \mathcal{M}_J\, \mathcal{M}_L^*\ \mathrm{tr}\left[c^{(J)}_{abcd}\left(c^{(L)}_{abcd}\right)^\dagger\right] \equiv \mathrm{Tr}\Big[HS\Big]_0$$

A virtual soft gluon will reshuffle color and mix the components of this vector

$$\text{QED}: \quad \mathcal{M}_{\text{div}} = S_{\text{div}}\mathcal{M}_{\text{Born}}\,; \qquad \text{QCD}: \quad [\mathcal{M}_{\text{div}}]_J = [S_{\text{div}}]_{JL}\,[\mathcal{M}_{\text{Born}}]_L$$

Sudakov factorization: pictorial



A pictorial representation of Sudakov factorization for fixed-angle scattering amplitudes

Soft Matrices

The soft function S is a matrix, mixing the available color tensors. It is defined by a correlator of Wilson lines.

$$(c_L)_{\{\alpha_k\}} \mathcal{S}_{LK}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) = \sum_{\{\eta_k\}} \langle 0| \prod_{i=1}^{n} \left[\Phi_{\beta_i}(\infty, 0)_{\alpha_k, \eta_k} \right] |0\rangle \, (c_K)_{\{\eta_k\}} \, ,$$

The soft function S obeys a matrix RG evolution equation

$$\mu \frac{d}{d\mu} \mathcal{S}_{IK}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) = - \mathcal{S}_{IJ}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) \Gamma^{\mathcal{S}}_{JK}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right)$$

- Γ^S is singular due to overlapping UV and collinear poles.

S is a pure counterterm. In dimensional regularization, using $\alpha_s(\mu^2 = 0, \varepsilon < 0) = 0$,

$$\mathcal{S}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) = P \exp\left[-\frac{1}{2} \int_0^{\mu^2} \frac{d\xi^2}{\xi^2} \Gamma^{\mathcal{S}}\left(\beta_i \cdot \beta_j, \alpha_s(\xi^2, \epsilon), \epsilon\right) \right] .$$

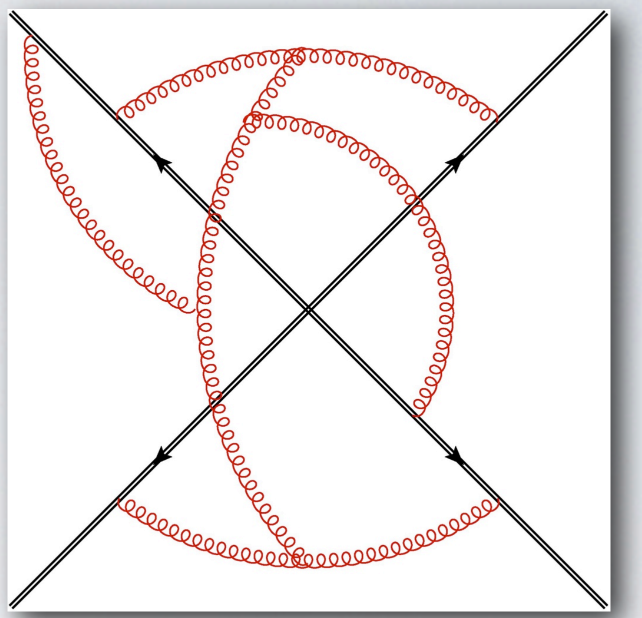
The determination of the soft anomalous dimension matrix Γ^S is the keystone of the resummation program for multiparton amplitudes and cross sections.

- It governs the interplay of color exchange with kinematics in multiparton processes.
- It is the only source of multiparton correlations for singular contributions.
- Collinear effects are `color singlet' and can be extracted from two-parton scatterings.

RECENT DEVELOPMENTS

Surprising Simplicity

- The matrix Γ_S can be computed from the UV poles of S.
- Computations can be performed directly for the exponent: the relevant diagrams are called "webs".
- Γ_S appears highly complex at high orders.
- g-loop webs directly correlate color and kinematics of up to g+1 Wilson lines.



A web contributing to the soft anomalous dimension matrix

The two-loop calculation (Aybat, Dixon, Sterman 06) leads to a surprising result: for any number of external massless partons

$$\Gamma_S^{(2)} = \frac{\kappa}{2}\,\Gamma_S^{(1)} \qquad \kappa = \left(\frac{67}{18} - \zeta(2)\right) C_A - \frac{10}{9} T_F C_F \,.$$

➡ No new kinematic dependence; no new matrix structure.

➡ κ is the two-loop coefficient of $\gamma_K(\alpha_s)$, rescaled by the appropriate quadratic Casimir,

$$\gamma_K^{(i)}(\alpha_s) = C^{(i)}\left[2\,\frac{\alpha_s}{\pi} + \kappa\left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}\left(\alpha_s^3\right)\right] .$$

The Dipole Formula

The two-loop result led to an all-order understanding. For massless partons, the soft matrix obeys a set of exact equations that correlate color exchange with kinematics.

The simplest solution to these equations is a sum over color dipoles (Becher, Neubert; Gardi, LM, 09). It leads to an ansatz for the all-order singularity structure of all multiparton fixed-angle massless scattering amplitudes: the dipole formula.

- All soft and collinear singularities can be collected in a multiplicative operator Z

$$\mathcal{M}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon\right) = Z\left(\frac{p_i}{\mu_f}, \alpha_s(\mu_f^2), \epsilon\right) \mathcal{H}\left(\frac{p_i}{\mu}, \frac{\mu_f}{\mu}, \alpha_s(\mu^2), \epsilon\right) ,$$

- Z contains both soft singularities from S, and collinear ones from the jet functions. It must satisfy its own matrix RG equation

$$\frac{d}{d\ln\mu} Z\left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon\right) = - Z\left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon\right) \Gamma\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) .$$

The matrix Γ inherits the dipole structure from the soft matrix. It reads

$$\boxed{\Gamma_{\rm dip}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) = -\frac{1}{4}\widehat{\gamma}_K\left(\alpha_s(\mu^2)\right)\sum_{j\neq i}\ln\left(\frac{-2\,p_i\cdot p_j}{\mu^2}\right)\mathbf{T}_i\cdot\mathbf{T}_j + \sum_{i=1}^{n}\gamma_{J_i}\left(\alpha_s(\mu^2)\right) .}$$

Note that all singularities are generated by integration over the scale of the coupling.

Features of the dipole formula

- All known results for IR divergences of massless gauge theory amplitudes are recovered.
- The absence of multiparton correlations implies remarkable diagrammatic cancellations.
- The color matrix structure is fixed at one loop: path-ordering is not needed.
- All divergences are determined by a handful of anomalous dimensions.
- The cusp anomalous dimension plays a very special role: a universal IR coupling.

Can this be the definitive answer for IR divergences in massless non-abelian gauge theories?

- There are precisely two sources of possible corrections.
  - Quadrupole correlations may enter starting at three loops: they must be tightly constrained functions of conformal cross ratios of parton momenta.

$$\Gamma\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) = \Gamma_{\rm dip}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) + \Delta\left(\rho_{ijkl}, \alpha_s(\mu^2)\right) \, , \qquad \rho_{ijkl} = \frac{p_i \cdot p_j \, p_k \cdot p_l}{p_i \cdot p_k \, p_j \cdot p_l}$$

  - The cusp anomalous dimension may violate Casimir scaling beyond three loops.

$$\gamma_K^{(i)}(\alpha_s) = C_i \, \widehat{\gamma}_K(\alpha_s) + \widetilde{\gamma}_K^{(i)}(\alpha_s)$$

  - The functional form of Δ is further constrained by: collinear limits, Bose symmetry and transcendentality bounds (Becher, Neubert; Dixon, Gardi, LM, 09).
  - A four-loop analysis indicates that Casimir scaling holds (Vernazza,11).

Messages from the dipole formula

- The structure of virtual corrections is a simple generalization of planar color flow:
  - ➡ Sum over all dipoles instead of just the adjacent ones!
  - For virtual corrections the result is accurate at least to NNLL for massless partons.
  - Virtual corrections must equal integrated real emission.
  - At some level this structure must match un-integrated real emission.

