Hadron Resonance Gas (HRG) model

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Study of QCD thermodynamics

Theoretical investigations of QCD thermodynamics make use of different methods and tools

From first principles:

- Lattice QCD
- Perturbation theory (large T and/or μ)
- Functional methods (functional renormalization group - FRG, Dyson-Schwinger equations, etc...)

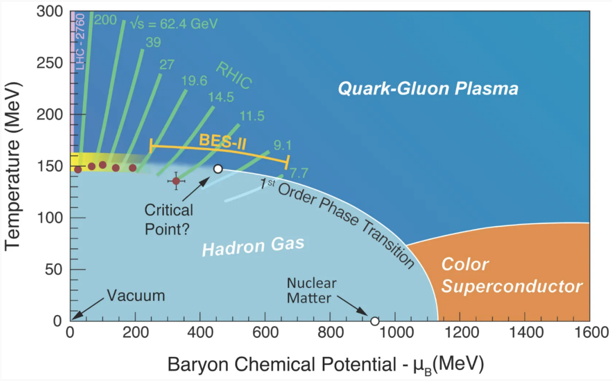
Models:

- Nambu-Jona-Lasinio (NJL) -type models (**Nambu and Jona-Lasinio, Phys. Rev. 122 (1961) 345, Phys. Rev. 124 (1961) 246**)
- **Hadron Resonance Gas (HRG)**-type models (**Hagedorn, Nuovo Cim. Suppl. 3 (1965), 147**)
- ...

The phase diagram of QCD

Different phases of QCD matter (in equilibrium) are depicted in (temperature vs baryo-chemical potential) phase diagram

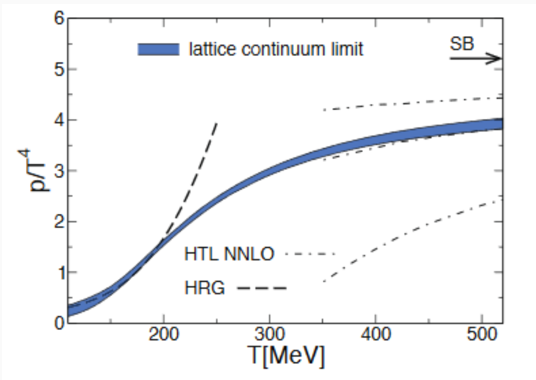
- **Hadron gas** at low-T and/or low-μ_B
- **Quark Gluon Plasma (QGP)** at large T and (possibly) at large μ_B
- **More exotic phases** proposed at low-T and high-μ_B (color superconductivity, etc...)



Equation of state at $\mu_B = 0$

A combination of methods gives us good understanding of the EoS at $\mu_B = 0$ at all temperatures

- Perturbative QCD at high temperature
  → "pure quark-gluon phase"
- **HRG model at low temperature**
  → "pure hadron phase"
- Lattice QCD bridges between regimes and captures the transition



Borsányi *et al.*, PLB 370 (2014) 99-104

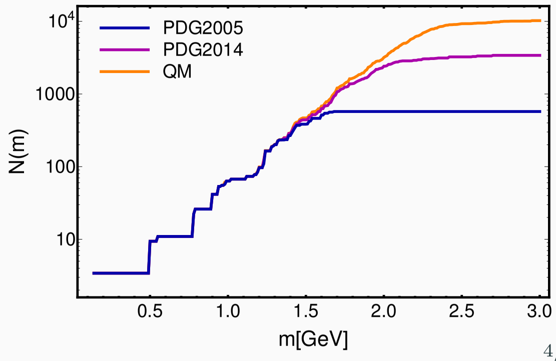
From Hagedorn to hadron resonance gas

- In its modern interpretation, the model does not have a continuous spectrum $\rho(m)$. Instead, the discrete set of known (or predicted) hadrons is considered, and summed over
- Nonetheless, hadronic states indeed seem to populate an exponentially increasing spectrum, when spin and isospin multiplicities are taken into account

Cumulative number of states

$$N(m) = \sum_i g_i \Theta(m - m_i)$$

S. Godfrey *et al.*, PRD 32, 189 (1985); S. Capstick *et al.*, PRD 34, 2809 (1986); Particle Data Group PTEP 2020, 083C01 (2020) and earlier versions



