Power Corrections in Perturbative QCD

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Abstract

Recent work on the theme of power corrections to QCD factorization theorems is briefly reviewed. Resummation of certain classes of contributions to IR–safe observables highlights the limitations of perturbation theory. The ambiguity of the perturbative answer points to nonperturbative, power suppressed corrections. Power corrections can be resummed into shape functions, for which theoretical models are available. Theoretical progress is closing in on the nonperturbative frontier.

Some references: M Beneke: hep-ph/9807443.
G. Korchemski, G. Sterman: hep-ph/9902341.
M. Dasgupta: hep-ph/0109220.
E. Gardi: hep-ph/0108222.

Outline

- The perturbative window on power corrections
  - "Large n_f" resummations: renormalons.
  - "Large N" resummations: the edge of phase space.
- Benchmarks: observables with OPE
  - The Adler function.
  - DIS structure functions.
  - Ultraviolet dominance of power corrections.
- Beyond OPE: two–jet observables
  - Factorization, resummation and shape functions.
  - Nonlocal operators, energy correlations.
  - Recovering renormalons.
- From renormalons to shape functions: DGE
  - Dressed gluon exponentiation.
  - Borel summation and power corrections.
- Phenomenology
  - Beyond universality: hadronization and mass effects.
  - Beyond the peak: thrust and jet masses.
  - α_s and nonperturbative parameters.
- Perspectives

Taking perturbative QCD to the limit

- FACT: The perturbative series for IR safe observables in QCD is at best asymptotic. This is good!
  - Nonperturbative QCD must be present and relevant.
  - Measuring the ambiguity of the perturbative answer gives information on the size of nonperturbative corrections.
- TOOL: Resummation of perturbation theory.
  - The edge of the perturbative domain: soft gluons.
  - Soft gluon emission is universal and factorizable.
  - Factorization implies resummation: soft gluon emission can be formally computed to all orders.
- RESULT: QCD resummations (renormalon, threshold ...) typically yield expressions of the form

$$f_a(Q^2) = \int_0^{Q^2} \frac{dk^2}{k^2} (k^2)^a \alpha_s(k^2) \ ,$$

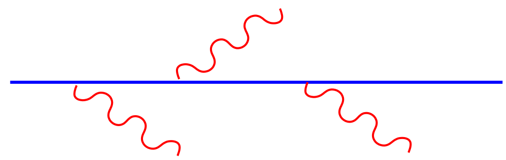
Such expressions are ill-defined, because of the Landau pole in the running coupling at $k^2 = \Lambda^2$. In fact, expanding

$$f_a(Q^2) = \sum_{n=0}^{\infty} c_n^{(a)} \left(\alpha_s(Q^2) \right)^n$$

one finds

$$c_n^{(a)} \propto n! \to \delta f_a(Q^2) \propto \left(\frac{\Lambda^2}{Q^2} \right)^a \ .$$

- EXAMPLE: soft gluon resummation.



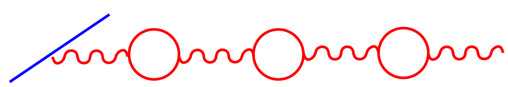
Thrust distribution $d\sigma/dt$ as $t \equiv 1 - T \to 0$.

$$\int_0^{t_{\max}} dt\, \mathrm{e}^{-\nu t} \frac{d\sigma}{dt} = \mathrm{e}^{-S(\nu,Q)} \, .$$

$$S(\nu, Q) = \int_0^1 \frac{d\alpha}{\alpha} \left(1 - \mathrm{e}^{-\nu\alpha}\right) \left[\int_{\alpha^2 Q^2}^{\alpha Q^2} \frac{dk_\perp^2}{k_\perp^2} \, \Gamma\left(\alpha_s(k_\perp^2)\right)\right] \; .$$

- Resums multiple soft gluon emission in Laplace space.
- Organizes $\log t$ singularities in $d\sigma/dt$.
- Integration over gluon $k_\perp$ exhibits Landau pole.

- EXAMPLE: renormalon resummation.



Event shapes or single particle distributions such as $\sigma_{T,L}$ can be represented as

$$\sigma(x) = \int \frac{dk^2}{k^2} \frac{1}{|1 + \Pi(k^2)|^2} \widehat{\sigma}\left(x, \frac{k^2}{Q^2}\right) \; .$$

- Resums multiple fermion bubble insertion.
- Identifies $n_f \to -3\beta_0/2$ ("naïve nonabelianization")
- Integration over gluon virtuality k^2 exhibits Landau pole.

Exploiting the consistency of QCD

- Regularizing the Landau pole

  A variety of regularizations have been proposed

  – Principal value/cutoff (Korchemsky, Sterman, Gardi)
  – Regular IR coupling (Dokshitzer, Marchesini, Webber)
  – Dimensional regularization (LM)
  – Choice of contour in Mellin/Laplace space (Catani *et al.*)
  * Result: the PT answer is ambiguous by a power–suppressed amount
  * Conclusion: a power–suppressed, non–PT contribution must exist, with a matching ambiguity.
  * Question: to what extent can we trust the PT ambiguity as a measure of the non–PT contribution?

- Benchmarks: observables with OPE

  – Nonlinear σ–model in $d = 2$ (David, 1982).
  – The Adler function $\pi'(q^2) = d\pi(q^2)/dq^2$, where $\sigma_{tot}(e^+e^-) \propto \mathrm{Im}\left[\pi(q^2)\right]$, (Muller, 1985)

$$\Pi_{\mu\nu}(q^2) \equiv \mathrm{i} \int d^4x \mathrm{e}^{\mathrm{i}qx} \langle 0|T\left[J_\mu(x)J_\nu(0)\right]|0\rangle$$

$$\pi(q^2) = \pi_0(q^2) + \frac{f^2}{Q^4}\pi_4(q^2) + \ldots$$

  – Deep inelastic structure functions (Beneke, 1998; Gardi *et al.*, 2002).

