Introducing perturbative QCD for hadron collider applications

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Signaling the Arrival of the LHC Era

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Plan of the lectures

1. Preface
2. Basics of perturbative QCD
3. Perturbative QCD at hadron colliders
4. Pointers to special topics

Preface

Motivation (I)

Preface

The workshop on 'HERA and the LHC' successfully brought together experimental and theory experts working on electron–proton and proton–proton collider physics. It offered a forum to discuss the impact of present and future measurements at HERA on the physics programme of the LHC. The workshop was launched with a meeting at CERN in March 2004 and its first phase was terminated with a summary meeting in April 2005 at DESY. The workshop was very timely with on the one hand HERA-II, expected to deliver more than 500 pb^{-1} per experiment by 2007, ramping up to full strength, and on the other hand three years before the first collisions at the LHC.

The following aims were defined as the charge to the workshop:

- To identify and prioritize those measurements to be made at HERA which have an impact on the physics reach of the LHC.
- To encourage and stimulate transfer of knowledge between the HERA and LHC communities and establish an ongoing interaction.
- To encourage and stimulate theory and phenomenological efforts related to the above goals.
- To examine and improve theoretical and experimental tools related to the above goals.
- To increase the quantitative understanding of the implication of HERA measurements on LHC physics.

Five working groups were formed to tackle the workshop charge. Results and progress were presented and discussed at six major meetings, held alternately at CERN and at DESY.

Working group one had a close look at the parton distribution functions (PDFs), their uncertainties and their impact on the LHC measurements. The potential experimental and theoretical accuracy with which various LHC processes such as Drell–Yan, the production of W's, Z's and dibosons, etc. can be predicted was studied. Cross-section calculations and differential distributions were documented and some of these processes are used as benchmark processes for PDF and other QCD uncertainty studies. In particular W and Z production at the LHC has been scrutinized in detail, since these processes will be important standard candles. It is even planned to use these for the luminosity determination at the LHC. The impact of PDFs on LHC measurements and the accuracy with which the PDFs can be extracted from current and forthcoming data, particular the HERA-II data, have been investigated, as well as the impact of higher order corrections, small-x and large-x resummations. Initial studies have been started to provide a combined data set on structure function measurements from the two experiments H1 and ZEUS. Arguments for running HERA at lower energies, to allow for the measurement of the longitudinal structure function, and with deuterons, have been brought forward.

The working group on multi-jet final states and energy flows studied processes in the perturbative and non-perturbative QCD region. One of the main issues of discussion during the workshop was the structure of the underlying event and of minimum-bias events. New models were completed and presented during the workshop, and new tunes on p–p data were discussed. A crucial test will be to check these generator tunes with e–p and γ–p data from HERA, and thus check their universality. Other important topics tackled by this working group concern the study of rapidity-gap events, multi-jet topologies and matrix-element parton-shower matching questions. The understanding of rapidity gaps and in particular their survival probability is of crucial importance to make reliable predictions for central exclusive processes at the LHC. HERA can make use of the virtuality of the photon to study in detail the onset of multiple interactions. Similarly HERA data, because of its handles on the event kinematics via the scattered electron, is an ideal laboratory to study multiple-scale QCD problems and improve our understanding in that area such that it can be applied with confidence to the LHC data. For example, the HERA data give strong indications that in order to get reliable and precise predictions, the use of unintegrated parton distributions will be necessary. The HERA data should be maximally exploited to extract those distributions.

The third group studied heavy flavours at HERA and the LHC. Heavy quark production, in particular at small momenta at the LHC, is likely to give new insight into low-x phenomena in general and saturation in particular. The possibilities for heavy quark measurements at LHC were investigated. The charm and bottom content of the proton are key measurements, and the anticipated precision achievable with HERA-II is very promising. Furthermore, heavy quark production in standard QCD processes may form an important background in searches for new physics at the LHC and has therefore to be kept as much as possible under control. Again, heavy quark production results from mostly multi-scale processes where topics similar to those discussed in working group two can be studied and tested. Important steps were taken for a better understanding of the heavy quark fragmentation functions, which are and will be measured at HERA. The uncertainties of the predicted heavy quark cross-section were studied systematically and benchmark cross-sections were presented, allowing a detailed comparison of different calculations.

Diffraction was the topic of working group four. A good fraction of the work in this group went into the understanding of the possibility of the exclusive central production of new particles such as the Higgs $pp \to p+H+p$ at the LHC. With measurable cross-sections, these events can then be used to pin down the CP properties of these new particles, via the azimuthal correlation of the two protons, and thus deliver an important added value to the LHC physics programme. The different theoretical approaches to calculate cross-sections for this channel have been confronted, and scrutinized. The Durham approach, though the one that gives the most conservative estimate of the event cross-section, namely in the order of a few femtobarns, has now been verified by independent groups. In this approach the generalized parton distributions play a key role. HERA can determine generalized parton distributions, especially via exclusive meson production. Other topics discussed in this group were the factorization breaking mechanisms and parton saturation. It appears that the present diffractive dijet production at HERA does not agree with a universal description of the factorization breaking, which is one of the mysteries in the present HERA data. Parton saturation is important for event rates and event shapes at the LHC, which will get large contributions of events at very low-x. Furthermore, the precise measurement of the diffractive structure functions is important for any calculation of the cross-section for inclusive diffractive reactions at the LHC. Additionally, this working group has really acted as a very useful forum to discuss the challenges of building and operating beam-line integrated detectors, such as Roman Pots, in a hadron storage ring. The experience gained at HERA was transferred in detail to the LHC groups which are planning for such detectors.

Finally, working group five on the Monte Carlo tools had very productive meetings on discussing and organizing the developments and tunings of Monte Carlo programs and tools in the light of the HERA–LHC connection. The group discussed the developments of the existing generators (e.g., PYTHIA, HERWIG) and new generators (e.g., SHERPA), or modifications of existing ones to include p–p scattering (e.g., RAPGAP, CASCADE). Many of the other studies like tuning to data, matrix-element and parton shower matching, etc., were done in common discussions with the other working groups. Validation frameworks have been compared and further developed, and should allow future comparisons with new and existing data to be facilitated.

In all it has been a very productive workshop, demonstrated by the content of these proceedings. Yet the ambitious programme set out from the start has not been fully completed: new questions and ideas arose in the course of this workshop, and the participants are eager to pursue these ideas. Also the synergy between the HERA and LHC communities, which has been built up during this workshop, should not evaporate. Therefore this initiative will continue and we look forward to further and new studies in the coming years, and the plan to hold a workshop once a year to provide the forum for communicating and discussion the new results.

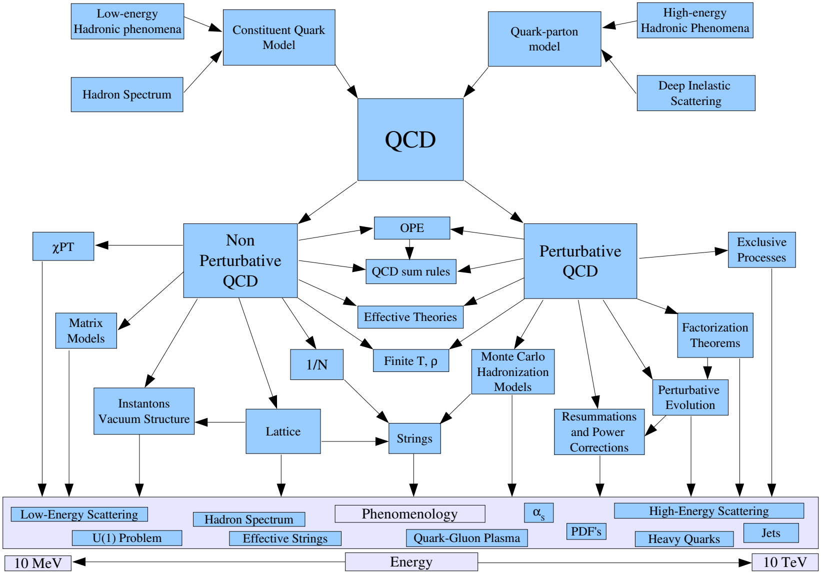
We thank all the convenors for the excellent organization of their working groups and all participants for their work and enthusiasm and contribution to these proceedings.

We are grateful to the CERN and DESY directorates for the financial support of this workshop and for the hospitality which they extended to all the participants. We are grateful to D. Denise, A. Grabowksi and S. Platz for their continuous help and support during all the meeting weeks. We would like to thank also B. Liebaug for the design of the poster for this first HERA–LHC workshop.

Hannes Jung and Albert De Roeck

Motivation (II)

LHC is a hadron collider


Low-energy Hadronic phenomena
Constituent Quark Model
Quark-parton model
High-energy Hadronic Phenomena
Hadron Spectrum
Deep Inelastic Scattering
QCD
Non Perturbative QCD
OPE
Perturbative QCD
χPT
QCD sum rules
Exclusive Processes
Effective Theories
Matrix Models
Factorization Theorems
1/N
Finite T, ρ
Monte Carlo Hadronization Models
Perturbative Evolution
Instantons Vacuum Structure
Lattice
Strings
Resummations and Power Corrections
Low-Energy Scattering
Hadron Spectrum
Phenomenology
αs
High-Energy Scattering
U(1) Problem
Effective Strings
Quark-Gluon Plasma
PDF's
Heavy Quarks
Jets
10 MeV
Energy
10 TeV

What we all know

- QCD is the quantum field theory of quarks and gluons. It exhibits unbroken $SU(3)$ non-abelian gauge invariance.
- QCD is renormalizable and works well in the ultraviolet. It is asymptotically free.
- QCD has a perturbative coupling that grows in the infrared. The theory generates its own dynamical scale, Λ_{QCD}.
- QCD exhibits color confinement and has a mass gap.

  `Note:` proving this point yields 10^6\$.
- QCD is the theory of strong interactions.

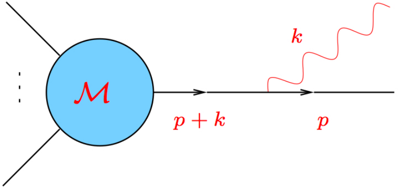
Basics of perturbative QCD

Mass divergences: qualitative discussion

- Fact: in quantum field theory, two kinds of divergences are associated with the presence of massless particles.
  - Infrared (IR): emission of particles with vanishing four-momentum ($\lambda_{DB} \rightarrow \infty$);
    - ★ present in gauge theories only;
    - ★ present also when matter particles are massive (QED).
  - Collinear (C): splitting of particles into parallel moving pairs
    - ★ present if all particles in the interaction vertex are massless.
- Origin: physical processes happening at large distances.
- Therapy: carefully sum over experimentally indistinguishable configurations.

Mass divergences: example

Emission of a massless gauge boson



$$\rightarrow \quad -\,\mathrm{i} g \overline{u}(p) \not{\epsilon}(k) t_a \frac{\mathrm{i}(\not{p} + \not{k})}{(p+k)^2 + \mathrm{i}\varepsilon} \mathcal{M} \ ,$$

Singularities: $2p \cdot k = 2p_0 k_0 (1 - \cos\theta_{pk}) = 0$,

$\rightarrow k_0 = 0$ (IR); $\cos\theta_{pk} = 1$ (C).

Note: $p_0 = 0$ singularity will be integrable.

Mass divergences: analysis

- In covariant perturbation theory:
  - p^μ is conserved in every vertex;
  - intermediate particles are generally off–shell;
  - the emitting fermion is on–shell: it can propagate indefinitely.
- In time-ordered perturbation theory:
  - all particles are on–shell;
  - energy is not conserved in the interaction vertices;
  - the IR/C emission vertex conserves energy: it can be placed at arbitrary distance.
- The matrix element is not suppressed at long distances.

Sickness and Therapy

- The sickness is serious. The S matrix does not exist in the Fock space of quarks and gluons.
  - No surprise ... quarks and gluons are not the correct asymptotic states!
- Observe. Mass divergences are associated with the existence of experimentally indistinguishable, energy degenerate states.
  - Physical detectors have finite resolution in energy and angle.
- KLN Theorem. Physically measurable quantities (transition probabilities, cross sections) are finite.
  - Mass divergences cancel, after summing coherently over all physically indistinguishable states.

