

*Soft gluon effects
for selected event shape distributions*

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Outline

Introduction

- Event shapes in e^+e^- annihilation
- Resummation of Sudakov logarithms
- Dressed gluon exponentiation

C parameter

- The Sudakov exponent
- Towards phenomenology

Angularities

- A family of event shapes
- Scaling of power corrections

Hadron collisions

- Impact of power corrections
- Observables: examples

Perspective





Event shapes in e^+e^- annihilation

Picturing the final state of high-energy collisions

- Event shape distributions probe QCD at *all scales* from the perturbative to the non-perturbative regime.

finite order \longrightarrow resummation \longrightarrow power corrections

- They provide a *global picture* of final state of hard collisions.

energy flow \longleftrightarrow hadronization \longleftrightarrow mass effects

- A large amount of data is *available* (LEP, HERA ...)

better theory \longleftrightarrow more analysis ?

- Studies are emerging for *hadron-hadron* collisions

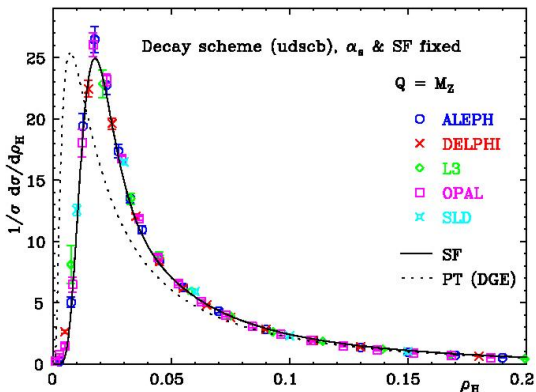
impact at LHC ?





A striking example

Fit of LEP data for *heavy jet mass* distribution (Gardi, Rathsman).



Note: “parameterless”, however *small* α_s .





Event shapes in e^+e^- annihilation

Examples

- Thrust: $T = \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{Q}$; $\tau = 1 - T$.
 → \hat{n} is used to define several other shape variables.
- C-parameter: $C = 3 - \frac{3}{2} \sum_{i,j} \frac{(p_i \cdot p_j)^2}{(p_i \cdot q)(p_j \cdot q)}$.
 → does not require maximization procedures.
- Angularity: $\tau_a = \frac{1}{Q} \sum_i (p_{\perp})_i e^{-|\eta_i|(1-a)}$.
 → recently introduced (Berger, Sterman)
- Transverse Thrust: $T_{\perp} = \frac{\sum_i |\vec{p}_{\perp i} \cdot \hat{n}_{\perp}|}{\sum_i \vec{p}_{\perp i}}$; $\tau_{\perp} = 1 - T_{\perp}$.
 → defined for *hadron-hadron* collisions



Resummation of Sudakov logarithms

Infrared and collinear emission dominates the two-jet limit

- Large *double* logarithms of the variable vanishing in the two-jet limit ($L = \log t$; $L = \log C$; ...) enhance finite orders \rightarrow *need to resum*.
- A pattern of *exponentiation* emerges

$$\sum_k \alpha_s^k \sum_p^{2k} c_{kp} L^p \rightarrow \exp \left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots \right]$$

- In general the *Laplace transform* exponentiates

$$\int_0^\infty d\tau e^{-\nu\tau} \frac{1}{\sigma} \frac{d\sigma}{d\tau} = \exp \left[\int_0^1 \frac{du}{u} (e^{-u\nu} - 1) \left(B(\alpha_s(uQ^2)) + \int_{u^2 Q^2}^{uQ^2} \frac{dq^2}{q^2} A(\alpha_s(q^2)) \right) \right].$$



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Resummation of Sudakov logarithms

Exponentiating power corrections

- The exponent is *ill-defined* because of the *Landau pole regularization* \rightarrow ambiguity \rightarrow power corrections
- Focus on *small* τ , *large* ν , set IR factorization scale μ , expand in powers of ν/Q (soft), *neglecting* ν/Q^2 (collinear).

$$S_{\text{NP}}(\nu/Q, \mu) = \int_0^{\mu^2} \frac{dq^2}{q^2} A(\alpha_s(q^2)) \int_{q^2/Q^2}^{q/Q} \frac{du}{u} (e^{-u\nu} - 1)$$

$$\simeq \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{\nu}{Q}\right)^n \lambda_n(\mu^2),$$

- Non-perturbative* parameters

$$\lambda_n(\mu^2) = \frac{1}{n} \int_0^{\mu^2} dq^2 q^{n-2} A(\alpha_s(q^2)).$$



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Resummation of Sudakov logarithms