DIS and UV dominance of power corrections

Introducing a factorization scale μ_F

$$F_a(N,Q^2) = C^i_{2,a}(N,\mu_F,Q^2)\langle O^{(2)}_i(N,\mu_F)\rangle + \frac{1}{Q^2}C^j_{4,a}(N,\mu_F,Q^2)\langle O^{(4)}_j(N,\mu_F)\rangle + \ldots$$

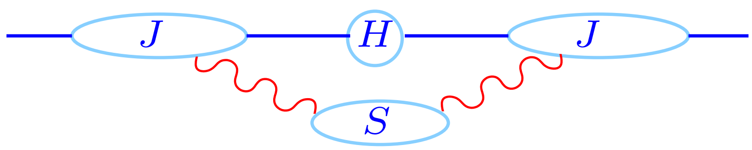
- μ_F acts as: IR cutoff for coefficient functions C^i, UV cutoff for operator matrix elements $\langle O_i\rangle$.
- Physical quantities (such as F_a) (or complete QCD predictions) must not depend on μ_F.
- Logarithmic dependence on μ_F cancels between C^i and $\langle O_i\rangle$ within a given twist, according to Altarelli–Parisi equations.
- C^i have a power–like ambiguity of IR origin, $\delta C^i = (\mu_f/Q)^p \lambda_p$, due to the divergence of PT.
- $\langle O_i\rangle$ have power–like UV divergences: they mix with leading twist, and make twist separation ambiguous.
- Once the same regularization is chosen for both C^i and $\langle O_i\rangle$ ($e.g.$ Borel representation of dressed gluon propagator), ambiguities cancel.
- Renormalon models rely on UV dominance of PC: assume $\langle O_i\rangle$ is well modelled by its UV divergences (M. Beneke, V. Braun, LM, 1997).
- UV dominance is verified in certain kinematic limits ($e.g.$ $W^2 = (1-x)Q^2/x \to 0$), where $\langle O^{4,6}_i\rangle$ are dominated by configurations mimicking twist 2 (Gardi *et al.*, 2002).

Beyond OPE: two–jet observables

- Generic IR safe observables in production processes cannot be described with OPE: they are weighted cross sections.
- Classic example: event shape distributions in e^+e^-.

$$\frac{d\sigma}{de} = \sum_N \; |\langle N \mid J(0) \mid 0 \rangle|^2 \; \delta\left(e - e(N)\right) \equiv \langle \delta\left(e - e(N)\right) \rangle \; ,$$

  where $e = 1 - T, \rho_J, C, \ldots$.
- Resummations are available for event shapes in the two–jet region, where matrix elements factorize



- In the two–jet region
  - Two scales: Qe, energy of soft gluons and $Q^2 e$, transverse momentum of jets.
  - Goal: organize corrections $(Qe)^{-n}$, neglect $(Q^2 e)^{-n}$.
  - Kinematics: $t = (M_R^2 + M_L^2)/Q^2$ as $t \to 0$.
- A useful tool: the radiation function

$$R(e) = \int_0^e de' \frac{d\sigma}{de'} = \langle \theta\left(e - e(N)\right) \rangle \; .$$

$$R_H(\rho) = \left\langle \theta\left(\rho - M_R^2/Q^2\right) \theta\left(\rho - M_L^2/Q^2\right) \right\rangle \; ,$$

$$R_T(t) = \left\langle \theta\left(t - (M_R^2 + M_L^2)/Q^2\right) \right\rangle \; .$$

Power corrections and shape functions

A view from perturbative resummation

- Sudakov exponent for thrust

$$S(\nu, Q) \equiv S_{PT}(\nu, Q, \mu) + S_{NP}(\nu, Q, \mu) \ .$$

- Focus on $t \longrightarrow 0$, large ν, expand formally in powers of ν/Q, neglect ν/Q^2. Then

$$S_{NP}(\nu/Q, \mu) = \int_0^{\mu^2} \frac{dk_\perp^2}{k_\perp^2} \Gamma\left(\alpha_s(k_\perp^2)\right) \int_{k_\perp^2/Q^2}^{k_\perp/Q} \frac{d\alpha}{\alpha} \left(1 - \mathrm{e}^{-\nu\alpha}\right)$$

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{\nu}{Q}\right)^n \lambda_n(\mu^2) \ ,$$

- Nonperturbative parameters:

$$\lambda_n(\mu^2) = \frac{1}{n} \int_0^{\mu^2} dk_\perp^2 \, k_\perp^{n-2} \Gamma\left(\alpha_s(k_\perp^2)\right) \ .$$

- Shape function:

$$\exp\left(-S_{NP}(\nu/Q, \mu)\right) \equiv \int_0^\infty d\epsilon \mathrm{e}^{-\nu\epsilon/Q} f_t(\epsilon, \mu) \ .$$

- Distribution, by inverse Laplace transform

$$\frac{d\sigma}{dt} \sim \int_0^{tQ} d\epsilon \, f_t(\epsilon, \mu) \frac{d}{dt} \sigma_{PT}\left(t - \frac{\epsilon}{Q}\right) \ .$$

- The distribution is given by the perturbative distribution, shifted by an amount proportional to the soft energy flow, and smeared by the shape function.

Features of the shape function

- More general event shapes (ρ_H) discriminate between hemispheres $\longrightarrow$ the shape function depends on two soft energy variables, ϵ_r, ϵ_l

$$M_{r,l}^2 = (M_{r,l}^2)_{PT} + \epsilon_{r,l} Q \; ; \quad \epsilon_{r,l} = \sum_{k=1}^{N_{r,l}^{(\mathrm{soft})}} E_k \left(1 - |\cos\theta_k|\right)$$

- For the heavy jet mass, for example

$$R_H(\rho) = \int_0^\mu d\epsilon_r d\epsilon_l \; f(\epsilon_r, \epsilon_l) \left\langle \theta\left(\rho - \frac{M_r^2}{Q^2} - \frac{\epsilon_r}{Q}\right) \theta\left(\rho - \frac{M_l^2}{Q^2} - \frac{\epsilon_l}{Q}\right) \right\rangle_{PT}$$