- A hierarchical structure emerges when taking collinear limits.
  - A colored splitting amplitude governs pairwise collinear limits

$$\mathcal{M}_n\left(p_1, p_2, p_j; \mu, \epsilon\right) \xrightarrow{1\|2} \mathbf{Sp}\left(p_1, p_2; \mu, \epsilon\right) \mathcal{M}_{n-1}\left(P, p_j; \mu, \epsilon\right)$$

  - Divergent terms of the splitting amplitude are determined by evolution

$$\mathbf{Sp}(p_1, p_2; \mu, \epsilon) = \mathbf{Sp}_{\mathcal{H}}^{(0)}(p_1, p_2; \mu, \epsilon) \exp\left[-\frac{1}{2}\int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \Gamma_{\mathbf{Sp}}(p_1, p_2; \lambda)\right]$$

  - The splitting anomalous dimension is dictated by the dipole formula

$$\Gamma_{\mathbf{Sp}}(p_1, p_2; \lambda) = -\frac{1}{2}\widehat{\gamma}_K\left(\alpha_s(\lambda^2)\right)\left[\ln\left(\frac{2\,|p_1 \cdot p_2|\;\mathrm{e}^{-\mathrm{i}\pi\lambda_{12}}}{\lambda^2}\right)\mathbf{T}_1\cdot\mathbf{T}_2 - \mathbf{T}_1\cdot(\mathbf{T}_1+\mathbf{T}_2)\ln z \right.$$
$$\left. - \mathbf{T}_2\cdot(\mathbf{T}_1+\mathbf{T}_2)\ln(1-z)\right] + \gamma_{J_1}\left(\alpha_s(\lambda^2)\right) + \gamma_{J_2}\left(\alpha_s(\lambda^2)\right) - \gamma_{J_P}\left(\alpha_s(\lambda^2)\right)$$

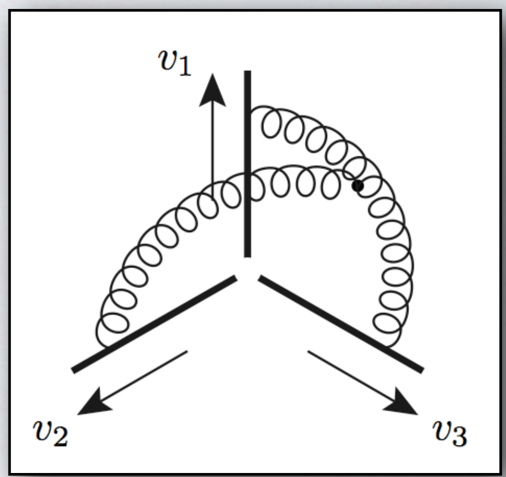
  - Beware! Full universality breaks down for space-like splitting (Catani et al., 11)

Massive particles

- The striking simplicity of the massless result does not carry over to massive partons.
  - The g-loop exponent will generally involve (g+1)-parton correlations.
  - An analytic calculation at two loops was carried out (Becher, Neubert; Ferroglia et al.; Mitov et al.; Kidonakis, 09) with interesting results.

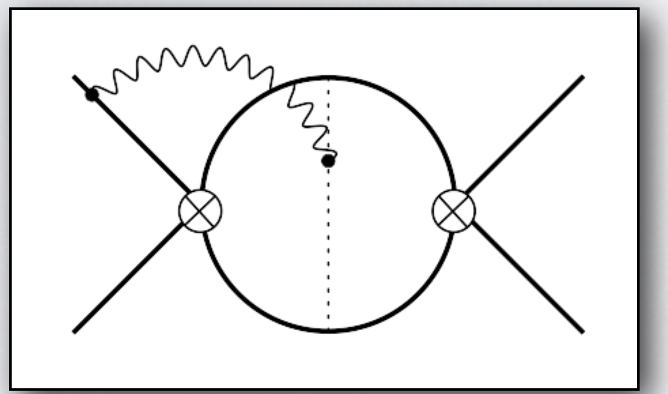
$$\Gamma\left(\frac{p_i}{\mu}, M_i, \alpha_s(\mu^2)\right) = \Gamma_{\rm dip}\left(\frac{p_i}{\mu}, M_i, \alpha_s(\mu^2)\right) + {\rm i}\sum_{i,j,k} f_{abc}\, \mathbf{T}_i^a\, \mathbf{T}_j^b\, \mathbf{T}_k^c\, F_1\left(\beta_{ij}, \beta_{jk}, \beta_{ik}\right) + \ldots$$

$$F_1^{(2)}\left(\beta_{ij}, \beta_{jk}, \beta_{ik}\right) = \frac{4}{3}\sum_{i,j,k} \epsilon_{ijk}\; g\left(\beta_{ij}\right) \beta_{ki} \coth \beta_{ki}$$



Three-parton correlations at two loops

  - The result still displays unexpected structure and simplicity: note the factorized dependence on cusp angles.



Soft and Coulomb gluons at two loops

- Another class of singularities of massive amplitudes is understood and resummed.
  - When massive particles are pair-produced near threshold, Coulomb singularities $\log^p \beta/\beta^k$ arise.
  - They can be organized using effective field theory (NRQCD).
  - A novel factorization theorem has been derived and applied to heavy colored particle production (Beneke et al., 09).

$$\hat{\sigma}_{pp'}\left(\hat{s}, \mu\right) = \sum_{KL} H_{KL}\left(M, \mu\right) \int dw \sum_{R_\alpha} J_{R_\alpha}\left(E - \frac{w}{2}\right)\, S_{KL}^{R_\alpha}\left(w, \mu\right)$$

The dipole formula at high energy

- Introducing `Mandelstam' color operators, and using color and momentum conservation

$$\begin{aligned} \mathbf{T}_s &= \mathbf{T}_1 + \mathbf{T}_2 = -(\mathbf{T}_3 + \mathbf{T}_4)\,, \\ \mathbf{T}_t &= \mathbf{T}_1 + \mathbf{T}_3 = -(\mathbf{T}_2 + \mathbf{T}_4)\,, \\ \mathbf{T}_u &= \mathbf{T}_1 + \mathbf{T}_4 = -(\mathbf{T}_2 + \mathbf{T}_3) \end{aligned}$$

$$s + t + u = 0$$

$$\mathbf{T}_s^2 + \mathbf{T}_t^2 + \mathbf{T}_u^2 = \sum_{i=1}^{4} C_i$$

it is easy to see that the infrared dipole operator Z factorizes in the high-energy limit

$$Z\left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon\right) = \widetilde{Z}\left(\frac{s}{t}, \alpha_s(\mu^2), \epsilon\right) Z_{\mathbf{1}}\left(\frac{t}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)$$

- The operator Z_1 is s-independent and proportional to the unit matrix in color space.
- Color dependence and s dependence are collected in the factor

$$\widetilde{Z}\left(\frac{s}{t}, \alpha_s(\mu^2), \epsilon\right) = \exp\left\{K\Big(\alpha_s(\mu^2), \epsilon\Big)\left[\ln\left(\frac{s}{-t}\right)\mathbf{T}_t^2 + i\pi\,\mathbf{T}_s^2\right]\right\},$$

where the coupling dependence is (once again!) completely determined by the cusp anomalous dimension and by the β function, through the function (Korchemsky 94-96)

$$K\Big(\alpha_s(\mu^2), \epsilon\Big) \equiv -\frac{1}{4}\int_0^{\mu^2}\frac{d\lambda^2}{\lambda^2}\,\widehat{\gamma}_K\left(\alpha_s(\lambda^2, \epsilon)\right)$$

- The simple structure of the high-energy operator governs Reggeization and its breaking.

Reggeization of leading logarithms

- At leading logarithmic accuracy, the (imaginary) s-channel contribution can be dropped, and the dipole operator becomes diagonal in a t-channel basis.