Shape functions

- The parameters $\lambda_n(\mu^2)$ build up a *shape function*

$$\exp \left[S_{\text{NP}}(\nu/Q, \mu) \right] \equiv \int_0^\infty d\epsilon e^{-\nu\epsilon/Q} f_\tau(\epsilon, \mu) .$$

- The physical *distribution* is recovered via inverse transform

$$\sigma(\tau) \sim \int_0^{\tau Q} d\epsilon f_\tau(\epsilon, \mu) \sigma_{\text{PT}}(\tau - \epsilon/Q) .$$

- One recovers the *perturbative* result *shifted* by the soft energy flow, and *smearred* by the shape function.
- Universality* of power corrections is in general *lost*, however *specific* observables still *related* ($1 - T$, ρ_J , C , ...).
- Assumption*: smooth transition to *nonperturbative* regime.



Dressed gluon exponentiation

It is possible to combine *renormalon* methods and *Sudakov resummation* to construct models of power corrections. One method is *dressed gluon exponentiation* (Gardi).

- *Step 1*: compute *characteristic function* $\mathcal{F}(k^2)$ of the dispersive method in the *Sudakov limit* (resum “*bubble graphs*”).



- *Step 2*: use dressed gluon distribution as *kernel of exponentiation*.

$$\ln \left(\frac{d\tilde{\sigma}}{d\nu} \Big|_{DGE} \right) = \int_0^\infty d\tau \frac{d\sigma}{d\tau} \Big|_{SDG} (1 - e^{-\nu\tau}) .$$

- *Step 3*: *Borel representation* of the exponent suggests pattern of *power corrections*.



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Dressed gluon exponentiation

Features of DGE

- *NLL* Sudakov resummation *reproduced* by using “*gluon bremsstrahlung*” definition of running coupling. *All* subleading logs computed in the “large n_f ” limit.
- *Factorial growth* of subleading logs detected: a *handle* on the range of applicability of $N^{\text{P}}LL$ resummation.
- *Definite prescription* for merging resummed PT with power corrections.
- *Phenomenology* of thrust, jet masses; models of power corrections in the Sudakov region for DIS, Drell-Yan, fragmentation.



The C parameter

Virtual gluon emission at one loop

Renormalon effects in the *inclusive approximation* can be extracted from the distribution computed with a “*massive*” gluon.

- Define the event shape with a gluon virtuality $\xi = k^2/Q^2$.

$$\mathbf{c}(x_1, x_2, \xi) \equiv \frac{C}{6} = \frac{(1-x_1)(1-x_2)(1-x_3+2\xi)-\xi^2}{x_1x_2x_3} .$$

- One-loop *matrix element* for virtual gluon emission

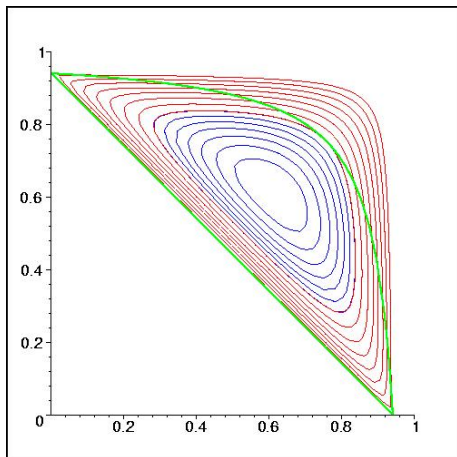
$$\mathcal{M}(x_1, x_2, \xi) = \frac{(x_1+\xi)^2+(x_2+\xi)^2}{(1-x_1)(1-x_2)} - \frac{\xi}{(1-x_1)^2} - \frac{\xi}{(1-x_2)^2} .$$

- *Characteristic function*

$$\mathcal{F}(\xi, c) = \int dx_1 dx_2 \mathcal{M}(x_1, x_2, \xi) \delta(\mathbf{c}(x_1, x_2, \xi) - c) .$$



Phase space with a 'massive' gluon



Phase space (green) and fixed c contours for $\xi = 0.06$

- Collinear limit:

$$x_{1,2} = 1 - \xi$$

- Soft limit:

$$x_1 = x_2 = 1 - \sqrt{\xi}$$

- $c_{\text{soft}} = \xi/x_3$

- *Soft region*

$$\frac{\xi}{1+\xi} < c < \frac{\sqrt{\xi}}{2}$$

- *Hard region*

$$\frac{\sqrt{\xi}}{2} < c < c_{\text{max}}$$



Constructing the Sudakov exponent

The single dressed gluon cross section

$\mathcal{F}(\xi, c)$ can be computed in closed form in terms of elliptic integrals (Gardi, LM). To perform *DGE* one must then