- For t and C, proportional to the sum of the hemisphere masses, one defines

$$f_{t,C}(\epsilon) = \int_0^\mu d\epsilon_r d\epsilon_l f(\epsilon_r, \epsilon_l, \mu) \delta(\epsilon - \epsilon_r - \epsilon_l)$$

- $f(\epsilon_r, \epsilon_l, \mu)$ does not depend on the hard scale Q. It is normalized by $\int d\epsilon_r d\epsilon_l f(\epsilon_r, \epsilon_l) = 1$.
- Like parton distributions, shape functions admit a nonlocal operator definition. Integer moments can be expressed in terms of correlators of the energy momentum tensor on the sphere at spatial infinity
- An example of phenomenological use

$$f(\epsilon_r, \epsilon_l) = \frac{\mathcal{N}(a,b)}{\Lambda^2} \left(\frac{\epsilon_r \epsilon_l}{\Lambda^2}\right)^{a-1} \exp\left(-\frac{\epsilon_r^2 + \epsilon_l^2 + 2b\epsilon_r\epsilon_l}{\Lambda^2}\right) \, .$$

Recovering renormalons

- Renormalon models (Dokshitzer *et al.*, Beneke *et al.*) predict moments of event shapes with one nonperturbative parameter

$$\alpha_0(\mu_F) \equiv \frac{1}{\mu_F} \int_0^{\mu_F} dk_\perp \alpha_s(k_\perp) \ .$$

- Event shapes are sensitive to radiation in different hemispheres (Nason, Seymour). A correction ("Milan") factor is necessary in the dispersive approach.
- Tube and renormalon models predict event shape distributions given at the NP level by shift of PT distribution.

$$\frac{d\sigma}{dt} \to \frac{d}{dt} \sigma_{PT} \left(t - \frac{\langle \epsilon \rangle}{Q} \right) \ .$$

* Shape functions recover dispersive results for average event shapes

$$\langle e \rangle = \langle e \rangle_{PT} + c_e \frac{\lambda_1}{Q} + \mathcal{O}\left(\frac{1}{Q^2}, \frac{\alpha_s}{Q} \right) \ .$$

Higher moments involve the new NP parameters λ_n.

* Noninclusive corrections are included in the proper definition of λ_1

$$\lambda_1 = \int d\epsilon_r d\epsilon_l (\epsilon_r + \epsilon_l) f(\epsilon_r, \epsilon_l) \ .$$

Inclusiveness assumption corresponds to $f(\epsilon_r, \epsilon_l) = g(\epsilon_r) g(\epsilon_l)$

* Shift in PT distribution is recovered with extra smearing by shape function. It is the leading correction when $e >> \Lambda/Q$.
* Shape functions provide a framework to study corrections to renormalon/dispersive results.

Dressed Gluon Exponentiation

DGE (Gardi, Gardi and Rathsman) combines Sudakov resummation with renormalon techniques, and yields a renormalon model for the shape function.

- STEP 1: Compute the characteristic function of the dispersive method (integrated probability for virtual gluon emission) for the observable at hand, in the Sudakov limit.

  Example: heavy jet mass ($\epsilon = k^2/Q^2$).

$$\dot{\mathcal{F}}(\rho, \epsilon)\Big|_{\log} = \frac{2}{\rho} - \frac{\epsilon}{\rho^2} - \frac{\epsilon^2}{\rho^3} \; .$$

- STEP 2: Dress the virtual gluon by turning to a Borel representation and integrating over gluon virtuality (Note: use "gluon bremsstrahlung" coupling to achieve NLL accuracy).

$$\frac{d\sigma}{d\rho} = \frac{C_F}{2\beta_0} \int_0^\infty du B(u, \rho) \exp\left(-u \ln Q^2/\bar{\Lambda}^2\right) \frac{\sin \pi u}{\pi u} \bar{A}_B(u)$$

- STEP 3: Use dressed gluon distribution as kernel of exponentiation.

$$\ln J(\nu, Q^2) = \int_0^1 \frac{d\sigma}{d\rho} \left(e^{-\nu\rho} - 1\right) d\rho \; ,$$

- STEP 4: Borel representation of the exponent suggests pattern of exponentiated power corrections.

$$R(\rho) = \int_C \frac{d\nu}{2\pi i \nu} \exp\left[\nu\rho + \ln J^{\rm PT}(\nu, Q^2) + \ln J^{\rm NP}(\nu, Q^2)\right] \ ,$$

$$\ln J^{\rm NP}(\nu\Lambda/Q) = - \sum_{n=1}^{\infty} \lambda_n \frac{1}{n!} \left(\frac{\nu\Lambda}{Q}\right)^n ,$$

Example: For the heavy jet mass: $\lambda_{2k} = 0$.

- STEP 5: Introduce shape function with moment structure derived by Borel representation.

$$J^{\rm NP}(\nu\Lambda/Q) = \int_0^\infty d\zeta \, f\left(\zeta\right) \, \exp\left(-\zeta\nu\Lambda/Q\right) \ .$$

- RESULTS
  - NLL Sudakov resummation reproduced. All subleading logs computed in the "large n_f" limit.
  - Factorial growth of subleading logs detected: power accuracy requires going beyond logarithms.
  - A definite prescription to handle resummed PT at power accuracy.
  - Phenomenology of thrust, jet masses; also DIS, Drell-Yan, fragmentation.
  - A renormalon implementation of the shape function.