Hadron Resonance Gas model

Basic idea: approximate a gas of interacting hadrons in their ground state through a *non-interacting* gas of hadrons and all their resonant states

Being non-interacting, it is formally extremely simple:

$$\ln \mathcal{Z}(T, V, \vec{\mu}) = \sum_{i \in \text{hadrons}} \ln \mathcal{Z}_i(T, V, \vec{\mu}) \ , \qquad (1)$$

where $\vec{\mu} = (\mu_B, \mu_Q, \mu_S)$.
The one-particle partition function reads

$$\ln \mathcal{Z}_i(T, V) = \frac{V \eta_i g_i}{(2\pi)^3} \int d^3\mathrm{p} \ln \left[1 + (-1)^{B_i+1} z_i e^{-\epsilon_i/T}\right] \ , \qquad (2)$$

where:

- g_i is spin degeneracy
- $\eta_i = (-1)^{B_i+1} = 1(-1)$ for baryons (mesons)
- relativistic particles $\epsilon_i = \sqrt{p^2 + m_i^2}$
- $z_i = \exp[\mu_i/T] = \exp[(\mu_B B_i + \mu_Q Q_i + \mu_S S_i)/T]$ is the fugacity

Hadron Resonance Gas model

The pressure follows trivially

$$P(T,\vec{\mu}) = -T\frac{\partial \ln \mathcal{Z}}{\partial V} = -\frac{T}{V}\ln \mathcal{Z} = \sum_i T\frac{\eta_i g_i}{2\pi^2}\int_0^\infty dp\, p^2 \ln\left[1+\eta_i e^{-\frac{\epsilon_i-\mu_i}{T}}\right] ,$$

the number density

$$n(T,\vec{\mu}) = \sum_i \frac{g_i}{2\pi^2}\int_0^\infty dp\, p^2 \frac{1}{\eta_i + e^{\frac{\epsilon_i-\mu_i}{T}}} ,$$

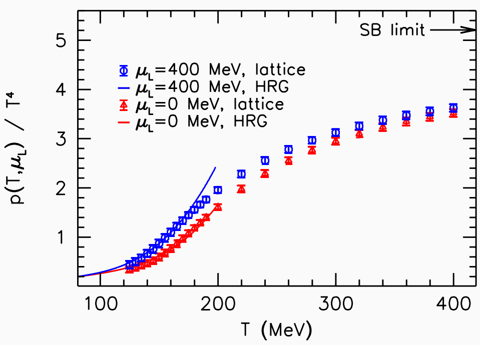
energy density

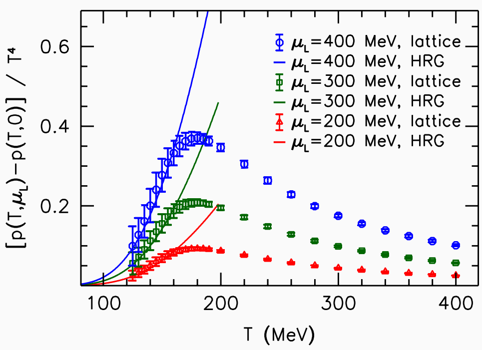
$$\epsilon(T,\vec{\mu}) = \sum_i \frac{g_i}{2\pi^2}\int_0^\infty dp\, p^2 \frac{\epsilon_i}{\eta_i + e^{\frac{\epsilon_i-\mu_i}{T}}} ,$$

and so on..

HRG model vs lattice QCD

Agreement between HRG model and lattice QCD is excellent up to around the transition temperature for virtually every observable





NOTE: the temperature where deviations start to occur decreases with increasing μ_B, as expected from the shape of the transition line

Borsányi *et al.*, JHEP 08 (2012) 053

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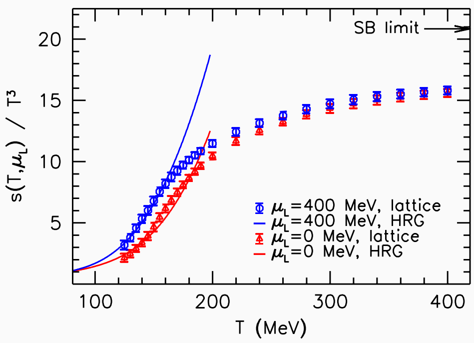