KLN Theorem

- Take any quantum theory with hamiltonian H
- Let $\mathcal{D}_\epsilon(E_0)$ be the set of exact eigenstates of H with energies $E_0 - \epsilon \le E \le E_0 + \epsilon$, with $\epsilon \neq 0$.
- Let $P(i \to j)$ be the transition probability per unit volume and per unit time between eigenstates i and j.
- Then the quantity

$$P(E_0, \epsilon) \equiv \sum_{i,j \in \mathcal{D}_\epsilon(E_0)} P(i \to j)$$

is finite as $m \to 0$ to all orders in perturbation theory

Note: in an asimptotically free theory $m(\mu) \to 0$ as $\mu \to \infty$.

Note: in QED ($m_e \neq 0$) summing over final states suffices.

Strategy of PQCD (I)

Infrared Safety

- Compute at partonic level, with an infrared regulator (*e. g.*: $\epsilon = 2 - d/2 < 0$), and at least one hard scale Q.

$$\sigma_{\rm part} = \sigma_{\rm part}\left(\frac{Q}{\mu}, \alpha_s(\mu), \left\{\frac{m(\mu)}{\mu}, \epsilon\right\}\right) .$$

- Select IR–safe quantities, with a finite limit when the IR regulator is removed ($\epsilon \to 0$, $m(\mu) \to 0$).

$$\sigma_{\rm part} = \sigma_{\rm part}\left(\frac{Q}{\mu}, \alpha_s(\mu), \{0, 0\}\right) + \mathcal{O}\left(\left\{\left(\frac{m}{\mu}\right)^p, \epsilon\right\}\right) .$$

- Interpret these partonic, inclusive quantities, expanded in powers of $\alpha_s(Q) \ll 1$, as estimates of hadronic quantities, valid up to $\mathcal{O}\left((\Lambda_{QCD}/Q)^p\right)$ corrections.

Strategy of PQCD (II)

Factorization

- Initial state hadrons break IR safety
  - Cancellation of IR divergences fails in QCD when summing over final states only.
  - The KLN theorem is not applicable when summing over initial states (we don't know the initial state wave function)
- Construct factorizable quantities, such that

$$\sigma_{\text{part}}\left(\frac{m}{\mu},\frac{Q}{\mu}\right)=\mathcal{F}\left(\frac{m}{\mu},\frac{\mu_F}{\mu}\right)*\widehat{\sigma}_{\text{part}}\left(\frac{Q}{\mu},\frac{\mu_F}{\mu}\right)+\mathcal{O}\left(\left(\frac{m}{\mu_F}\right)^p\right) .$$

  - Absorb divergences into initial state distributions $\mathcal{F}$.
  - Compute finite hard partonic cross section $\widehat{\sigma}_{\text{part}}$.
  - Fold perturbative $\widehat{\sigma}_{\text{part}}$ with measured $\mathcal{F}$.

IR Safety: $R_{e^+e^-}$

Simplest example: the total cross section in e^+e^- annihilation.

- It is insensitive to long distances.
- It can be expanded in a small parameter, $\alpha_s(Q^2)$.
- Partons will give hadrons with probability one.

Compute:

$$\sigma_{\rm tot}(q^2) = \frac{1}{2q^2} \sum_X \int d\Gamma_X \, \frac{1}{4} \sum_{\rm spin} |\mathcal{M}(k_1 + k_2 \to X)|^2 \ .$$

Normalize:

$$R_{e^+e^-} \equiv \frac{\sigma_{\rm tot}\left(e^+e^- \to \text{hadrons}\right)}{\sigma_{\rm tot}\left(e^+e^- \to \mu^+\mu^-\right)}$$

At tree level:

$$\sigma^{(0)}_{\rm tot} = \frac{4\pi\alpha^2}{3q^2} N_c \sum_f q_f^2 \quad \to \quad R^{(0)}_{e^+e^-} = N_c \sum_f q_f^2 \ .$$

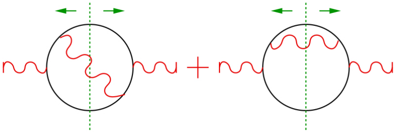
Radiative corrections

- Concentrate on photon decay (tree level in QED).
- Introduce an IR regulator, $d = 4 - 2\epsilon$ with $\epsilon < 0$.

$$\sigma_{\text{tot}}(q^2) = \frac{1}{2q^2} L_{\mu\nu}(k_1, k_2) H^{\mu\nu}(p_1, p_2) = \frac{e^2 \mu^{2\epsilon}}{2q^4} \frac{1-\epsilon}{3-2\epsilon} \left(-H^{\mu}_{\mu}(q^2) \right) .$$

- Compute diagrams for the squared matrix element

$$\left[-H^{\mu}_{\mu}\right]^{(1)} =$$



$+$ c.t.

- Summing over positions of the final state cut yields real gluon emission and virtual gluon exchange corrections.

Real emission

Integration of three-particle phase space in d dimensions

$$\left[H^{\mu}_{\mu}\right]^{(1,R)} = \int \frac{d^d p \, d^d k}{(2\pi)^{2d-3}} \, \delta_+(p^2) \, \delta_+(k^2) \, \delta_+((p+k-q)^2) \, \left[\mathcal{H}^{\mu}_{\mu}\right]^{(1)} \; ,$$

with $y \equiv (1-\cos\theta_{pk})/2$ and $z = 2k_0/\sqrt{s}$, gives

$$\left[H^{\mu}_{\mu}\right]^{(1,R)} = \left[H^{\mu}_{\mu}\right]^{(0)} K(\epsilon) \, \frac{\alpha_s}{\pi} \, C_F \int_0^1 dz \, dy \left[\frac{1}{z^{1+2\epsilon} \left[y(1-y)\right]^{1+\epsilon}} + \ldots \right] .$$

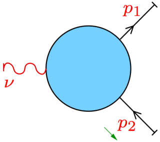
One recognizes the IR pole, $z \to 0$, and the two collinear poles, $y \to 0, 1$. Integration yields a typical double pole,

$$\left[H^{\mu}_{\mu}\right]^{(1,R)} = \left[H^{\mu}_{\mu}\right]^{(0)} \, \frac{\alpha_s}{\pi} C_F \left[\frac{2}{\epsilon^2} + \frac{5}{\epsilon} - \frac{5}{3}\pi^2 + \frac{33}{2} + \mathcal{O}(\epsilon) \right] \; .$$

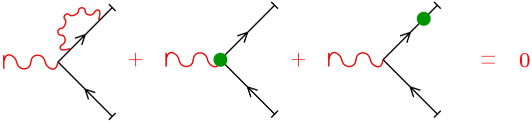
Virtual exchange

- Virtual contributions are given by the quark form factor

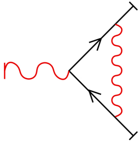
$$\Gamma_\nu\left(p_1, p_2; \mu^2, \epsilon\right) =$$



Dimensional regularization and QED gauge invariance imply



$$\Gamma_\nu^{(1)}\left(p_1, p_2; \mu^2, \epsilon\right) =$$



- One diagram gives the complete answer

Cancellation

Result for the form factor (after renormalization!)

$$\Gamma^{(1)} = -\frac{\alpha_s}{4\pi} C_F \left(\frac{4\pi\mu^2}{-q^2}\right)^{\epsilon} \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)} \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + 8 + \mathcal{O}(\epsilon)\right] .$$

Note! $(-q^2 + \mathrm{i}\varepsilon)^{-\epsilon} = (q^2)^{-\epsilon}\, \mathrm{e}^{-\mathrm{i}\pi\epsilon}$.

Finally: IR and collinear poles cancel.

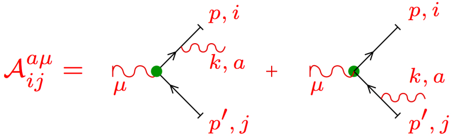
$$\sigma_{\mathrm{tot}} = \frac{4\pi\alpha^2}{3q^2} N_c \sum_f q_f^2 \left(1 + \frac{\alpha_s}{\pi}\frac{3}{4} C_F + \mathcal{O}(\alpha_s^2)\right) ,$$

For $SU(3)$, where $C_F = 4/3$, the (classical) result is

$$R_{e^+e^-} = N_c \sum_f q_f^2 \left(1 + \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right) .$$

Soft approximation

Universality: soft emission factorizes form the Born amplitude



The exact amplitude probes spin and energy of hard partons

$$\mathcal{A}_{ij}^{a\mu} = g\, t_{ij}^a\, \overline{u}(p) \left[\frac{\not\varepsilon(k)(\not p + \not k)\Gamma_\mu}{2p \cdot k} - \frac{\Gamma_\mu(\not p' + \not k)\not\varepsilon(k)}{2p' \cdot k} \right] v(p') \, .$$

Neglecting $\not k$, and using the Dirac equation, the soft amplitude factorizes: a scale-invariant soft factor multiplies the amplitude with no radiation.

$$\mathcal{A}_{ij}^{a\mu}\Big|_{\text{soft}} = g\, t_{ij}^a \left[\frac{p \cdot \varepsilon}{p \cdot k} - \frac{p' \cdot \varepsilon}{p' \cdot k} \right] \mathcal{A}_0^\mu \, ,$$

Soft approximation

- The soft amplitude is gauge–invariant (it vanishes if $\varepsilon \propto k$).
- Soft gluon emission has universal characters.
  - Long-wavelength gluons cannot analyze the short-distance properties of the emitter (spin, internal structure), they only detect the global color charge and the direction of motion
- The result generalizes to multiple gluon emission.
- The result generalizes to gluon emission from gluons.
- The soft approximation can be applied to virtual diagrams, with some care (eikonal approximation).
  - When $k_\mu \ll \sqrt{q^2}$, $\forall \mu$, one can neglect k^2 with respect to $p_i \cdot k$ in denominators, as well as k in numerators.
  - Beware: the approximation is not uniformly valid in Minkowsky space! (May need to deform integration contours, may break down).

Soft cross section

Soft gluon phase space also factorizes (hard partons do not recoil). Therefore the cross section also factorizes.

$$\sigma_{q\bar{q}g}^{\text{soft}} = g^2\, C_F\, \sigma_{q\bar{q}} \int \frac{d^3k}{2|\mathbf{k}|(2\pi)^3} \frac{2p\cdot p'}{p\cdot k\; p'\cdot k} \;.$$

In the center-of-mass frame ($\mathbf{q} = \mathbf{0}$) and in the soft approximation the quark and the antiquark are still back to back. One recovers

$$\sigma_{q\bar{q}g}^{\text{soft}} = \sigma_{q\bar{q}}\, C_F\, \frac{\alpha_s}{\pi} \int_{-1}^{1} d\cos\theta_{pk} \int_0^\infty \frac{d|\mathbf{k}|}{|\mathbf{k}|} \frac{2}{(1-\cos\theta_{pk})(1+\cos\theta_{pk})}.$$

Displaying the expected soft and collinear singularities.

Angular ordering

The soft approximation displays a general feature.

- Consider the gluon emission probability from a boosted $q\bar{q}$ dipole (small $\theta_{pp'}$).

$$d\sigma^{\text{soft}}_{q\bar{q}g} = d\sigma_{q\bar{q}} C_F \frac{\alpha_s}{\pi} \frac{d|\mathbf{k}|}{|\mathbf{k}|} d\cos\theta_k \frac{d\phi_k}{2\pi} \frac{1-\cos\theta_{pp'}}{(1-\cos\theta_{pk})(1-\cos\theta_{p'k})} .$$

- Split the positive definite emission probability in two terms, assigned to the quark and the antiquark.

$$d\sigma^{\text{soft}}_{q\bar{q}g} = d\sigma_{q\bar{q}} C_F \frac{\alpha_s}{\pi} \frac{d|\mathbf{k}|}{|\mathbf{k}|} d\cos\theta_k \frac{d\phi_k}{2\pi} \frac{1}{2} \left(W_q + W_{\bar{q}}\right) .$$

- Choose

$$W_q = \frac{1-\cos\theta_{pp'}}{(1-\cos\theta_{pk})(1-\cos\theta_{p'k})} + \frac{1}{(1-\cos\theta_{pk})} - \frac{1}{(1-\cos\theta_{p'k})} .$$

Angular ordering

The Radiation factors W_q and $W_{\bar{q}}$ have important properties.

