Factorization and Universality for massless gauge theory amplitudes

Lorenzo Magnea

Università di Torino – INFN, Sezione di Torino

Work in collaboration with L. Dixon and G. Sterman

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Outline

Ancient History

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Note on the Radiation Field of the Electron

F. Bloch and A. Nordsieck*
Stanford University, California

(Received May 14, 1937)

Previous methods of treating radiative corrections in non-stationary processes such as the scattering of an electron in an atomic field or the emission of a β-ray, by an expansion in powers of $e^2/\hbar c$, are defective in that they predict infinite low frequency corrections to the transition probabilities. This difficulty can be avoided by a method developed here which is based on the alternative assumption that $e^2\omega/mc^3$, $\hbar\omega/mc^2$ and $\hbar\omega/c\Delta p$ (ω=angular frequency of radiation, Δp=change in momentum of electron) are small compared to unity. In contrast to the expansion in powers of $e^2/\hbar c$, this permits the transition to the classical limit $\hbar=0$. External perturbations on the electron are treated in the Born approximation. It is shown that for frequencies such that the above three parameters are negligible the quantum mechanical calculation yields just the directly reinterpreted results of the classical formulae, namely that the total probability of a given change in the motion of the electron is unaffected by the interaction with radiation, and that the mean number of emitted quanta is infinite in such a way that the mean radiated energy is equal to the energy radiated classically in the corresponding trajectory.

A remarkable achievement, before quantum field theory was born.

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Modern History

Factorization

$$\mathcal{M}^{[\mathrm{f}]}_{\{r_i\}}\left(\beta_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \sum_{L=1}^{N^{[\mathrm{f}]}} \mathcal{M}^{[\mathrm{f}]}_{L}\left(\beta_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) (c_L)_{\{r_i\}}$$

$$\mathcal{M}^{[\mathrm{f}]}_{L}\left(\beta_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \prod_{i=1}^{n+2} J^{[i]}\left(\frac{Q'^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)$$
$$\times \; S^{[\mathrm{f}]}_{LI}\left(\beta_j, \frac{Q'^2}{\mu^2}, \frac{Q'^2}{Q^2}, \alpha_s(\mu^2), \epsilon\right) \; H^{[\mathrm{f}]}_{I}\left(\beta_j, \frac{Q^2}{\mu^2}, \frac{Q'^2}{Q^2}, \alpha_s(\mu^2)\right),$$

Progress

- Exponentiation applies to *non-abelian* gauge theories.
- Exponentiation extends to *collinear* divergences.
- Exponentiation is performed at the *amplitude* level.
- An optimal *regularization scheme* is used.

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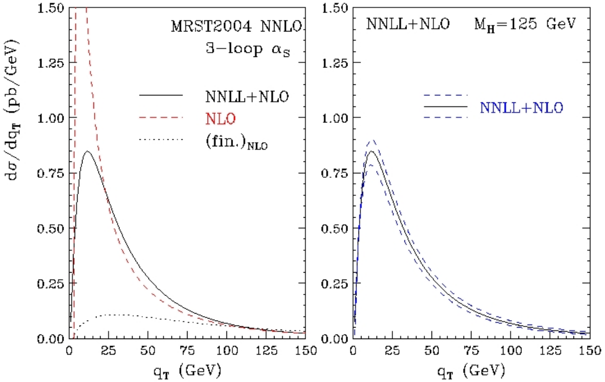
Motivation: LHC phenomenology

Higgs boson spectrum at LHC *(M. Grazzini,* hep-ph/0512025*)*



Predictions for the q_T spectrum of Higgs bosons produced via gluon fusion at the LHC, with and without resummation.

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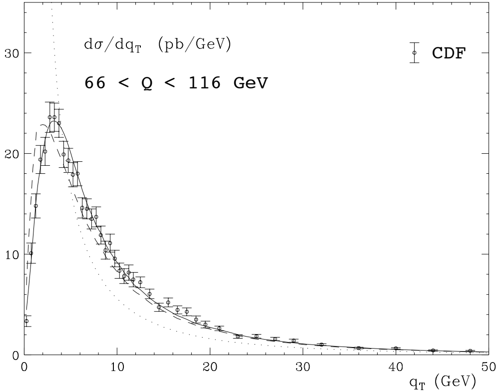
Motivation: LHC phenomenology

Z boson spectrum at Tevatron (A. Kulesza et al., hep-ph/0207148)



CDF data on Z production compared with QCD predictions at fixed order (dotted), with resummation (dashed), and with the inclusion of power corrections (solid).