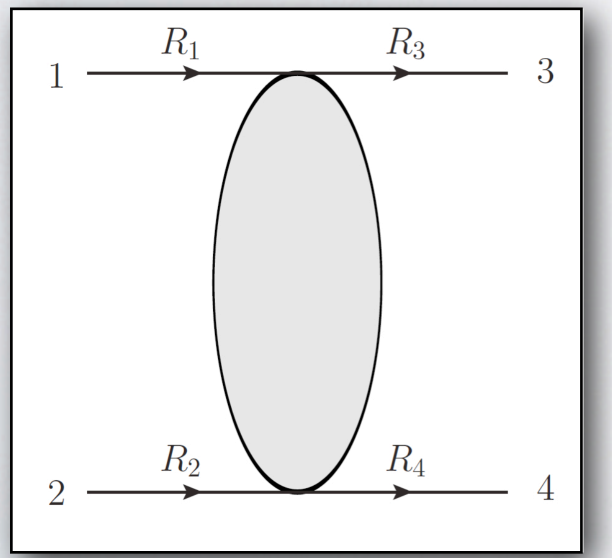
$$\mathcal{M}\left(\frac{p_i}{\mu},\alpha_s(\mu^2),\epsilon\right) = \exp\left\{K\big(\alpha_s(\mu^2),\epsilon\big)\,\ln\left(\frac{s}{-t}\right)\,\mathbf{T}_t^2\right\}\,Z_{\mathbf{1}}\,\mathcal{H}\left(\frac{p_i}{\mu},\alpha_s(\mu^2),\epsilon\right)$$

- If, at LO and at leading power in t/s, the scattering is dominated by t-channel exchange, then the hard function is an eigenstate of the color operator $\mathbf{T}_t^2$

$$\mathbf{T}_t^2\,\mathcal{H}^{gg\to gg} \xrightarrow{|t/s|\to 0} C_t\,\mathcal{H}_t^{gg\to gg}$$

- Leading-logarithmic Reggeization for arbitrary t-channel color representations follows

$$\mathcal{M}^{gg\to gg} = \left(\frac{s}{-t}\right)^{C_A\,K\left(\alpha_s(\mu^2),\epsilon\right)} Z_{\mathbf{1}}\,\mathcal{H}_t^{gg\to gg}$$



Scattering for generic color exchange

- The LL Regge trajectory is universal and obeys Casimir scaling.
- Scattering of arbitrary color representations can be analyzed Example: let 1 and 2 be antiquarks, 4 a gluon and 3 a sextet; use

$$\overline{\mathbf{3}}\otimes\mathbf{6} = \mathbf{3}\oplus\mathbf{15} \qquad\qquad \overline{\mathbf{3}}\otimes\mathbf{8}_a = \overline{\mathbf{3}}\oplus\mathbf{6}\oplus\overline{\mathbf{15}}$$

LL Reggeization of the **3** and **15** t-channel exchanges follows.

Beyond leading logarithms

- The high-energy infrared operator can be systematically expanded beyond LL, using the Baker-Campbell-Hausdorff formula. At NLL one finds a series of commutators

$$\widetilde{Z}\left(\frac{s}{t},\alpha_s,\epsilon\right)\Big|_{\rm NLL} = \left(\frac{s}{-t}\right)^{K(\alpha_s,\epsilon)\,\mathbf{T}_t^2}\Bigg\{1+{\rm i}\,\pi K\left(\alpha_s,\epsilon\right)\Bigg[\mathbf{T}_s^2-\frac{K\left(\alpha_s,\epsilon\right)}{2!}\ln\left(\frac{s}{-t}\right)\left[\mathbf{T}_t^2,\mathbf{T}_s^2\right] + \frac{K^2\left(\alpha_s,\epsilon\right)}{3!}\ln^2\left(\frac{s}{-t}\right)\left[\mathbf{T}_t^2,\left[\mathbf{T}_t^2,\mathbf{T}_s^2\right]\right]+\ldots\Bigg]\Bigg\}$$

- The real part of the amplitude Reggeizes also at NLL for arbitrary t-channel exchanges.
- At NNLL Reggeization generically breaks down also for the real part of the amplitude.
  - At two loops, terms that are non-logarithmic and non-diagonal in a t-channel basis arise

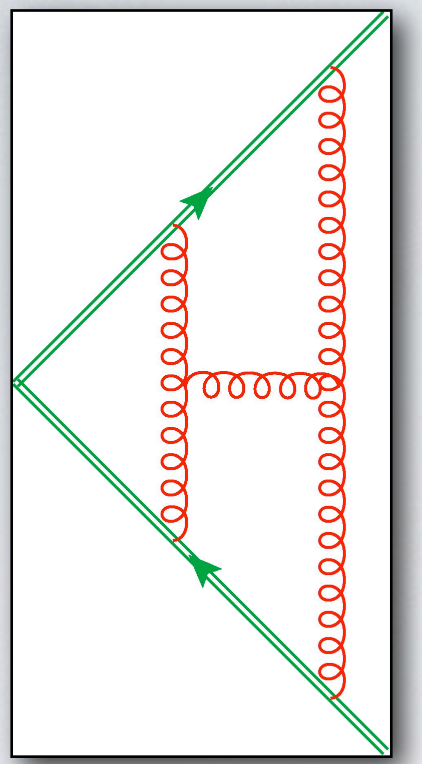
$$\mathcal{E}_0\left(\alpha_s,\epsilon\right) \equiv -\frac{1}{2}\pi^2\,K^2\left(\alpha_s,\epsilon\right)\,\left(\mathbf{T}_s^2\right)^2$$

  - At three loops, the first Reggeization-breaking logarithms of s/t arise, generated by

$$\mathcal{E}_1\left(\frac{s}{t},\alpha_s,\epsilon\right) \equiv -\frac{\pi^2}{3}\,K^3(\alpha_s,\epsilon)\,\ln\left(\frac{s}{-t}\right)\left[\mathbf{T}_s^2,\left[\mathbf{T}_t^2,\mathbf{T}_s^2\right]\right]$$

- **NOTE**
  - In the planar limit ($N_C \to \infty$) all commutators vanish and Reggeization holds also beyond NLL (as perhaps expected from string theory).
  - Possible quadrupole corrections to the dipole formula cannot come to the rescue.

Multiparton webs

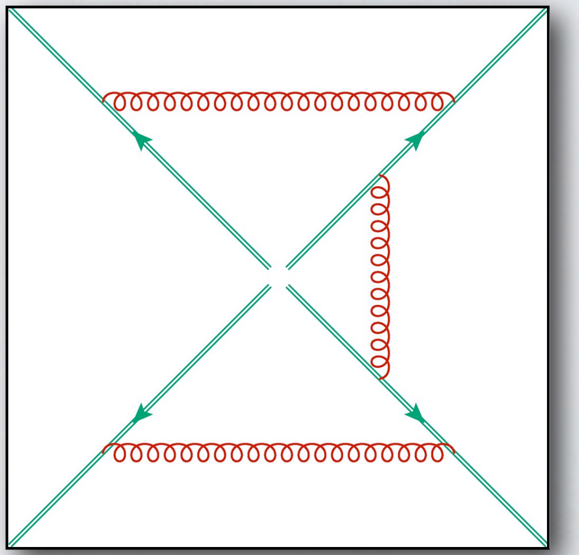


A web

- Infrared divergences of gauge scattering amplitudes exponentiate.
- The exponent can be computed directly in terms of a subset of the original diagrams with modified color factors, called `webs'. (Gatheral 83; Frenkel, Taylor 84)
- For amplitudes with two hard partons (color singlet), webs have a precise topological characterization and special properties.
  - Webs are `two-eikonal-irreducible' diagrams.
  - Webs have modified color factors that can be computed recursively.
  - Webs have no nested UV subdivergences.

- We are now understanding the structure of the multileg exponent. (Gardi et al. 10-11; Mitov, Sterman, Sung 10)

$$\mathcal{Z} \equiv \int [\mathcal{D}A_{\rm s}^{\mu}] \, {\rm e}^{{\rm i}S(A_{\rm s}^{\mu})} \left[\Phi^{(1)} \otimes \cdots \otimes \Phi^{(L)}\right] = \exp\left[\sum_{D} \tilde{C}(D) \, \mathcal{F}(D)\right] .$$



Is this a web?

