Phenomenology course: Problem Sheet 1

1. In the lecture I approximated the quark mass threshold by a step function. What actually happens at the threshold ?
2. Since $R(e^+e^-)$ is the only quantity for which we have NNNLO results, it is our only chance to calibrate how good different schemes for fixing μ are. Looking at the figure for the μ scale dependence, $K^{(n)}$ vs μ, discuss the relative merits of the three schemes defined there.
3. Convince yourself, without any algebra, that the thrust of a three-parton configuration is given by $\max(x_1, x_2, x_3)$, where $x_i = 2E_i/\sqrt{s}$, with E_i being the parton energy and $\sqrt{s}$ the collider energy.
4. Draw all the Feynman diagrams for the following three processes

$$1. \qquad e^+(1)e^-(2) \to q(3)\bar{q}(4)g(5)g(6)$$

$$2. \qquad e^+(1)e^-(2) \to q(3)\bar{q}(4)Q(5)\bar{Q}(6) \ [q \neq Q]$$

$$3. \qquad e^+(1)e^-(2) \to q(3)\bar{q}(4)Q(5)\bar{Q}(6) \ [q = Q]$$

   (Include only diagrams of order $\alpha^2\alpha_s^2$ and ignore Z propagators, only use photon ones.) Note down the relative signs among all diagrams in each case (beyond those that emerge from adopting the QED and QCD Feynman rules). Justify the relative signs in terms of Bose-Einstein and Fermi-Dirac statistics.

Solutions to Problem Sheet 1

1. The width of the quark would model the threshold behaviour more smoothly. More importantly, you have the formation of hadronic resonances ! This cannot be modelled in perturbative QCD. There is a plot in page 19 of the lecture notes showing some resonances explicitely.

2. The answer to this was actually given in the lecture, so that the purpose of this exercise is to fix in the mind of the students, at least qualitatively, the issues which emerge in higher order calculations, that they may never have to tackle in the remainder of the PhD course.

3. It suffices to project the three-momentum of the two lowest energy particles along the axis of the most energetic one to convince oneself.

4. The Feynman diagrams can be found in Figs. 1–3. For processes 1. and 2. all diagrams have the same relative sign. In process 3, diagrams 1, 4, 5, 6 have opposite sign relative to 2, 3, 7, 8. The latter account for the fact that, when $p_3 = p_5$ (for four-momenta), $s_3 = s_5[\lambda_3 = \lambda_5]$ (for spin[helicity]) and the colour is the same for quarks 3 and 5, the amplitude should be zero, as two fermions cannot be produced in the same quantum mechanical state, according to Fermi-Dirac statistics: this is indeed a manifestation of Pauli's Principle. (The argument can equally be formulated for antiquarks 4 and 6.) The relative sign among all diagrams in process 2. is always the same, as there can never be two identical quantum-mechanical states here. In process 1, despite the two gluons 5 and 6 can be in an identical quantum mechanical state (when $p_5 = p_6$, $s_5 = s_6[\lambda_5 = \lambda_6]$ and their colour is the same), no relative minus sign is required between graphs 2, 4, 6 (on the one hand) and 5, 1, 8 (on the other hand), the two sets being one-to-one related by swapping $5 \leftrightarrow 6$, in accordance with Bose-Einstein statistics. Finally, notice that to swap $5 \leftrightarrow 6$ in diagrams 3 and 7 of process 1 does not generate new graphs, because of the symmetry of the triple gluon vertex. Recall that in verifying the last statement, you ought to permutate not only the four-momenta, but also the Lorentz and colour indices associated to each gluon entering the ggg (triple-gluon) Feynman rule. (Besides, do not forget that $g^{\alpha\beta} = g^{\beta\alpha}$ and $f^{ABC} = -f^{BAC}$.)

   Hence, always recall that:

- Any two diagrams which differ in the exchange of two identical external (anti)particles have a relative plus(minus) sign, if the two identical (anti)particles are bosons(fermions).