Motivation: gauge field theories

- *Remarkable progress* has been achieved in *techniques* to compute *finite order* gauge theory amplitudes.
- *Supersymmetric versions* of *Yang-Mills theory* and *QCD* have remarkable properties.

  Example: $\mathcal{N} = 4$ SYM is *conformal invariant*: $\beta_{\mathcal{N}=4}(\alpha_s) = 0$.

  - *Exponentiation* of IR/C poles in QCD amplitudes *simplifies*

    Note: at most *double* poles in the exponent.
  - *AdS/CFT* suggests that $\mathcal{N} = 4$ SYM must 'be simple' *at strong coupling*. Can this be seen in *perturbation theory*?
  - *Exponentiation* has been observed for MHV amplitudes with up to *five legs* (Z. Bern *et al.*).
  - A *stringy* calculation at *strong coupling* is consistent with the *perturbative result* (L. Alday and J. Maldacena).

Tools: dimensional regularization

Nonabelian exponentiation of IR poles requires d-*dimensional* evolution equations. The *running coupling* in $d = 4 - 2\epsilon$ obeys

$$\mu \frac{\partial \overline{\alpha}}{\partial \mu} \equiv \beta(\epsilon, \overline{\alpha}) = -2\,\epsilon\,\overline{\alpha} + \hat{\beta}(\overline{\alpha}) \quad , \quad \hat{\beta}(\overline{\alpha}) = - \frac{\overline{\alpha}^2}{2\pi} \sum_{n=0}^{\infty} b_n \left(\frac{\overline{\alpha}}{\pi} \right)^n \quad .$$

The *one-loop* solution is

$$\overline{\alpha}\left(\mu^2\right) = \alpha_s(\mu_0^2) \left[\left(\frac{\mu^2}{\mu_0^2} \right)^{\epsilon} - \frac{1}{\epsilon} \left(1 - \left(\frac{\mu^2}{\mu_0^2} \right)^{\epsilon} \right) \frac{b_0}{4\pi} \alpha_s(\mu_0^2) \right]^{-1} \quad .$$

The β function develops an *IR free* fixed point, so that $\overline{\alpha}(0, \epsilon) = 0$ for $\epsilon < 0$. The Landau pole is at

$$\mu^2 = \Lambda^2 \equiv Q^2 \left(1 + \frac{4\pi\epsilon}{b_0 \alpha_s(Q^2)} \right)^{-1/\epsilon} \quad .$$

Tools: factorization

All *factorizations* separating dynamics at different energy scales lead to *resummation* of logarithms of the ratio of scales.

- *Renormalization group* logarithms.

  Renormalization *factorizes* cutoff dependence

$$G_0^{(n)}\left(p_i, \Lambda, g_0\right) = \prod_{i=1}^{n} Z_i^{1/2}\left(\Lambda/\mu, g(\mu)\right)\; G_R^{(n)}\left(p_i, \mu, g(\mu)\right) \;,$$

$$\frac{dG_0^{(n)}}{d\mu} = 0 \quad \rightarrow \quad \frac{d\log G_R^{(n)}}{d\log\mu} = -\sum_{i=1}^{n} \gamma_i\left(g(\mu)\right) \;.$$

- *RG* evolution *resums* $\alpha_s^n(\mu^2)\log^n\left(Q^2/\mu^2\right)$ into $\alpha_s(Q^2)$.

Note: *Factorization* is the *difficult* step . . .

Tools: factorization

- *Collinear factorization* logarithms.

  Mellin moments of partonic *DIS structure functions* factorize

$$\widetilde{F}_2\left(N,\frac{Q^2}{m^2},\alpha_s\right)=\widetilde{C}\left(N,\frac{Q^2}{\mu_F^2},\alpha_s\right)\widetilde{f}\left(N,\frac{\mu_F^2}{m^2},\alpha_s\right)$$

$$\frac{d\widetilde{F}_2}{d\mu_F}=0\quad\rightarrow\quad\frac{d\log\widetilde{f}}{d\log\mu_F}=\gamma_N\left(\alpha_s\right)\ .$$

- *Altarelli-Parisi* evolution *resums* collinear logarithms into *evolved* parton distributions.

Note: *Sudakov* logarithms are more difficult. Ordinary renormalization group is not sufficient. Gauge invariance plays a key role. *Or:* use effective field theory (SCET).