- W_q ($W_{\bar{q}}$) is singular only when $\cos\theta_{pk} \to 1$ ($\cos\theta_{p'k} \to 1$).
- W_q and $W_{\bar{q}}$ are not positive definite.
- The azimuthal average of W_q (with respect to the axis defined by **p**) vanishes if $\theta_{pk} > \theta_{pp'}$.

$$\frac{1}{2\pi}\int_0^{2\pi} d\phi \; W_q(\phi) = \frac{2}{1-\cos\theta_{pk}} \, \Theta\left(\theta_{pp'} - \theta_{pk}\right) \; ,$$

  - It can be proven using

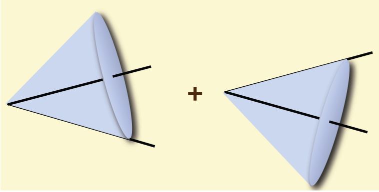
$$\cos\theta_{p'k} = \cos\theta_{pk}\cos\theta_{pp'} + \sin\theta_{pk}\sin\theta_{pp'}\cos\phi \; .$$

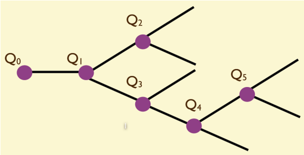
- Azimuthal averages are positive definite.
- Interpret as probability distributions for independent emission from the quark and the antiquark.

Towards hadronization

Angular ordering generalizes to multiple emissions to leading power in $1/N_c^2$.

- Emission is inside cones.
- Further emissions have smaller cones.
- Hadronization is local in phase space.





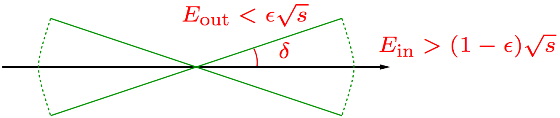
- Hadronization is approximately a Markov chain.
- After branching daughter partons have splitting probability.
- Leads to shower Monte Carlo's.

Sterman-Weinberg jets

Can one construct less inclusive IR-C finite observables?

Prototype: Sterman-Weinberg jet cross section

- An event is a two-jet event iff $\exists$ two cones with opening angle δ, such that all energy, up to at most a fraction ϵ, flows in the cones.



- All events are two-jet events at leading order.
- At $\mathcal{O}(\alpha_s)$ two-jet events have
  - an IR gluon (emitted in any direction), or
  - a collinear gluon (with any energy).
- Virtual corrections are two-jet events. Therefore, the partonic two-jet cross section is finite.

Three-jet cross section

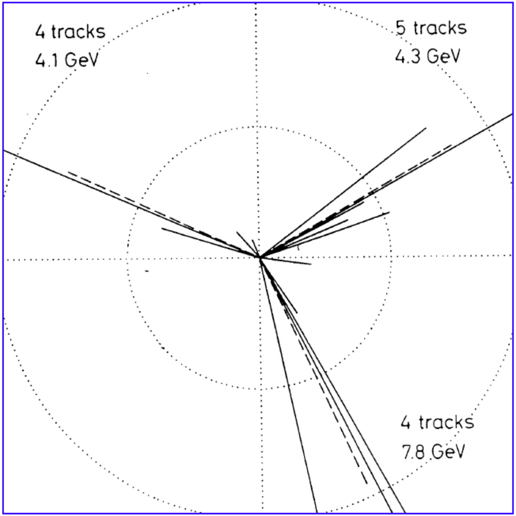
- At leading order (LO) one finds simply $\sigma_{2j}^{(0)}(\epsilon, \delta) = \sigma_{tot}^{(0)}$.
- At next-to-leading order one finds only two- or three-jet events, so that

$$\sigma_{2j}^{(1)}(\epsilon, \delta) = \sigma_{tot}^{(1)} - \sigma_{3j}^{(1)}(\epsilon, \delta) \ ,$$

- $\sigma_{3j}^{(1)}$ is easily computed at tree-level. The dominant contributions as $\epsilon, \delta \to 0$ are

$$\sigma_{3j}^{(1)}(\epsilon, \delta) = \sigma_{tot}^{(0)} C_F \frac{\alpha_s}{\pi} \left[4 \log(\delta) \log(2\epsilon) + 3 \log(\delta) + \frac{\pi^2}{3} - \frac{7}{4} \right] \ .$$

- Observe:
  - The total cross section is dominated by two-jet events at large q^2 (asymptotic freedom for jets!).
  - The angular distribution of two-jet events $d\sigma_{2j}/d\cos\theta \propto 1 + \cos^2\theta$ is typical of spin $1/2$ quarks.



QCD history in the making: TASSO at PETRA “sees the gluons” (1979!)

Event shapes

A further generalization: pick observables assigning equal weights to events differing only by IR or C emissions.

- Given m partons, and the observable $E_m(p_1, \ldots, p_m)$, let

$$\frac{d\sigma}{de} = \frac{1}{2q^2} \sum_m \int d\mathrm{LIPS}_m \, \overline{|\mathcal{M}_m|^2} \, \delta\left(e - E_m(p_1, \ldots, p_m)\right) ,$$

- Different final states contribute: at order α_s^{m-1}

$$\sigma(e)\Big|_{\mathcal{O}\left(\alpha_s^{m+1}\right)} = \int d\sigma_{m+1}^{(R)} + \int d\sigma_m^{(1V)} + \ldots .$$

- IR-C safety: cancellation is preserved if

$$\lim_{p_j^\mu \to 0} E_{m+1}(p_1, \ldots, p_j, \ldots) = E_m(p_1, \ldots, p_{j-1}, p_{j+1}, \ldots) ,$$

$$\lim_{p_k^\mu \to \alpha p_j^\mu} E_{m+1}(p_1, \ldots, p_j, \ldots, p_k, \ldots) = E_m(p_1, \ldots, p_j + p_k, \ldots) .$$

Event shapes: examples

Thrust

$$T_m = \max_{\hat{\mathbf{n}}} \frac{\sum_{i=1}^{m} |\mathbf{p_i} \cdot \hat{\mathbf{n}}|}{\sum_{i=1}^{m} |\mathbf{p_i}|}$$

- $0 < T_m \leq 1$
- $T_m = 1$: two back to back pencil-like jets.

C parameter

$$C_m = 3 - \frac{3}{2} \sum_{i,j=1}^{m} \frac{(p_i \cdot p_j)^2}{(p_i \cdot q)\,(p_j \cdot q)}$$

- $0 < C_m \leq 1$
- $C_m = 0$: two back to back pencil-like jets.
- $C = 3(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3)$

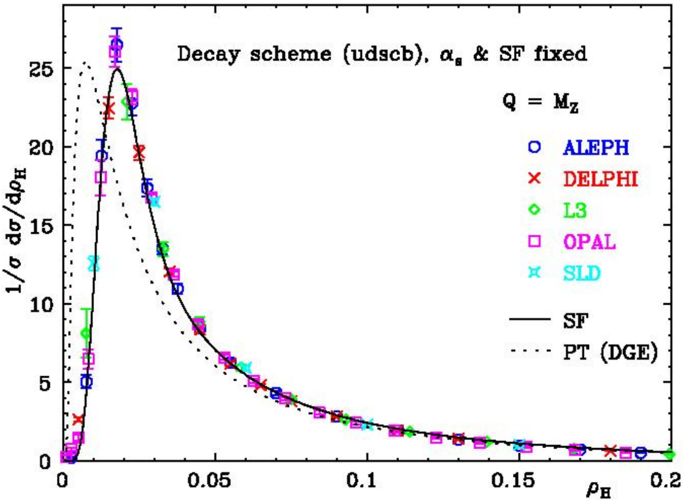
Jet masses

$$\rho_m^{(H)} = \frac{1}{q^2} \left(\sum_{p_i \in H} p_i \right)^2$$

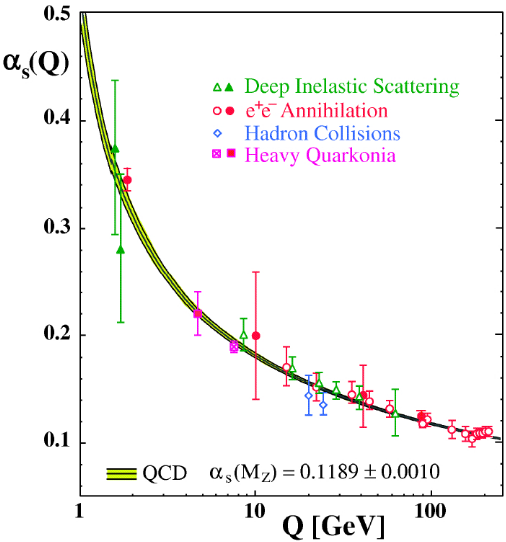
- H: hemisphere defined by thrust axis.
- $\rho_m^{(H)} = 0$: massless jet in H.

Event shapes: phenomenology

- At leading order distributions are $\delta(e)$, unlike data ...
- NNLO calculation recently completed
- At higher orders distributions are singular in the two-jet limit, behaving as $\alpha_s^n \log^{2n-1} e/e$.
  - Sudakov logarithms are tied to IR-C poles.
  - They can be resummed to all orders.
- Moments of the distributions are finite.
- Great phenomenological relevance (for example: determination of α_s, study of hadronization corrections).
- Jet algorithms can be seen as particular event shapes.
- Generalizations exist to a hadron collider environment.



A sample fit of LEP data (Gardi and Rathsman) for the jet mass ρ_H, with NLL resummation and power corrections.



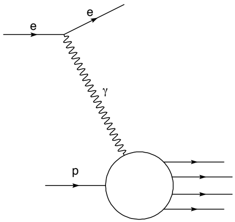
Mesurements of $\alpha_s(Q)$ from various processes, compared to four-loop QCD (Bethke).

Perturbative QCD at hadron colliders

DIS: kinematics

Kinematic variables:

- $q = k - k' \;\rightarrow\; Q^2 = -q^2$
- $x = \frac{Q^2}{2p\cdot q}\,, \quad y = \frac{p\cdot q}{p\cdot k}$
- $W^2 = (p+q)^2 = Q^2\,\frac{1-x}{x}$



Cross section (for electromagnetic DIS):

$$\frac{d^2\sigma}{dxdy} = \frac{\alpha^2 y}{2Q^4} L^{\mu\nu}(k,k')H_{\mu\nu}(p,q) = \frac{4\pi\alpha^2}{Q^2}\left[y\,F_1(x,Q^2) + \frac{1-y}{y}\frac{F_2(x,Q^2)}{x}\right]$$

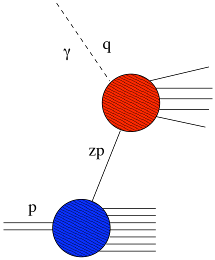
Bjorken scaling:

$$Q^2 \rightarrow \infty\,, \text{ with } x \text{ finite :} \qquad \frac{\partial F_i(x,Q^2)}{\partial Q^2} \rightarrow 0$$

as expected for scattering on pointlike free fermions

DIS: parton model

Relativity and asymptotic freedom combine in the parton picture



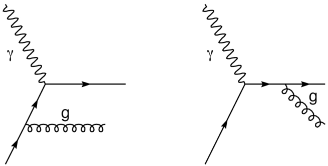
- At large Q^2, the hadron is a loosely bound collection of partons.
- Parton scatterings do not interfere.
- Each parton is characterized by a probability distribution in longitudinal momentum, $f_{q/H}(z)$.

$$\sigma(p) = \sum_q e_q^2 \int_0^1 dz\, f_{q/H}(z)\, \hat{\sigma}\,(z\,p) \quad \Rightarrow \quad F_2(x) = 2\,x\,F_1(x) = \sum_q e_q^2\, x\, f_{q/H}(x)$$

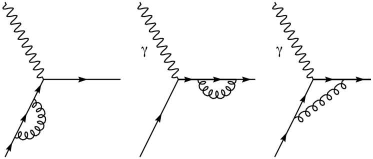
- The fast hadron is seen as a flattened disk with slowly interacting constituents.
- The effective coupling at short distances is small.

DIS: radiative corrections

The parton picture survives radiative corrections.



Real emission



Virtual corrections

- Inclusive final state: IR-C divergences cancel.
- One parton in the initial state: uncancelled collinear divergence.
  Note: it must be so: kinematics is different.
- Reabsorb collinear divergence in the parton distribution.
  Note: it is a long-distance effect!
- Parton distributions acquire scale dependence.

DIS: factorization

Factorization of initial state collinear singularities into parton distributions can be proven to all orders in perturbation theory.

