

# RESUMMATIONS IN QCD

## - A PROGRESS REPORT -

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# Outline

- Introducing resummations
- Soft gluons and large logs
- Recent developments: from theory ...
- ... to phenomenology
- Outlook



# A BIT OF HISTORY





# A subject with a long history ...

JULY 15, 1937

PHYSICAL REVIEW

VOLUME 52

## Note on the Radiation Field of the Electron

F. BLOCH AND A. NORDSIECK\*

*Stanford University, California*

(Received May 14, 1937)

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External perturbations on the electron are treated in the Born approximation. It is shown that for frequencies such that the above three parameters are negligible the quantum mechanical calculation yields just the directly reinterpreted results of the classical formulae, namely that the total probability of a given change in the motion of the electron is unaffected by the interaction with radiation, and that the mean number of emitted quanta is infinite in such a way that the mean radiated energy is equal to the energy radiated classically in the corresponding trajectory.

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CERN-TH-6640-92, CAVENDISH-HEP-91-11. Nov 1992. 39 pp.

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# A FIRST LOOK





# Large logs

• **Multi-scale** problems in **renormalizable** quantum field theories have perturbative corrections of the general form  $\alpha_s^n \log^k (Q_i^2/Q_j^2)$ , which may **spoil** the reliability of the perturbative expansion.

- **Renormalization** and **factorization** logs:  $\alpha_s^n \log^n (Q^2/\mu^2)$
- **High-energy logs**:  $\alpha_s^n \log^{n-1} (s/t)$
- **Sudakov** logs:  $\alpha_s^n \log^{2n-1} (1-z)$ ,  $1-z = W^2/Q^2, 1-M^2/\hat{s}, Q_\perp^2/Q^2, \dots$

• **Sudakov** logs are **universal**: they originate from **infrared and collinear singularities**: they **exponentiate** and can be **resummed**

$$\underbrace{\frac{1}{\epsilon}}_{\text{virtual}} + \underbrace{(Q^2)^\epsilon \int_0^{m^2} \frac{dk^2}{(k^2)^{1+\epsilon}}}_{\text{real}} \implies \ln(m^2/Q^2)$$

- For **inclusive** observables: **analytic** resummation to high logarithmic accuracy.
- For **exclusive** final states: **parton shower** event generators, (N)LL accuracy.

• **Resummation** probes the **all-order structure** of perturbation theory.

- **Power-suppressed** corrections to QCD cross sections can be studied.
- Links to the **strong coupling** regime can be established for SUSY gauge theories.



# The perturbative exponent

A classic way to **organize** Sudakov logarithms is in terms of the **Mellin (Laplace) transform** of the momentum space cross section (**Catani et al. 93**),

$$\begin{aligned} d\sigma(\alpha_s, N) &= \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{k=0}^{2n} c_{nk} \log^k N + \mathcal{O}(1/N) \\ &= H(\alpha_s) \exp \left[ \log N g_1(\alpha_s \log N) + g_2(\alpha_s \log N) + \alpha_s g_3(\alpha_s \log N) + \dots \right] + \mathcal{O}(1/N) \end{aligned}$$

This displays the main **features of Sudakov resummation**

- 📌 **Predictive:** a **k**-loop calculation determines **g<sub>k</sub>** and thus a whole **tower** of logarithms to all orders in perturbation theory.
- 📌 **Effective:**
  - the **range of applicability** of perturbation theory is **extended** (finite order: **α<sub>s</sub> log<sup>2</sup>N** small. NLL resummed: **α<sub>s</sub>** small);
  - the renormalization **scale dependence** is naturally **reduced**.
- 📌 **Theoretically interesting:** resummation **ambiguities** related to the **Landau pole** give access to non-perturbative **power-suppressed corrections**.
- 📌 **Well understood:**
  - **NLL** Sudakov resummations **exist** for most **inclusive** observables at hadron colliders, **NNLL** and approximate **N<sup>3</sup>LL** in simple cases.
  - Different **'schools'** (**USA, Italian, SCET ...**) compete, complacency is not an option, active and lively debate.



# Color singlet hard scattering

A well-established formalism exists for **distributions** in processes that are **electroweak at tree level** (Gardi, Grunberg 07). For an observable **r vanishing in the two-jet limit**

$$\frac{d\sigma}{dr} = \delta(r) [1 + \mathcal{O}(\alpha_s)] + C_R \frac{\alpha_s}{\pi} \left\{ \left[ -\frac{\log r}{r} + \frac{b_1 - d_1}{r} \right]_+ + \mathcal{O}(r^0) \right\} + \mathcal{O}(\alpha_s^2)$$

The Mellin (Laplace) transform,  $\sigma(N) = \int_0^1 dr (1-r)^{N-1} \frac{d\sigma}{dr}$

exhibits **log N** singularities that can be organized in **exponential form**

$$\sigma(\alpha_s, N, Q^2) = H(\alpha_s) \mathcal{S}(\alpha_s, N, Q^2) + \mathcal{O}(1/N)$$

where the exponent of the **'Sudakov factor'** is in turn a Mellin transform

$$\mathcal{S}(\alpha_s, N, Q^2) = \exp \left\{ \int_0^1 \frac{dr}{r} \left[ (1-r)^{N-1} - 1 \right] \mathcal{E}(\alpha_s, r, Q^2) \right\}$$

and the general form of the **kernel** is

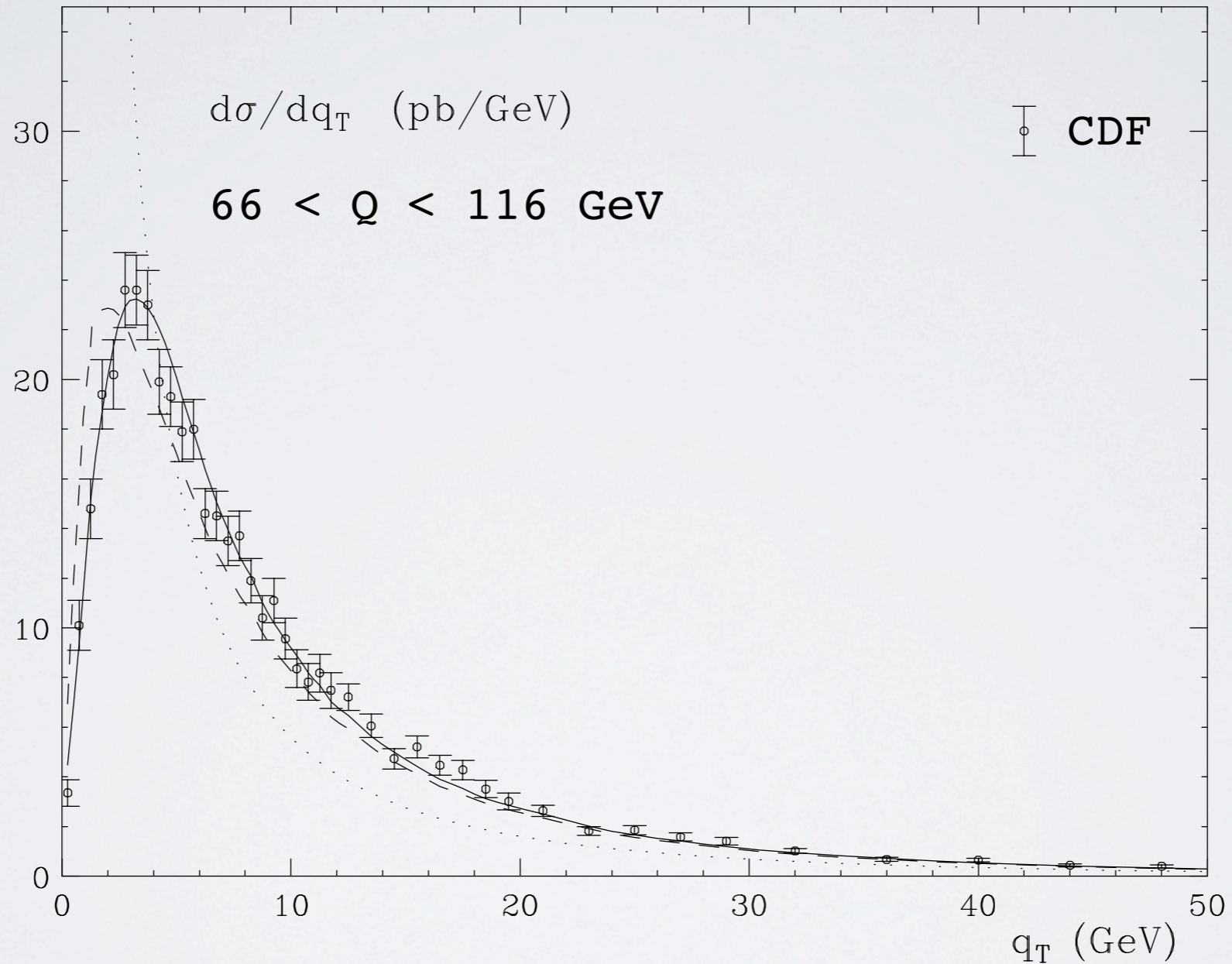
$$\mathcal{E}(\alpha_s, r, Q^2) = \int_{r^2 Q^2}^{rQ^2} \frac{d\xi^2}{\xi^2} A(\alpha_s(\xi^2)) + B(\alpha_s(rQ^2)) + D(\alpha_s(r^2 Q^2))$$

where **A** is the **cusp** anomalous dimension, and **B** and **D** have **distinct physical characters**.