Gauge theory form factors

Consider as an example the *quark form factor*

$$\Gamma_\mu(p_1, p_2; \mu^2, \epsilon) \equiv \langle 0 | J_\mu(0) | p_1, p_2 \rangle = \overline{v}(p_2) \gamma_\mu u(p_1) \; \Gamma\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) .$$

- The form factor obeys the *evolution equation*

$$Q^2 \frac{\partial}{\partial Q^2} \log\left[\Gamma\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)\right] = \frac{1}{2}\left[K\left(\epsilon, \alpha_s(\mu^2)\right) + G\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)\right] ,$$

- *Renormalization group invariance* requires

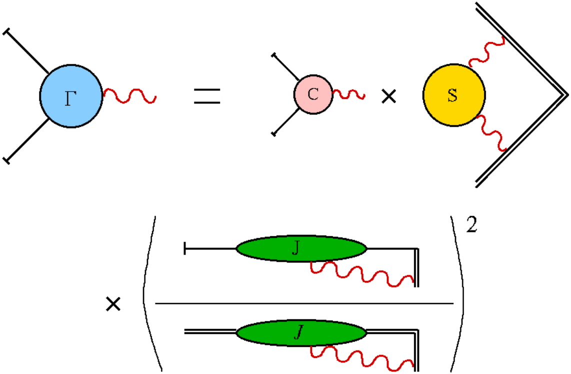
$$\mu \frac{dG}{d\mu} = -\mu \frac{dK}{d\mu} = \gamma_K\left(\alpha_s(\mu^2)\right) ,$$

  Note: $\gamma_K(\alpha_s)$ is the *cusp anomalous dimension*.

- *Dimensional regularization* provides a *trivial initial condition* for evolution if $\epsilon < 0$ (for *IR* regularization).

$$\overline{\alpha}(\mu^2 = 0, \epsilon < 0) = 0 \quad \rightarrow \Gamma\left(0, \alpha_s(\mu^2), \epsilon\right) = \Gamma\left(1, \overline{\alpha}\left(0, \epsilon\right), \epsilon\right) = 1 .$$

Detailed factorization



Operator factorization of the Sudakov form factor, with subtractions.

Operator definitions

The *functional form* of this graphical factorization is

$$\Gamma\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = C\left(\frac{Q^2}{\mu^2}, \frac{(p_i \cdot n_i)^2}{n_i^2 \mu^2}, \alpha_s(\mu^2), \epsilon\right) \times \mathcal{S}\left(\beta_1 \cdot \beta_2, \alpha_s(\mu^2), \epsilon\right) \times \prod_{i=1}^{2} \left[\frac{J\left(\frac{(p_i \cdot n_i)^2}{n_i^2 \mu^2}, \alpha_s(\mu^2), \epsilon\right)}{\mathcal{J}\left(\frac{(\beta_i \cdot n_i)^2}{n_i^2}, \alpha_s(\mu^2), \epsilon\right)} \right] .$$

We introduced *factorization vectors* n_i^μ, with $n_i^2 \neq 0$, to define the *jets*,

$$J\left(\frac{(p \cdot n)^2}{n^2 \mu^2}, \alpha_s(\mu^2), \epsilon\right) u(p) = \langle 0 \,| \Phi_n(\infty, 0)\, \psi(0) \,| p \rangle .$$

where Φ_n is the *Wilson line* operator along the direction n_i^μ.

$$\Phi_n(\lambda_2, \lambda_1) = P \exp\left[ig \int_{\lambda_1}^{\lambda_2} d\lambda\, n \cdot A(\lambda n) \right] ,$$

The jet J has *collinear* divergences only along p.

Operator definitions

The *soft function* $\mathcal{S}$ is the *eikonal limit* of the massless form factor

$$\mathcal{S}\left(\beta_1 \cdot \beta_2, \alpha_s(\mu^2), \epsilon\right) = \langle 0 | \Phi_{\beta_2}(\infty, 0)\, \Phi_{\beta_1}(0, -\infty) | 0 \rangle \, .$$

Soft-collinear regions are *subtracted* dividing by *eikonal jets* $\mathcal{J}$.

$$\mathcal{J}\left(\frac{(\beta_1 \cdot n_1)^2}{n_1^2}, \alpha_s(\mu^2), \epsilon\right) = \langle 0 | \Phi_{n_1}(\infty, 0)\, \Phi_{\beta_1}(0, -\infty) | 0 \rangle \, ,$$