- Strategies:
  - Use OPE and dispersion relations on the hadronic tensor
  - Analyze DIS on a parton, define parton-in-parton distributions, match divergences to all orders.
- Result:

$$F_2^{(H)}(x, Q^2) = \sum_a \int_x^1 d\xi \; f_{a/H}(\xi, \mu_F) \; \mathcal{F}_2^{(a)}\left(\frac{x}{\xi}, \frac{Q}{\mu_F}; \alpha_s(\mu)\right) + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

- Interpretation:
  - Parton distributions $f_{a/H}$ are universal, non-perturbative, depend on μ_F but not on Q; they must be measured.
  - Coefficient functions $\mathcal{F}_2^{(a)}$ are process-dependent, finite in perturbation theory, depend on Q; they must be computed.

Factorization and evolution

Factorizations separate dynamics at different energy scales. They lead to evolution equations. Solving evolution leads to resummations of logarithms of the ratio of scales.

- Renormalization group logarithms.

  Renormalization factorizes cutoff dependence

$$G_0^{(n)}(p_i, \Lambda, g_0) = \prod_{i=1}^{n} Z_i^{1/2}(\Lambda/\mu, g(\mu))\ G_R^{(n)}(p_i, \mu, g(\mu))\ ,$$

$$\frac{dG_0^{(n)}}{d\mu} = 0 \ \rightarrow\ \frac{d \log G_R^{(n)}}{d \log \mu} = -\sum_{i=1}^{n} \gamma_i(g(\mu))\ .$$

- Renormalization group evolution resums $\alpha_s^n(\mu^2)\log^n(Q^2/\mu^2)$ into $\alpha_s(Q^2)$, and $\log^n(s_{ij}/\mu^2)$ using anomalous dimensions γ_i.

Note: Factorization is the difficult step!

Parton evolution

- Collinear factorization logarithms.

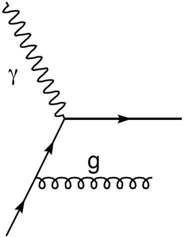
  Mellin moments of partonic DIS structure functions factorize

$$\widetilde{F}_2\left(N, \frac{Q^2}{m^2}, \alpha_s\right) = \widetilde{\mathcal{F}}_2\left(N, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \widetilde{f}\left(N, \frac{\mu_F^2}{m^2}, \alpha_s\right)$$

$$\frac{d\widetilde{F}_2}{d\mu_F} = 0 \quad \rightarrow \quad \frac{d\log \widetilde{f}}{d\log \mu_F} = \gamma_N\left(\alpha_s\right) \ .$$

- Altarelli-Parisi evolution resums collinear logarithms into evolved parton distributions.
- Result: while parton distributions are not computable in perturbation theory, their scale dependence is.
- In practice: evolution kernels are the coefficients of collinear singularities in diagrams with parton splitting.

Altarelli-Parisi kernels



- The struck quark has momentum fraction z.
- Phase space integration is IR-C divergent
- The IR divergence is canceled by the virtual correction, as $z \to 1$.
- The collinear divergence gives the splitting function: it is a distribution in z.

Define a plus distribution $[g(z)]_+$ by

$$\int_0^1 dz\, f(z)\, [g(z)]_+ \equiv \int_0^1 dz \left[f(z) - f(1) \right] g(z)$$

The classic result for quark $\to$ quark splitting is then

$$P_{qq}^{(1)}(z) = C_F \left[\frac{1+z^2}{1-z} \right]_+$$

which must be generalized to all other parton $\to$ parton splittings.

Altarelli-Parisi kernels

Parton evolution acts as a matrix of kernels on parton flavors.

$$\frac{\partial q_f(z,Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi}\int_z^1 \frac{dy}{y}\left[P_{qq}\left(\frac{z}{y},\alpha_s(\mu)\right) q_f(y,Q^2) + P_{qg}\left(\frac{z}{y},\alpha_s(\mu)\right) g(y,Q^2)\right]$$

$$\frac{\partial g(z,Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi}\int_z^1 \frac{dy}{y}\left[P_{gq}\left(\frac{z}{y},\alpha_s(\mu)\right) \sum_f q_f(y,Q^2) + P_{gg}\left(\frac{z}{y},\alpha_s(\mu)\right) g(y,Q^2)\right]$$

Splitting functions are easily computed at leading order

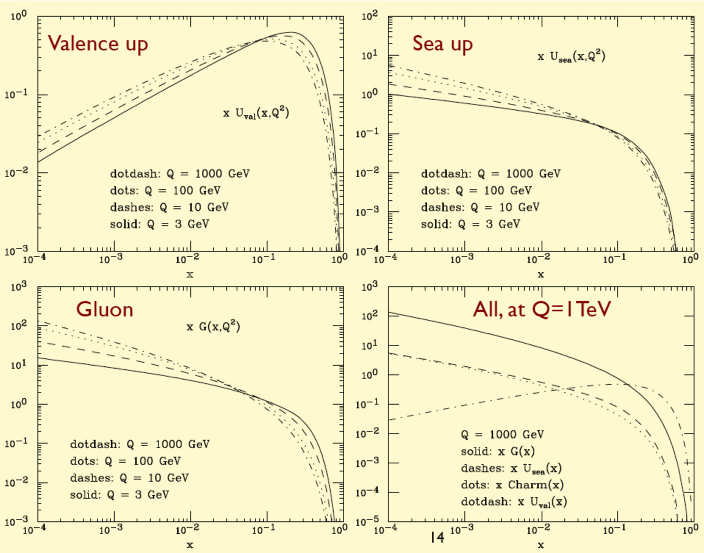
$$P_{qg}^{(1)}(z) = \frac{1}{2}\left(z^2 + (1-z)^2\right) , \quad P_{gq}^{(1)}(z) = \frac{1}{2}\left(\frac{1+(1-z)^2}{z}\right) ,$$

$$P_{gg}^{(1)}(z) = 2C_A\left(\frac{z}{[1-z]_+} + \frac{1-z}{z} + z(1-z)\right) + \delta(1-x)\left(\frac{11C_A - 2n_f}{6}\right) .$$

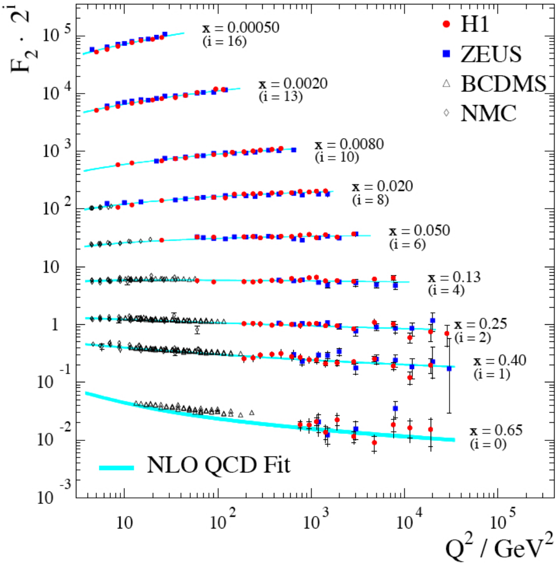
Their Mellin moments are the anomalous dimensions $\gamma_N(\alpha_s)$

`Note`: Splitting functions are known to three loops (!)

PDF's and their evolution



DIS: a success story

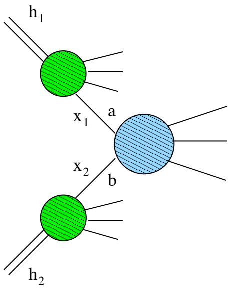


Factorization for hadron colliders

A factorization formula for hadron-hadron scattering replicates the reasoning of DIS, with two partons in the initial state.

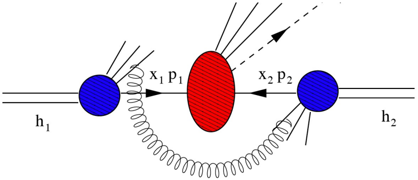
$$\sigma_H(S,Q^2) = \sum_{a,b} \int_0^1 dx_1\, dx_2\, f_{a/h_1}(x_1,\mu_F)\, f_{b/h_2}(x_2,\mu_F)\, \widehat{\sigma}_P^{a,b}\left(x_1 x_2 S, Q^2, \mu_F\right)$$

The universality of $f_{a/h}$, with computable μ_F dependence, suggests a strategy.



- Choose a factorization scheme.
- Compute $\widehat{\sigma}_P^{a,b}(\mu_0)$ for process A.
- Measure $\sigma_H(Q \sim \mu_0)$ for process A.
- Determine $f_{a/h}(\mu_0)$.
- Evolve $f_{a/h}(\mu)$ to the scale μ_1.
- Compute $\widehat{\sigma}_P^{a,b}(\mu_1)$ for process B.
- Predict $\sigma_H(Q \sim \mu_1)$. for process B.

Factorization for hadron colliders?



- Do soft gluons rearrange partons before the collision?
- Is pdf universality lost?
- Are there uncancelled IR divergences?

$$A^\mu = \frac{(1,0,0,v)}{\left[(z-vt)^2 + (1-v^2)(x^2+y^2)\right]^{1/2}}$$

- As $v \to 1$, A^μ does not vanish! However, $A_\mu \propto \partial_\mu \log|z - vt|$
- A^μ is a pure gauge, $F_{\mu\nu}$ vanishes as $v \to 1$, except at $z = t$.
- Factorization proofs are hard for hadron- hadron scattering: need to enforce gauge invariance.
- Uncancelled IR divergences are suppressed by Λ^2/Q^2.

Electroweak annihilation

Annihilation of QCD partons into electroweak final states is of great interest and widely studied.

- Clean ($q\bar{q} \to \mu^+\mu^-$) or interesting ($gg \to \mathrm{Higgs}$) final state.
- Relatively simple computationally.
  - Completes the 'trio' of processes with an electroweak side.
  - No initial-final state interference ('few' QCD legs).
- Therefore: computed to high accuracy: *NNLO* QCD, *NNLL* soft resummation available.
- Many interesting physics measurements.
  - Main W, Z production channel (possible luminometry).
  - Dominant Higgs production channel (via top loop).
  - Useful to constrain pdf's: typically up/down from $W^\pm$ production asymmetries.
  - Access new physics channels: heavy gauge bosons, contact interactions, Kaluza-Klein modes ...

EWA kinematics

Assume you require the production of an electroweak state $\mathcal{S}$ of mass Q^2. At Born level

$\mathcal{S}$ is produced by partons	$Q^2 = \hat{s} = x_1 x_2 s$
$\mathcal{S}$ is moving in hadronic CM	$Q^\mu_{\rm cm} = \left((x_1 + x_2)\sqrt{s}, 0, 0, (x_1 - x_2)\sqrt{s}\right)$
Measure the rapidity y of $\mathcal{S}$	$y = \frac{1}{2}\log\frac{Q^0_{\rm cm}+Q^3_{\rm cm}}{Q^0_{\rm cm}-Q^3_{\rm cm}} = \frac{1}{2}\log\frac{x_1}{x_2}$
or the pseudorapidity η	$\eta = -\log\tan\frac{\theta_{\rm cm}}{2}$
Parton momentum fractions are then fixed	$x_1 = \sqrt{\frac{Q^2}{s}}\mathrm{e}^{y}\,,\ x_2 = \sqrt{\frac{Q^2}{s}}\mathrm{e}^{-y}$

The rapidity distribution of the state $\mathcal{S}$ gives direct access to parton distributions at correlated values of momentum fraction.