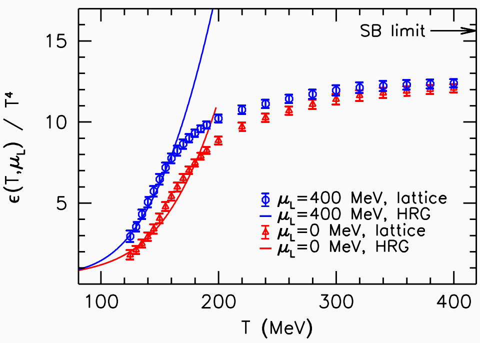

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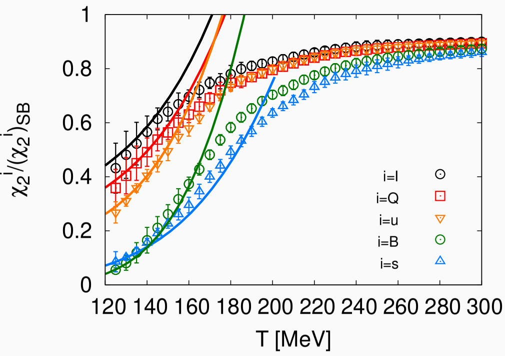


Borsanyi *et al.* JHEP 1201 (2012) 138

Why the HRG model?

Ok, nice, the HRG model provides an accurate description of the thermodynamics in the confined region. **But why do we care?**

I. Theory-experiment comparison: HRG offers a number of advantages over lattice QCD results

- No sign problem: can be used at finite chemical potential
- Lattice QCD gets increasingly demanding at lower T
- Individual particle contributions can be isolated
- Additional effects typical of experimental setup can be included:
  - Resonance decays
  - Finite acceptance in detectors
  - etc.

$\Rightarrow$ Better for comparison to experimental results

II. Theory-theory comparison: HRG can be used to understand results from lattice QCD at low T in terms of hadronic degrees of freedom, and to reverse-engineer from lattice results

Fluctuations of conserved charges

Looking at observables which are particularly sensitive to NLO effects (interactions, exotic states), one can test the HRG model description in detail

Fluctuations of conserved charges

They are defined as:

$$\chi_{ijk}^{BQS}(T,\mu_B,\mu_Q,\mu_S) = \frac{\partial^{i+j+k} P\left(T,\mu_B,\mu_Q,\mu_S\right)/T^4}{\partial\left(\mu_B/T\right)^i \partial\left(\mu_Q/T\right)^j \partial\left(\mu_S/T\right)^k}$$

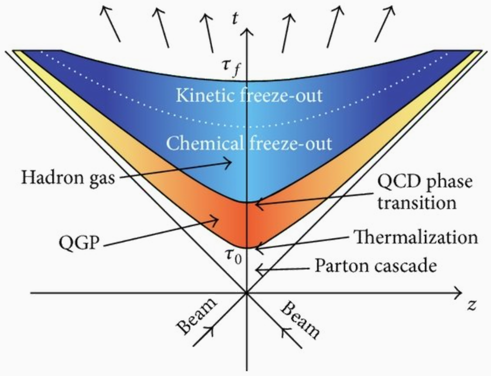
and are related to the moments of net-particle distributions:

mean:	$M = \chi_1$	variance:	$\sigma^2 = \chi_2$
skewness:	$S = \chi_3/\left(\chi_2\right)^{3/2}$	kurtosis:	$\kappa = \chi_4/\left(\chi_2\right)^2$

Freeze-out in heavy-ion collisions

The stages of a heavy-ion collision

- **Thermalization**: after a short time τ_0 the system thermalizes to a QGP (if the energy density is sufficient)
- **Hadronization**: when the system reaches T_C, hadrons are formed
- **Chemical freeze-out**: all inelastic collision cease and chemical composition is fixed (**yields, fluctuations**)
- **Kinetic freeze-out**: elastic collisions cease and spectra are fixed → free streaming to the detectors



Hui Wang's PhD thesis [Wang:2012jua]

Freeze-out parameters extraction

Thanks to measurements of net-particle moments (mean, variance, etc.), given two numbers, one can get T, μ_B