  - Multiparton webs are sets of diagrams whose kinematic and color structures mix. They are not all irreducible.
  - Modified color factors are given by web mixing matrices

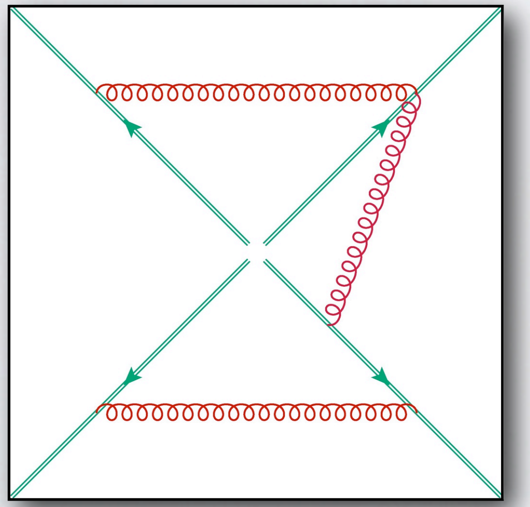
$$\widetilde{C}(D) = \sum_{D'} R(D, D') \, C(D')$$

  - All subleading poles are determined by lower-order webs.

Beyond the eikonal

- Hadronic cross sections near partonic threshold receive non-singular logarithmic corrections $\alpha_s^p \log^k(1 - z)$, or $\alpha_s^p \log^k N/N$, which may be relevant for phenomenology. Can they also be organized and resummed? (Kraemer et al.; Vogt et al.; Grunberg, ...)
  - For two-parton processes, $O(N^0)$ contributions exponentiate (Laenen, LM, 03).
  - Phenomenological evidence indicates that also `sub-eikonal' logs partly exponentiate.
  - An ansatz summarizes the resummable for Drell-Yan (and DIS) (Laenen et al., 06).

$$
\begin{aligned}
\ln\left[\widehat{\omega}(N)\right] \;=\; & \mathcal{F}_{\rm DY}\left(\alpha_s(Q^2)\right) + \int_0^1 dz\, z^{N-1}\left\{\frac{1}{1-z}\, D\left[\alpha_s\left(\frac{(1-z)^2 Q^2}{z}\right)\right]\right. \\
& \left. + 2\int_{Q^2}^{(1-z)^2Q^2/z} \frac{dq^2}{q^2}\, P_s\left[z, \alpha_s(q^2)\right]\right\}_+
\end{aligned}
$$



Is THIS a web?

- A systematic study of soft-gluon dynamics beyond the eikonal approximation has been undertaken (Laenen et al. 08, 10).
  - A class of factorizable contributions exponentiate via NE webs

$$
\mathcal{M} = \mathcal{M}_0 \exp\left[\sum_{D_{\rm eik}} \tilde{C}(D_{\rm eik})\,\mathcal{F}(D_{\rm eik}) + \sum_{D_{\rm NE}} \tilde{C}(D_{\rm NE})\,\mathcal{F}(D_{\rm NE})\right] .
$$

  - "Feynman rules" for the NE exponent, including "seagull" vertices.
  - Non-factorizable contribution can be studied using Low's theorem.

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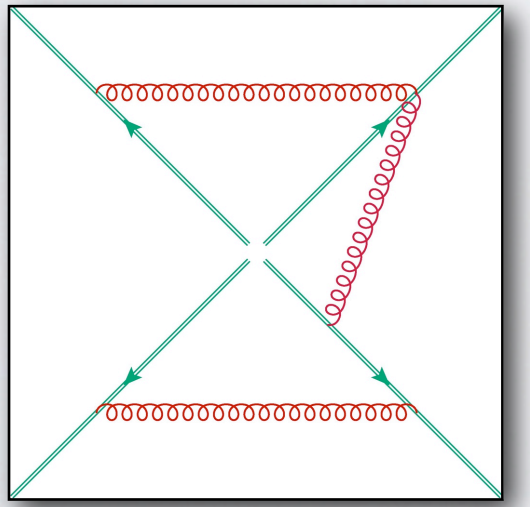
$$\ln\left[\widehat{\omega}(N)\right] = \mathcal{F}_{\rm DY}\left(\alpha_s(Q^2)\right) + \int_0^1 dz\, z^{N-1} \left\{ \frac{1}{1-z} D\left[\alpha_s\left(\frac{(1-z)^2 Q^2}{z}\right)\right] + 2\int_{Q^2}^{(1-z)^2 Q^2/z} \frac{dq^2}{q^2} P_s\left[z, \alpha_s(q^2)\right] \right\}_+$$

$$\frac{2}{1-z} \longrightarrow \frac{2\,z}{1-z}$$



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PHENOMENOLOGY

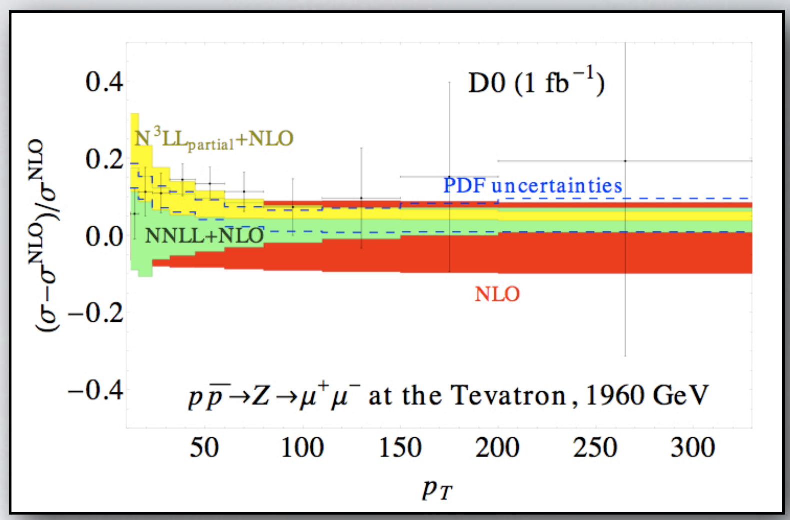
PHENOMENOLOGY

No. 28—"Three Log" Load of Sugar Pine at the Mill Pond.

Electroweak annihilation

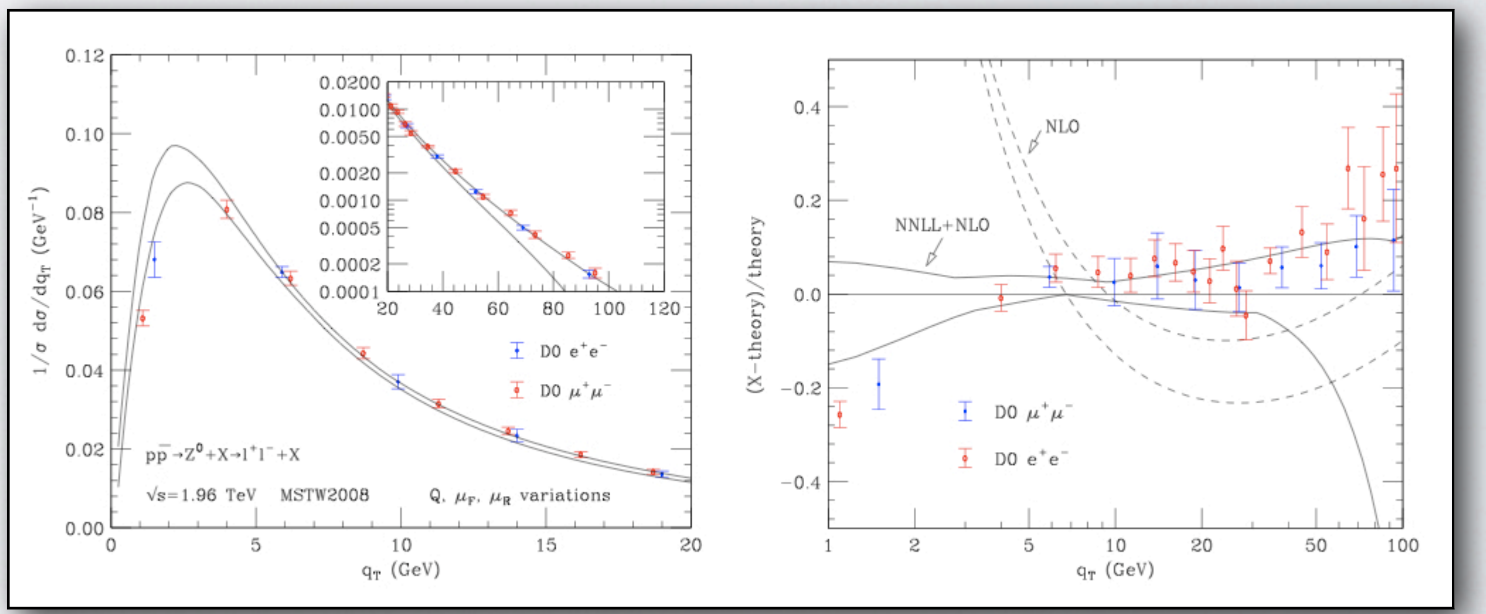
- Classic threshold resummation (for σ_{TOT}) is possible to `well-approximated' N^3LL.