EWA: tree level

The classic result for the parton model Drell-Yan cross section is

$$Q^2 \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{3N_c s} \sum_q e_q^2 \int_0^1 \frac{dx_1}{x_1} \int_0^1 \frac{dx_2}{x_2} f_{q/h_1}(x_1)\, f_{\bar{q}/h_2}(x_2)\, \delta\left(1 - \frac{Q^2}{x_1 x_2 s}\right)$$

at fixed rapidity, defining $\tau = Q^2/s$

$$Q^2 \frac{d^2\sigma}{dQ^2 dy} = \frac{4\pi\alpha^2}{3N_c s} \sum_q e_q^2\; f_{q/h_1}\left(\sqrt{\tau}\, \mathrm{e}^{y}\right)\; f_{\bar{q}/h_2}\left(\sqrt{\tau}\, \mathrm{e}^{-y}\right)\; .$$

The W production cross section at LHC is similarly given by

$$\sigma\,(pp \to W) = \frac{\pi\tau}{m_W^2} \sum_{ab} K_{ab} \int_\tau^1 \frac{dx}{x}\, f_{a/p}(x)\, f_{b/p}\left(\frac{\tau}{x}\right) \equiv \frac{\pi}{m_W^2} \sum_{ab} K_{ab}\, \tau\, L_{ab}(\tau)$$

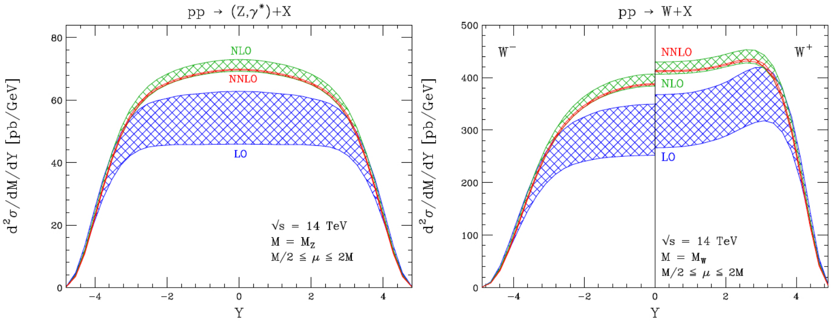
Substituting a typical small-x behavior $f_{a/p}(x) \sim x^{-1-\delta}$ one finds that σ grows at least as $\log s$.

Higher orders: status

Inclusive QCD cross sections which are electroweak at tree level are known to great accuracy.

- DIS structure functions: the best-known observable in PQCD.
  - Analytic result at three loops (N^3LO).
  - Soft gluons corrections resummed at NNLL ('almost' N^3LL).
  - Solid results on power corrections ($\mathcal{O}(\Lambda^2/Q^2)$ terms).
- e^+e^- annihilation: complex observables, hard calculations.
  - Total cross section ($R_{e^+e^-}$) known to four loops.
  - Event shapes distributions known at NNLO (numerically).
  - Soft gluon resummation at NLL.
  - Power corrections ($\mathcal{O}(\Lambda/Q)$!) important and well studied.
- Electroweak annihilation
  - Inclusive cross sections known at NNLO.
  - Soft gluon effects at NNLL. Power corrections at $\mathcal{O}(\Lambda^2/Q^2)$.
  - `New!` Exclusive distributions available at NNLO.

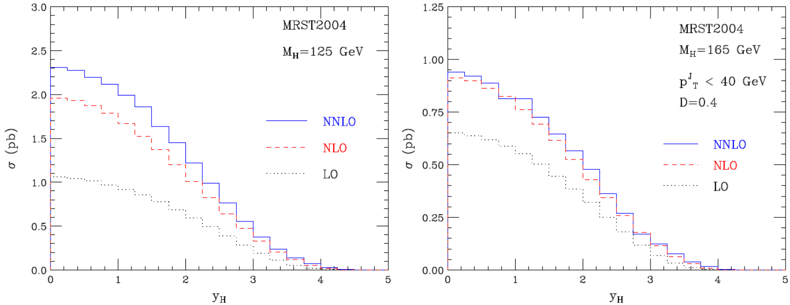
Drell-Yan: rapidity distribution



NNLO rapidity distributions for Z, $W^{\pm}$ production at LHC (Anastasiou et al.).

- Even for inclusive σ's, $50 - 100\%$ QCD corrections are common.
- K-factors are not factors in general.
- Theoretical uncertainties are greatly reduced.

Higgs production: jet veto



NNLO rapidity distributions for Higgs production at LHC, without and with jet veto (Catani, Grazzini).

- QCD corrections over 100% at central rapidity (not a *K*-factor).
- Jet veto selects Higgs from QCD background in *WW* decays.
- QCD corrections are reduced with jet veto.

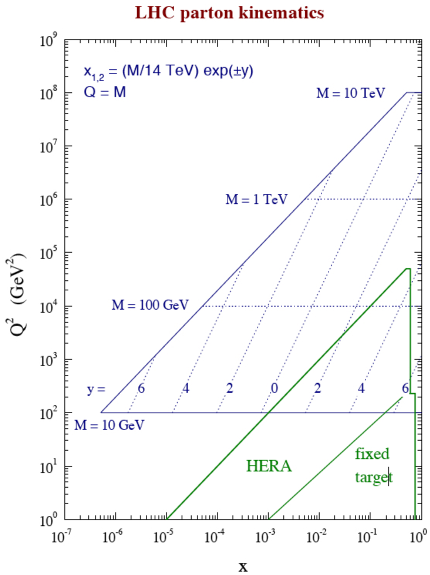
Pointers to special topics

Parton Distribution Factories

The determination of PDF's: a near-industrial effort.

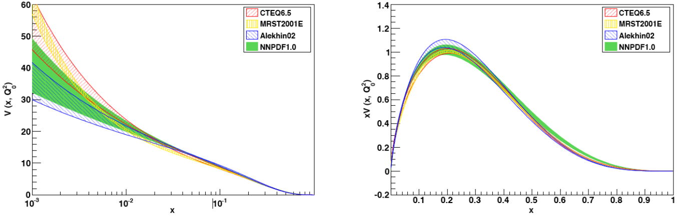
- Strategy: global fits. Consider data from many different QCD processes, machines, experiments.
  - Data: DIS (γ, ν); Drell-Yan; prompt photon; jet production ...
  - Positive: constraining; processes select parton combinations.
  - Negative: must combine errors, data sets are incompatible.
- Method: constrained parametrizations.
  - Select a functional form: $f_{a/h}(x, Q_0^2) = x^\alpha (1-x)^\beta P(x, \gamma_i)$.
  - Impose symmetry and dynamical constraints, sum rules ...
  - Fit to data with selected accuracy in PQCD (LO, NLO, ...)
  - Apply precise evolution code.
- Players: CTEQ, MRST $\rightarrow$ MSTW, NNPDF, Alekhin, Zeus, ...
- PDF uncertainties: a difficult statistical problem.
  - Collaborations provide multiple sets; need inflated χ^2.
  - Radical approach by NNPDF: Monte Carlo replicas, neural network parametrization.

The reach of LHC

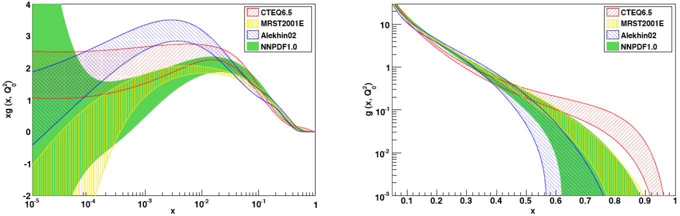


- Large mass states are made at large x and central rapidities.
- Small x means limited Q^2.
- Altarelli-Parisi evolution is up, feeding from the left.
- Precise evolution codes are needed.
- LHC will measure PDF's on its own.

Parton distributions: a sample

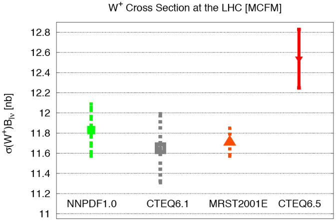


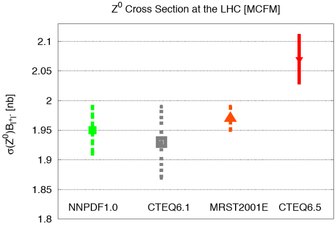
Valence quark PDF's with uncertainties, log and linear scale (NNPDF)



Gluon PDF's with uncertainties, log and linear scale (NNPDF)

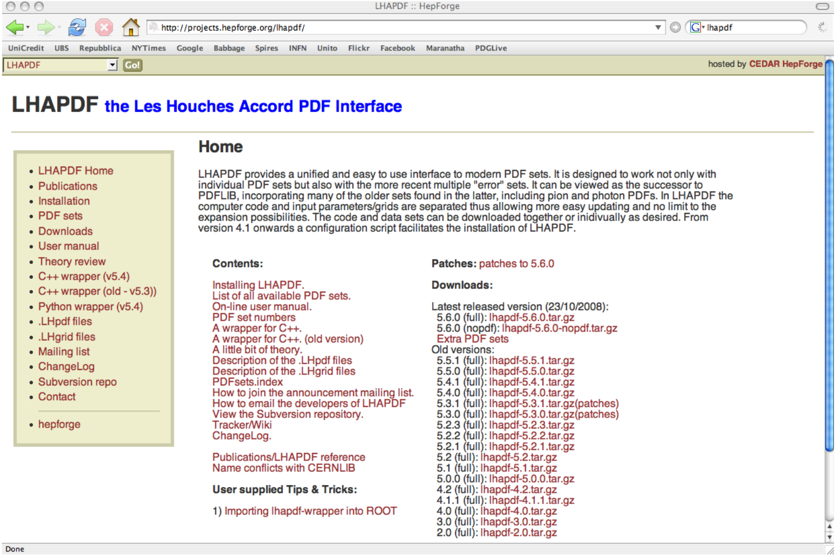
Caveat emptor





- PDF sets used to compute standard candle cross sections: W and Z production, with PDF uncertainties.
- At LHC, expected uncertainties: a few percent.
- A technical change by CTEQ in the treatment of quark mass thresholds (“ZM−VFS” $\longrightarrow$ “GM−ACOT”) moved the cross section by 2.5σ.
- Explanation: smaller heavy quark PDF’s by sum rules imply larger light quark PDF’s (which make W’s).
- More recent MRST fit reported to be close to high value of CTEQ.
- NNPDF expected to catch up after move to “GM−ACOT”.

A parton distribution interface



LHAPDF the Les Houches Accord PDF Interface

- LHAPDF Home
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- C++ wrapper (old - v5.3))
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Home

LHAPDF provides a unified and easy to use interface to modern PDF sets. It is designed to work not only with individual PDF sets but also with the more recent multiple "error" sets. It can be viewed as the successor to PDFLIB, incorporating many of the older sets found in the latter, including pion and photon PDFs. In LHAPDF the computer code and input parameters/grids are separated thus allowing more easy updating and no limit to the expansion possibilities. The code and data sets can be downloaded together or inidivually as desired. From version 4.1 onwards a configuration script facilitates the installation of LHAPDF.

Contents:

Installing LHAPDF.
List of all available PDF sets.
On-line user manual.
PDF set numbers
A wrapper for C++.
A wrapper for C++. (old version)
A little bit of theory.
Description of the .LHpdf files
Description of the .LHgrid files
PDFsets.index
How to join the announcement mailing list.
How to email the developers of LHAPDF
View the Subversion repository.
Tracker/Wiki
ChangeLog.

Publications/LHAPDF reference
Name conflicts with CERNLIB

User supplied Tips & Tricks:

1) Importing lhapdf-wrapper into ROOT

Patches: patches to 5.6.0

Downloads:

Latest released version (23/10/2008):
5.6.0 (full): lhapdf-5.6.0.tar.gz
5.6.0 (nopdf): lhapdf-5.6.0-nopdf.tar.gz
Extra PDF sets
Old versions:
5.5.1 (full): lhapdf-5.5.1.tar.gz
5.5.0 (full): lhapdf-5.5.0.tar.gz
5.4.1 (full): lhapdf-5.4.1.tar.gz
5.4.0 (full): lhapdf-5.4.0.tar.gz
5.3.1 (full): lhapdf-5.3.1.tar.gz(patches)
5.3.0 (full): lhapdf-5.3.0.tar.gz(patches)
5.2.3 (full): lhapdf-5.2.3.tar.gz
5.2.2 (full): lhapdf-5.2.2.tar.gz
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3.0 (full): lhapdf-3.0.tar.gz
2.0 (full): lhapdf-2.0.tar.gz

Order by order: LO

or: when is a problem 'solved'?

Computing tree amplitudes in gauge theories is a nontrivial problem.

Njets	2	3	4	5	6	7	8
# diag's	4	25	220	2485	34300	$5\text{x}10^5$	10^7

Quantum number management helps.

$$\mathcal{A}^{\text{tree}}(1,2,\ldots,n) = g^{n-2} \sum_{\text{ncp}} \text{Tr}(T_{a_1} T_{a_2} \ldots T_{a_n})\, A^{\text{tree}}(1,2,\ldots,n)$$

$$A^{\text{tree}}(-,-,+,\ldots,+) = \mathrm{i}\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

The problem has a recursive solution.