Several effects are taken into account

- **Resonance decays**, considering (strongly) stable hadrons:

$$N_i = N_i^0 + \sum_R P_{R\to i} N_R$$

  where $P_{R\to i} = \mathrm{BR}_{R\to i}\, n_i^R$ is the average number of particles i produced by a particle R

- Include **acceptance cuts** on the kinematics of measured particles:

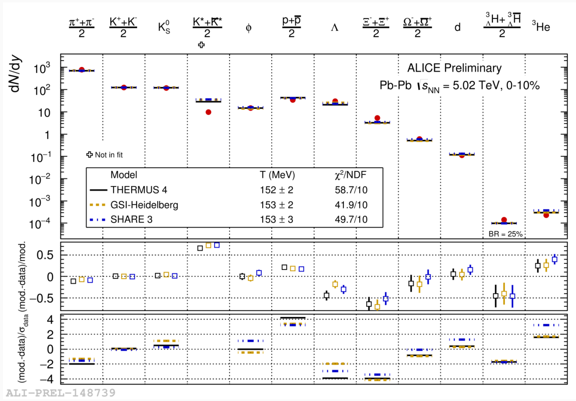
$$p_T^{\mathrm{m}} \le p_T \le p_T^{\mathrm{M}} \qquad |y| < y^* \quad (\text{or } |\eta| < \eta^*)$$

- Impose **strangeness neutrality** to constrain chemical potentials:

$$\langle n_S \rangle = 0 \qquad \langle n_Q \rangle = 0.4\, \langle n_B \rangle$$

Thermal fits to particle yields: a success story

The simplest case: number of particles (1^{st} moment). With one parameter (the temperature) yields are reproduced over 7 orders of magnitude

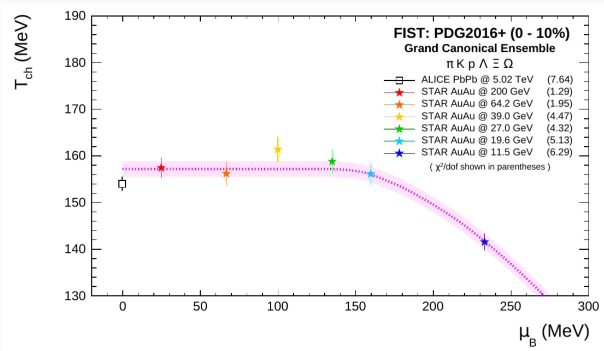


ALICE Collaboration: Nucl.Phys. A971 (2018); Nucl.Phys. A982 (2019)

Thermal fits to yields across energies

Systematic analysis of yields over energy scan (freeze-out parameters are now T, μ_B):

- ALICE, Pb-Pb at $\sqrt{s} = 5.02\,\text{TeV}$ (**Nucl.Phys. A982 (2019)**)
- STAR, Au-Au at $\sqrt{s} = 200 - 11.5\,\text{GeV}$ (**PRC. 96 (2017) 044904; arXiv: 1906.03732**)

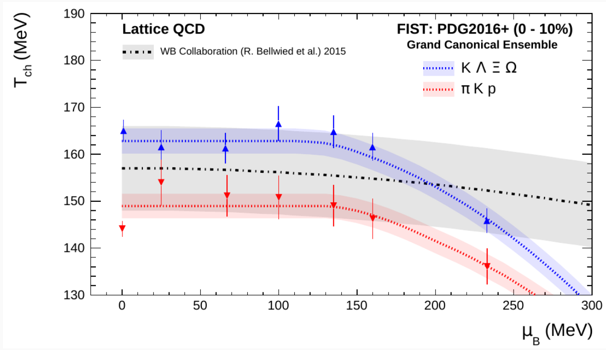


Freeze-out at $\mu_B = 0$ at $T_{\text{FO}} = 158.0 \pm 3.8\,\text{MeV}$

Flor *et al.* Phys.Lett.B 814 (2021) 136098

Thermal fits: a double freeze-out scenario

It's been hypothesized that strange and non-strange particles freeze-out at different stages (thus temperatures) during the system's evolution: **flavour hierarchy**



In 2FO scenario: $T_{\rm FO}^{\rm light} = 150.0 \pm 2.5\,{\rm MeV}$, $T_{\rm FO}^{\rm strange} = 163.0 \pm 4.0\,{\rm MeV}$

