Fisica della Materia allo Stato Fluido e di Plasma

3. Descrizione Cinetica

Kinetic Theory

- Bellan:
  - Vlasov, collisions (intro): Sec. 2.1, 2.2, 2.3.1 [page 30-36]
  - Fokker-Planck: Sec. 13.1, 13.2
- Sturrock:
  - Chap 10, page 145
- Boyd:
  - sec. 7.1
  - Transport coefficients: sec. 8.1, 8.2
- Goedbloeds & Poedts:
  - Kinetic plasma theory: sec. 2.3, page 47, 2.3.1 page 48-51
  - Collision terms: sec.3.1 (page 83), 3.2.1, 3.2.2

2.1 Phase-space

Consider a particle moving in a one-dimensional space and let its position be described as $x = x(t)$ and its velocity as $v = v(t)$. A way to visualize the x and v trajectories simultaneously is to plot them on a 2-dimensional graph where the horizontal coordinate is given by $x(t)$ and the vertical coordinate is given by $v(t)$. This $x - v$ plane is called *phase-space.* The trajectory (or orbit) of several particles can be represented as a set of curves in phase-space as shown in Fig.2.1. Examples of a few qualitatively different phase-space orbits are shown in Fig.2.1.

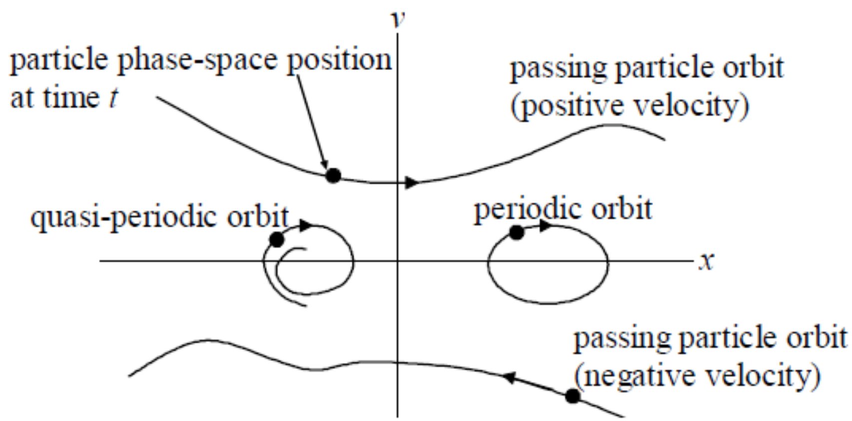

Figure 2.1: Phase space showing different types of possible particle orbits.

Particles in the upper half plane always move to the right since they have a positive velocity while those in the lower half plane always move to the left. Particles having exact periodic motion [e.g., $x = A\cos(\omega t)$, $v = -\omega A\sin(\omega t)$] alternate between moving to the right and the left and so describe an ellipse in phase-space. Particles with nearly periodic (quasi-periodic) motions will have near-ellipses or spiral orbits. A particle that does not reverse direction is called a passing particle, while a particle confined to a certain region of phase-space (e.g., a particle with periodic motion) is called a trapped particle.

2.2 Distribution function and Vlasov equation

At any given time, each particle has a specific position and velocity. We can therefore characterize the instantaneous configuration of a large number of particles by specifying the density of particles at each point x, v in phase-space. The function prescribing the instantaneous density of particles in phase-space is called the *distribution function* and is denoted by $f(x,v,t)$. Thus, $f(x,v,t)\mathrm{d}x\mathrm{d}v$ is the number of particles at time t having positions in the range between x and $x+\mathrm{d}x$ and velocities in the range between v and $v+\mathrm{d}v$. As time progresses, the particle motion and acceleration causes the number of particles in these x and v ranges to change and so f will change. This temporal evolution of f gives a description of the system more detailed than a fluid description, but less detailed than following the trajectory of each individual particle. Using the evolution of f to characterize the system does not keep track of the trajectories of individual particles, but rather characterizes classes of particles having the same x, v.

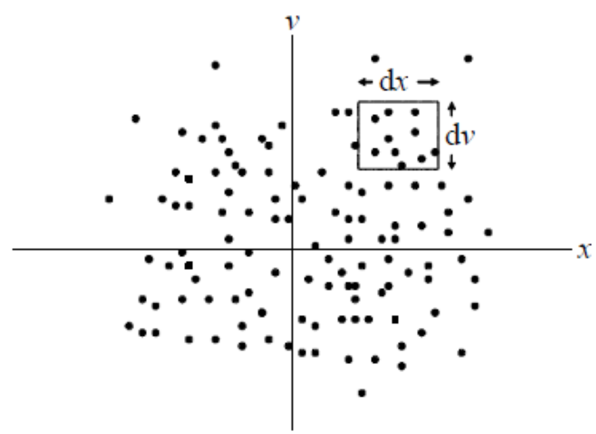

Figure 2.2: A box with in phase space having width $\mathrm{d}x$ and height $\mathrm{d}v$.