- At large measured transverse momentum one is again close to partonic threshold.
  - p_T-threshold resummation now performed to approximate N^3LL (Becher, Schwartz, 11).



Large p_T D0 data vs. `threshold' resummation

- Transverse momentum resummation is available at NNLL (Bozzi et al., 10, Becher, Neubert 11-12).
  - Favorable comparison to Tevatron data.
  - Small theoretical uncertainty.
  - Awaiting LHC data comparison.

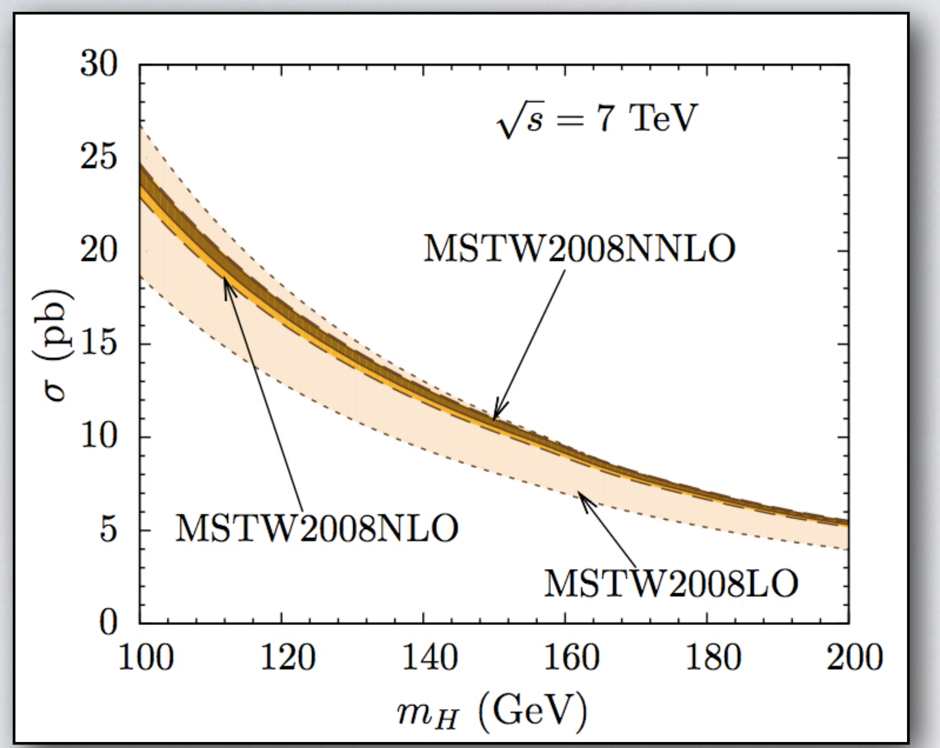


q_T spectrum of Z bosons at Tevatron (D0) compared to NNLL resummation

- Caveat: detailed SCET analysis (Becher and Neubert, 11) indicates (large) modification of 3-loop coefficient!
  - Theoretically interesting `collinear anomaly', transverse momentum pdf issues.

- NLL predictions for p_T spectrum also from SCET (Mantry, Petriello, 10)

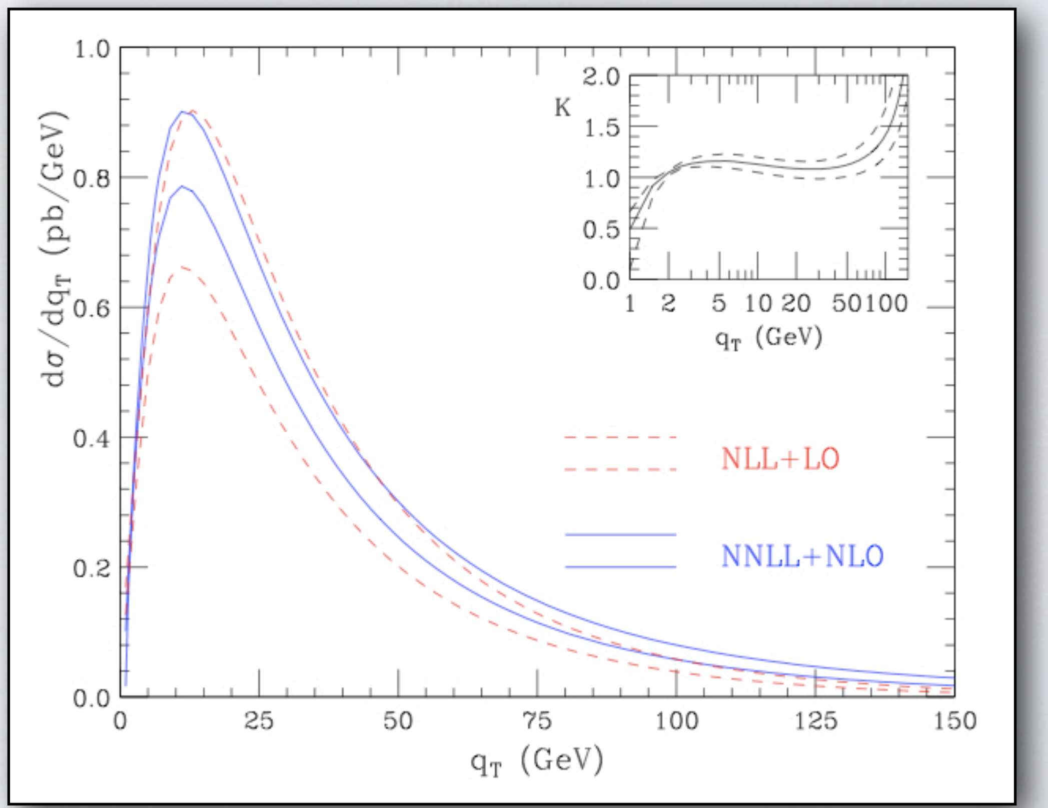
Higgs production



N^{3}LL resummed cross section for Higgs production via gluon fusion at LHC

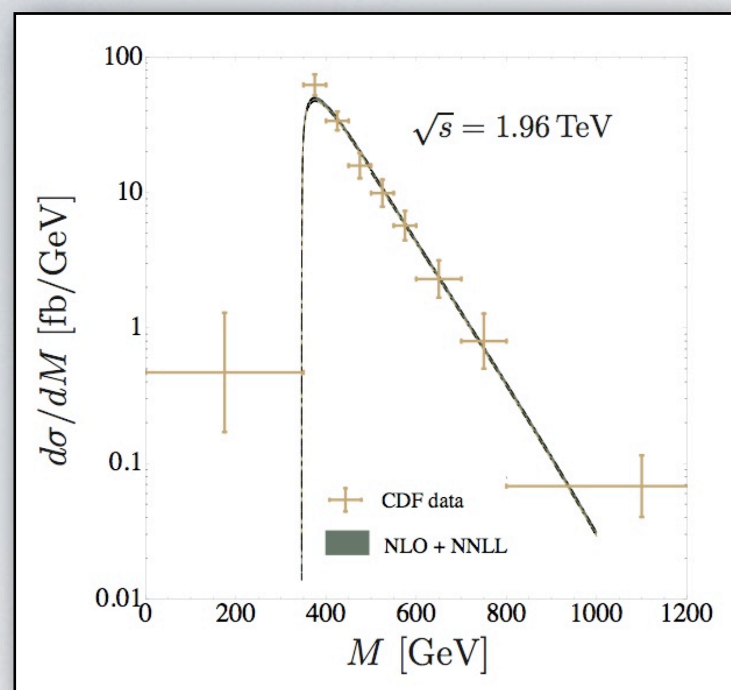
- The **total cross section** for **gg->H** is known to **N^{3}LL** and **NNLO**, with **NLO EW** corrections.
  - One of the **best-known** observables in the SM.
  - A combined analysis (Ahrens et al. 11) gives a 3% (th) + 8% (pdf) + 1% (mq) **uncertainty.**
  - Ongoing **debate** on theoretical and pdf uncertainty (Baglio et al. 11).