# Impact of resummation

Z-boson  $q_T$  spectrum at Tevatron (Kulesza et al. 03)

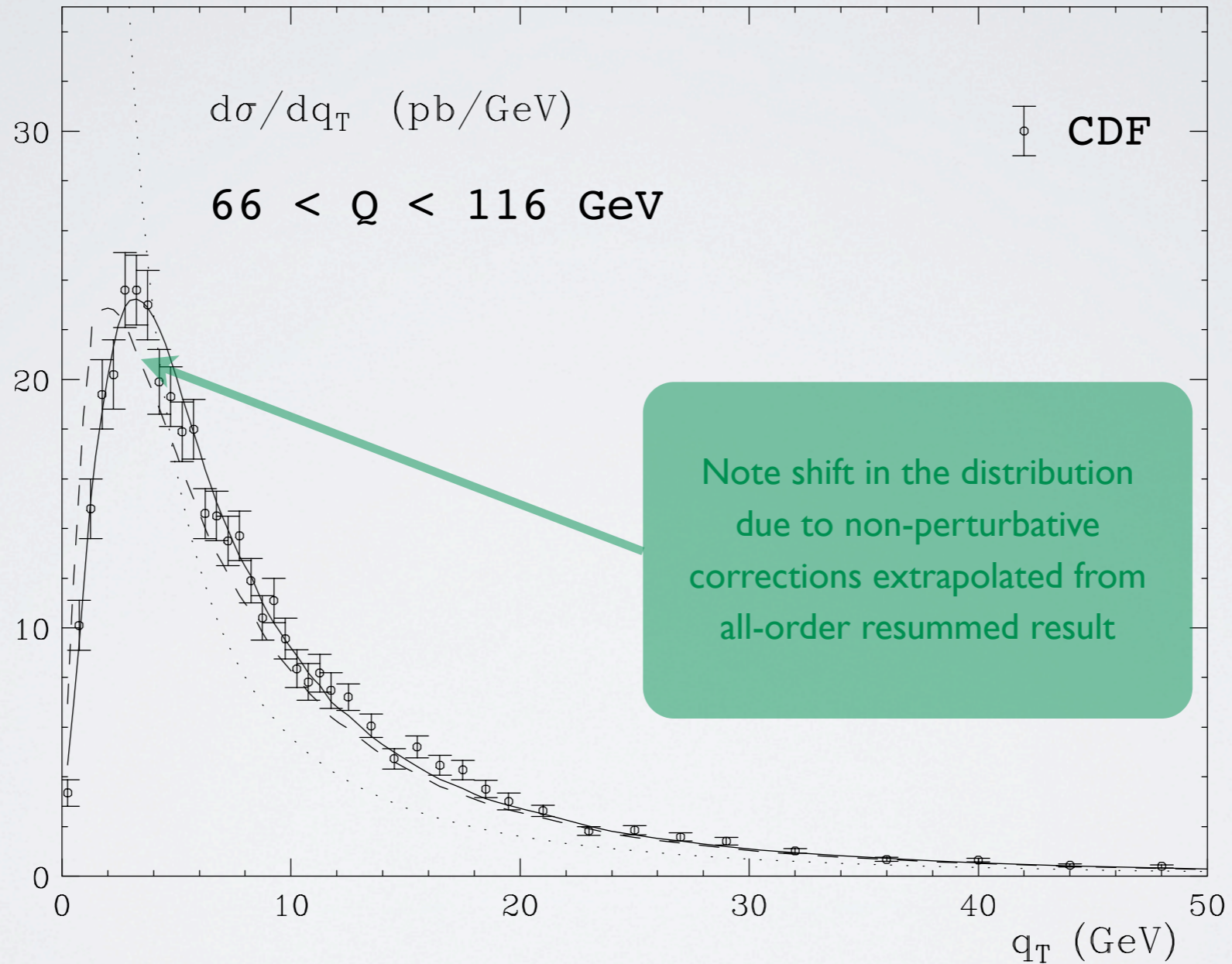


CDF data on  $Z$  production compared with QCD predictions at fixed order (dotted), with joint resummation (dashed), and with the inclusion of power corrections (solid).



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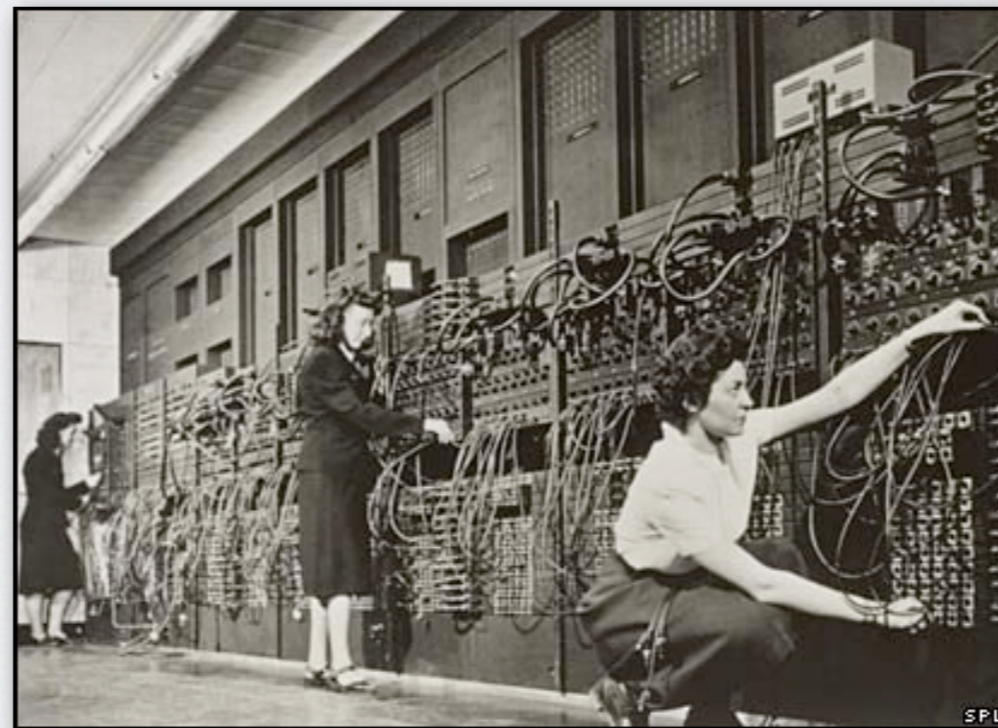
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# A SECOND LOOK





# Factorization

All factorizations separating dynamics at different energy scales lead to **resummation** of logarithms of the ratio of scales.

**Renormalization** is a textbook example.

- Renormalization **factorizes** cutoff dependence.

$$G_0^{(n)}(p_i, \Lambda, g_0) = \prod_{i=1}^n Z_i^{1/2}(\Lambda/\mu, g(\mu)) G_R^{(n)}(p_i, \mu, g(\mu))$$

- Factorization requires the introduction of an **arbitrarily chosen** scale  $\mu$ .

- Results must be **independent** of the arbitrary choice of  $\mu$ .

$$\frac{dG_0^{(n)}}{d\mu} = 0 \quad \rightarrow \quad \frac{d \log G_R^{(n)}}{d \log \mu} = - \sum_{i=1}^n \gamma_i(g(\mu)) .$$

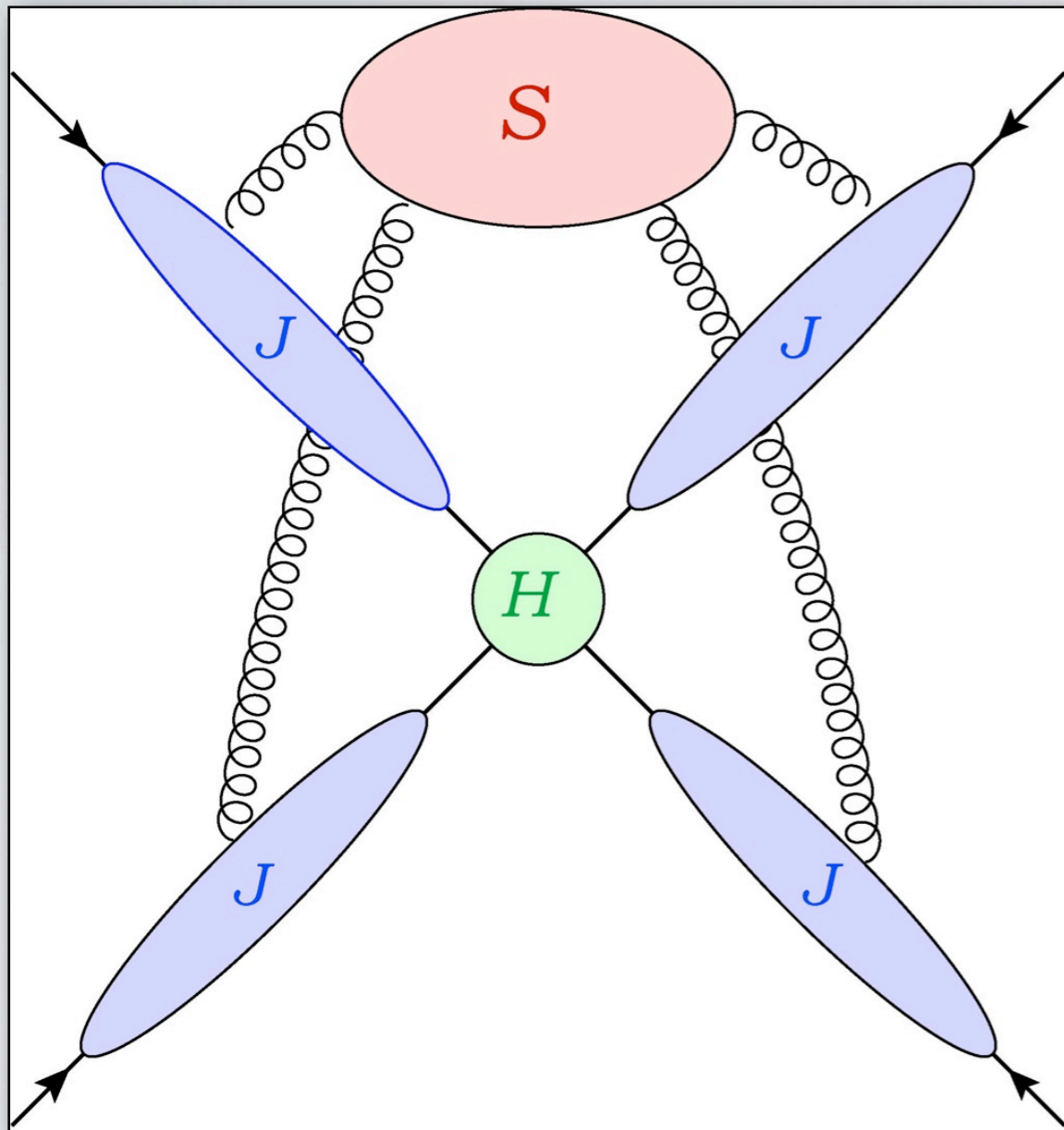
- The simple **functional dependence** of the factors is dictated by **separation of variables**.

- Proving **factorization** is the **difficult** step: it requires all-order diagrammatic analyses. **Evolution** equations **follow** automatically.

- Solving RG evolution **resums** logarithms of  $Q^2/\mu^2$  into  $\alpha_s(\mu^2)$ .