The Feynman diagram formalism contains lots of real physics !

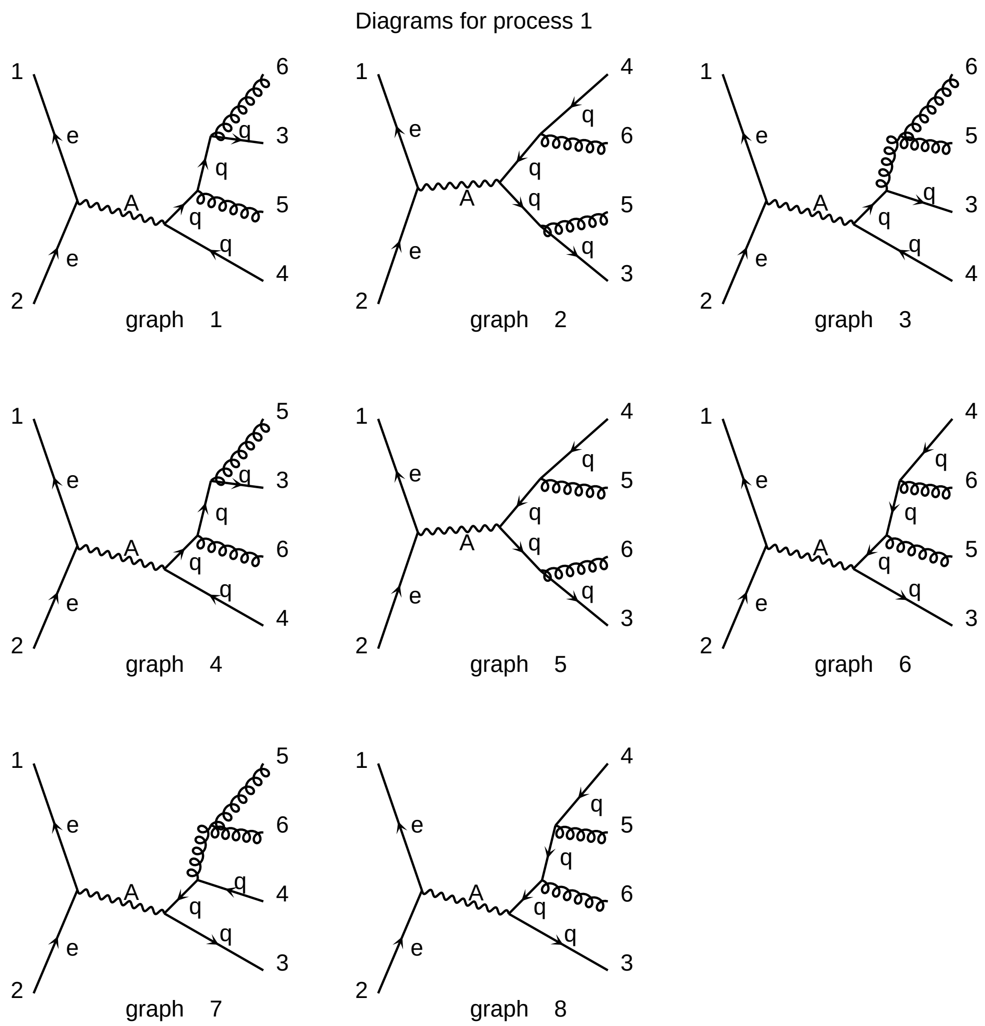


Figure 1: Diagrams for $e^{+}(1)e^{-}(2) \to q(3)\bar{q}(4)g(5)g(6)$.