We have already considered the Vlasov equation that describes the evolution of a distribution function when the effects of collisions may be ignored. In this chapter, we consider the effects of collisions on the distribution functions of the various species that comprise a plasma. If the plasma contained neutral particles, it would be necessary to consider the effects of large-angle collisions. This is normally carried out by a procedure due originally to Boltzmann. However, if neutral particles do not play a significant role in the plasma, the collision processes are due to the long-range Coulomb force. As we found in Chapter 2, Coulomb interactions give rise mainly to many small-angle collisions. The appropriate equation to use in this case is the Fokker–Planck equation, that we shall now introduce.

$$\frac{\partial f}{\partial t} + v_r \frac{\partial f}{\partial x_r} + \frac{q}{m}\left(E_r + \frac{1}{c}\varepsilon_{rst} v_s B_t\right)\frac{\partial f}{\partial v_r} =$$

$$-\frac{\partial}{\partial v_r}\left(\left\langle\frac{\Delta v_r}{\Delta t}\right\rangle f\right) + \frac{1}{2}\frac{\partial^2}{\partial v_r \partial v_s}\left(\left\langle\frac{\Delta v_r \Delta v_s}{\Delta t}\right\rangle f\right)$$

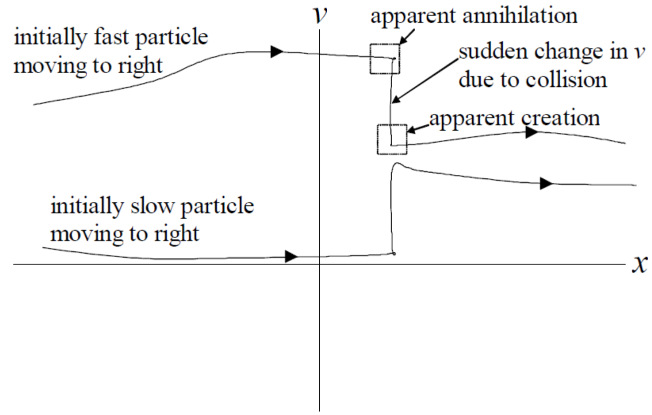

Figure 2.4: Detailed view of collisions causing 'jumps' in phase space

2.3.1 Treatment of collisions in the Vlasov equation

It was shown in Sec. 1.8 that the cumulative effect of grazing collisions dominates the cumulative effect of the more infrequently occurring large angle collisions. In order to see how collisions affect the Vlasov equation, let us now temporarily imagine that the grazing collisions are replaced by an equivalent sequence of abrupt large scattering angle encounters as shown in Fig.2.4. Two particles involved in a collision do not significantly change their positions during the course of a collision, but they do substantially change their velocities. For example, a particle making a head-on collision with an equal mass stationary particle will stop after the collision, while the target particle will assume the velocity of the incident particle. If we draw the detailed phase-space trajectories characterized by a collision between two particles we see that each particle has a sudden change in its vertical coordinate (i.e., velocity) but no change in its horizontal coordinate (i.e., position). The collision-induced velocity jump occurs very fast so that if the phase-space trajectories were recorded with a "movie camera" having insufficient framing rate to catch the details of the jump the resulting movie would show particles being spontaneously created or annihilated within given volumes of phase-space (e.g., within the boxes shown in Fig. 2.4).

The details of these individual jumps in phase-space are complicated and yet of little interest since all we really want to know is the cumulative effect of many collisions. It is therefore both efficient and sufficient to follow the trajectories on the slow time scale while accounting for the apparent "creation" or "annihilation" of particles by inserting a *collision operator* on the right hand side of the Vlasov equation. In the example shown here it is seen that when a particle is apparently "created" in one box, another particle must be simultaneously "annihilated" in another box at the same x coordinate but a different v coordinate (of course, what is actually happening is that a single particle is suddenly moving from one box to the other). This coupling of the annihilation and creation rates in different boxes constrains the form of the collision operator. We will not attempt to derive collision operators in this chapter but will simply discuss the constraints on these operators. From a more formal point of view, collisions are characterized by constrained sources and sinks for particles in phase-space and inclusion of collisions in the Vlasov equation causes the Vlasov equation to assume the form

$$\frac{\partial f_\sigma}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot (\mathbf{v} f_\sigma) + \frac{\partial}{\partial \mathbf{v}} \cdot (\mathbf{a} f_\sigma) = \sum_\alpha C_{\sigma\alpha}(f_\sigma) \tag{2.12}$$

where $C_{\sigma\alpha}(f_\sigma)$ is the rate of change of f_σ due to collisions of species σ with species α.

Let us now list the constraints which must be satisfied by the collision operator $C_{\sigma\alpha}(f_\sigma)$ are as follows:

- (a) Conservation of particles – Collisions cannot change the total number of particles at a particular location so

$$\int d\mathbf{v} C_{\sigma\alpha}(f_\sigma) = 0. \tag{2.13}$$

  (b) Conservation of momentum – Collisions between particles of the same species cannot change the total momentum of that species so

$$\int d\mathbf{v} m_\sigma \mathbf{v} C_{\sigma\sigma}(f_\sigma) = 0 \tag{2.14}$$

  while collisions between different species must conserve the total momentum of both species together so

$$\int d\mathbf{v} m_i \mathbf{v} C_{ie}(f_i) + \int d\mathbf{v} m_e \mathbf{v} C_{ei}(f_e) = 0. \tag{2.15}$$

  (c) Conservation of energy –Collisions between particles of the same species cannot change the total energy of that species so

$$\int d\mathbf{v} m_\sigma v^2 C_{\sigma\sigma}(f_\sigma) = 0 \tag{2.16}$$

  while collisions between different species must conserve the total energy of both species together so

$$\int d\mathbf{v} m_i v^2 C_{ie}(f_i) + \int d\mathbf{v} m_e v^2 C_{ei}(f_e) = 0. \tag{2.17}$$