- *Extract* terms responsible for Sudakov logs

$$\mathcal{F}_1(\xi, c) = \frac{1}{c} \left[-4 \ln \left(\frac{\xi}{c} \right) - 3 + 2 \left(\frac{\xi}{c} \right) + \left(\frac{\xi}{c} \right)^2 - 8 \ln \left(\frac{1}{2} \left(1 + \sqrt{1 - 4c^2/\xi} \right) \right) \right].$$

- *Compute* the dispersive integral

$$\frac{1}{\sigma} \frac{d\sigma}{dc} (c, Q^2) = -\frac{C_F}{2\beta_0} \int_0^1 d\xi \frac{d\mathcal{F}(\xi, c)}{d\xi} A(\xi Q^2).$$

- *Use* the Borel representation for the coupling

$$A(\xi Q^2) = \int_0^\infty du (Q^2/\Lambda^2)^{-u} \frac{\sin \pi u}{\pi u} e^{\kappa u} \xi^{-u}.$$

Note: κ defines the *renormalization* scheme.



Constructing the Sudakov exponent

Exponentiating the dressed gluon

- *Define* the Borel representation of the *SDG* cross section.

$$\frac{1}{\sigma} \frac{d\sigma}{dc} \Big|_{\text{SDG}} = \frac{C_F}{2\beta_0} \int_0^\infty du (Q^2/\Lambda^2)^{-u} B(c, u).$$

Note: the Borel integral is always left unperformed

- *Exponentiate* in Laplace space.

$$\frac{1}{\sigma} \frac{d\sigma}{dc} \Big|_{\text{DGE}} = \int_{k-i\infty}^{k+i\infty} \frac{d\nu}{2\pi i} e^{\nu c} \exp [S(\nu, Q^2)],$$

using the *single gluon* result as kernel

$$S(\nu, Q^2) = \int_0^\infty dc \frac{1}{\sigma} \frac{d\sigma}{dc} \Big|_{\text{SDG}} (e^{-\nu c} - 1).$$

- *Results* are summarized by the Borel exponent.

$$S(\nu, Q^2) = \frac{C_F}{2\beta_0} \int_0^\infty du (Q^2/\Lambda^2)^{-u} B_c(\nu, u).$$



The Borel exponent

Although the *single gluon* result $B(c, u)$ has *no renormalons*, upgrading to *multi-gluon* emission via exponentiation generates *towers* of renormalon poles in $B_c(\nu, u)$.

- For the *C-parameter*

$$B_c(\nu, u) = 2e^{5u/3} \frac{\sin \pi u}{\pi u} \left[\Gamma(-2u) (\nu^{2u} - 1) 2^{1-2u} \frac{\sqrt{\pi} \Gamma(u)}{\Gamma(\frac{1}{2} + u)} - \Gamma(-u) (\nu^u - 1) \left(\frac{2}{u} + \frac{1}{1-u} + \frac{1}{2-u} \right) \right].$$

- Compare with the *thrust*

$$B_\tau(\nu, u) = 2e^{5u/3} \frac{\sin \pi u}{\pi u} \left[\Gamma(-2u) (\nu^{2u} - 1) \frac{2}{u} - \Gamma(-u) (\nu^u - 1) \left(\frac{2}{u} + \frac{1}{1-u} + \frac{1}{2-u} \right) \right].$$



Towards phenomenology

Borel functions contains *perturbative* and *non-perturbative* informations (in part *common* to t and C).

- Coefficients of subleading logarithms (in the **large n_f** limit): expand in powers of u , with $u^n \rightarrow n!(b_0\alpha_s/\pi)^{n+1}$.
- *IR safety* is expressed by cancellation of singularities at $u = 0$.
- Renormalons of *collinear* origin at $u = 1, 2$, corresponding to power corrections of the form $(\nu/Q^2)^p$, $p = 1, 2$.
NOTE: only relevant at $\tau, C \sim \Lambda^2/Q^2$, may depend on massive definition of observable.
- Renormalons from *wide angle soft* radiation $u = m/2$, m odd, corresponding to leading power corrections $(\nu/Q)^m$.
NOTE: Even powers absent in the **large n_f** inclusive approximation.