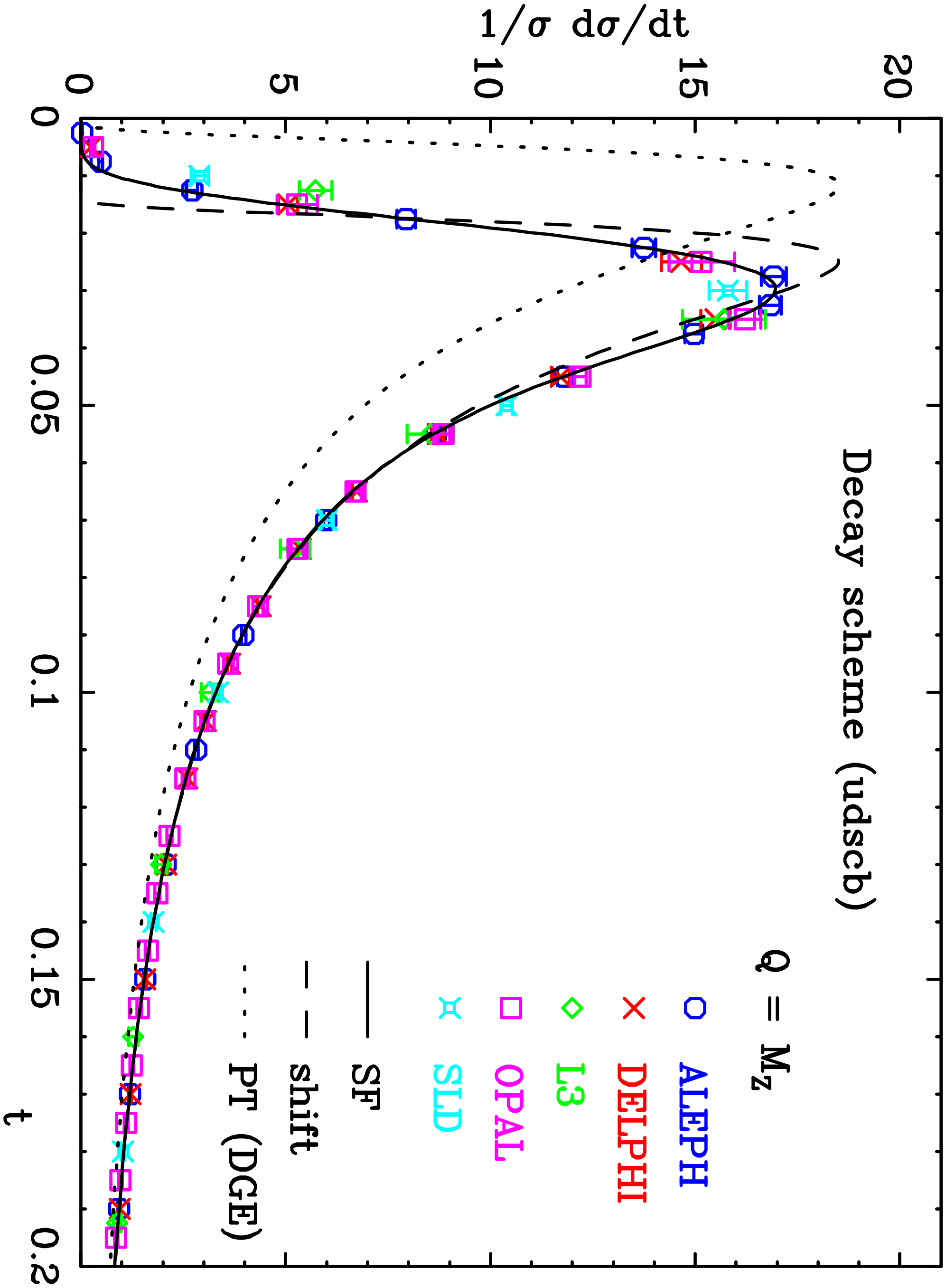
Hadronization and hadron masses

All models of power corrections are derived in massless QCD. However event shapes are measured using massive particles produced in hadronization.

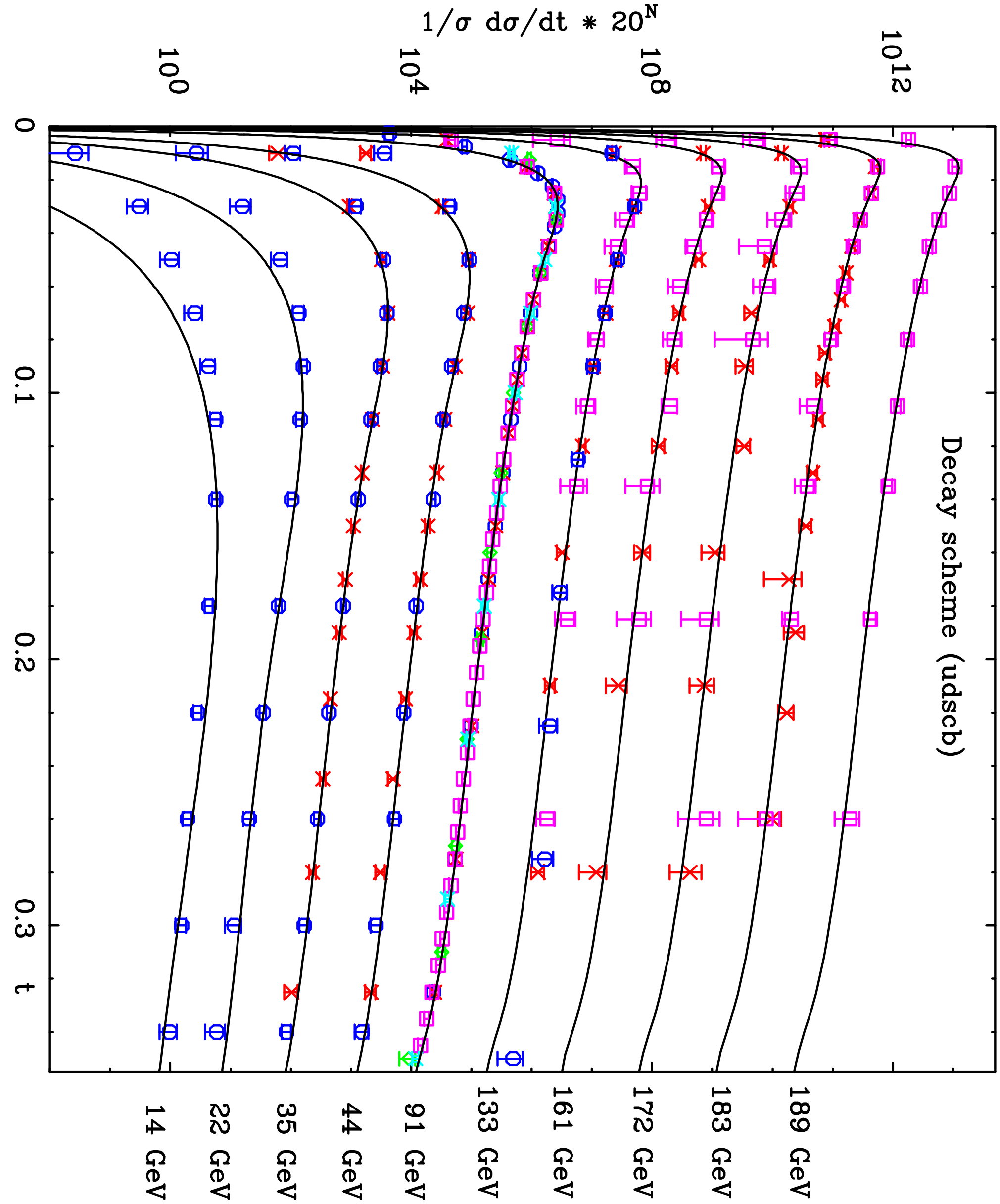
The difference between massless and massive definitions of event shapes induces non–universal power corrections of the same parametric size (Λ/Q) as conventional ones (Salam, Wicke).