# Sudakov Factorization



Leading integration regions in loop momentum space for Sudakov factorization

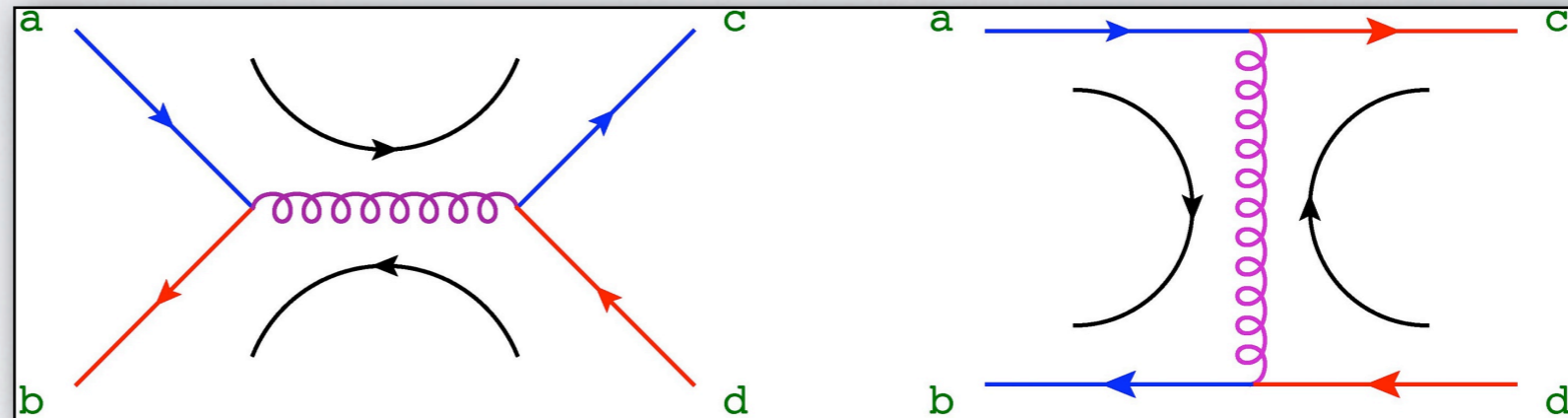
- **Sudakov logarithms** are **remainders** of infrared and collinear **divergences**.
- **Divergences** arise in **scattering** amplitudes from **leading regions** in loop momentum space.
- **Soft gluons** factorize both from **hard** (easy) and from **collinear** (intricate) virtual exchanges.
- **Jet functions J** represent **color singlet** evolution of **external** hard partons.
- The **soft function S** is a **matrix** mixing the available **color representations**.
- In the **planar limit** soft exchanges are confined to **wedges**: **S** is proportional to the **identity**.
- In the **planar limit S** can be **reabsorbed** defining **jets** as square roots of **elementary form factors**.
- **Beyond** the planar limit **S** is determined by an **anomalous dimension matrix**  $\Gamma_S$ .



# Color flow

In order to understand the **matrix structure** of the **soft function** it is sufficient to consider the simple case of **quark-antiquark** scattering.

At tree level



Tree-level diagrams and color flows for quark-antiquark scattering

For this process only **two color structures** are possible. A **basis** in the space of available color tensors is

$$c_{abcd}^{(1)} = \delta_{ab}\delta_{cd}, \quad c_{abcd}^{(2)} = \delta_{ac}\delta_{bd}$$

The **matrix element** is a **vector** in this space, and the Born cross section is

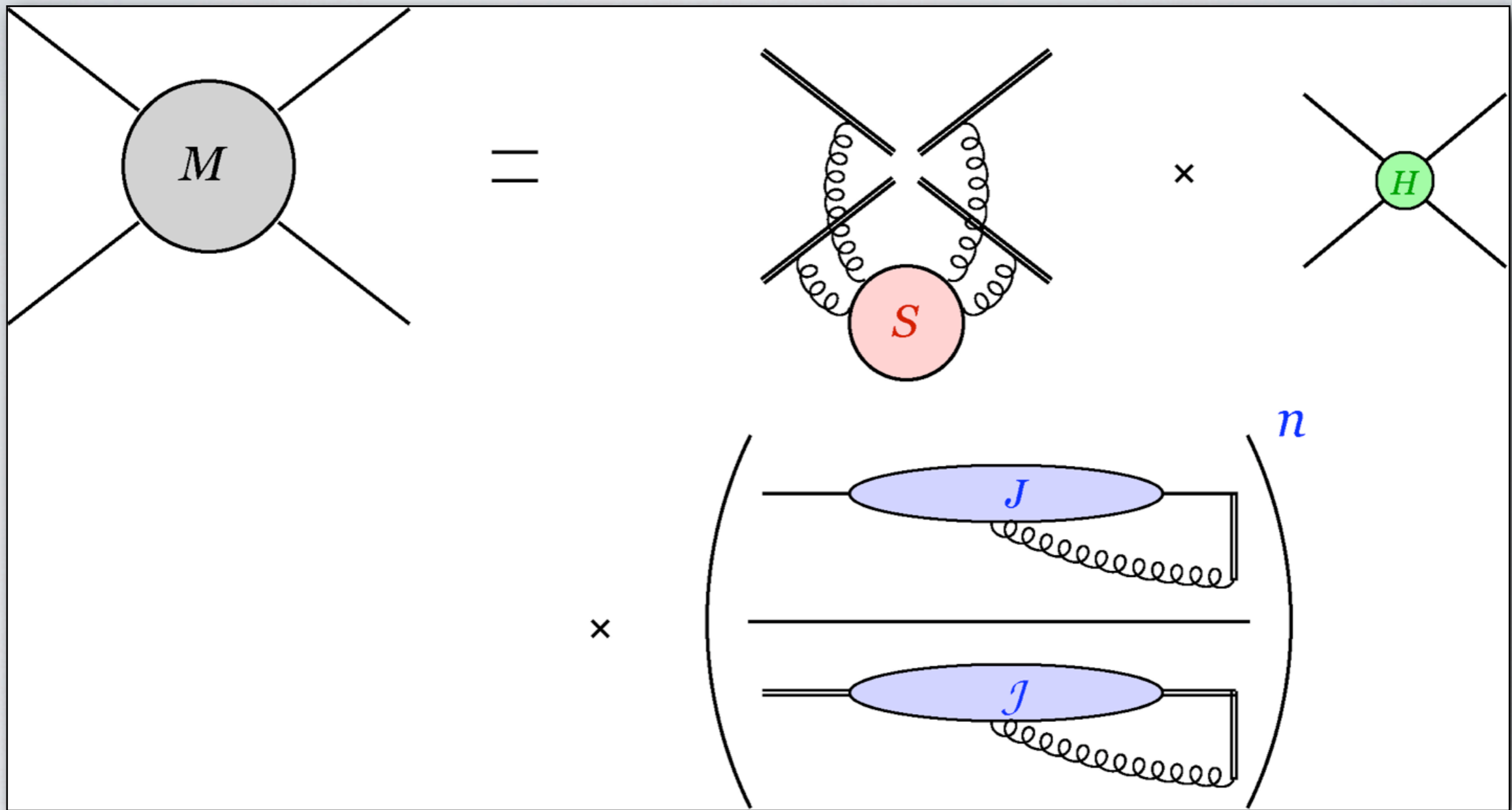
$$\mathcal{M}_{abcd} = \mathcal{M}_1 c_{abcd}^{(1)} + \mathcal{M}_2 c_{abcd}^{(2)} \longrightarrow \sum_{\text{color}} |\mathcal{M}|^2 = \sum_{J,L} \mathcal{M}_J \mathcal{M}_L^* \text{tr} \left[ c_{abcd}^{(J)} \left( c_{abcd}^{(L)} \right)^\dagger \right] \equiv \text{Tr} [HS]_0$$

A virtual **soft gluon** will **reshuffle** color and mix the components of this vector

$$\text{QED} : \mathcal{M}_{\text{div}} = S_{\text{div}} \mathcal{M}_{\text{Born}} ; \quad \text{QCD} : [\mathcal{M}_{\text{div}}]_J = [S_{\text{div}}]_{JL} [\mathcal{M}_{\text{Born}}]_L$$



# Sudakov factorization: pictorial



A pictorial representation of Sudakov factorization for fixed-angle scattering amplitudes



# Soft Matrices

The **soft function**  $\mathcal{S}$  is a **matrix**, mixing the available color tensors. It is defined by a correlator of **Wilson lines**.

$$(c_L)_{\{\alpha_k\}} \mathcal{S}_{LK}(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon) = \sum_{\{\eta_k\}} \langle 0 | \prod_{i=1}^n \left[ \Phi_{\beta_i}(\infty, 0)_{\alpha_k, \eta_k} \right] | 0 \rangle (c_K)_{\{\eta_k\}} ,$$

The soft function  $\mathcal{S}$  obeys a **matrix** RG evolution equation

$$\mu \frac{d}{d\mu} \mathcal{S}_{IK}(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon) = - \mathcal{S}_{IJ}(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon) \Gamma_{JK}^{\mathcal{S}}(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon)$$

📌  $\Gamma^{\mathcal{S}}$  is **singular** due to overlapping **UV** and **collinear** poles.

$\mathcal{S}$  is a **pure counterterm**. In dimensional regularization, using  $\alpha_s(\mu^2 = 0, \epsilon < 0) = 0$ ,

$$\mathcal{S}(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon) = P \exp \left[ -\frac{1}{2} \int_0^{\mu^2} \frac{d\xi^2}{\xi^2} \Gamma^{\mathcal{S}}(\beta_i \cdot \beta_j, \alpha_s(\xi^2), \epsilon), \epsilon \right] .$$

The determination of the **soft anomalous dimension matrix**  $\Gamma^{\mathcal{S}}$  is the **keystone** of the resummation program for multiparton **amplitudes** and **cross sections**.

- 📌 It **governs** the interplay of **color** exchange with **kinematics** in multiparton processes.
- 📌 It is the only **source** of multiparton **correlations** for singular contributions.
- 📌 **Collinear** effects are **'color singlet'** and can be extracted from **two-parton** scatterings.



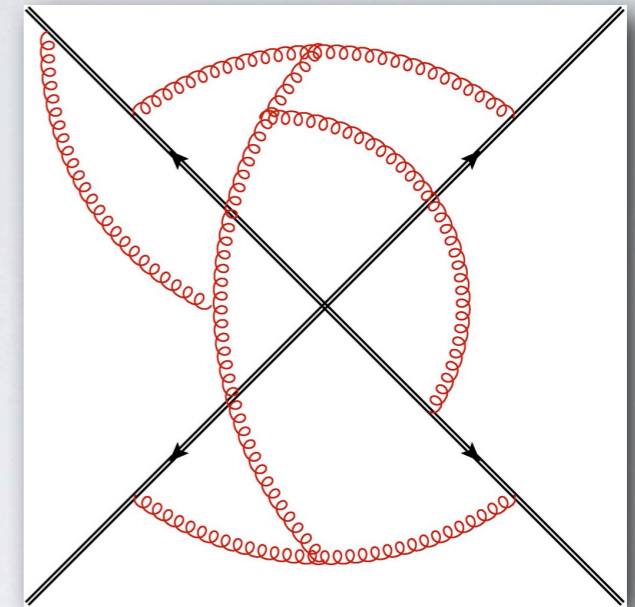
# RECENT DEVELOPMENTS





# Surprising Simplicity

- The matrix  $\Gamma_S$  can be computed from the **UV poles** of  $S$ .
- **Computations** can be performed directly **for the exponent**: the relevant diagrams are called “**webs**”.
- $\Gamma_S$  appears **highly complex** at high orders.
- **g-loop** webs directly **correlate** color and kinematics of up to **g+1** Wilson lines.