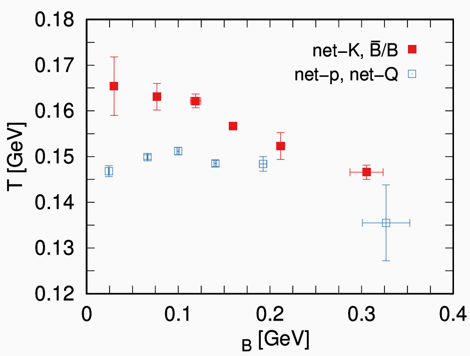
Flor *et al.* Phys.Lett.B 814 (2021) 136098

Freeze-out parameters from fluctuations

Thanks to measurements of higher order flutuations (variance, etc.), these quantities too could be used to extract freeze-out parameters

- Blue points: net-proton and net-charge (p,π,K) with M/σ^2
- Red points: net-kaon M/σ^2 and strange antibaryon-baryon ratios

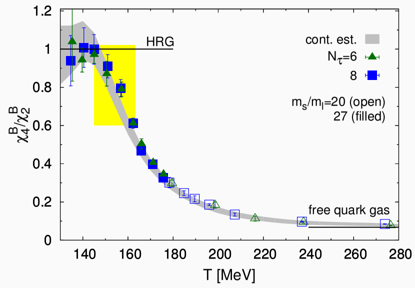
NOTE: fit to ratios are common because the leading volume dependence $\sim V$ is trivially removed (all moments are extensive quantities)

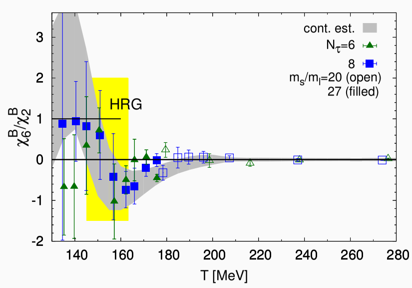


Alba *et al.* Phys.Lett.B 738 (2014) 305-310; Bluhm *et al.*, EPJ C79 (2019) no.2, 155

Theory-theory comparison

- The simplicity of the HRG model is also in the fact that it does not have free parameters. One "free parameter" is the hadronic spectrum utilized to sum over. Although "bulk" quantities like pressure, entropy etc. depend on it fairly mildly, more specific quantities can really test its content





The agreement is pushed to lower temperatures. Interactions, which play a larger role in higher moments, are likely the reason of the disagreement

Bazavov *et al.*, PRD 95 (2017) 054504

Theory-theory comparison

Different improvements have been proposed to restore agreement with lattice QCD at larger temperatures for more specific observables:

- **Different hadron spectra:** new hadron states are discovered routinely, their inclusion does improve agreement for some observables
  Bazavov *et al.*, PRL 113 (2014) 072001; Alba *et al.*, PRD 96 (2017) 034517; Alba *et al.*, PRC 101 (2020) 054905
- **Excluded volume and van der Waals interactions:** repulsive (excluded volume) and attractive interactions have been introduced. Agreement is slightly improved
  Vovchenko *et al.*, PRL 118 (2017) 182301; Vovchenko, IJMPE 29 (2020) 05, 2040002
- **S-matrix formalism:** inclusion of information from scattering experiments on the partial waves expansion of hadron-hadron scatterings
  Dashen, Ma, Bernstein, Phys. Rev. 187 (1969) 345; Pok Man Lo, EPJC 77 (2017) 8, 533