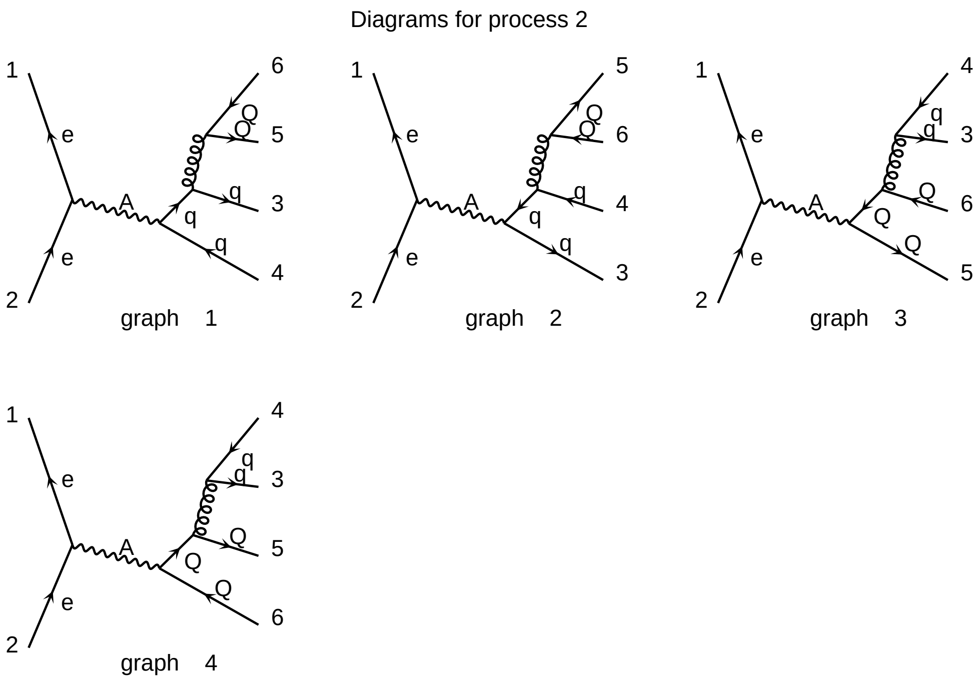


Figure 2: Diagrams for $e^+(1)e^-(2) \to q(3)\bar{q}(4)Q(5)\bar{Q}(6)$ $[q \neq Q]$.