- The **p_T distribution** for **gg->H** is known to **NNLL** and **NNLO** (M. Grazzini et al. 07, 10 Ahrens et al. 11)
  - Resummation **reduces** scale uncertainty
  - A subtle **polarization effect** uncovered but not implemented yet (Catani, Grazzini, 10)
  - Impact of **revised three-loop** coefficient **must** be gauged



NNLL resummed pT distribution for Higgs production via gluon fusion at LHC

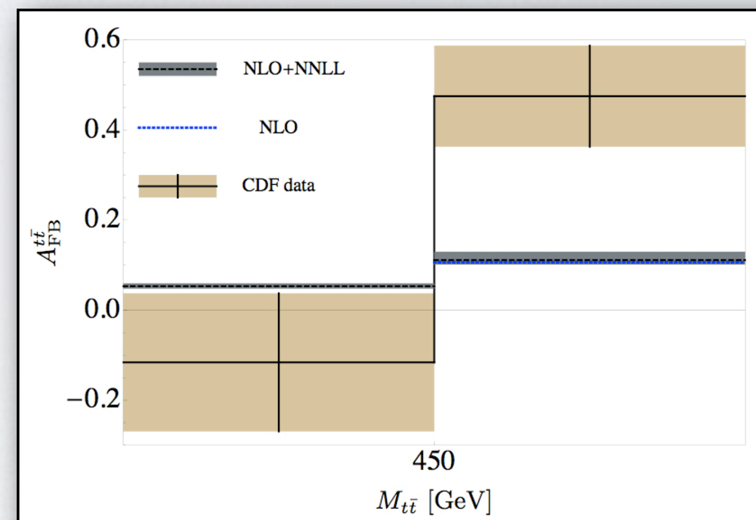
Top distributions



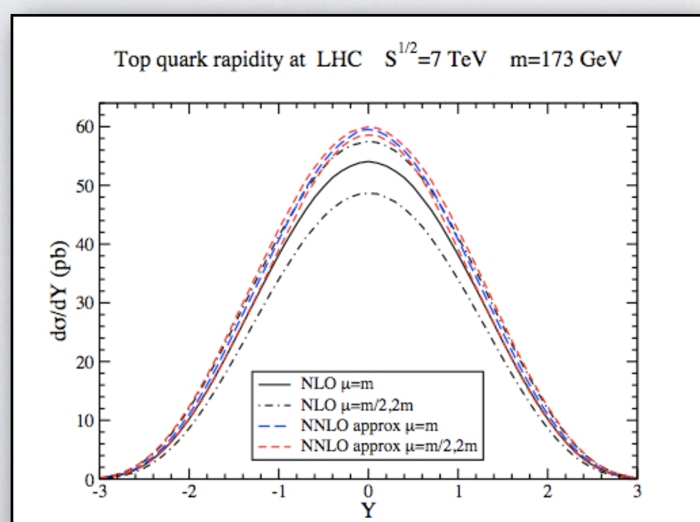
NNLL top-antitop invariant mass spectrum compared to CDF data

- The calculation of the two-loop massive anomalous dimension matrix makes it possible to perform NNLL resummation for generic distributions (Ahrens et al., 09).
  - Invariant pair mass distribution shows remarkable agreement with CDF data (LHC awaited).
  - Negligible theoretical uncertainty.
  - Different choices of kinematics and frame possible, vast menu of distributions available.

- The Tevatron top-antitop FB asymmetry can be computed in QCD at NNLL+NLO (Ahrens et al., 09).
  - Negligible impact on NLO result: the solution to the Tevatron puzzle is not QCD higher orders.



NNLL top-antitop FB asymmetry compared to CDF data



Top rapidity distribution at approximate NNLO

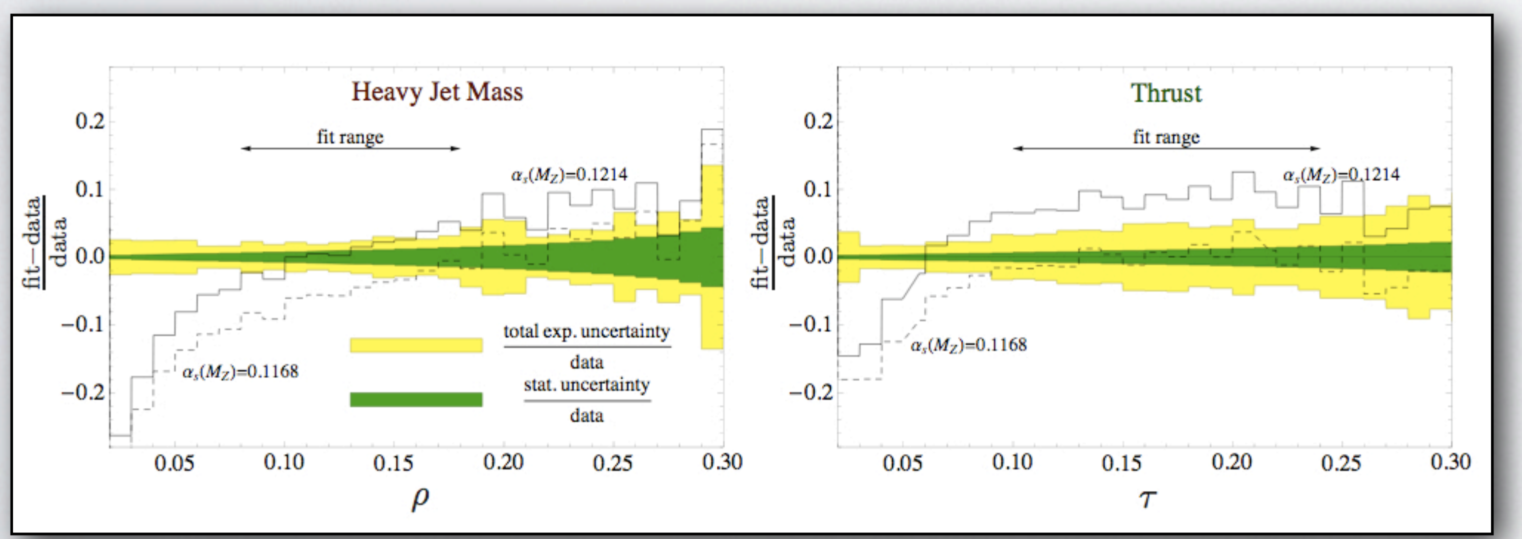
- Resummation can also be used in a simplified way to compute approximate higher order corrections and distributions (Kidonakis, 10-11)
  - Some conceptual and technical issues avoided; partial reduction in scale uncertainties.

Event shapes

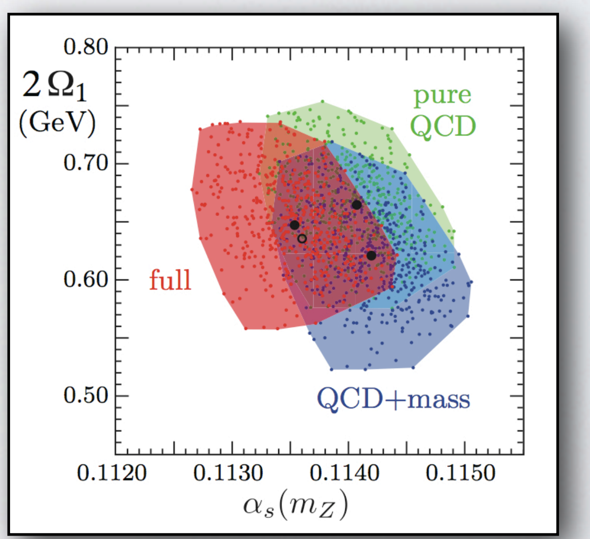
- First studies of event shapes with exact NNLO information and (well) approximated N^3LL resummation have appeared (Becher, Schwartz, 08; Schwartz, Cien; Abbate et al. 10).
- The studies deploy neat tricks (Padé approximants, numerical determination of 2-loop soft coefficients) and great care (hadronization, b-mass, QED corrections).
- Perturbative agreement between SCET and standard resummation (Gehrmann et al., 11).
- Significant differences remain in the final results for the strong coupling.

$$\begin{array}{rcll} \alpha_s(M_Z^2) & = & 0.1172 \pm 0.0022 & \text{thrust (BS)} \\ \alpha_s(M_Z^2) & = & 0.1220 \pm 0.0031 & \text{jet mass (SC)} \\ \alpha_s(M_Z^2) & = & 0.1135 \pm 0.0010 & \text{thrust (AFHMS)} \end{array}$$

- Many possible sources of discrepancy, the main suspect remains hadronization/MC.
- The problem is still not fully understood: do we really know α_s to percent accuracy?