- Berends-Giele recursion relations 20 years old and still fastest.
- Twistor-inspired methods lead to new insights, new recursions (BCFW).
- Factorial complexity degraded to power law: $t_n \sim n^4$.

Order by order: NLO

light after the bottlenecks

- Bottleneck #1: computing loop integrals
  - Obstacles: analytic structure; tensor integral decomposition.
  - State of the art: generic 5-points 'standard', 6-points 'frontier'.
  - Spectacular progress with twistor-inspired + unitarity techniques. For gluons: factorial complexity degraded to power law: $t_n \sim n^9$.
- Bottleneck #2: subtracting IR-C poles
  - Combine $(n+1)$-parton trees with n-parton one-loop amplitudes.
  - Compute singular phase-space integrals for generic observables.
  - General methods exist: slicing, subtraction, dipole subtraction.
- Bottleneck #3: interfacing with shower MC's
  - Practical usage of a theory calculation requires four steps. `ME` $\longrightarrow$ `generator` $\longrightarrow$ `shower` $\longrightarrow$ `hadronization MC`
  - New problem at NLO: double counting of first IR-C emission.
  - Methods available (`MC@NLO`, `POWHEG` ...), implementation in progress.

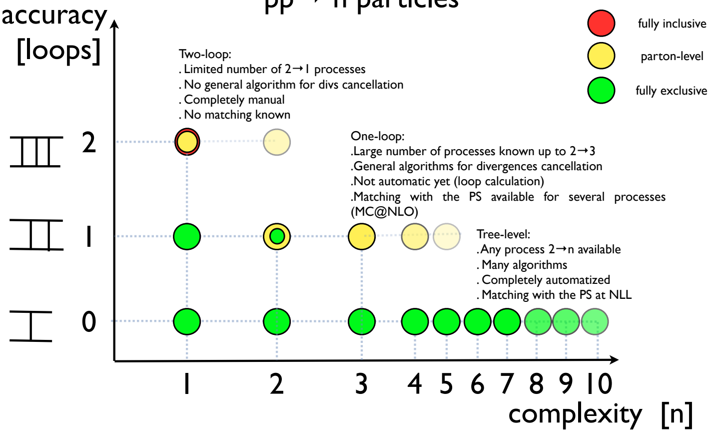
Order by order: NNLO

deep in the dark bottlenecks

- Bottleneck #1: computing loop integrals
  - Obstacles: analytic structure; tensor integral decomposition; a basis of scalar integrals is not known.
  - State of the art: only 'nearly massless' virtual 4-point amplitudes computed (ingredients for NNLO jets).
  - Only fully inclusive quantities with one particle in final state are computed at NNLO.
- Bottleneck #2: subtracting IR-C poles
  - Combine $(n+2)$-parton trees, $n+1$-parton one-loop amplitudes, n-parton two-loop amplitudes.
  - Several groups working on a general subtraction method.
  - Only one calculation completed to date: NNLO $e^+e^- \to 3\text{jets}$.
- Bottleneck #3: interfacing with shower MC's
  - `Hic sunt leones`.

Status

pp→ n particles



All orders: the boundaries of PQCD

Multi–scale problems can have large perturbative corrections of the general form $\alpha_s^n \log^k \left(Q_i^2 / Q_j^2 \right)$, with $k \leq n$ (single logs) or $k < 2n$ (double logs). Examples include

- Renormalization logs: $\alpha_s^n \log^n \left(Q^2 / \mu_R^2 \right)$.
- Collinear factorization logs: $\alpha_s^n \log^n \left(Q^2 / \mu_F^2 \right)$.
- High-energy logs: $\alpha_s^n \log^{n-2} \left(s/t \right)$.
- Sudakov logs in DIS : $\alpha_s^n \log^{2n-1} \left(Q^2 / W^2 \right)$.

  in EWA processes: $\alpha_s^n \log^{2n-1} \left(1 - Q^2 / \hat{s} \right)$.
- Transverse momentum logs: $\alpha_s^n \log^{2n-1} \left(Q_\perp^2 / Q^2 \right)$.

`Note:` Sudakov logs originate from mass singularities: they are universal and can/must be resummed.

Beyond the boundaries of PQCD

- Factorization theorems apply up to non-perturbative corrections suppressed by $\mathcal{O}\left((\Lambda/Q)^p\right)$.

  `Impact:` p is important to validate perturbative calculations.

- In the presence of several hard scales, power corrections can be enhanced (the smallest scale dominates).

  `Example:` DIS as $x \sim 1 \quad \Rightarrow \quad \mathcal{O}\left(\Lambda^2/\left(Q^2(1-x)\right)\right)$.

- Power corrections can affect phenomenology, even at LHC.

  `Compare:` compete with NLO (at LEP) or NNLO (at LHC) perturbative corrections.

- All-order results in perturbation theory encode information on the parametric size of power corrections.

  `Techniques:` OPE, Renormalons, Sudakov resummations.

Sudakov resummation: facts

The problem: a large Sudakov logarithm L implies an expansion in powers of $\alpha_s L^2$, valid only if $\alpha_s L^2 \ll 1$.

The answer: Sudakov logarithm can be computed to all orders in perturbation theory: they exponentiate.

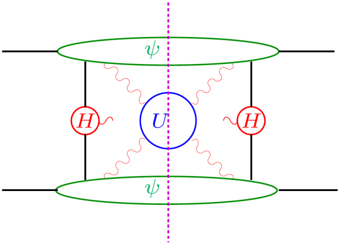
Some facts about the resummation:

- Non–trivial. Reorganizes perturbation theory in a predictive way.

$$\sum_k \alpha_s^k \sum_p^{2k} c_{kp} L^p \to \exp\left[\sum_k \alpha_s^k \sum_p^{k+1} d_{kp} L^p\right] = \exp\left[L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s\, g_3(\alpha_s L) + \ldots\right]$$

- Predictive. With NLL resummation $\alpha_s \ll 1$ suffices to apply perturbative methods. Scale dependence is reduced.
- Widespread. NLL available for main inclusive cross sections at colliders (NNLL for processes which are EW at tree level).
- Non–perturbative aspects of QCD become accessible. Integrals in the exponent run into the Landau pole.

Sudakov resummation: EWA



Threshold logarithms:

$$z = Q^2/\hat{s} \to 1$$

$$\left[\frac{\log^p(1-z)}{1-z}\right]_+ \to \log^{p+1} N$$

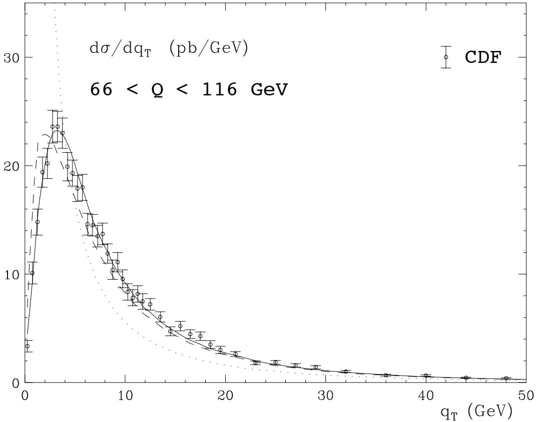
Factorization leads to resummation:

$$\omega(N,\epsilon) = |H_{\rm DY}|^2 \; \psi(N,\epsilon)^2 \; U(N) + \mathcal{O}(1/N) \;\Rightarrow$$

$$\Rightarrow \; \widehat{\omega}_{\overline{\rm MS}}(N) \;=\; \exp\left[\int_0^1 dz\, \frac{z^{N-1}-1}{1-z}\left\{2\int_{Q^2}^{(1-z)^2Q^2}\frac{d\mu^2}{\mu^2}\,A\left[\alpha_s(\mu^2)\right] + D\left[\alpha_s\left((1-z)^2Q^2\right)\right]\right\} + \mathcal{F}_{\overline{\rm MS}}(\alpha_s)\right] + \mathcal{O}\left(\frac{1}{N}\right).$$

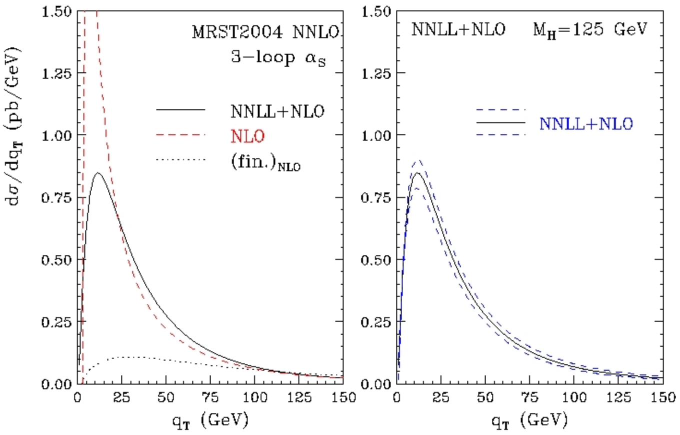
- The functions A and D are known to three loops (almost N^3LL).
- The expansion in towers of logs is well behaved to this order.

Z production at Tevatron



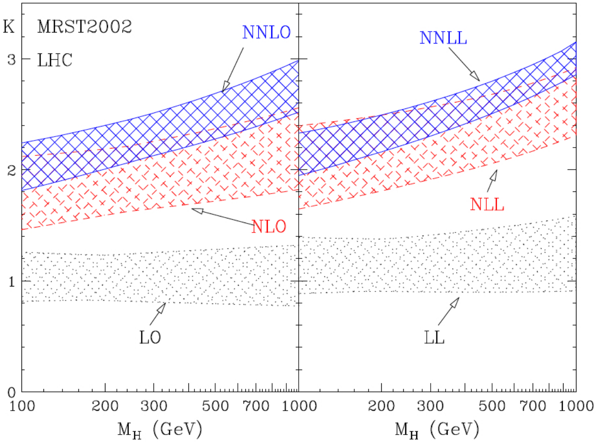
CDF data on Z production compared with QCD predictions at fixed order (dotted), with resummation (dashed), and with power corrections (solid) (A. Kulesza et al.).

Higgs production at LHC



Predictions for the q_T spectrum of Higgs bosons produced via gluon fusion at the LHC, with and without resummation (M. Grazzini).

Higgs production at LHC



Fixed-order and resummed K-factors for Higgs production at the LHC (S. Catani and M. Grazzini).

Jets at Tevatron and LHC

- Jets are ubiquitous at hadron colliders

  $\longrightarrow$ the most common high-p_t final state

- Jets need to be understood in detail

  $\longrightarrow$ top mass, Higgs searches, QCD studies, new particle cascades

- Jets at LHC will be numerous and complicated

  $\longrightarrow$ $t\bar{t}H \rightarrow 8\,\mathrm{jets}$ ... , underlying event, pileup ...

- Jets are inherently ambiguous in QCD

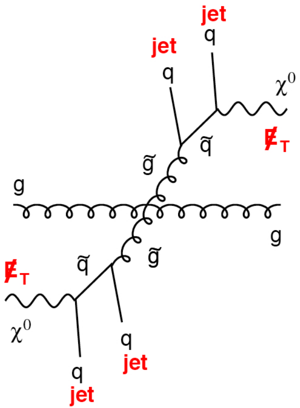
  $\longrightarrow$ no unique link hard parton $\rightarrow$ jet

- Jets are theoretically interesting

  $\longrightarrow$ IR/C safety, resummations, hadronization ...