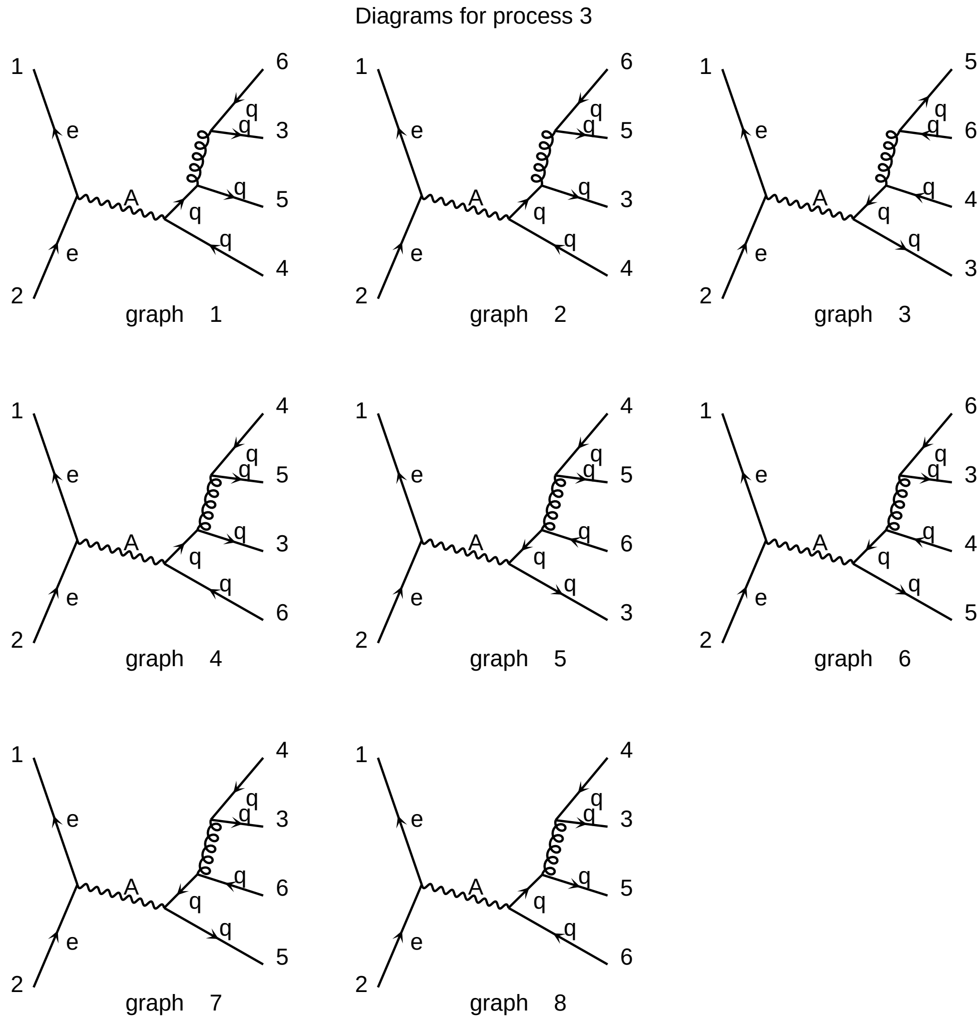


Figure 3: Diagrams for $e^{+}(1)e^{-}(2) \to q(3)\bar{q}(4)Q(5)\bar{Q}(6)$ $[q = Q]$.

Phenomenology course: Problem Sheet 2

1. Solve the differential equation

$$\mu^2 \frac{d\alpha_s(\mu^2)}{d\mu^2} = \beta(\alpha_s(\mu^2)) = -\beta_0 \alpha_s^2(\mu^2) + \mathcal{O}(\alpha_s^3)$$

(also called the α_s evolution equation) through first order: i.e., ignore the higher order terms $\mathcal{O}(\alpha_s^3)$. Recall that in QCD:

$$\beta_0 \equiv \frac{11N_C - 2n_f}{12\pi} = \frac{33 - 2n_f}{12\pi}.$$

Determine the integration constant by setting $\alpha_s(\mu^2 = M_Z^2) \equiv \alpha_s(M_Z^2) = 0.12$ and calculate the value of α_s at $\mu = 10$ GeV. (Use $n_f = 5$ and $M_Z = 91.1$ GeV.)

2. In the lecture, I said that F_2 and F_L depend only on x and Q^2, and that the cross section is given by the linear combination:

$$\frac{d^2\sigma}{dx\, dQ^2} = \frac{4\pi\alpha^2}{xQ^4} \left[(1 + (1-y)^2)F_2(x, Q^2) + y^2 F_L(x, Q^2)\right].$$

How would you measure F_2 and F_L separately?

3. The most general form of the DIS cross section has three non-zero structure functions, F_1, F_2 and F_3, while the e^+e^- cross section only has one (which I called B). Why the difference ?

4. Given that the +-distribution is defined by:

$$\int_0^1 \frac{f(x)}{(1-x)_+} dx = \int_0^1 \frac{f(x) - f(1)}{(1-x)} dx,$$

and

$$\frac{1}{(1-x)_+} = \frac{1}{1-x} \quad \text{for } 0 \leq x < 1,$$

show that

$$\int_0^1 P_{qq}^{(0)}(x) dx = 0,$$

where

$$P_{qq}^{(0)}(x) = C_F \left[\frac{1+x^2}{(1-x_+)} + \frac{3}{2}\delta(1-x)\right].$$

What is the significance of this result ?