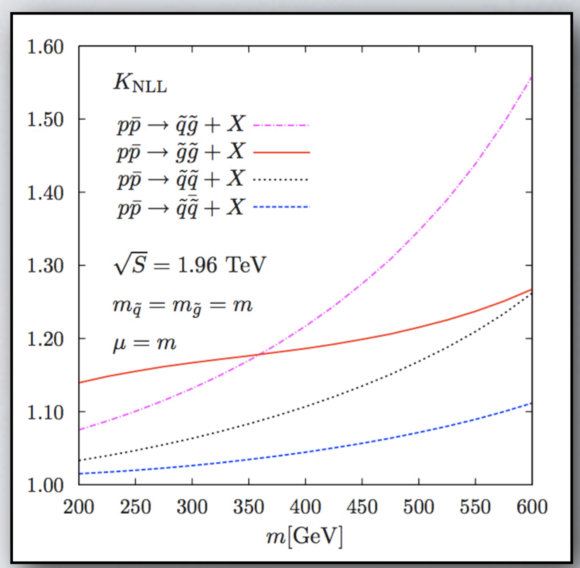
Comparing the α_s fit quality for thrust and heavy jet mass at N^3LL (SC)



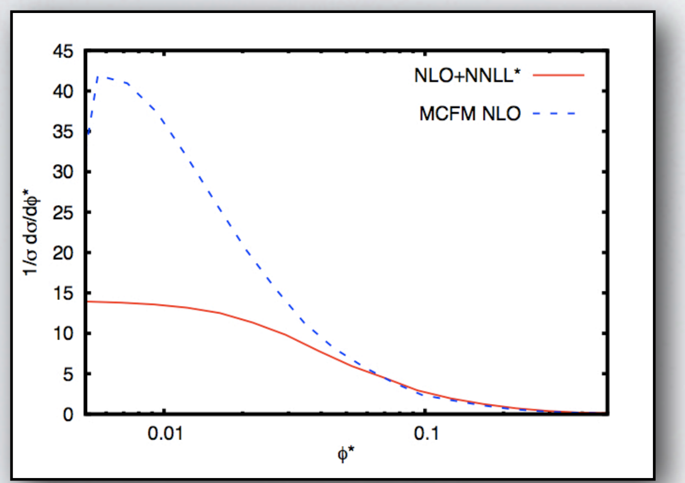
Joint fit of α_s and hadronization parameter Ω_1 from N^3LL thrust (AFHMS)

Miscellanea

- Soft gluon resummations are being applied to SUSY particles.
  - SUSY particles are heavy (and getting heavier ...), close to threshold: corrections useful for exclusion limits.
  - Gaugino and slepton production (singlets) (Klasen 06-11).
  - Colored sparticle production (requires soft matrices) (Kulesza et al. 09-11; Beneke et al., general color, 10).



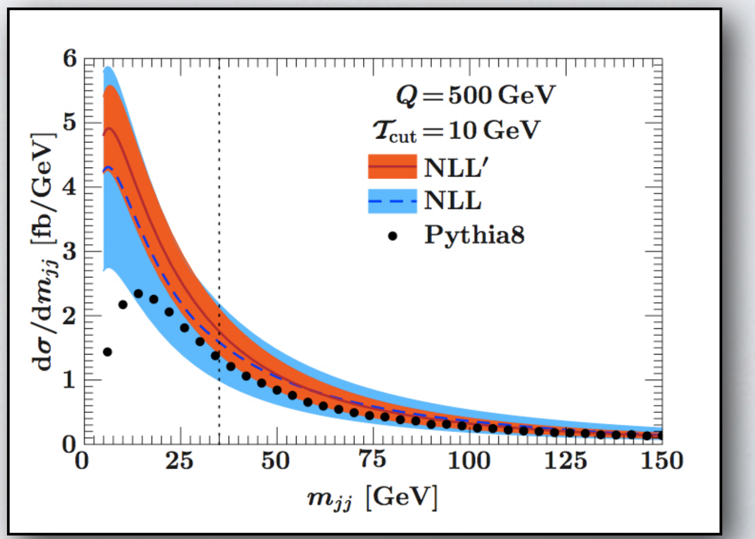
NLL K-factors for squarks and gluinos



NNLL vs. NLO φ^* distribution

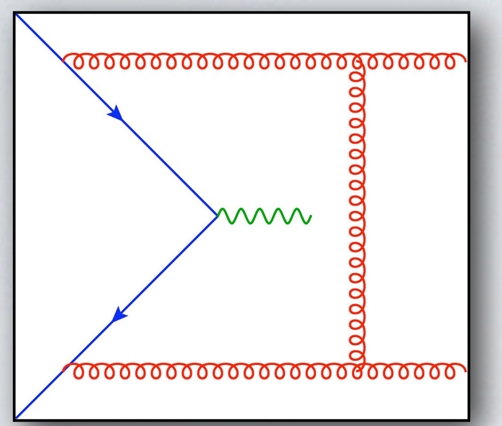
- New observables, designed by experiments, require (and get) soft gluon resummation (Banfi et al. 09-11).
  - Variables related to the angle between leptons are more accurately measured than the p_T of the lepton pair.
  - Resummation is crucial close to Born configuration.

- Complex jet observables are designed and resummed.
  - Jet shapes to study internal structure of jets, useful for boosted heavy particle production (Ellis et al. 09-11).
  - Dijet mass distribution with fixed \`background' event shape (\`N-jettiness'), extension of SCET (Bauer et al. 11).