EXTRA - Example: hadron spectrum

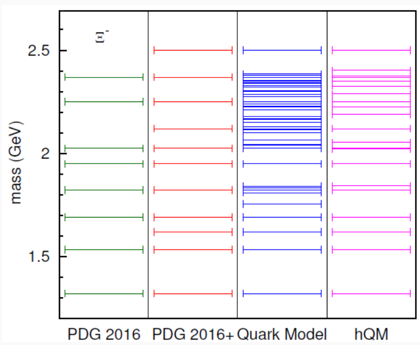
hadron	m_i (GeV)	d_i	B_i	S_i	I_i	hadron	m_i (GeV)	d_i	B_i	S_i	I_i
π	0.140	3	0	0	1	N (1535)	1.530	4	1	0	1/2
K	0.496	2	0	1	1/2	π_1 (1600)	1.596	9	0	0	1
$\overline{K}$	0.496	2	0	-1	1/2	Δ (1600)	1.600	16	1	0	3/2
η	0.543	1	0	0	0	Λ (1600)	1.600	2	1	-1	0
ρ	0.776	9	0	0	1	Δ (1620)	1.630	8	1	0	3/2
ω	0.782	3	0	0	0	η_2 (1645)	1.617	5	0	0	0
K^*	0.892	6	0	1	1/2	N (1650)	1.655	4	1	0	1/2
$\overline{K}^*$	0.892	6	0	-1	1/2	ω (1650)	1.670	3	0	0	0
N	0.939	4	1	0	1/2	Σ (1660)	1.660	6	1	-1	1
η'	0.958	1	0	0	0	Λ (1670)	1.670	2	1	-1	0
f_0	0.980	1	0	0	0	Σ (1670)	1.670	2	1	-1	1
a_0	0.980	3	0	0	1	ω_3 (1670)	1.667	7	0	0	0
ϕ	1.020	3	0	0	0	π_2 (1670)	1.672	15	0	0	1
Λ	1.116	2	1	-1	0	Ω^-	1.672	4	1	-3	0
h_1	1.170	3	0	0	1	N (1675)	1.675	12	1	0	1/2
Σ	1.189	6	1	-1	1	ϕ (1680)	1.680	3	0	0	0
a_1	1.230	9	0	0	1	K^* (1680)	1.717	6	0	1	1/2
b_1	1.230	9	0	0	1	$\overline{K}^*$ (1680)	1.717	6	0	-1	1/2
Δ	1.232	16	1	0	3/2	N (1680)	1.685	12	1	0	1/2
f_2	1.270	5	0	0	0	ρ_3 (1690)	1.688	21	0	0	1
K_1	1.273	6	0	1	1/2	Λ (1690)	1.690	4	1	-1	0
$\overline{K}_1$	1.273	6	0	-1	1/2	Ξ (1690)	1.690	8	1	-2	1/2
f_1	1.285	3	0	0	1	ρ (1700)	1.720	9	0	0	1
η (1295)	1.295	1	0	0	0	N (1700)	1.700	8	1	0	1/2
π (1300)	1.300	3	0	0	1	Δ(1700)	1.700	16	1	0	3/2

Hadron spectrum: additional states?

Strangeness-related observables show conflicting results when further states are added to the spectrum: $\Rightarrow$ Systematic analysis of hadron spectrum

Different lists

- **Particle Data Group** (PDG) lists particle according to experimental evidence
- **Quark models** predict many additional states (especially in the strange sector)
- Lattice QCD can help determine which states exist
- In figure:
  - **PDG 2016** (**, *** and **** states): 608 states
  - **PDG 2016+** (*, **, *** and **** states): 738 states
  - **Quark Model**: 1517 states
  - **Hypercentral QM (hQM)**: 985 states



Alba *et al.* PRD 96 (2017) 034517

Hadron spectrum: partial pressures

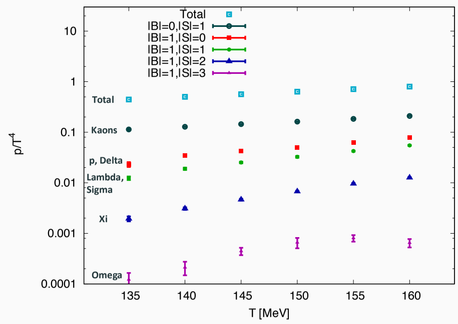
Separate contribution to the pressure from different quantum numbers (in Boltzmann approximation):

$$\frac{P}{T^4} = \sum_i (-1)^{B_i+1} \frac{d_i}{2\pi^2 T^3} \int dk\, k^2 \ln\Big(1 + (-1)^{B_i+1} \exp\left[-\left(\epsilon_i - \mu_i\right)/T\right]\Big) \simeq$$

$$\simeq \sum_i \frac{d_i}{2\pi^2 T^3} \int dk\, k^2 e^{-(\epsilon_i - \mu_i)/T} \simeq \sum_i e^{\mu_i/T} \frac{d_i}{2\pi^2 T^3} \int dk\, k^2 e^{-\epsilon_i/T}$$