Phenomenology course: Problem Sheet 3

1. On proving gauge invariance (referred to several times in the lectures): show that in the so-called Landau gauge,

$$\sum_{\lambda} \epsilon^{\mu}(k,\lambda)\epsilon^{\nu *}(k,\lambda) = -g^{\mu\nu} + \frac{k^{\mu}k^{\nu}}{k^2 + i\varepsilon},$$

   the term $\propto k^{\mu}k^{\nu}$ added to the photon spin sum $-g^{\mu\nu}$ of the Feynman gauge does not contribute to QED Compton scattering $e^{-}\gamma \to e^{-}\gamma$. Neglect the electron mass.

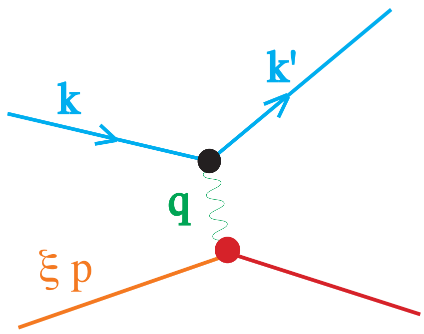
(*Hint:* To prove gauge invariance is sufficient to replace, e.g., $\epsilon^{\mu}(k,\lambda) \to k^{\mu}$ in the expressions you derived (i.e., prior to any squaring, tracing, etc.) and show, after some manipulations, that the *sum* of the two amplitudes is zero. Do it for just one photon.)

Phenomenology course: Problem Sheet 4

1. Consider the process (photon exchange only)

$$e^-(k) + Q(\xi p) \to e^-(k') + Q(\xi p + q)$$

where Q represents a generic massless quark of EM charge e_q and e^- a massless electron. Schematically:



Show that

$$\frac{d^2\hat{\sigma}}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[1 + (1-y)^2\right]\frac{1}{2}e_q^2\delta(x-\xi).$$

You may use the expression

$$|\overline{\mathcal{M}}|^2 = 2e_q^2 e^4 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$$

for the matrix element squared summed/averaged over final/initial colours and spins. (Hint: express $\hat{s}$, $\hat{t}$ and $\hat{u}$ in terms of the standard DIS variables

$$Q^2 = -q^2, \qquad x = \frac{-q^2}{2p\cdot q} \qquad \text{and} \qquad y = \frac{q\cdot p}{k\cdot p}.)$$

2. In calculating the Z^0 lineshape I neglected the interference between the γ and Z contributions (i.e. the correct result should have been obtained by computing $|\mathcal{M}_\gamma + \mathcal{M}_{Z^0}|^2$ instead of $|\mathcal{M}_\gamma|^2$ and $|\mathcal{M}_{Z^0}|^2$ separately. Estimate the interference contribution. What is its value on the peak ($\sqrt{s} = M_{Z^0}$) ?

3. Given that neutrinos cannot be detected, how is Γ_ν, the partial decay width of a Z^0 into neutrinos, measured ?

Phenomenology course: Problem Sheet 5

1. Show that in the Standard Model the decay width Γ_{WW} of the Higgs boson into two W bosons is

$$\Gamma_{WW} = \frac{\sqrt{2} G_F M_W^2 M_H}{8\pi} \frac{\sqrt{1-x_W}}{2x_W} (3x_W^2 - 4x_W + 4),$$

where

$$G_F = \sqrt{2} \frac{4\pi\alpha}{8 \sin^2\theta_W M_W^2}$$

and

$$x_W = \frac{4M_W^2}{M_H^2}.$$

Use the expression

$$\Gamma_{WW} = \int d\Omega \frac{|\overline{\mathcal{M}}|^2}{32\pi^2} \frac{|\mathbf{q}_{\rm CM}|}{M_H^2},$$

where $|\mathbf{q}_{\rm CM}|$ is the modulus of the three-momentum of either W boson in the Centre-of-Mass (CM) frame (i.e., where the Higgs boson is at rest).