- $\mathcal{S}$ and $\mathcal{J}$ are *pure counterterms* in dimensional regularization.
- $\mathcal{S}$ *only* depends on kinematics through the *cusp anomaly*.
- A *single pole* function where the cusp anomaly *cancels* is

$$\overline{\mathcal{S}}\left(\rho_{12}, \alpha_s(\mu^2), \epsilon\right) \equiv \frac{\mathcal{S}\left(\beta_1 \cdot \beta_2, \alpha_s(\mu^2), \epsilon\right)}{\prod_{i=1}^{2} \mathcal{J}\left(\frac{(\beta_i \cdot n_i)^2}{n_i^2}, \alpha_s(\mu^2), \epsilon\right)}$$

It can *only* depend on the *scaling variable*

$$\rho_{12} \equiv \frac{(-\beta_1 \cdot \beta_2)^2 \; n_1^2 \; n_2^2}{(-\beta_1 \cdot n_1)^2 \, (-\beta_2 \cdot n_2)^2} \, .$$

Jet evolution

The *full form factor* does not depend on the *factorization vectors* n_i^μ.
Defining $x_i \equiv (-\beta_i \cdot n_i)^2 / n_i^2$,

$$x_i \frac{\partial}{\partial x_i} \log \Gamma \left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) = 0 \,.$$

This *dictates* the evolution of the jet J

$$\begin{aligned} x_i \frac{\partial}{\partial x_i} \log J_i \quad &= \quad - x_i \frac{\partial}{\partial x_i} \log C \, + \, x_i \frac{\partial}{\partial x_i} \log \mathcal{J}_i \\ &\equiv \quad \frac{1}{2} \left[\mathcal{G}_i \Big(x_i, \alpha_s(\mu^2), \epsilon \Big) + \mathcal{K} \Big(\alpha_s(\mu^2), \epsilon \Big) \right] \,, \end{aligned}$$

Imposing *RG invariance* of the form factor

$$\gamma_{\overline{\mathcal{S}}} \left(\rho_{12}, \alpha_s \right) + \gamma_C \left(\rho_{12}, \alpha_s \right) + 2\gamma_J \left(\alpha_s \right) = 0 \,.$$

leads to the final *evolution equation*

$$Q \frac{\partial}{\partial Q} \log \Gamma = \beta(\epsilon, \alpha_s) \frac{\partial}{\partial \alpha_s} \log C - \gamma_{\overline{\mathcal{S}}} - 2 \, \gamma_J + \sum_{i=1}^{2} \left(\mathcal{G}_i + \mathcal{K} \right) \,.$$

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Collinear evolution

It is *useful* to establish a connection to conventional *collinear factorization*. Define a *parton-in-parton* distribution as

$$\phi_{q/q}\left(x,\epsilon\right) = \frac{1}{4N_c}\int \frac{d\lambda}{2\pi}\,\mathrm{e}^{-\mathrm{i}\lambda x p\cdot\beta}\,\langle p|\overline{\psi}_q(\lambda\beta)\,\gamma\cdot\beta\,\Phi_\beta(\lambda,0)\,\psi_q(0)|p\rangle\,,$$

The *virtual contribution* can be isolated. At the *amplitude level* it is

$$\overline{\Gamma}_{q/q}\left(\frac{p\cdot\beta}{\mu},\alpha_s(\mu^2),\epsilon\right) \equiv \langle 0|\Phi_\beta(\infty,0)\,\psi_q(0)\,|p\rangle\,,$$

Comparing *factorizations* for this amplitude and the jet J, and enforcing *Altarelli-Parisi* evolution one finds

$$\frac{\left[J\left(\frac{(\beta_p\cdot n)^2}{n^2},\alpha_s(\mu^2),\epsilon\right)\right]_{\rm pole}}{\mathcal{J}\left(\frac{(\beta_p\cdot n)^2}{n^2},\alpha_s(\mu^2),\epsilon\right)} = \frac{\overline{\Gamma}_{q/q}\left(\beta_p\cdot\beta,\alpha_s(\mu^2),\epsilon\right)}{\overline{\Gamma}^{\rm eik}_{q/q}\left(\beta_p\cdot\beta,\alpha_s(\mu^2),\epsilon\right)}$$

$$= \exp\left[\frac{1}{2}\int_0^{\mu^2}\frac{d\xi^2}{\xi^2}\,B_\delta^{[q]}\left(\overline{\alpha}(\xi^2,\epsilon)\right)\right].$$

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Results for Sudakov form factors