A web contributing to the soft anomalous dimension matrix

The **two-loop** calculation (Aybat, Dixon, Sterman 06) leads to a **surprising result**: for **any number** of external **massless** partons

$$\Gamma_S^{(2)} = \frac{\kappa}{2} \Gamma_S^{(1)} \quad \kappa = \left( \frac{67}{18} - \zeta(2) \right) C_A - \frac{10}{9} T_F C_F .$$

- ➔ **No** new kinematic dependence; **no** new matrix structure.
- ➔  $\kappa$  is the two-loop coefficient of  $\gamma_K(\alpha_s)$ , rescaled by the appropriate **quadratic Casimir**,

$$\gamma_K^{(i)}(\alpha_s) = C^{(i)} \left[ 2 \frac{\alpha_s}{\pi} + \kappa \left( \frac{\alpha_s}{\pi} \right)^2 + \mathcal{O}(\alpha_s^3) \right] .$$



# The Dipole Formula

The two-loop result led to an **all-order understanding**. For **massless** partons, the soft matrix obeys a set of **exact equations** that **correlate color** exchange with **kinematics**.

The **simplest solution** to these equations is a **sum over color dipoles** (Becher, Neubert; Gardi, LM, 09). It leads to an **ansatz** for the all-order singularity structure of **all** multiparton fixed-angle **massless** scattering amplitudes: the **dipole formula**.

📌 All **soft** and **collinear** singularities can be **collected** in a multiplicative operator **Z**

$$\mathcal{M} \left( \frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon \right) = Z \left( \frac{p_i}{\mu_f}, \alpha_s(\mu_f^2), \epsilon \right) \mathcal{H} \left( \frac{p_i}{\mu}, \frac{\mu_f}{\mu}, \alpha_s(\mu^2), \epsilon \right),$$

📌 **Z** contains both soft singularities from **S**, and collinear ones from the jet functions. It must **satisfy** its own matrix **RG equation**

$$\frac{d}{d \ln \mu} Z \left( \frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon \right) = - Z \left( \frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon \right) \Gamma \left( \frac{p_i}{\mu}, \alpha_s(\mu^2) \right).$$

The matrix **Γ** **inherits** the **dipole structure** from the soft matrix. It reads

$$\Gamma_{\text{dip}} \left( \frac{p_i}{\mu}, \alpha_s(\mu^2) \right) = -\frac{1}{4} \hat{\gamma}_K(\alpha_s(\mu^2)) \sum_{j \neq i} \ln \left( \frac{-2 p_i \cdot p_j}{\mu^2} \right) \mathbf{T}_i \cdot \mathbf{T}_j + \sum_{i=1}^n \gamma_{J_i}(\alpha_s(\mu^2)).$$

Note that **all singularities** are **generated by integration** over the scale of the coupling.



# Features of the dipole formula

- All known results for IR divergences of massless gauge theory amplitudes are recovered.
- The absence of multiparton correlations implies remarkable diagrammatic cancellations.
- The color matrix structure is fixed at one loop: path-ordering is not needed.
- All divergences are determined by a handful of anomalous dimensions.
- The cusp anomalous dimension plays a very special role: a universal IR coupling.

Can this be the definitive answer for IR divergences in massless non-abelian gauge theories?

► There are precisely two sources of possible corrections.

- Quadrupole correlations may enter starting at three loops: they must be tightly constrained functions of conformal cross ratios of parton momenta.

$$\Gamma\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) = \Gamma_{\text{dip}}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) + \Delta(\rho_{ijkl}, \alpha_s(\mu^2)) \quad , \quad \rho_{ijkl} = \frac{p_i \cdot p_j \, p_k \cdot p_l}{p_i \cdot p_k \, p_j \cdot p_l}$$

- The cusp anomalous dimension may violate Casimir scaling beyond three loops.

$$\gamma_K^{(i)}(\alpha_s) = C_i \hat{\gamma}_K(\alpha_s) + \tilde{\gamma}_K^{(i)}(\alpha_s)$$

- The functional form of  $\Delta$  is further constrained by: collinear limits, Bose symmetry and transcendentality bounds (Becher, Neubert; Dixon, Gardi, LM, 09).
- A four-loop analysis indicates that Casimir scaling holds (Vernazza, EPS 2011).



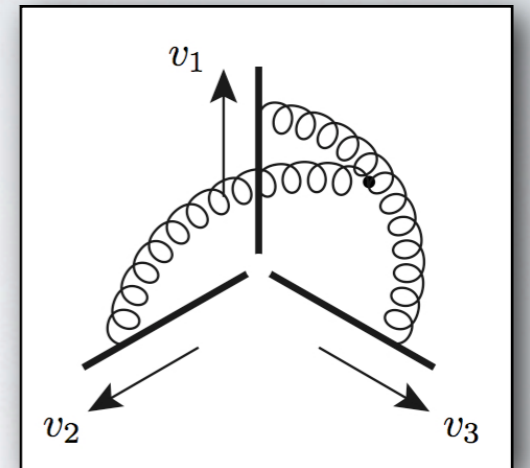
# Massive particles

- The striking **simplicity** of the massless result **does not carry over** to massive partons.
  - The **g-loop exponent** will generally involve **(g+1)-parton correlations**.
  - An **analytic** calculation at **two loops** was carried out (**Becher, Neubert; Ferroglia et al.; Mitov et al.; Kidonakis, 09**) with interesting results.

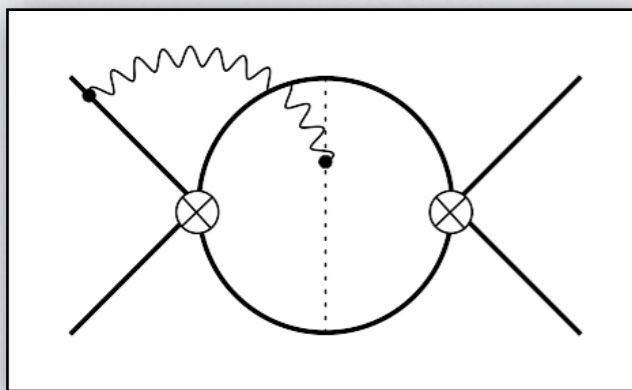
$$\Gamma\left(\frac{p_i}{\mu}, M_i, \alpha_s(\mu^2)\right) = \Gamma_{\text{dip}}\left(\frac{p_i}{\mu}, M_i, \alpha_s(\mu^2)\right) + i \sum_{i,j,k} f_{abc} \mathbf{T}_i^a \mathbf{T}_j^b \mathbf{T}_k^c F_1(\beta_{ij}, \beta_{jk}, \beta_{ik}) + \dots$$

$$F_1^{(2)}(\beta_{ij}, \beta_{jk}, \beta_{ik}) = \frac{4}{3} \sum_{i,j,k} \epsilon_{ijk} g(\beta_{ij}) \beta_{ki} \coth \beta_{ki}$$

- The result still displays **unexpected** structure and **simplicity**.



Three-parton correlations at two loops



Soft and Coulomb gluons at two loops

- Another class of singularities of massive amplitudes is understood and **resummed**.

- When **massive** particles are **pair-produced** near **threshold**, **Coulomb** singularities  $\log^p \beta / \beta^k$  arise.
- They can be **organized** using effective field theory (**NRQCD**).
- A novel **factorization theorem** has been derived and applied to **heavy** colored particle production (**Beneke et al., 09**).

$$\hat{\sigma}_{pp'}(\hat{s}, \mu) = \sum_{KL} H_{KL}(M, \mu) \int dw \sum_{R_\alpha} J_{R_\alpha}\left(E - \frac{w}{2}\right) S_{KL}^{R_\alpha}(w, \mu)$$



# The high-energy limit



# The high-energy limit

The **dipole formula** has recently been applied to the '**Regge limit**',  $s \gg t$ , providing a **novel viewpoint** on the phenomenon of **Reggeization** (Del Duca et al. II).

📌 **Reggeization**: logarithms  $\log(s/t)$  are generated by the **Reggeized t-channel propagator**

$$\frac{1}{t} \longrightarrow \frac{1}{t} \left( \frac{s}{-t} \right)^{\alpha(t)} \quad \alpha(t) = \frac{\alpha_s(-t, \epsilon)}{4\pi} \alpha^{(1)} + \left( \frac{\alpha_s(-t, \epsilon)}{4\pi} \right)^2 \alpha^{(2)} + \mathcal{O}(\alpha_s^3)$$

📌 The infrared matrix **Z factorizes** and acts as a **Reggeization operator**

$$Z(p_i, \alpha_s, \epsilon) = \tilde{Z}\left(\frac{s}{t}, \alpha_s, \epsilon\right) Z_1(t, \alpha_s, \epsilon) \quad \tilde{Z}\left(\frac{s}{t}, \alpha_s, \epsilon\right) = \exp \left\{ K(\alpha_s, \epsilon) \left[ \ln\left(\frac{s}{-t}\right) \mathbf{T}_t^2 + i\pi \mathbf{T}_s^2 \right] \right\},$$

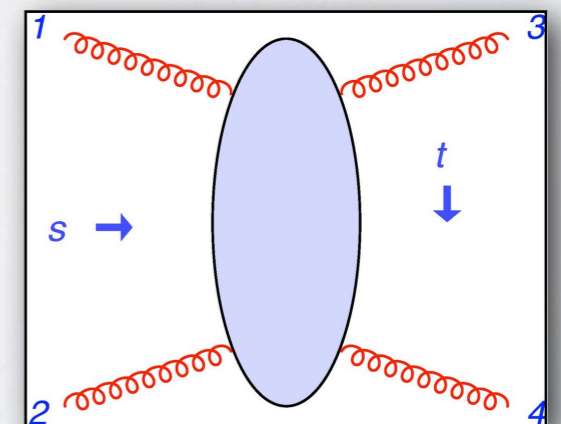
where we defined the **s**- and **t**-channel color operators  $\mathbf{T}_s = \mathbf{T}_1 + \mathbf{T}_2$ ,  $\mathbf{T}_t = \mathbf{T}_1 + \mathbf{T}_3$ .