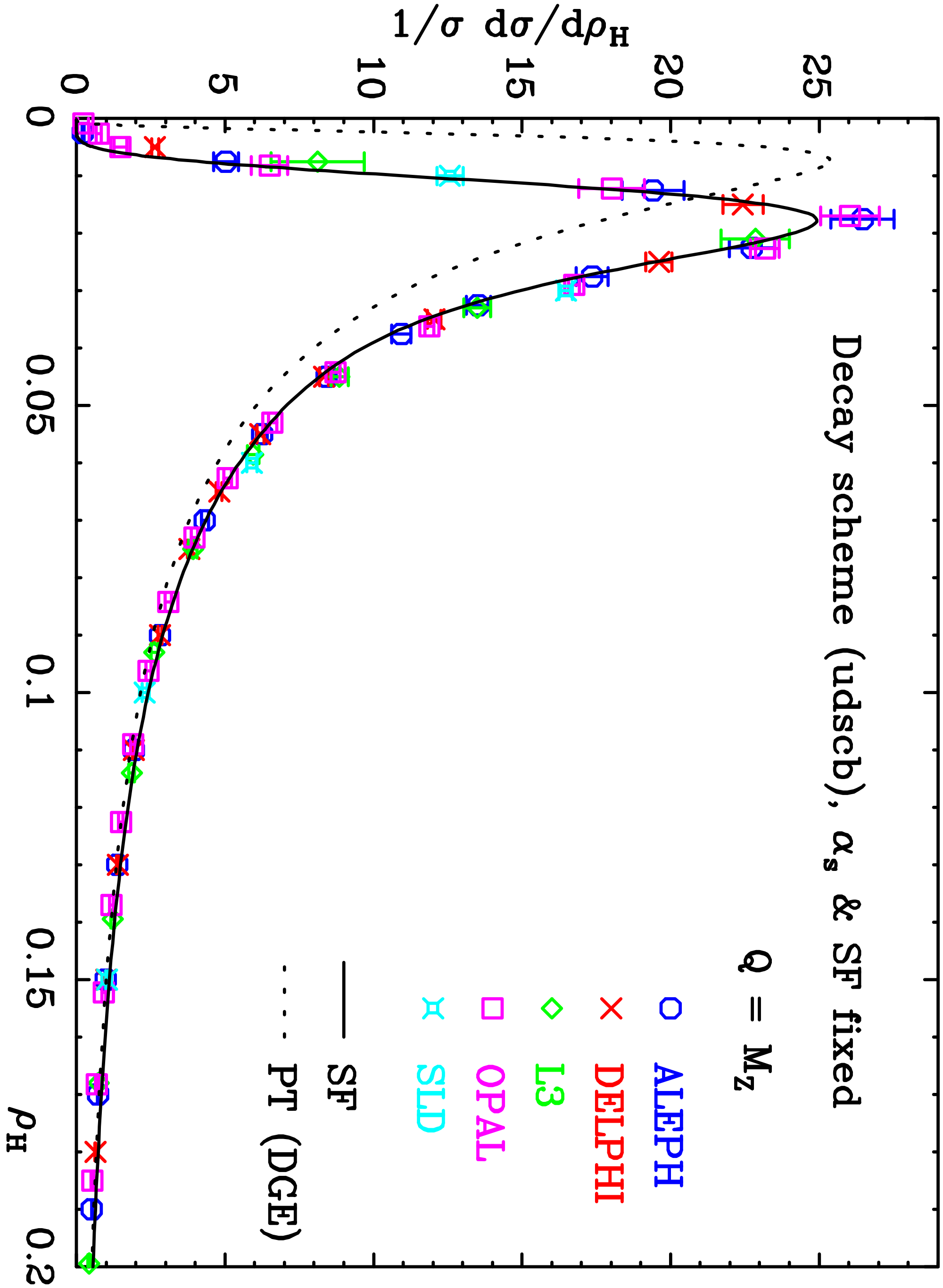
- When fitting nonperturbative parameters it is necessary to specify a scheme to connect massless QCD computation and measured event shapes
  - Conventional scheme: ignore mass effects in the definition of events shapes. Different shapes are treated differently.
  - p–scheme: use measured three–momenta $\mathbf{p}_i$, set $E_i = |\mathbf{p}_i|$. Energy is not conserved.
  - E–scheme: use measured energies E_i and rescale three–momenta by E_i/p_i. Three–momentum is not conserved. Non–universal PC vanish in a tube model.
  - Decay scheme: let all particles decay into massless particles ($e.g.$ via MC). Not all decays are strong, nor realistic.
- Mass effects are enhanced by hadron multiplicity. Model calculations using LPHD suggest

$$\delta_m \langle e \rangle = c_e \frac{\Lambda}{Q} \left(\log \frac{Q}{\Lambda} \right)^A \quad ; \quad A = \frac{4 N_c}{b_0} \sim 1.6$$