The pressure then becomes:

$$P(T, \frac{\mu_B}{T}, \frac{\mu_S}{T}) = P_{00}^{BS} + P_{10}^{BS} \cosh(\frac{\mu_B}{T}) +$$
$$+ P_{01}^{BS} \cosh(\frac{\mu_S}{T}) +$$
$$+ P_{11}^{BS} \cosh(\frac{\mu_B}{T} - \frac{\mu_S}{T}) +$$
$$+ P_{12}^{BS} \cosh(\frac{\mu_B}{T} - 2\frac{\mu_S}{T}) +$$
$$+ P_{13}^{BS} \cosh(\frac{\mu_B}{T} - 3\frac{\mu_S}{T})$$



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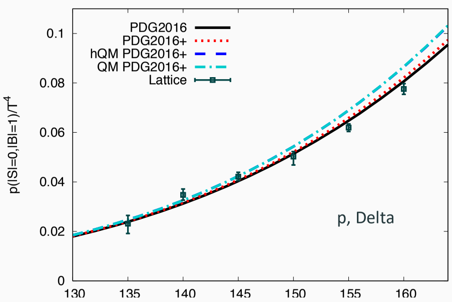
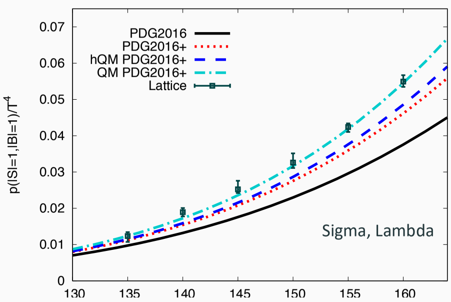
Comparison between lattice QCD and HRG for the different lists:





- No significant difference between lists for $|S| = 0$ baryons
- All lists except QM list contain too few $|S| = 1$ states

Alba *et al.* PRD 96 (2017) 034517

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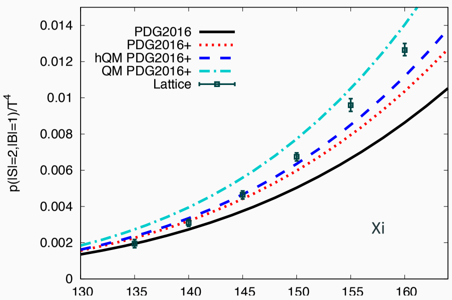

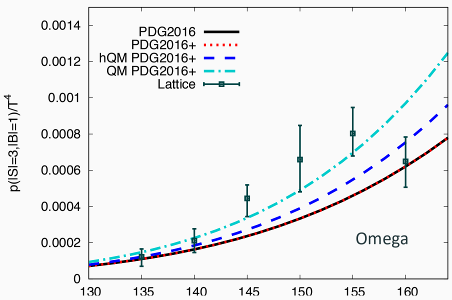

- The QM list contains too many $|S| = 2$ states, but works well for $|S| = 3$ baryons
- Again the PDG2016 list is not enough in both cases

Alba *et al.* PRD 96 (2017) 034517

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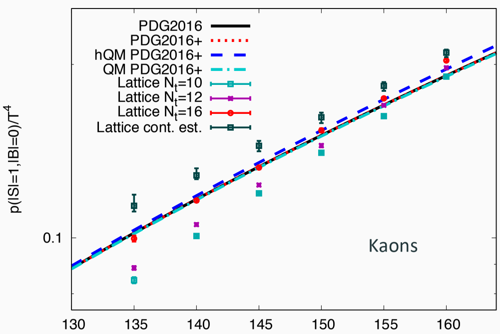

- All lists underestimate lattice data for strange mesons

Alba *et al.* PRD 96 (2017) 034517

Summary

- Hadron resonance gas model proves excellent at describing low-T thermodynamics results from lattice QCD
- At $T \leq 120\,\text{MeV}$ lattice results are expensive (thus rare) $\rightarrow$ HRG effectively trusted to be the correct description
- HRG can give insight in what degrees of freedom contribute to certain observables. Example: analysis of hadron spectra from comparison to lattice QCD results
- Comparison with experiment have proven fruitful for a long time. Fits to yields and fluctuations give us knowledge on freeze-out locations at different collision energies
- Many experimental effects can be at least partially incorporated
- Many improvements to the model have been proposed and are being studied and utilized