📌 **Leading logarithmic** Reggeization immediately **follows** for **generic** representations.

$$\mathcal{M}|_{\text{LL}} = \left( \frac{s}{-t} \right)^{K \mathbf{T}_t^2} Z_1 \mathcal{H} \quad K(\alpha_s, \epsilon) \equiv -\frac{1}{4} \int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \hat{\gamma}_K(\alpha_s(\lambda^2, \epsilon))$$

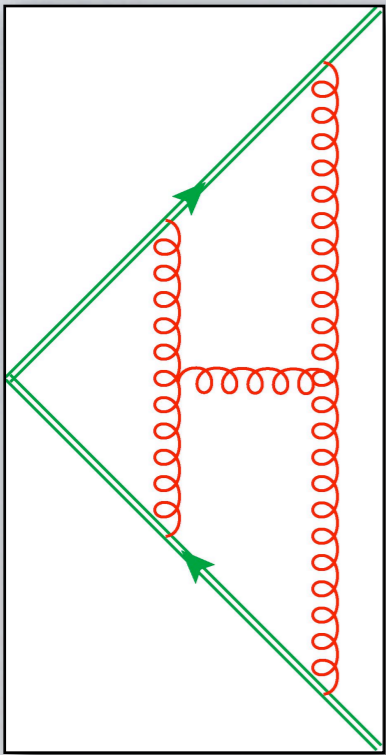
📌 **Reggeization** (of singular contributions) can be studied for **subleading logs** and **generic color** configurations.

- Results for **Regge trajectories** at LL and NLL are **recovered**.
- Reggeization is seen to generically **break down at NNLL**.
- The formalism applies in **Multi-Regge kinematics**.





# Multiparton webs



A web

- Infrared divergences of gauge scattering amplitudes **exponentiate**.
- The **exponent** can be **computed directly** in terms of a subset of the original diagrams with modified color factors, called **'webs'**.  
(Gatheral 83; Frenkel, Taylor 84)
- For amplitudes with **two** hard partons (**color singlet**), webs have a precise **topological characterization** and special properties.
  - Webs are **'two-eikonal-irreducible'** diagrams.
  - Webs have **modified color factors** that can be computed recursively.
  - Webs have no nested UV **subdivergences**.

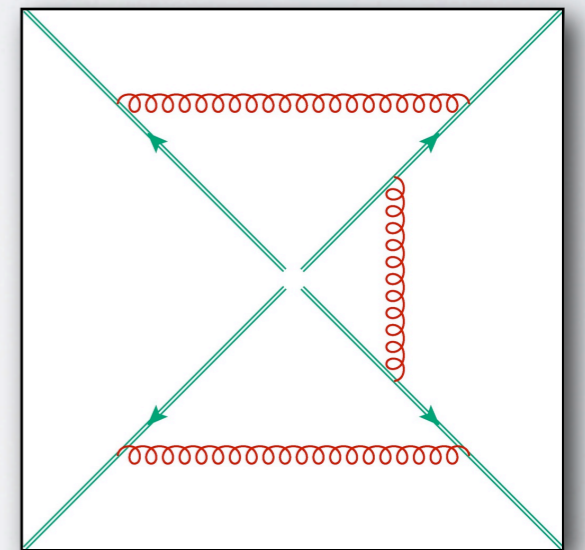
- We are now **understanding** the structure of the **multileg exponent**.  
(Gardi et al. '10-'11; Mitov, Sterman, Sung '10)

$$\mathcal{Z} \equiv \int [\mathcal{D}A_s^\mu] e^{iS(A_s^\mu)} \left[ \Phi^{(1)} \otimes \dots \otimes \Phi^{(L)} \right] = \exp \left[ \sum_D \tilde{C}(D) \mathcal{F}(D) \right].$$

- Multiparton **webs** are **sets** of diagrams whose kinematic and color structures **mix**. They are **not all irreducible**.
- Modified color factors are given by **web mixing matrices**

$$\tilde{C}(D) = \sum_{D'} R(D, D') C(D')$$

- All subleading poles are **determined** by lower-order webs.



Is this a web?

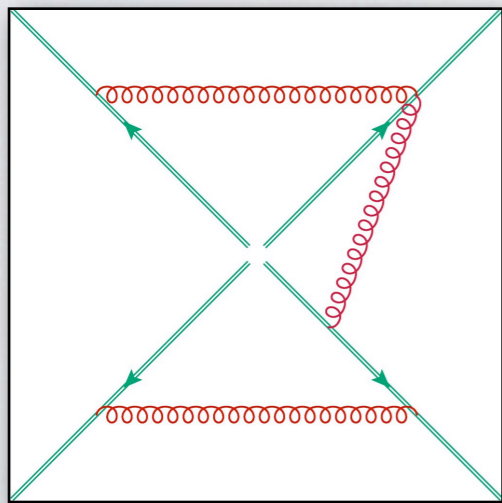


# Beyond the eikonal

Hadronic cross sections **near partonic threshold** receive **non-singular** logarithmic corrections  $\alpha_s^p \log^k(1 - z)$ , or  $\alpha_s^p \log^k N/N$ , which may be **relevant** for phenomenology. Can they also be organized and **resummed**? (Kraemer et al.; Vogt et al.; Grunberg, ...)

- For **two-parton** processes,  $\mathcal{O}(N^0)$  contributions **exponentiate** (Laenen, LM, 03).
- **Phenomenological** evidence indicates that also ‘**sub-eikonal**’ logs partly **exponentiate**.
- An **ansatz** summarizes the resumable for **Drell-Yan** (and **DIS**) (Laenen et al., 06).

$$\ln \left[ \widehat{w}(N) \right] = \mathcal{F}_{\text{DY}}(\alpha_s(Q^2)) + \int_0^1 dz z^{N-1} \left\{ \frac{1}{1-z} D \left[ \alpha_s \left( \frac{(1-z)^2 Q^2}{z} \right) \right] + 2 \int_{Q^2}^{(1-z)^2 Q^2 / z} \frac{dq^2}{q^2} P_s \left[ z, \alpha_s(q^2) \right] \right\}_+$$



Is THIS a web?

A **systematic** study of soft-gluon dynamics **beyond the eikonal** approximation has been undertaken (Laenen et al. '08, 10).

- A class of **factorizable** contributions **exponentiate** via **NE webs**

$$\mathcal{M} = \mathcal{M}_0 \exp \left[ \sum_{D_{\text{eik}}} \tilde{C}(D_{\text{eik}}) \mathcal{F}(D_{\text{eik}}) + \sum_{D_{\text{NE}}} \tilde{C}(D_{\text{NE}}) \mathcal{F}(D_{\text{NE}}) \right].$$

- “**Feynman rules**” for the NE exponent, including “**seagull**” vertices.
- **Non-factorizable** contribution can be studied using **Low’s theorem**.



# PHENOMENOLOGY



# PHENOMENOLOGY





# Electroweak annihilation

Classic **threshold** resummation (for  $\sigma_{\text{TOT}}$ ) is possible to **'well-approximated'**  $N^3\text{LL}$ .

At **large** measured **transverse momentum** one is again **close** to partonic **threshold**.

- **$p_T$ -threshold** resummation now performed to approximate  $N^3\text{LL}$  (Becher, Schwartz, 11).

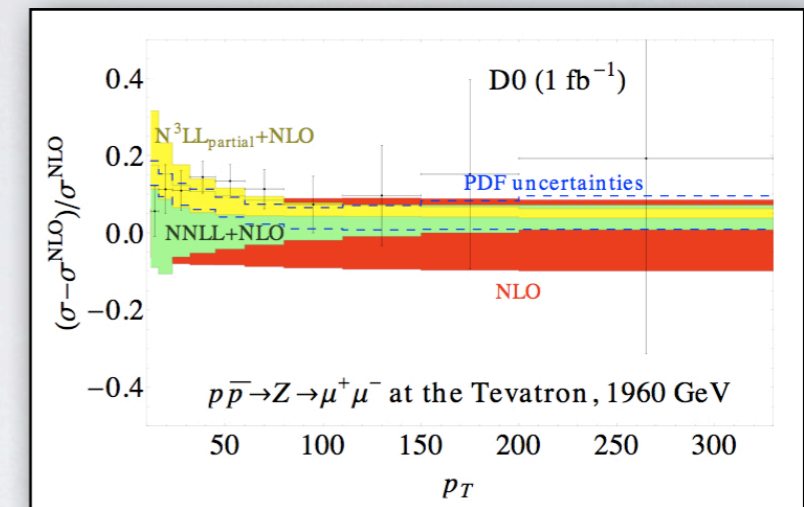
**Transverse momentum** resummation is available at **NNLL** (Bozzi et al., 10).

- **Favorable** comparison to Tevatron data.
- **Small** theoretical uncertainty.
- **Awaiting** LHC data comparison.

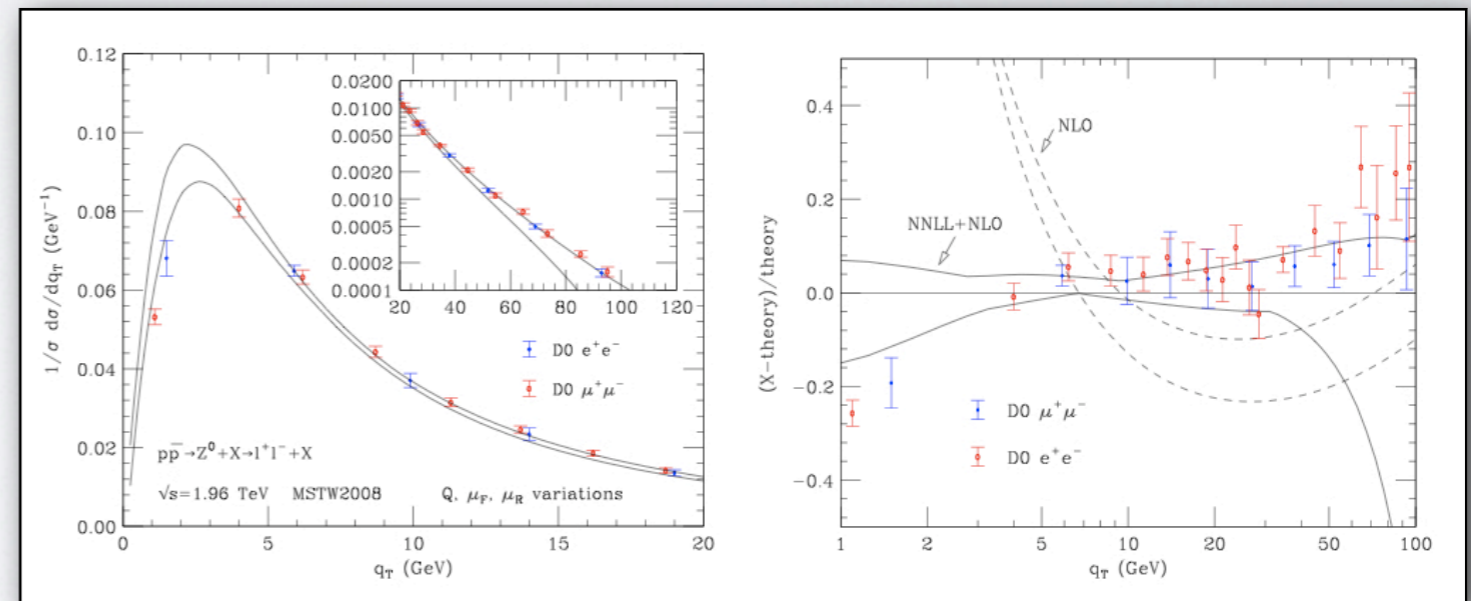
**Caveat:** detailed **SCET** analysis (Becher and Neubert, 11) indicates (large) **modification** of **3-loop** coefficient!