Decay scheme (udscb)
1/σ dσ/dt
t
Q = M_Z
ALEPH
DELPHI
L3
OPAL
SLD
SF
shift
PT (DGE)
0
5
10
15
20
0.05
0.1
0.15
0.2


1/σ dσ/dt * 20^N
10^0
10^4
10^8
10^12
0
0.1
0.2
0.3
t
Decay scheme (udscb)
14 GeV
22 GeV
35 GeV
44 GeV
91 GeV
133 GeV
161 GeV
172 GeV
183 GeV
189 GeV


Decay scheme (udscb), α_s & SF fixed
$Q = M_Z$
ALEPH
DELPHI
L3
OPAL
SLD
SF
PT (DGE)
$1/\sigma\ d\sigma/d\rho_H$
ρ_H
0
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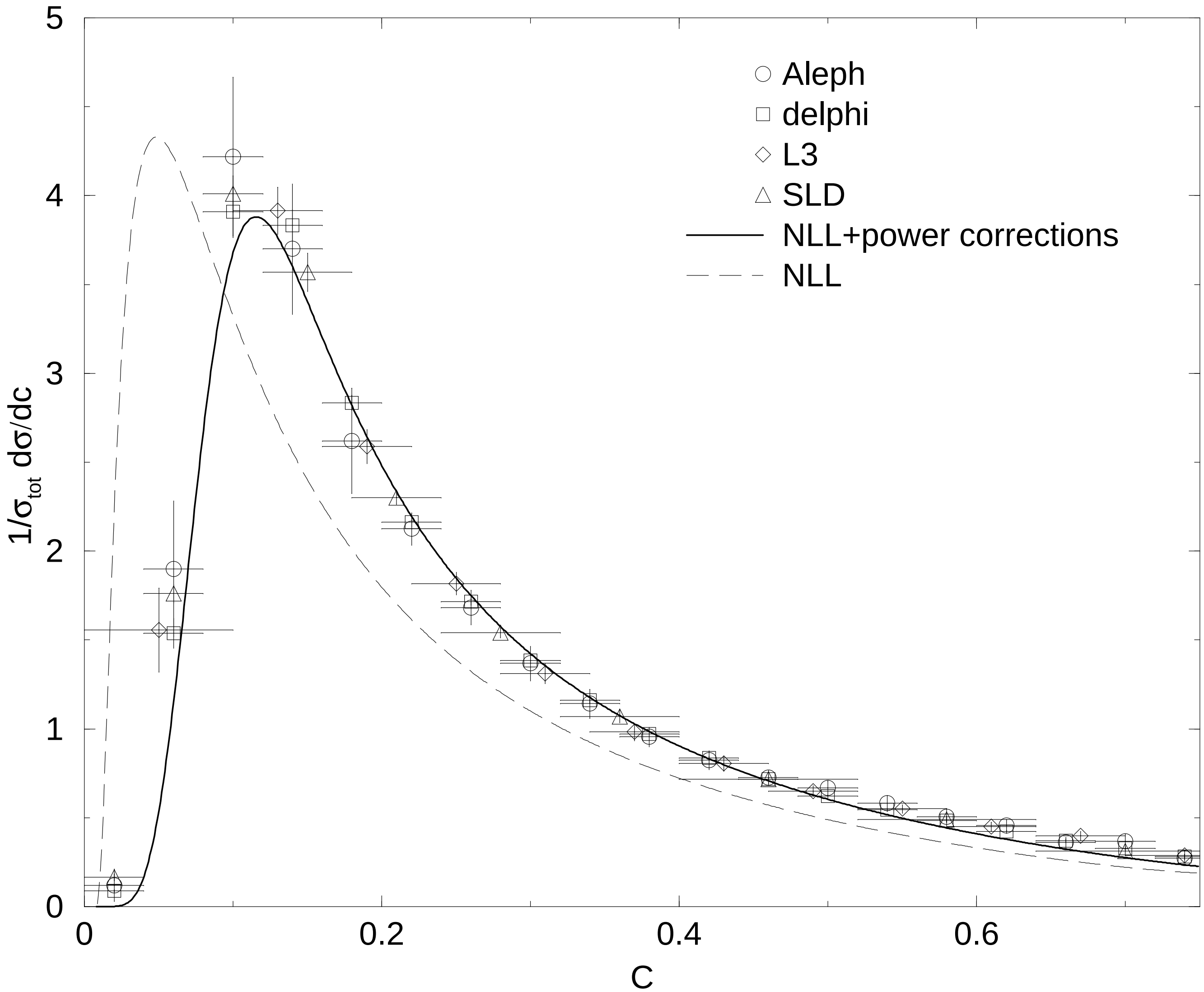
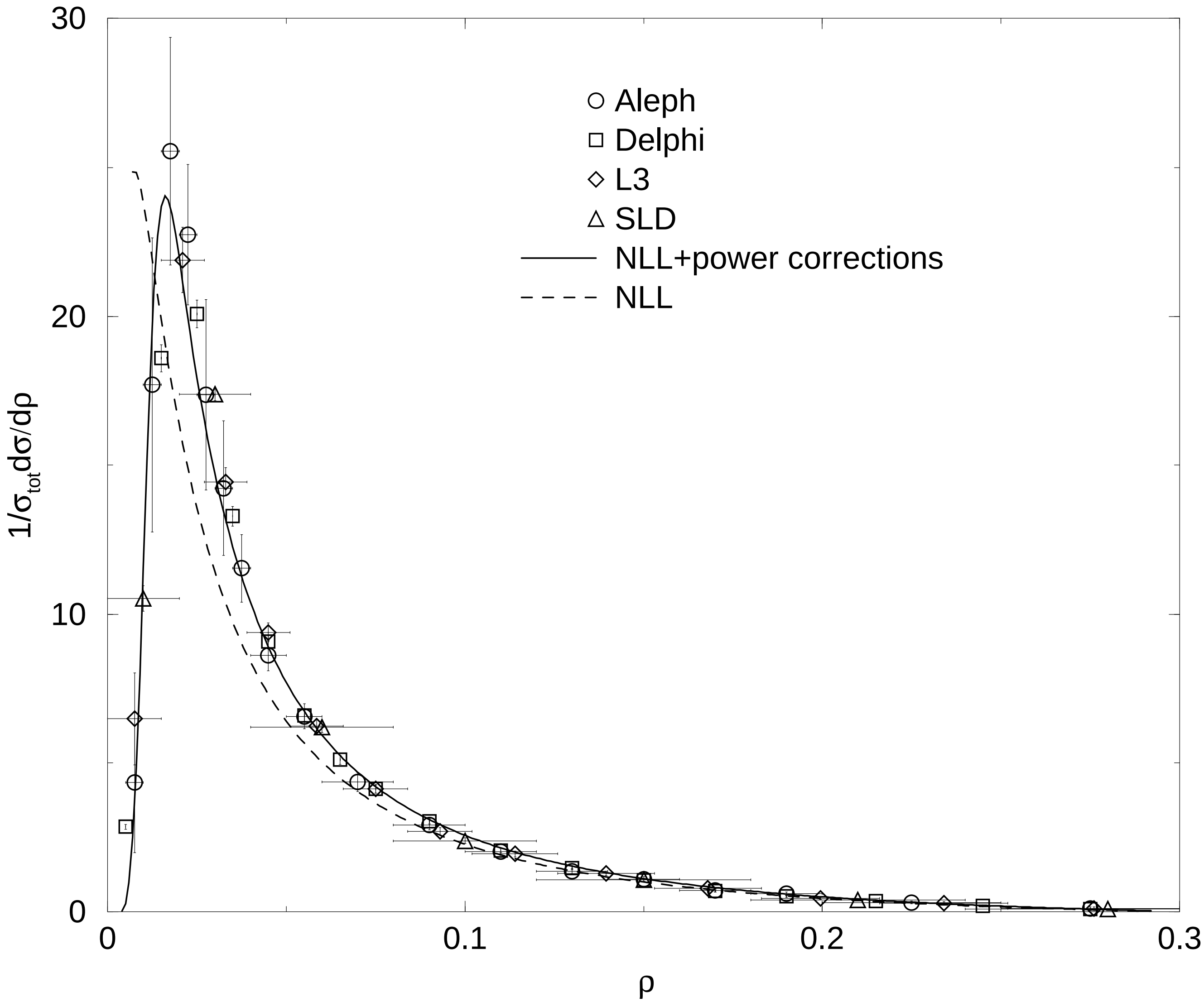
10
15
20
25
0
0.05
0.1
0.15
0.2