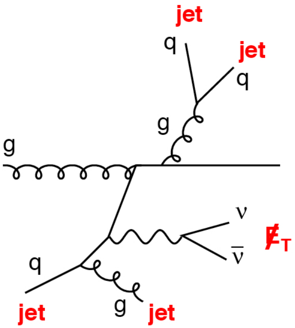
Signal and background jets

Signal



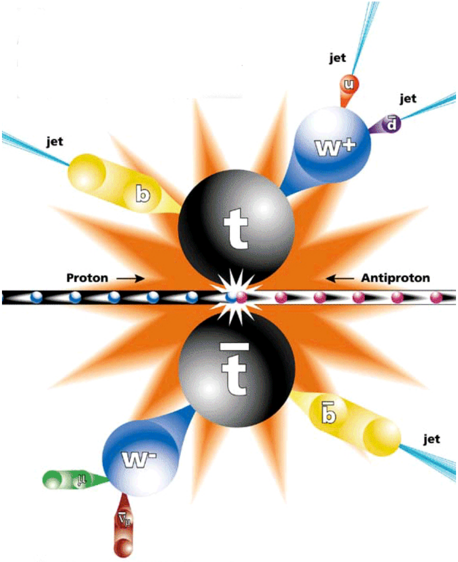
Generic SUSY cascade event

Background

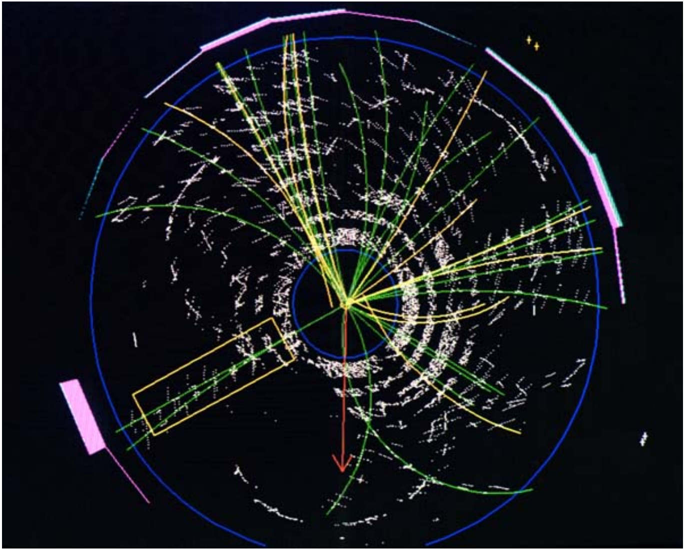


Standard model event with same signature

$t\bar{t} \rightarrow 4\,\mathrm{jets} + \mathrm{lepton} + \not{E}_t$: a cartoon



$t\bar{t} \rightarrow 4\,\text{jets} + \text{lepton} + \not{E}_t$: real life at CDF



From hard partons to jets

Hard scattering provides us with high-p_t partons initiating the jets. Jet momenta receive several PT and NP corrections.

- Perturbative radiation $+$ parton showering

  $\longrightarrow$ expensive: $5 \cdot 10^2\,\mathrm{p \cdot y} \sim \$\,5 \cdot 10^7$ at NNLO ...

- Universal hadronization, induced by soft radiation

  $\longrightarrow$ from hard scattering, as in DIS, e^+e^-

- Underlying event, colored fragments from proton remnants

  $\longrightarrow$ no perturbative control, large at LHC

- Pileup, multiple proton scatterings per bunch crossing

  $\longrightarrow$ experimental issue, up to $10^2\,\mathrm{GeV}$ per unit rapidity at LHC

Jet algorithms

- Requirements.

  IR/C safe, for theoretical stability; fast, for implementation; limited hadronization corrections.

- Algorithm structures.
  - Cone. Top-down, intuitive, Sterman-Weinberg inspired.

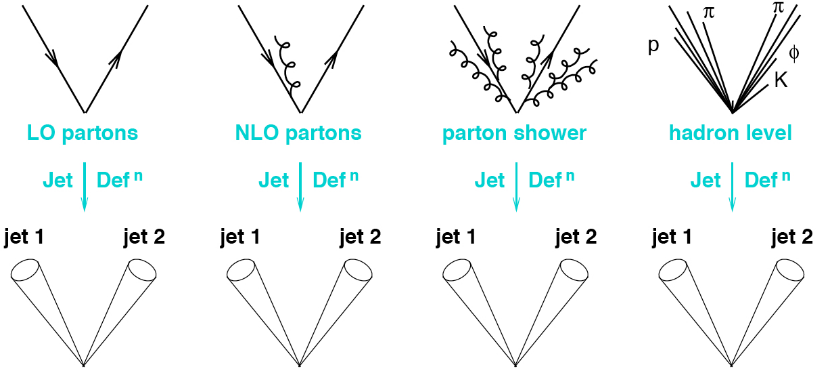
    $\longrightarrow$ IR/C safety issues $\longrightarrow$ SISCone
  - Sequential recombination. Bottom-up, clustering, adapted from e^+e^- collisions.

    Metric: $d_{ij}^{(p)} \equiv \min\left(k_{t,i}^{2p}, k_{t,j}^{2p}\right) \frac{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}{R^2}$, $d_{iB}^{(p)} \equiv k_{t,i}^{2p}$.

    Choices: $p = 1$: k_t ; $p = 0$: Cambridge ; $p = -1$: Anti-k_t .

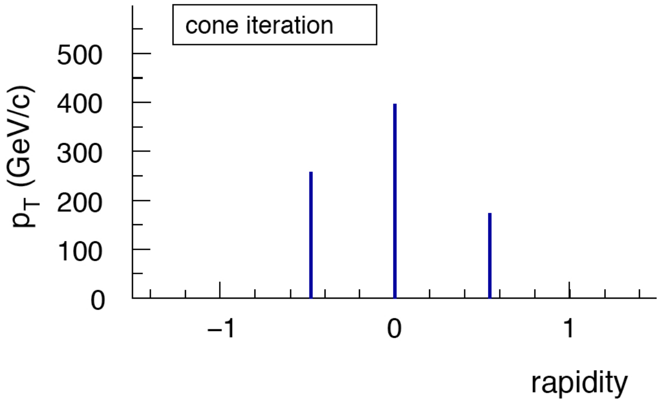
- Recent progress.
  - G. Salam *et al.*: `FastJet`, SISCone, Anti-k_t, Jet Area, Jet Flavor, Hadronization.
  - S. Ellis *et al.*: `SpartyJet`.

Stability of jet definitions



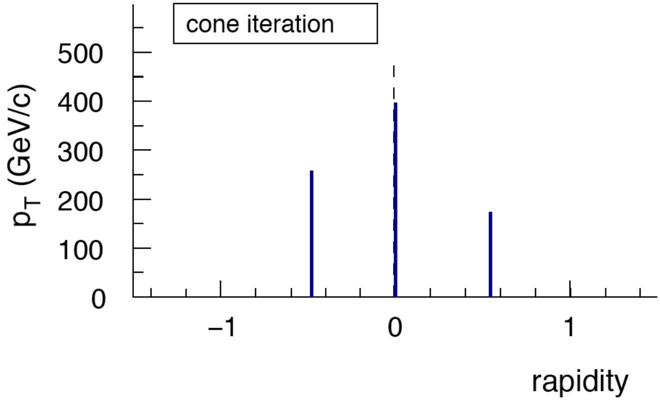
Projection to jets should be resilient to QCD effects

Safety of jet algorithms: a cartoon



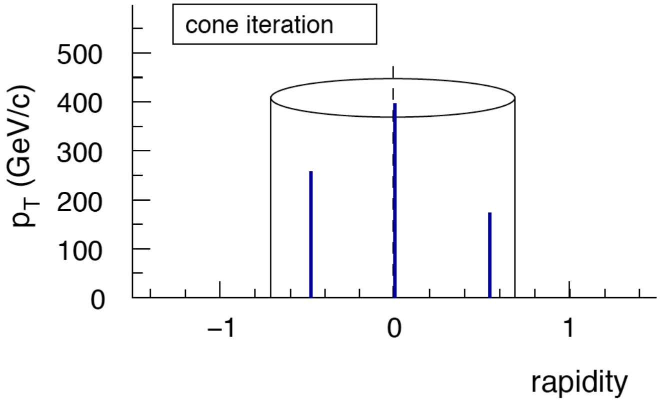
Three hard partons

Safety of jet algorithms: a cartoon



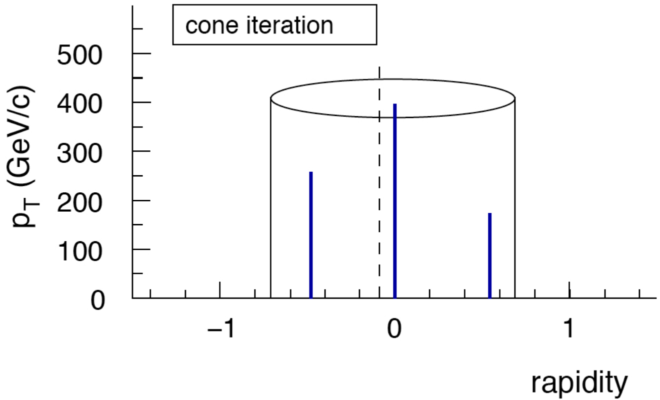
Pick the hardest as seed

Safety of jet algorithms: a cartoon



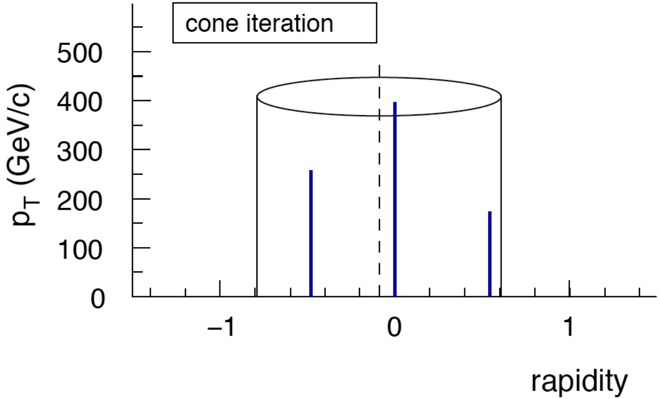
Draw a cone

Safety of jet algorithms: a cartoon



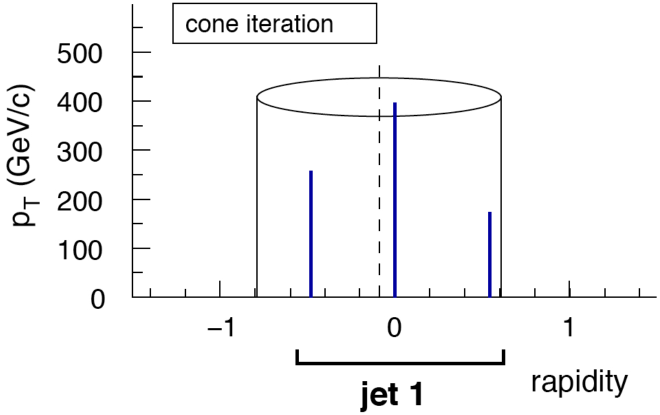
Momentum sum gives new seed

Safety of jet algorithms: a cartoon



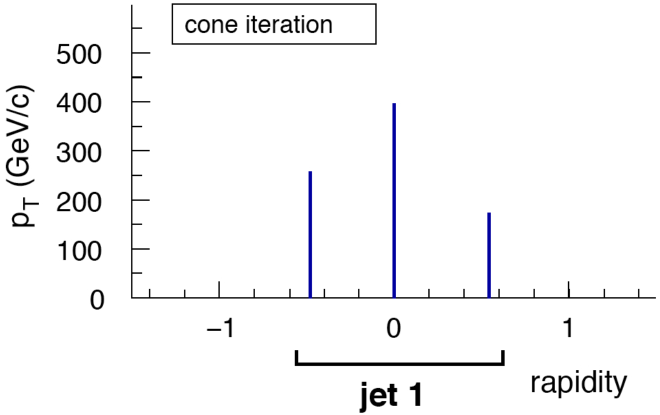
Draw a new cone

Safety of jet algorithms: a cartoon



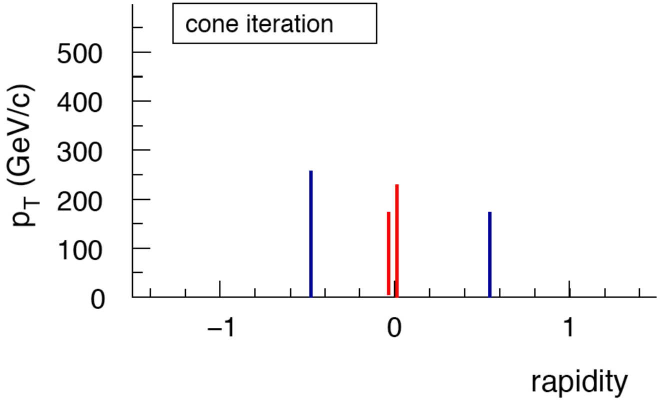
It is stable: call it a jet

Safety of jet algorithms: a cartoon



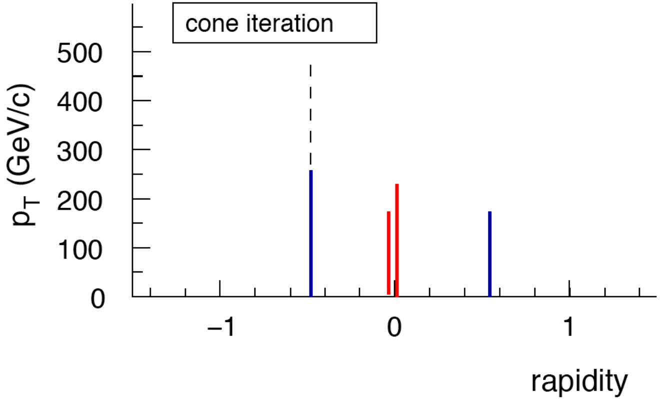
No more partons: end

Safety of jet algorithms: a cartoon



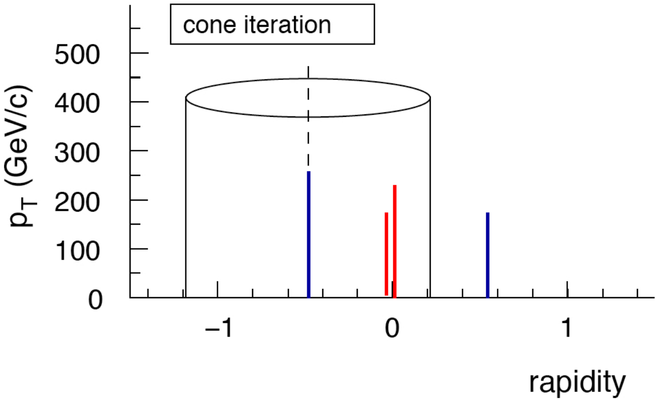
There was a collinear splitting!