- Theoretically **interesting** **'collinear anomaly'**, transverse momentum pdf issues.

**NLL** predictions for  **$p_T$**  spectrum also from **SCET** (Mantry, Petriello, 10)



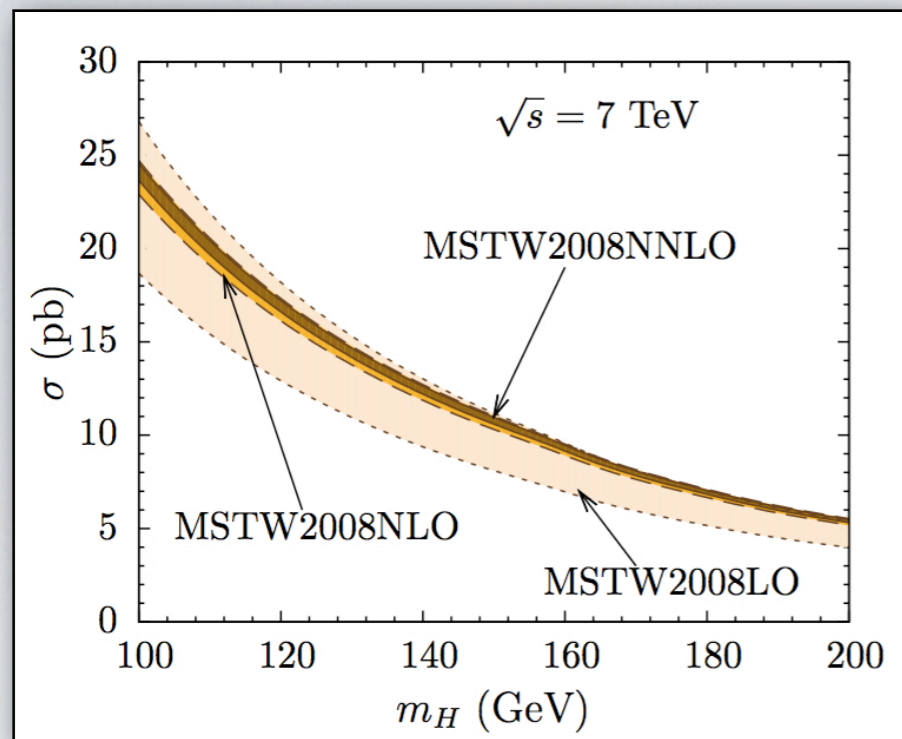
Large  $p_T$  D0 data vs. 'threshold' resummation



$q_T$  spectrum of Z bosons at Tevatron (D0) compared to NNLL resummation



# Higgs production



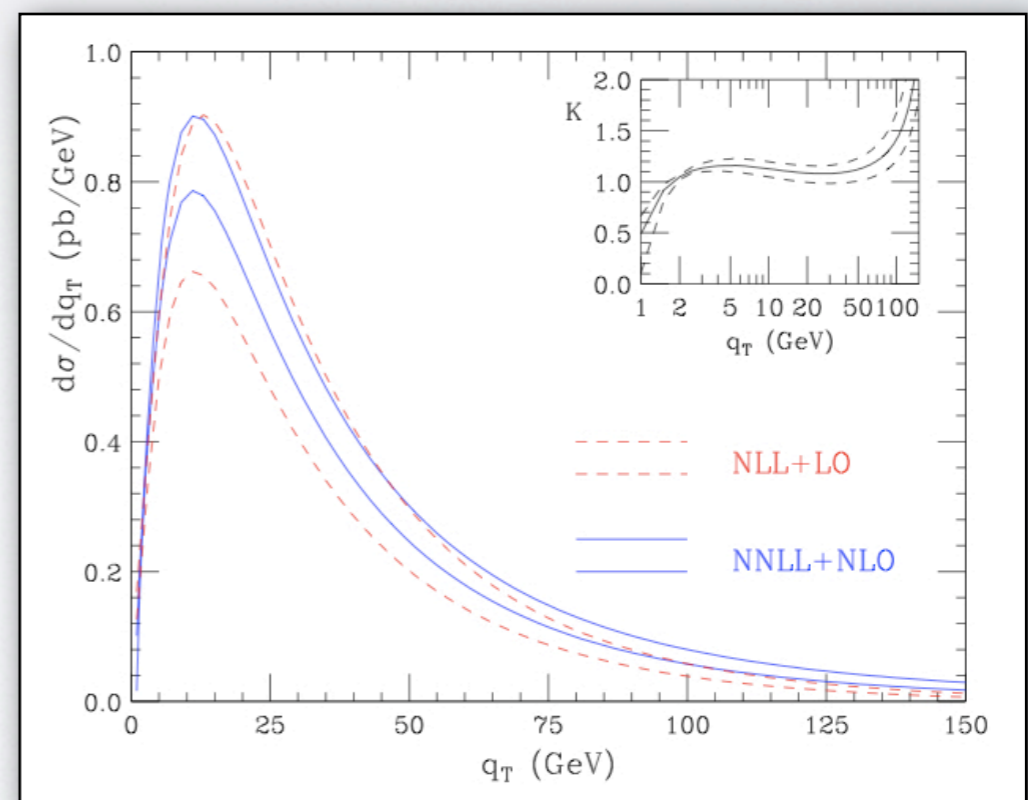
$N^3$ LL resummed cross section for Higgs production via gluon fusion at LHC

The  $p_T$  distribution for  $gg \rightarrow H$  is known to NNLL and NNLO (M. Grazzini et al. '07, '10)

- Resummation **reduces** scale uncertainty
- A subtle **polarization effect** uncovered but not implemented yet (Catani, Grazzini, 10)
- Impact of **revised three-loop** coefficient **must** be gauged

The **total cross section** for  $gg \rightarrow H$  is known to  $N^3$ LL and NNLO, with NLO EW corrections.

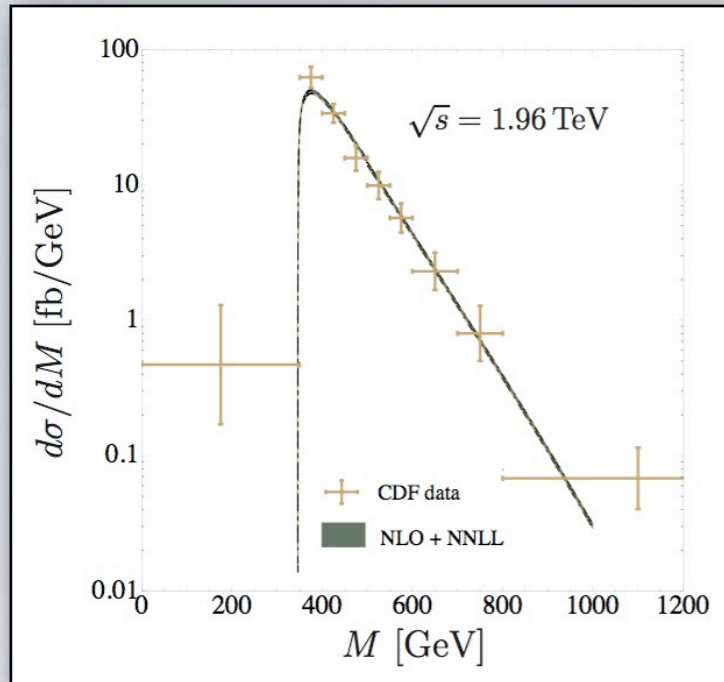
- One of the **best-known** observables in the SM.
- A combined analysis (Ahrens et al. 11) gives a **3%** (th) + **8%** (pdf) + **1%** (mq) **uncertainty**.
- Ongoing **debate** on theoretical and pdf uncertainty (Baglio et al. 11).



NNLL resummed  $p_T$  distribution for Higgs production via gluon fusion at LHC



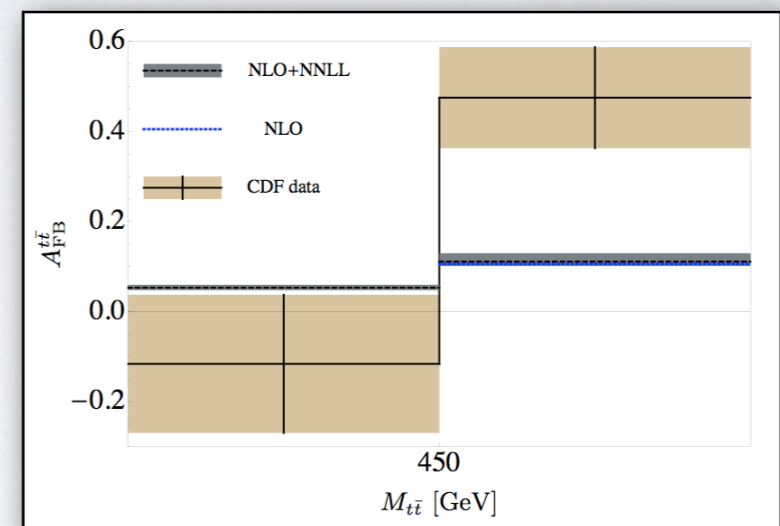
# Top distributions



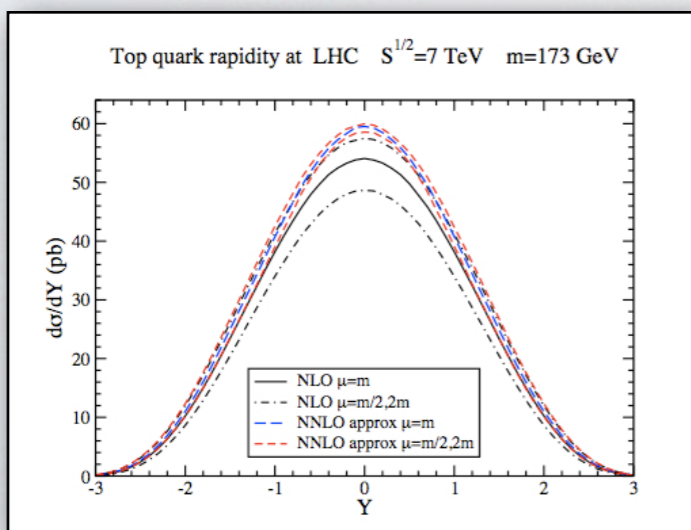
NNLL top-antitop invariant mass spectrum compared to CDF data

- The calculation of the **two-loop massive** anomalous dimension matrix makes it possible to perform **NNLL** resummation for **generic distributions** (Ahrens et al., 09).
  - Invariant pair mass** distribution shows remarkable **agreement** with **CDF** data (LHC awaited).
  - Negligible** theoretical uncertainty.
  - Different choices of **kinematics** and **frame** possible, vast **menu** of distributions available.