Aleph
delphi
L3
SLD
NLL+power corrections
NLL
1/σ_tot dσ/dc
C
0
0.2
0.4
0.6
1
2
3
4
5


Aleph
Delphi
L3
SLD
NLL+power corrections
NLL
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20
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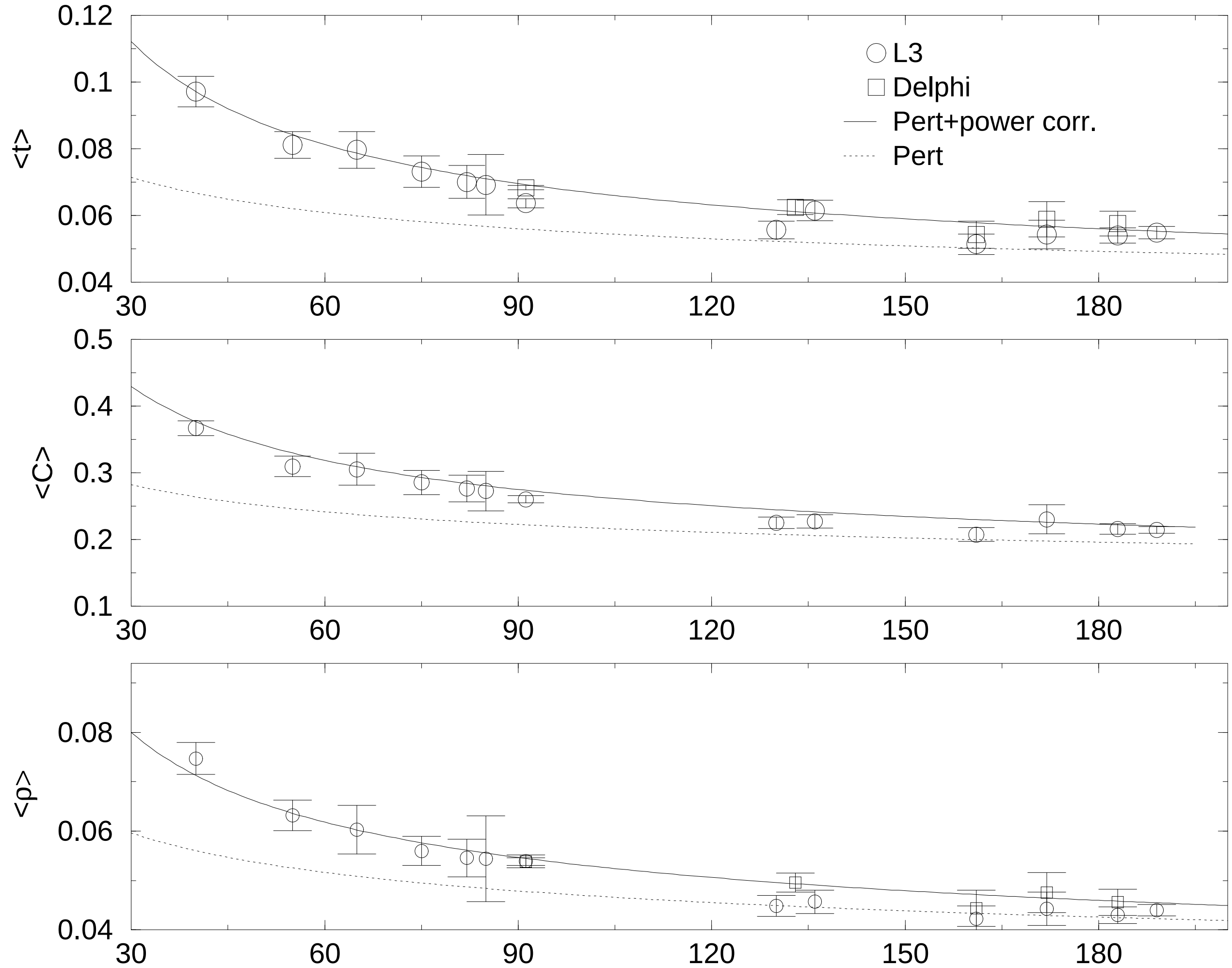
0.1
0.2
0.3
ρ
1/σ_tot dσ/dρ