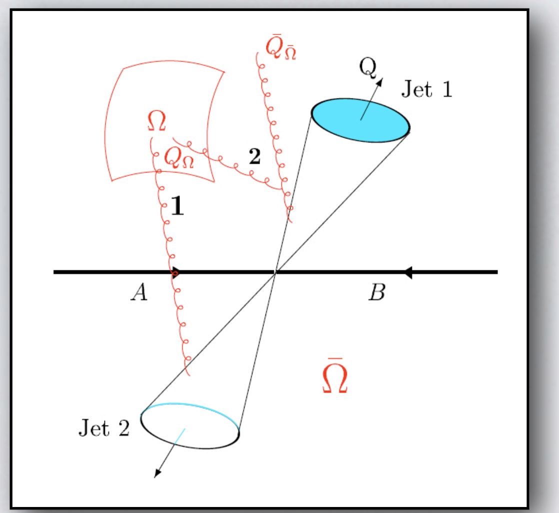


NLL dijet mass distribution for fixed N-jettiness

Jactuum caveat emptor



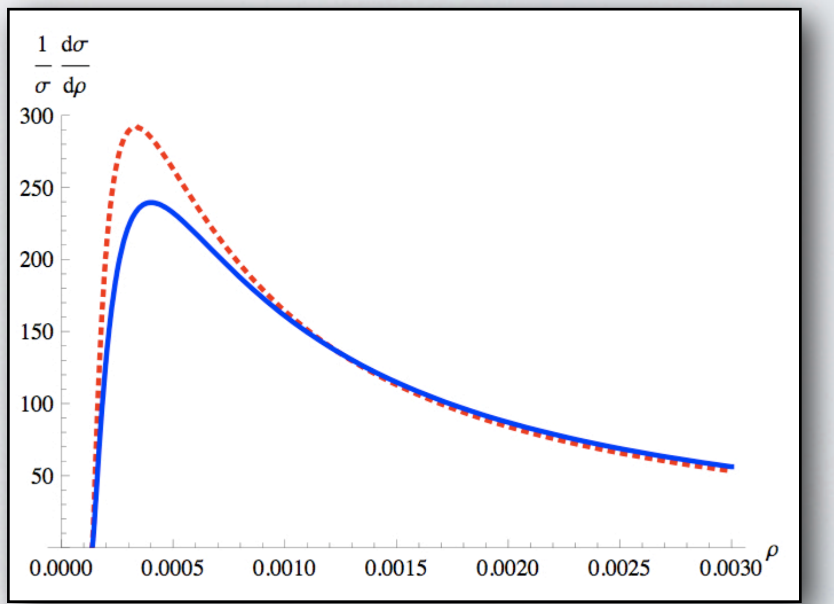
Spectator interaction via Glauber gluon



Non-global logarithms for energy flow

- As the jet observables proliferate and are resummed, several caveats must be kept in mind.
  - Glauber gluons: they cancel in inclusive jet cross sections (Aybat, Sterman 09), but no proof if jets are opened up. They are not in SCET, might be added (Bauer et al., 10).
  - Non-global logarithms: arise whenever gluon emission phase space is cut up (Dasgupta, Salam, 01); affect observables at single logarithmic level; resummable only at large N_C.
  - Jet algorithms: the choice of jet algorithm affects both non-global and ordinary Sudakov logarithms. Clustering correlates \`independent' gluons, except for anti-kT (Banfi et al., 05-06).

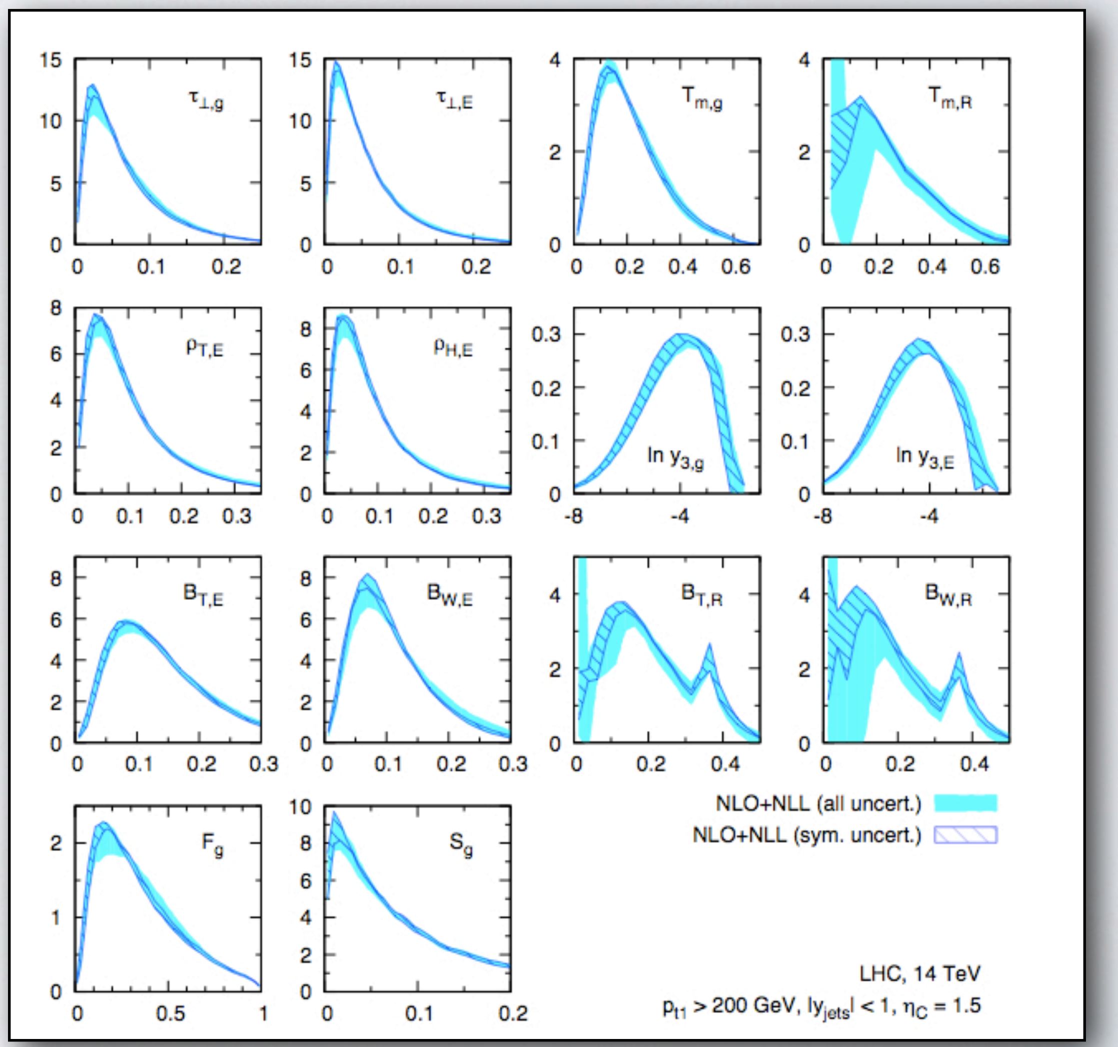
- A striking example of the impact of NG logs on jet shapes.
  - Jet shapes measure properties of a single jet in a multijet event. They are generically affected by NG logs.
  - For a typical jet shape (\`in-jet angularity') NG logs change the height of the (formally NLL) distribution by 15-20% in the small-R limit (Banfi et al., 10).



The impact of non-global logarithms on jet shapes

Hadronic event shapes

- An interesting alternative to the use of jets, which bypasses the need for an algorithm, is to introduce global event shapes, in analogy to those used in e^+e^- annihilation.
  - The hadronic environment requires suppressing the beam region.
- NLL+NLO resummation can be performed numerically with the program `Caesar` recently generalized to hadron collisions (Banfi, Salam, Zanderighi, 10).
- Numerically resummable event shapes are carefully characterized:
  - Functional constraints.
  - Continuous globalness.
  - Recursive IR safety.
- A vast variety of event shapes is introduced, categorized and resummed.
  - Simple example: transverse thrust.
- Relevant issues for NLL+NLO resummation of event shapes are dealt with in detail.
  - Control of non-global logs.
  - Transition particle-jet (algorithm issues).
  - Possible superleading logs.
  - Matching to NLOJET++.
  - Power corrections (analytic and MC).
  - Impact of underlying event.



A menu of NLL-resummed hadronic event shapes

OUTLOOK

Summary

- Resummations are a powerful tool both for theory and for phenomenology.
  - ✓ Explore the boundary between perturbative and non-perturbative physics.
  - ✓ Are necessary for precision phenomenology.
- Resummations have a long history, but
  - ✓ past few years have seen very intense LHC-motivated activity and theoretical progress.
- Factorization theorems ⇒ Evolution equations ⇒ Exponentiation.
  - ✓ Sudakov factorization ⇒ soft-gluon resummation (also formalized by SCET).
  - ✓ Multiparton processes require anomalous dimension matrices.
- Remarkable progress on the theory side.
  - ✓ We are understanding the all-order structure of the perturbative exponent.
  - ✓ For massless partons the dipole formula may give the definitive answer.
  - ✓ For massive partons the general two-loop anomalous dimension matrix is known.
  - ✓ SCET provides new insights: momentum space resummation, `collinear anomaly'.
  - ✓ Ongoing efforts to go beyond the eikonal approximation.
- A vast array of phenomenological applications, many vital for LHC precision physics.
  - ✓ Electroweak annihilation processes are known to high logarithmic accuracy.
  - ✓ Top distributions can be computed with unprecedented theoretical precision.
  - ✓ The strong coupling can be precisely determined from resummed event shapes.
  - ✓ Hadronic event shapes provide a flexible alternative tool for hadron collisions.
- We have come a long way, but each step forward brings new insight and new questions ...

THANK YOU