- The **Tevatron** top-antitop **FB asymmetry** can be computed in QCD at **NNLL+NLO** (Ahrens et al., 09).
  - Negligible** impact on NLO result: the **solution** to the Tevatron puzzle is **not QCD higher orders**.



NNLL top-antitop FB asymmetry compared to CDF data



Top rapidity distribution at approximate NNLO

- Resummation can also be used in a **simplified** way to compute **approximate higher order** corrections and distributions (Kidonakis, 10-11)
  - Some conceptual and **technical issues avoided**; **partial** reduction in scale uncertainties.

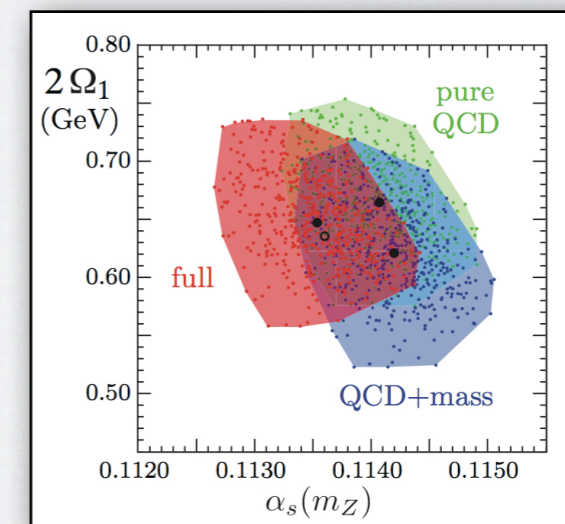
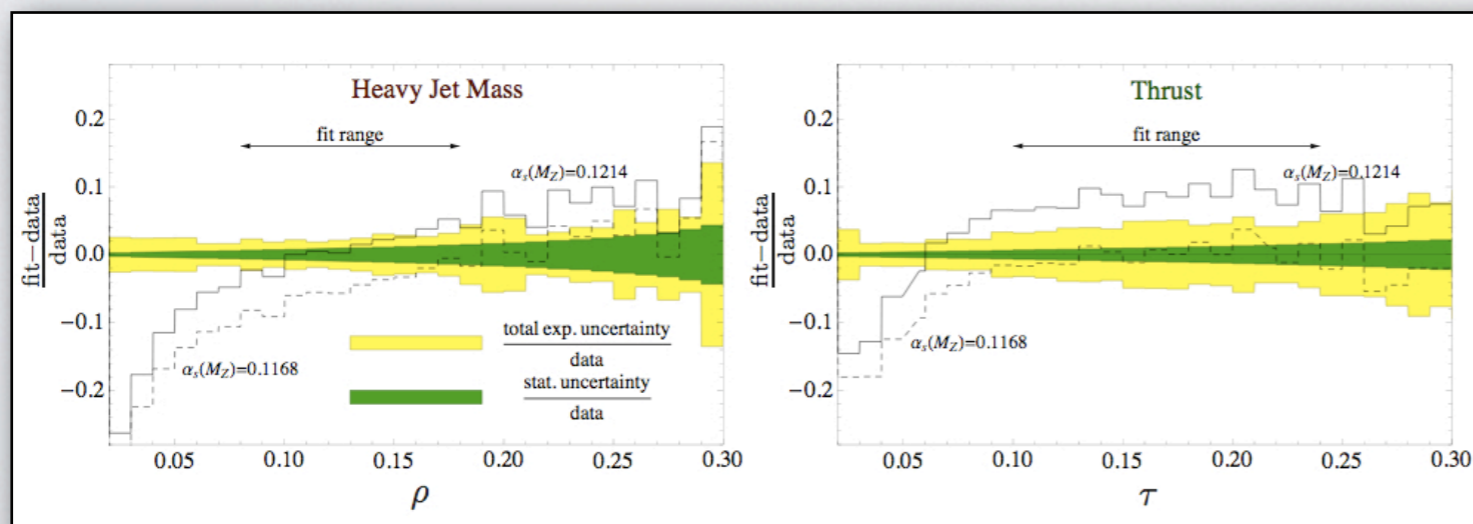


# Event shapes

- First studies of **event shapes** with exact **NNLO** information and (well) approximated **N<sup>3</sup>LL** resummation have **appeared** (Becher, Schwartz, 08; Schwartz, Cien; Abbate et al. 10).
- The studies deploy **neat tricks** (Padé approximants, numerical determination of 2-loop soft coefficients) and **great care** (hadronization, b-mass, QED corrections).
- Perturbative **agreement** between SCET and standard resummation (Gehrmann et al., 11).
- Significant differences remain** in the final results for the **strong coupling**.

$$\begin{aligned}\alpha_s(M_Z^2) &= 0.1172 \pm 0.0022 && \text{thrust (BS)} \\ \alpha_s(M_Z^2) &= 0.1220 \pm 0.0031 && \text{jet mass (SC)} \\ \alpha_s(M_Z^2) &= 0.1135 \pm 0.0010 && \text{thrust (AFHMS)}\end{aligned}$$

- Many** possible **sources** of discrepancy, the main suspect remains **hadronization/MC**.
- The problem is still **not fully understood**: do we really know  $\alpha_s$  to **percent** accuracy?



Comparing the  $\alpha_s$  fit quality for thrust and heavy jet mass at N<sup>3</sup>LL (SC)

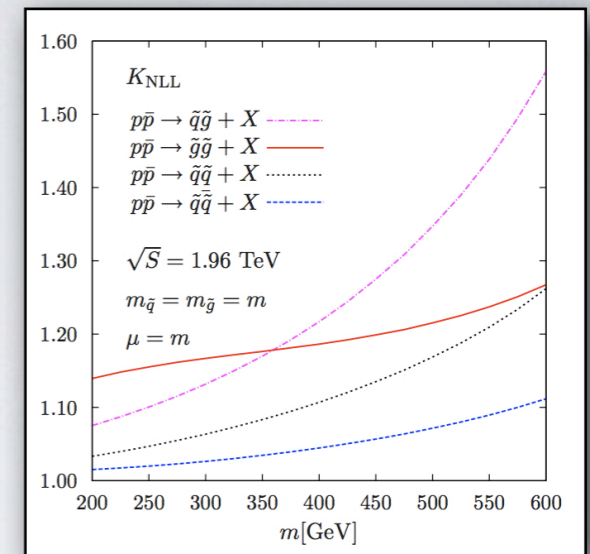
Joint fit of  $\alpha_s$  and hadronization parameter  $\Omega_1$  from N<sup>3</sup>LL thrust (AFHMS)



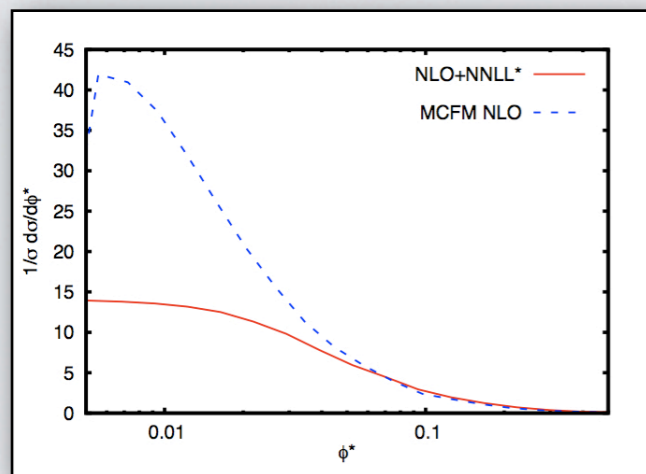
# Miscellanea

Soft gluon **resummations** are being **applied** to **SUSY particles**.

- SUSY particles are **heavy** (and getting heavier ...), close to **threshold**: corrections useful for **exclusion limits**.
- **Gaugino** and **slepton** production (singlets) (Klasen 06-11).
- **Colored** sparticle production (requires **soft matrices**) (Kulesza et al. 09-11; Beneke et al., general color, 10).



NLL K-factors for squarks and gluinos



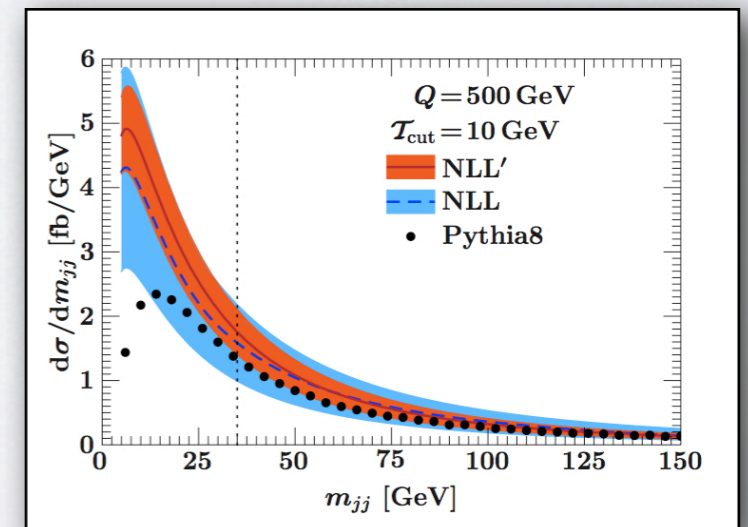
NNLL vs. NLO  $\phi^*$  distribution

New observables, designed by **experiments**, require (and get) soft gluon resummation (Banfi et al. 09-11).

- Variables related to the **angle between leptons** are more **accurately** measured than the  $p_T$  of the lepton **pair**.
- Resummation is **crucial** close to Born configuration.

Complex jet observables are **designed** and **resummed**.

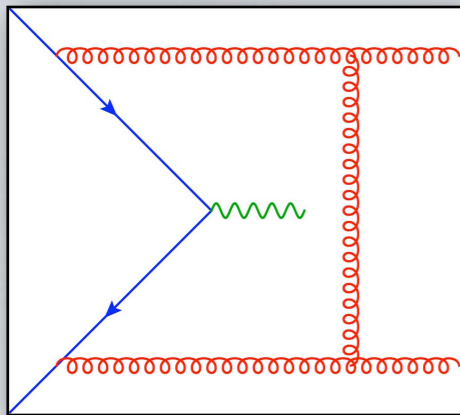
- **Jet shapes** to study internal structure of jets, useful for **boosted** heavy particle production (Ellis et al. 09-11).
- **Dijet mass** distribution with fixed 'background' event shape ('N-jettiness'), extension of **SCET** (Bauer et al. 11).