Safety of jet algorithms: a cartoon



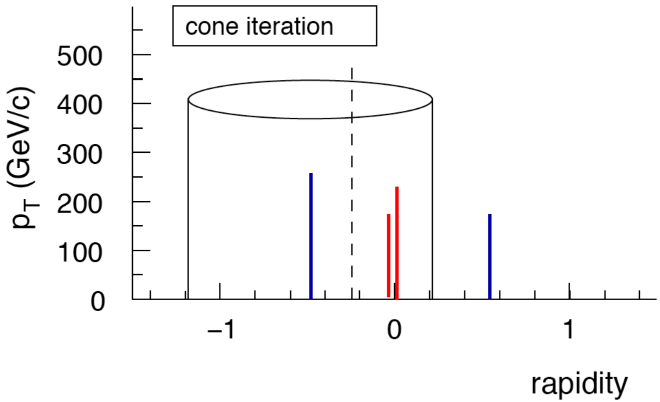
Pick the hardest as seed

Safety of jet algorithms: a cartoon



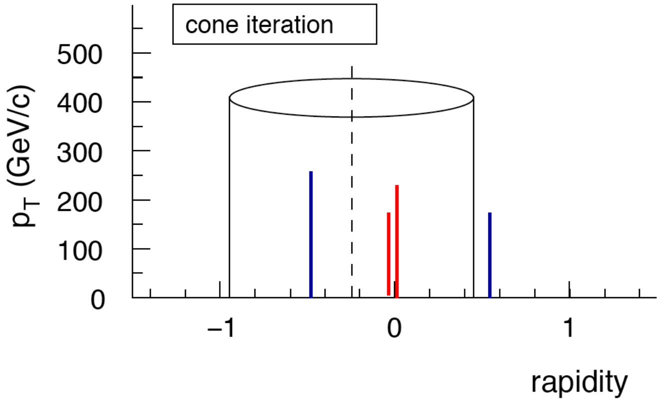
Draw a cone

Safety of jet algorithms: a cartoon



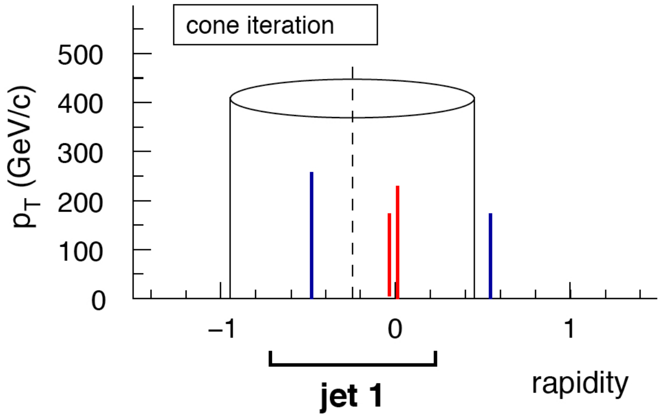
Momentum sum gives a new seed

Safety of jet algorithms: a cartoon



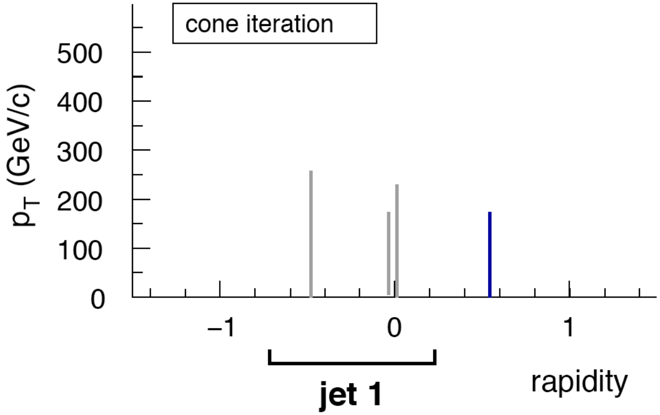
Draw a new cone

Safety of jet algorithms: a cartoon



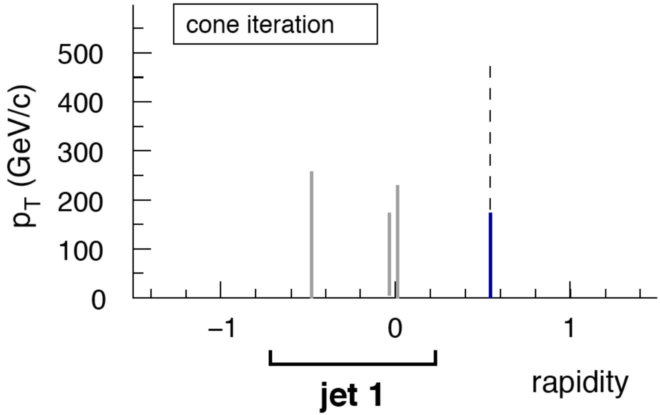
It is stable: call it a jet

Safety of jet algorithms: a cartoon



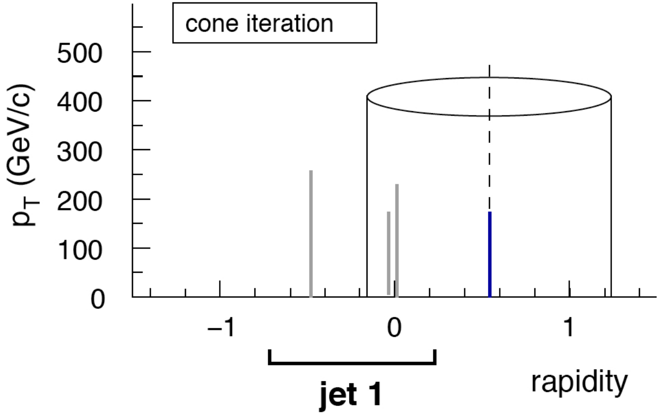
Erase the jet partons

Safety of jet algorithms: a cartoon



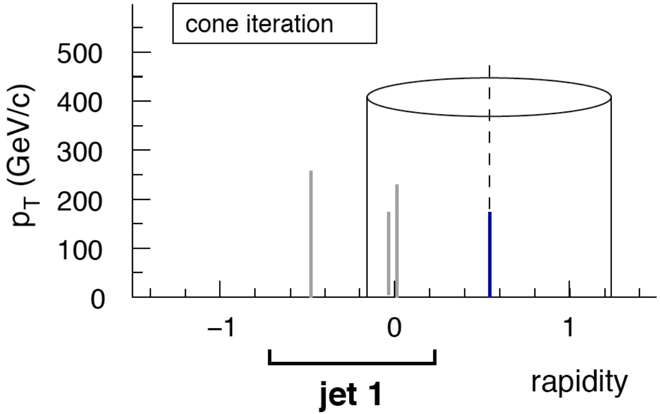
Pick the hardest remaining as seed

Safety of jet algorithms: a cartoon



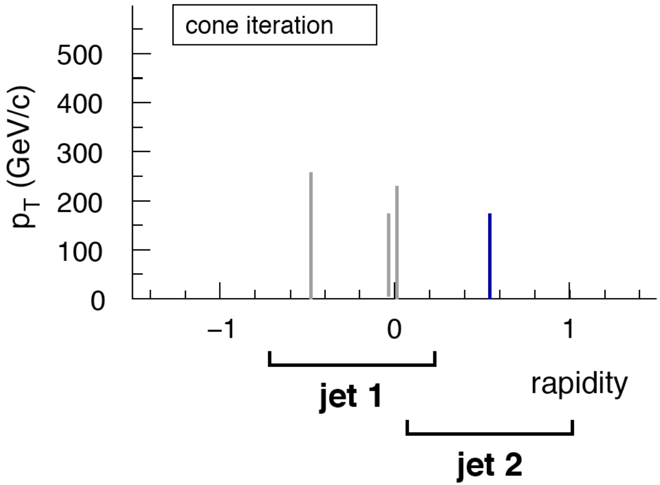
Draw a cone

Safety of jet algorithms: a cartoon



Momentum sum gives a new seed

Safety of jet algorithms: a cartoon



It is stable: call it a jet

Comparing jet algorithms

Algorithm	Type	IRC status	Ref.	Notes
inclusive k_t	$SR_{p=1}$	OK	[130–132]	also has exclusive variant
flavour k_t	$SR_{p=1}$	OK	[133]	d_{ij} and d_{iB} modified when i or j is “flavoured”
Cambridge/Aachen	$SR_{p=0}$	OK	[134, 135]	
anti-k_t	$SR_{p=-1}$	OK	[125]	
SISCone	SC-SM	OK	[128]	multipass, with optional cut on stable cone p_t
CDF JetClu	IC_r-SM	IR_{2+1}	[136]	
CDF MidPoint cone	IC_{mp}-SM	IR_{3+1}	[127]	
CDF MidPoint searchcone	$IC_{se,mp}$-SM	IR_{2+1}	[129]	
D0 Run II cone	IC_{mp}-SM	IR_{3+1}	[127]	no seed threshold, but cut on cone p_t
ATLAS Cone	IC-SM	IR_{2+1}		
PxCone	IC_{mp}-SD	IR_{3+1}		no seed threshold, but cut on cone p_t,
CMS Iterative Cone	IC-PR	$Coll_{3+1}$	[137, 138]	
PyCell/CellJet (from Pythia)	FC-PR	$Coll_{3+1}$	[85]	
GetJet (from ISAJET)	FC-PR	$Coll_{3+1}$		

A Les Houches compilation of jet algorithms, see arXiv:0803.0678.

Unsafe jet algorithms

Unsafe algorithms correspond to theoretical predictions that become meaningless beyond a given order.

$$\sigma = \sigma_0 \left(1 + c_1\alpha_s + c_2\alpha_s^2 + \dots\right) \qquad \dots \quad c_2 = \infty\,!$$

$$\sigma = \sigma_0 \left(1 + c_1\alpha_s + K \log\left(\frac{\Lambda}{Q}\right)\alpha_s^2 + \dots\right) = \sigma_0\left(1 + (c_1 + K)\alpha_s + \dots\right) .$$

IR-C sensitivity at N^pLO destroys predictivity of N^{p-1}LO calculation.

Impact depends on specific algorithm and observable.

- The inclusive jet cross section is least affected: $\delta\sigma/\sigma \sim 5\%$ comparing SIScone and Midpoint cone.
- Multi-jet observables can be severely affected.
  - $W + 2\,\mathrm{jets}$ existing NLO prediction is not applicable to Midpoint cone algorithms.
  - For jet mass studies, the overall normalization is affected.

Thanks

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Gavin Salam

Thank You for Your Attention!