Towards phenomenology

Thrust and *C-parameter* have common features and differences.

- Sudakov logarithms *coincide* up to NLL (Catani, Webber), but *differ* beyond, due to soft emission.
 - ⇒ Numerical growth of coefficients *milder* for *C*.
- Similar *pattern* of corrections: *only odd* powers of ν/Q .
 - ⇒ May *explain* success of *shifting* perturbative distribution.
 - ⇒ However: *smaller residues* of renormalon poles for *C*.
- *Origin* of differences: scale of soft emission is $2Qc$ for $c = C/6$, while it is $Q\tau$ for $\tau = 1 - T$.
- *Consequences*: resummed *perturbative* prediction, and approximation of shape function by *shift* expected to *work better* for *c* than for τ .



Angularities

- *Definition:* $\tau_a = \frac{1}{Q} \sum_i (p_\perp)_i e^{-|\eta_i|(1-a)}$.

Also: $\tau_a = \frac{1}{Q} \sum_i \omega_i (\sin \theta_i)^a (1 - |\cos \theta_i|)^{1-a}$,

- *Some properties*

- $\tau_0 = 1 - T$; $\tau_1 = B$.
- $a \leq 2$ for IR safety.
- $a \leq 1$ for feasibility of resummation.
- For *negative* a , high rapidity particles (w.r.t. the thrust axis) are weighted less: *better* collinear behavior.
- At *one loop*, with the thrust axis given by particle i ,

$$\tau_a = \frac{(1-x_i)^{1-a/2}}{x_i} \left[(1-x_j)^{1-a/2} (1-x_k)^{a/2} + (j \leftrightarrow k) \right] .$$



Resummation for angularities

Berger, Kucs, Sterman, hep-ph/0303051

- Sudakov logs at one loop have *simple scaling* with a .

$$\left. \frac{d\sigma}{d\tau_a} \right|_{\log}^{(1)} = \frac{2}{2-a} \frac{2}{\tau_a} C_F \frac{\alpha_s}{\pi} \ln \left(\frac{1}{\tau_a} \right) = \frac{2}{2-a} \left. \frac{d\sigma}{d\tau} \right|_{\log}^{(1)}.$$

- Resummation is *intricate*.

$$\ln [\sigma_{\text{LL}}(\nu, a)] = 2 \int_0^1 \frac{du}{u} \left[\int_{u^2 Q^2}^{u Q^2} \frac{dp_T^2}{p_T^2} A(\alpha_s(p_T)) \left(e^{-u^{1-a} \nu \left(\frac{p_T}{Q} \right)^a} - 1 \right) \right].$$

- General a -dependence of Sudakov logs is *nontrivial*.

$$g_1(x, a) = -\frac{4}{\beta_0} \frac{1}{1-a} \frac{A^{(1)}}{x} \left[\left(\frac{1}{2-a} - x \right) \ln(1 - (2-a)x) - (1-x) \ln(1-x) \right].$$



Scaling of power corrections

An analysis of power corrections for angularities has been carried out using *resummation* (Berger, Sterman) and using *DGE* (Berger, LM). Remarkably

- A *simple scaling* holds. DGE yields

$$B_a^{\text{soft}}(\nu, u) = \frac{1}{1-a} \left[2 e^{5u/3} \frac{\sin \pi u}{\pi u} \Gamma(-2u) (\nu^{2u} - 1) \frac{2}{u} \right]$$

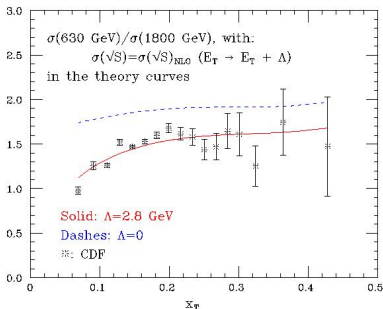
- *Collinear* contribution shows an *intricate* structure of *fractional* power corrections in DGE, but they are suppressed by either ν/Q^{2-a} or ν^b/Q^2 with $0 < b < 1$.
- In the *shape function* language

$$S_{\text{NP}}^{(a)}(\nu/Q, \mu) = \frac{1}{1-a} S_{\text{NP}}^{(0)}(\nu/Q, \mu) ,$$

a *testable prediction*.



What impact at Tevatron/LHC?



Fit of CDF data with NLO QCD assuming

E_T -independent shift Λ in jet energy (Mangano,

hep-ph/9911256).