- In dimensional regularization $(\epsilon < 0)$ one has the *boundary value* $\Gamma(0,\epsilon) = 1$. Then

$$\log\left[\Gamma\left(Q^2,\epsilon\right)\right] = \frac{1}{2}\int_0^{-Q^2}\frac{d\xi^2}{\xi^2}\left[K(\epsilon) + G\left(\overline{\alpha}\left(\xi^2\right),\epsilon\right) + \frac{1}{2}\int_{\xi^2}^{\mu^2}\frac{d\lambda^2}{\lambda^2}\gamma_K\left(\overline{\alpha}\left(\lambda^2\right)\right)\right]$$

- The functions K and γ_K are *not independent*

$$\mu\frac{d}{d\mu}K(\epsilon,\alpha_s) = -\gamma_K(\alpha_s) \quad\Longrightarrow\quad K\left(\epsilon,\alpha_s(\mu^2)\right) = -\frac{1}{2}\int_0^{\mu^2}\frac{d\lambda^2}{\lambda^2}\gamma_K\left(\bar{\alpha}(\lambda^2,\epsilon)\right) .$$

- The *form factor* can be written in terms of just G and γ_K,

$$\begin{aligned}\Gamma\left(Q^2,\epsilon\right) \quad &= \quad \exp\left\{\frac{1}{2}\int_0^{-Q^2}\frac{d\xi^2}{\xi^2}\left[G\Big(-1,\overline{\alpha}\left(\xi^2,\epsilon\right),\epsilon\Big)\right.\right.\\ &\left.\left. - \frac{1}{2}\gamma_K\left(\overline{\alpha}\left(\xi^2,\epsilon\right)\right)\log\left(\frac{-Q^2}{\xi^2}\right)\right]\right\} .\end{aligned}$$

Implications

- The exponent is *not affected* by the *Landau pole* for $\epsilon < 0$. Γ is an *analytic function* of the coupling and ϵ. At *one loop* in QCD

$$\log \Gamma\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \log \Gamma\left(1, \alpha_s(Q^2), \epsilon\right)$$
$$= -\frac{1}{b_0}\left\{\frac{\gamma_K^{(1)}}{\epsilon}\,\mathrm{Li}_2\left[\frac{a(Q^2)}{a(Q^2)+\epsilon}\right] + 2\,G^{(1)}(\epsilon)\,\log\left[1+\frac{a(Q^2)}{\epsilon}\right]\right\}.$$

- The *ratio* of the *timelike* to the *spacelike* form factor admits a simple representation

$$\log\left[\frac{\Gamma(Q^2,\epsilon)}{\Gamma(-Q^2,\epsilon)}\right] = \mathrm{i}\frac{\pi}{2}K(\epsilon) + \frac{\mathrm{i}}{2}\int_0^\pi \left[G\left(\overline{\alpha}\left(\mathrm{e}^{\mathrm{i}\theta}Q^2\right),\epsilon\right) - \frac{\mathrm{i}}{2}\int_0^\theta d\phi\,\gamma_K\left(\overline{\alpha}\left(\mathrm{e}^{\mathrm{i}\phi}Q^2\right)\right)\right]$$

which is *physically relevant* for resummed EW annihilation processes.

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Form factors in $\mathcal{N} = 4$ *SYM*

- In $d = 4 - 2\epsilon$ conformal invariance is *broken* and $\beta(\alpha_s) = -2\,\epsilon\,\alpha_s$.
- All integrations are trivial. The exponent has *only double* and *single* poles to *all orders*.

$$
\begin{aligned}
\log\left[\Gamma\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)\right] &= -\frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n \left(\frac{\mu^2}{-Q^2}\right)^{n\epsilon}\left[\frac{\gamma_K^{(n)}}{2n^2\epsilon^2} + \frac{G^{(n)}(\epsilon)}{n\epsilon}\right] \\
&= -\frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{\alpha_s(Q^2)}{\pi}\right)^n \mathrm{e}^{-\mathrm{i}\pi n\epsilon}\left[\frac{\gamma_K^{(n)}}{2n^2\epsilon^2} + \frac{G^{(n)}(\epsilon)}{n\epsilon}\right] ,
\end{aligned}
$$

- In the *planar limit* this captures *all singularities* of fixed-angle amplitudes in $\mathcal{N} = 4$ SYM.
- The *analytic continuation* yields a *finite* result in *four dimensions*, arguably *exact*.

$$
\left|\frac{\Gamma(Q^2)}{\Gamma(-Q^2)}\right|^2 = \exp\left[\frac{\pi^2}{4}\,\gamma_K\left(\alpha_s(Q^2)\right)\right] .
$$