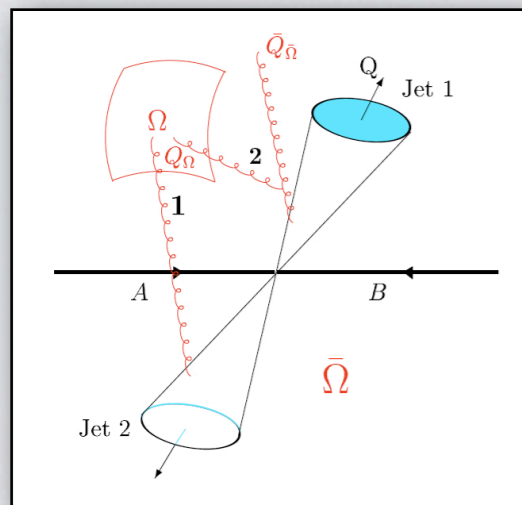
NLL dijet mass distribution for fixed N-jettiness



# Jactuum caveat emptor



Spectator interaction via Glauber gluon



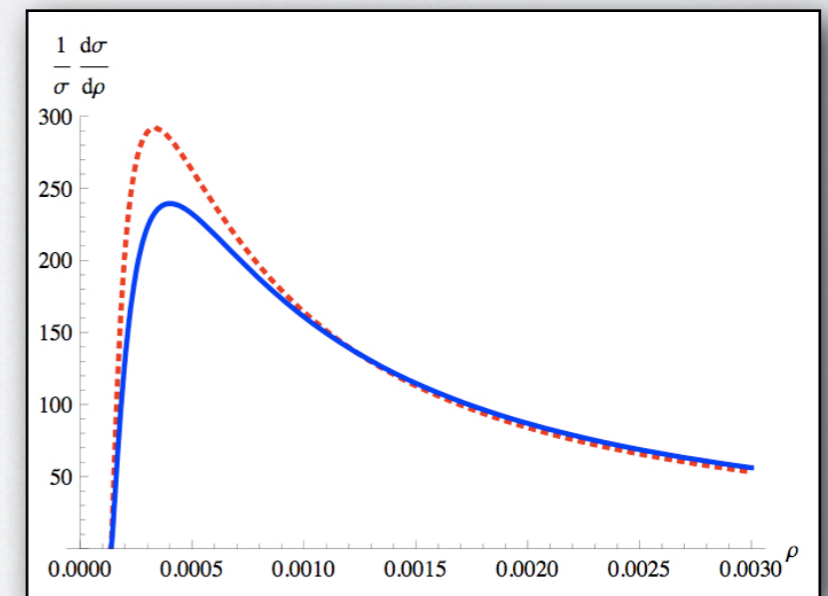
Non-global logarithms for energy flow

As the jet observables **proliferate** and are **resummed**, several **caveats** must be kept in mind.

- **Glauber gluons**: they cancel in **inclusive** jet cross sections (Aybat, Sterman 09), but no proof if jets are **opened up**. They are **not** in **SCET**, might be **added** (Bauer et al., 10).
- **Non-global logarithms**: arise whenever gluon emission phase space is **cut up** (Dasgupta, Salam, 01); affect observables at **single logarithmic** level; **resummable** only at **large  $N_c$** .
- **Jet algorithms**: the choice of jet algorithm **affects** both **non-global** and ordinary **Sudakov** logarithms. **Clustering correlates** 'independent' gluons, **except** for anti-kT (Banfi et al., 05-06).

A striking **example** of the impact of **NG logs** on **jet shapes**.

- Jet shapes measure properties of a **single jet** in a **multijet** event. They are **generically** affected by NG logs.
- For a **typical** jet shape ('in-jet angularity') NG logs change the **height** of the (formally **NLL**) distribution by **15-20%** in the small-**R** limit (Banfi et al., 10).

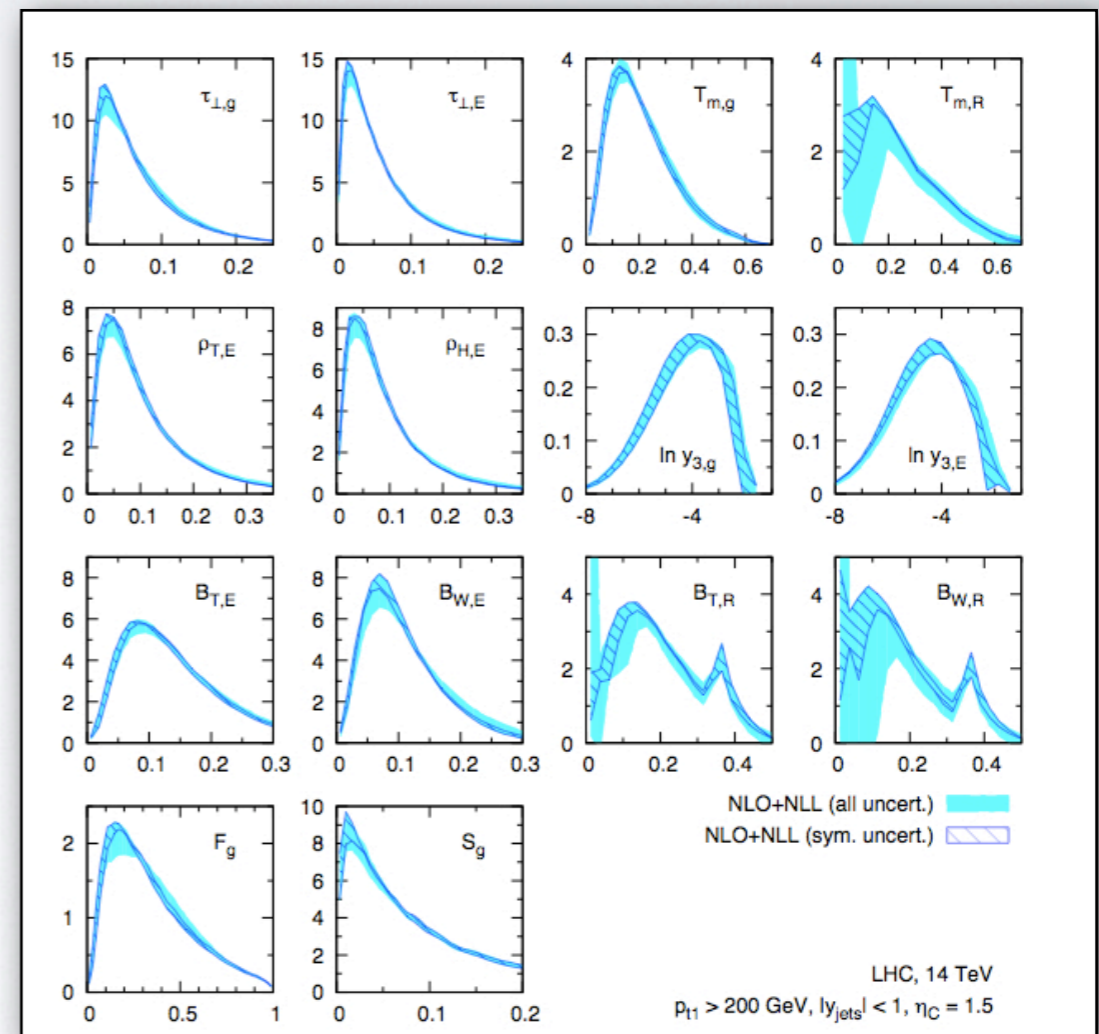


The impact of non-global logarithms on jet shapes



# Hadronic event shapes

- An interesting **alternative** to the use of **jets**, which bypasses the need for an algorithm, is to introduce **global event shapes**, in analogy to those used in  $e^+e^-$  annihilation.
  - The hadronic environment requires **suppressing the beam region**.
- NLL+NLO resummation** can be performed numerically with the program **Caesar** recently generalized to hadron collisions (**Banfi, Salam, Zanderighi, 10**).
- Numerically** resumable event shapes are carefully **characterized**:
  - Functional** constraints.
  - Continuous **globalness**.
  - Recursive **IR safety**.
- A **vast variety** of event shapes is introduced, categorized and resummed.
  - Simple example: **transverse thrust**.
- Relevant issues** for **NLL+NLO** resummation of event shapes are dealt with in detail.
  - Control of **non-global logs**.
  - Transition **particle-jet** (algorithm issues).
  - Possible **superleading logs**.
  - Matching** to NLOJET++.
  - Power corrections** (analytic and MC).
  - Impact of **underlying event**.



A menu of NLL-resummed hadronic event shapes



# OUTLOOK





# Summary

- 🔊 **Resummations** are a powerful tool both for **theory** and for **phenomenology**.
  - ✓ Explore the **boundary** between **perturbative** and **non-perturbative** physics.
  - ✓ Are **necessary** for precision phenomenology.
- 🔊 **Resummations** have a **long history**, but
  - ✓ past few years have seen **very intense** LHC-motivated activity and theoretical progress.
- 🔊 **Factorization** theorems  $\Rightarrow$  **Evolution** equations  $\Rightarrow$  **Exponentiation**.
  - ✓ Sudakov **factorization**  $\Rightarrow$  soft-gluon **resummation** (also formalized by **SCET**).
  - ✓ Multiparton processes require **anomalous dimension matrices**.
- 🔊 Remarkable progress on the **theory** side.
  - ✓ We are understanding the **all-order structure** of the perturbative **exponent**.
  - ✓ For massless partons the **dipole formula** may give the **definitive** answer.
  - ✓ For massive partons the **general two-loop** anomalous dimension matrix is **known**.
  - ✓ **SCET** provides new insights: **momentum space** resummation, '**collinear anomaly**'.
  - ✓ Ongoing efforts to go **beyond the eikonal** approximation.
- 🔊 A vast array of **phenomenological** applications, many vital for LHC precision physics.
  - ✓ **Electroweak annihilation** processes are known to high logarithmic accuracy.
  - ✓ **Top distributions** can be computed with unprecedented theoretical precision.
  - ✓ The **strong coupling** can be precisely determined from resummed event shapes.
  - ✓ **Hadronic event shapes** provide a flexible alternative tool for hadron collisions.
- 🔊 We have come **a long way**, but each step forward brings **new insight** and **new questions** ...



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"And if you enjoyed my summation you're going to love my new CD,  
'The Very Best Summations of Walter J. Prescott!'"



THANK YOU