- Cross section ratio should *scale* up to *PDF* and α_s effects.
- Data can be fitted with *shift* in distribution.
- *Small* Λ has impact at *high* E_T .
- $\sigma(E_T) \sim E_T^{-n} \rightarrow \frac{\delta\sigma}{\sigma} \sim -n\delta E_T$
- *Several* sources of energy flow *in* and *out* of jets.



Power corrections and other problems ...

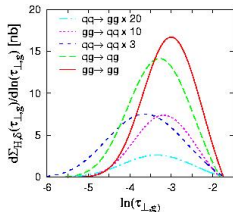
- *Sources* of power corrections
 - Soft radiation from *hard antenna* \Rightarrow *resummation*.
 - * *Calculable* in perturbative QCD.
 - * Partly *localized* in phase space.
 - Soft radiation from *underlying event* \Rightarrow *models*.
 - * *Not calculable* in perturbative QCD.
 - * *Fills* phase space (*minijets?*)
- Experimental *issues*.
 - Detector *coverage* and event *cuts* \Rightarrow *constraints* on global event shapes.
 - *Observable-specific* problems \Rightarrow *jet* algorithms, *non-global* logarithms.
- Need *discriminating* observables ...



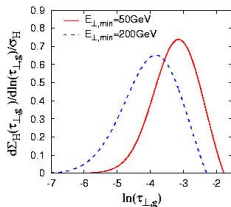


An example: the transverse thrust

A. Banfi et al. hep-ph/0407287



τ_{\perp} distribution separated into different hard scattering channels.

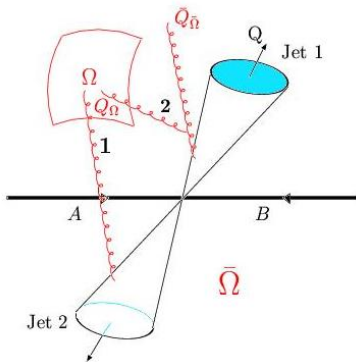


τ_{\perp} distribution summed over channels for different $E_{\perp, \min}$ cuts.

- Numerically *resummable* with **CAESAR**. Resummation applicable to $\log \tau_{\perp} \sim \eta_{\max}$.
- *Discriminates* parton channels.
- Underlying event modeled by *shift*.



Event shape/energy flow correlations



Energy flow into Ω and origin of nonglobal logs

- Gluon 1: $\log(Q_\Omega/Q)$.
- Gluon 2: $\log(Q_\Omega/Q_{\bar{\Omega}})$.
- *Resummation* of nonglobal logs *under study*.
- Non-global \rightarrow Non-Sudakov \rightarrow Non-linear.
- Can one *suppress* them?
- Study soft radiation *without* hard antenna?

Event shape/energy flow correlations

- In e^+e^- annihilation *suppress* nonglobal logs via

$$\sigma(\epsilon_1, \epsilon_2) = \frac{1}{2s} \sum_N \overline{|M(N)|^2} \delta(\epsilon_1 - f_\Omega(N)) \delta(\epsilon_2 - \tau_a^{(1)}(N) - \tau_a^{(2)}(N))$$

with $f_\Omega(N) = (\sum_{i \in \Omega} \omega_i) / s$ and $\tau_a^{(n)}$ the angularity of jet n .

- At small ϵ_1, ϵ_2 , with $\epsilon_1 \sim \epsilon_2$ radiation into $\bar{\Omega}$ is forced to the *two-jet limit*.
- Possible *generalization* to hadron-hadron collisions: introduce ϵ_3 for beam jets. $\{\epsilon_i, a\}$ serve as *handles* to *tune* soft radiation.
 - $\epsilon_i \ll 1, a > 0$: narrow jets, wide-angle radiation suppressed.
 - $\epsilon_i \ll 1, a < 0$: inclusive on soft radiation.
 - $\epsilon_3 \sim 1, a < 0$: suppress high rapidity.



Perspective

- *Event shape distributions* map the transition between **perturbative** and **non-perturbative** QCD (also in *DIS* ...)
- *Theoretical advances* lead to **testable** QCD-motivated **models** of power corrections (*shape functions*).
- *DGE* combines resummation with renormalon calculus.
 - All available **perturbative** information is **retained**.
 - A well-defined **matching** between PT and NP is **provided**.
 - Models for shape functions yield testable **predictions**:
C-parameter, angularities ...
- Detailed *phenomenology* awaits destiny of **data**
- Extension to *hadron collisions* desirable, *flexible* observables required.