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Characterizing $G(\alpha_s, \epsilon)$

The *single pole* function $G(\alpha_s, \epsilon)$ is a sum of *anomalous dimensions*

$$G(\alpha_s) = \beta(\epsilon, \alpha_s) \frac{\partial}{\partial \alpha_s} \log C - \gamma_{\bar{S}} - 2\gamma_J + \sum_{i=1}^{2} \mathcal{G}_i \,,$$

In $d = 4 - 2\epsilon$ *finite remainders* can be *neatly exponentiated*

$$C\left(\alpha_s(Q^2), \epsilon\right) = \exp\left[\int_0^{Q^2} \frac{d\xi^2}{\xi^2} \left\{ \frac{d \, \log \overline{C}\left(\overline{\alpha}\left(\xi^2, \epsilon\right), \epsilon\right)}{d \ln \xi^2} \right\}\right] \equiv \exp\left[\frac{1}{2} \int_0^{Q^2} \frac{d\xi^2}{\xi^2} G_{\overline{C}}\left(\overline{\alpha}\left(\xi^2, \epsilon\right), \epsilon\right)\right]$$

The *soft function* exponentiates *like* the full form factor

$$\mathcal{S}\left(\alpha_s(\mu^2), \epsilon\right) = \exp\left\{\frac{1}{2} \int_0^{\mu^2} \frac{d\xi^2}{\xi^2} \left[G_{\rm eik}\left(\overline{\alpha}\left(\xi^2, \epsilon\right)\right) - \frac{1}{2}\gamma_K\left(\overline{\alpha}\left(\xi^2, \epsilon\right)\right) \log\left(\frac{\mu^2}{\xi^2}\right)\right]\right\} .$$

$G(\alpha_s, \epsilon)$ is then *simply related* to *collinear splitting functions* and to the *eikonal approximation*

$$G(\alpha_s, \epsilon) = 2\, B_\delta\left(\alpha_s\right) + G_{\rm eik}\left(\alpha_s\right) + G_{\overline{C}}\left(\alpha_s, \epsilon\right) \,,$$

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Single logarithms in resummation

A deeper characterization of infrared and collinear *single poles* in *amplitudes* should be reflected in *single logarithms* in *cross sections*.

For a resummation as in the *Drell-Yan* cross section

$$\widehat{\omega}_{\overline{\rm MS}}(N) = \left| \frac{\Gamma(Q^2,\epsilon)}{\phi_V(Q^2,\epsilon)} \right|^2 \exp\left[F_{\rm DY}(\alpha_s) + \int_0^1 dz \, \frac{z^{N-1}-1}{1-z} \left\{ 2 \int_{Q^2}^{(1-z)^2 Q^2} \frac{d\mu^2}{\mu^2} A\left(\alpha_s(\mu^2)\right) + D\left(\alpha_s\left((1-z)^2 Q^2\right)\right) \right\} \right] .$$

the function $D(\alpha_s)$ should be *related* to $B_\delta(\alpha_s)$ and to $G(\alpha_s)$. Indeed,

$$\begin{aligned} A(\alpha_s) &= \gamma_K(\alpha_s)/2 \, , \\ D(\alpha_s) &= 4\, B_\delta(\alpha_s) - 2\, \widetilde{G}(\alpha_s) + \hat{\beta}(\alpha_s) \frac{d}{d\alpha_s} F_{\rm DY}(\alpha_s) \, , \\ B(\alpha_s) &= B_\delta(\alpha_s) - \widetilde{G}(\alpha_s) + \hat{\beta}(\alpha_s) \frac{d}{d\alpha_s} F_{\rm DIS}(\alpha_s) \, , \end{aligned}$$

where $B(\alpha_s)$ is the *single log* function for *DIS* resummation.

Perspective

- The *all-order* analysis of infrared and collinear divergences in gauge theories has a *long history*.
- This history is *entering a new phase*
    - New motivations from *LHC phenomenology*
    - New *(non)-perturbative* input from $\mathcal{N} = 4$ SYM.
- We are beginning to unravel *all-order structure* in the resummed *exponent*.
- *Exact results* are in sight in $\mathcal{N} = 4$ SYM.
- For infrared and collinear divergences in *fixed-angle* massless gauge theory *amplitudes* only *three functions* play a role: $\gamma_K(\alpha_s)$, $G_{\rm eik}(\alpha_s)$ and $B_\delta(\alpha_s)$, possibly even *beyond* the *planar limit*.