0.12
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0.08
0.06
0.04
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L3
Delphi
Pert+power corr.
Pert
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150
180
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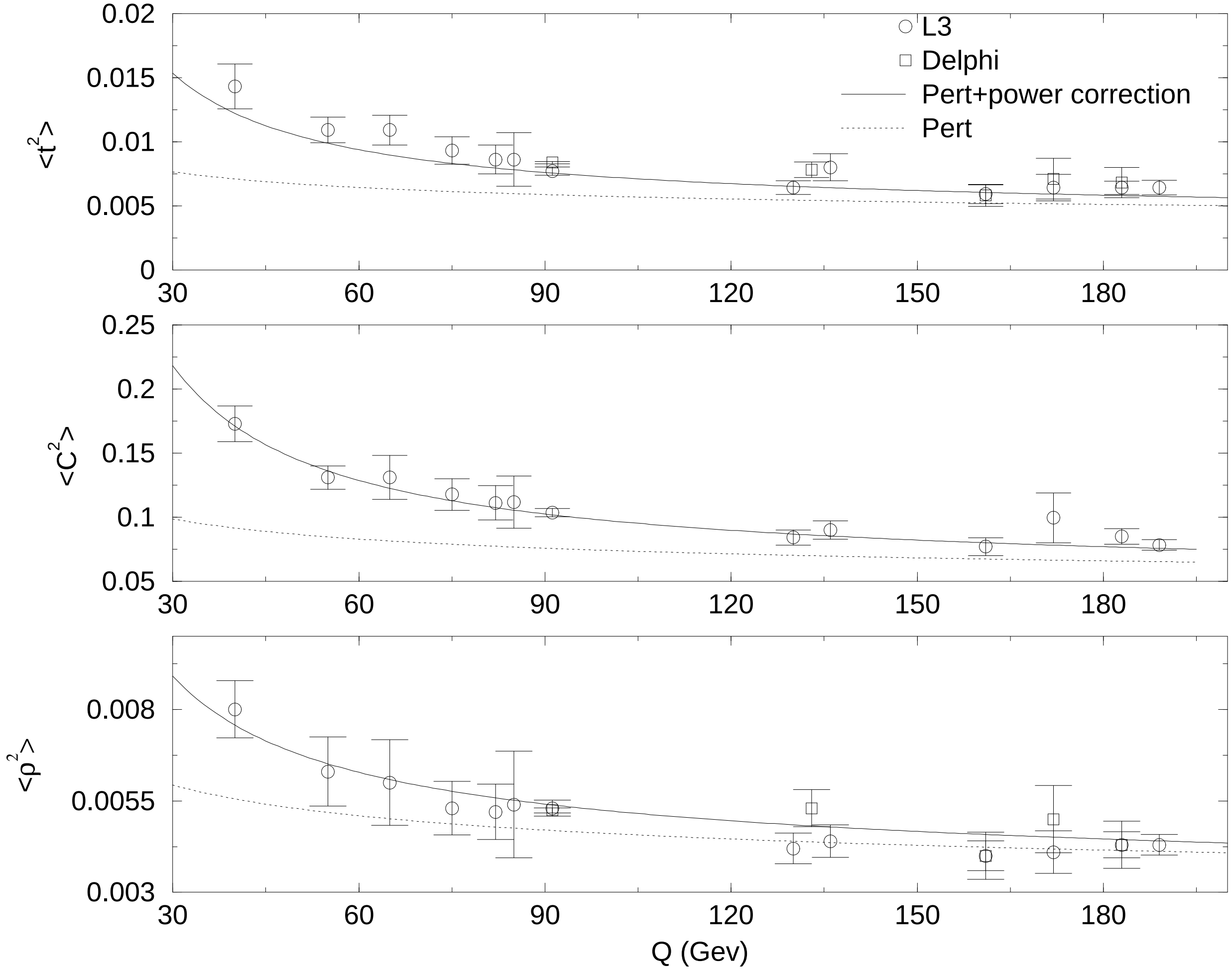
180


L3
Delphi
Pert+power correction
Pert
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<C²>
<ρ²>
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0.015
0.01
0.005
0
0.25
0.2
0.15
0.1
0.05
0.008
0.0055
0.003
30
60
90
120
150
180
Q (Gev)

Perspectives

- Resummed QCD amplitudes point beyond perturbation theory.
- Renormalon models agree with OPE, where available, as to the size of nonperturbative corrections. UV dominance applies in certain kinematic domains.
- Shape functions provide a general framework for studies of power corrections. Like PDF's, they must be fitted from data. Different models suggest different functional forms. Renormalon models are recovered.
- Dressed gluon exponentiation implements renormalon calculus in Sudakov resummation. It provides a model for shape functions.
- Hadronization generates mass–related log–enhanced power corrections. A defining scheme for event shapes must be chosen.
- Phenomenology is possible in the peak region for event shape distributions. A cautionary tale: shape function fits yield $\alpha_s(M_Z) \sim 0.110$; theoretical error?
- The future for theory: NNLO; beyond one chain; beyond two jets; theoretically motivated choices of shape functions.
- The future for experiment: discriminate between models; preserve past data; devise methods to preserve future data!