

Heavy Quark bound states in a deconfined Quark-Gluon Plasma

Stefano Carignano

December 2008

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Plasma

The deconfinement
transition

Experimental
signatures

A suppression model

Heavy mesons in the
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Results

Spectral functions

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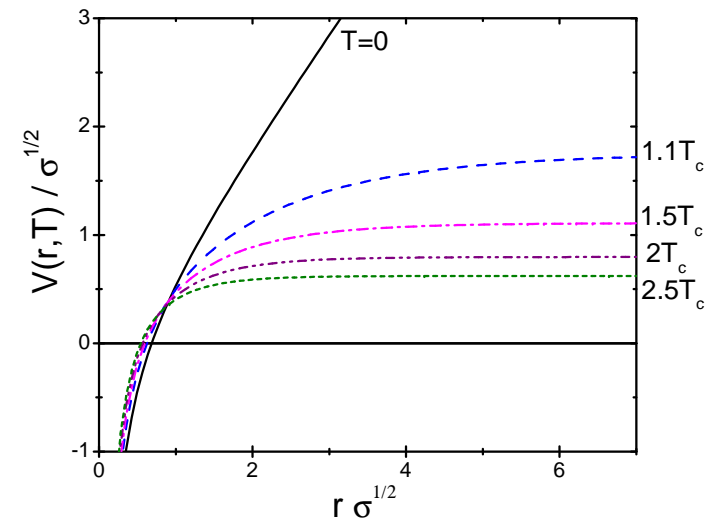
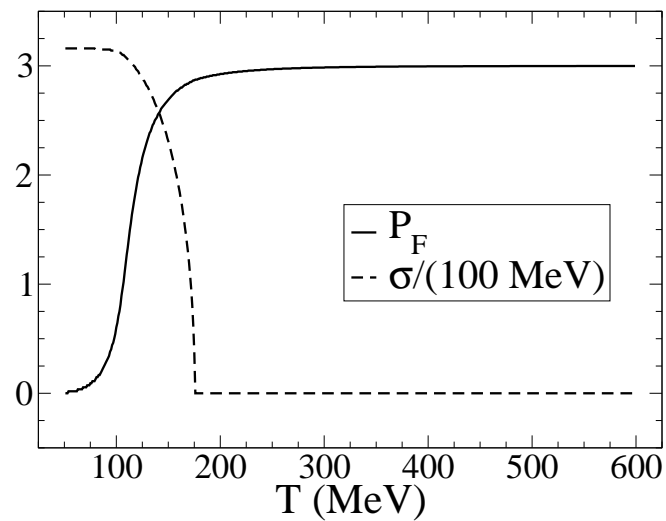
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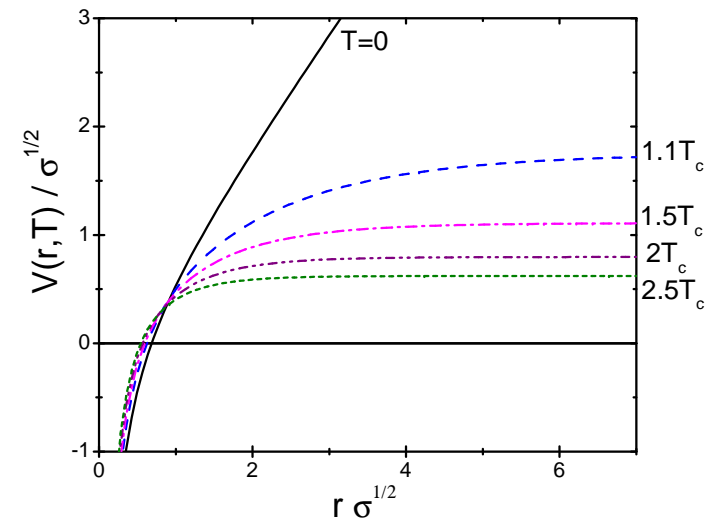
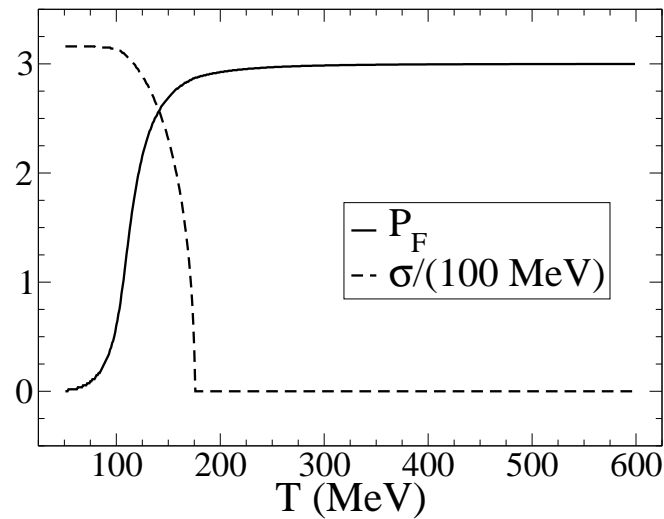
■ Lattice QCD: phase transition



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The deconfinement transition

■ Lattice QCD: phase transition



Critical temperature T_c

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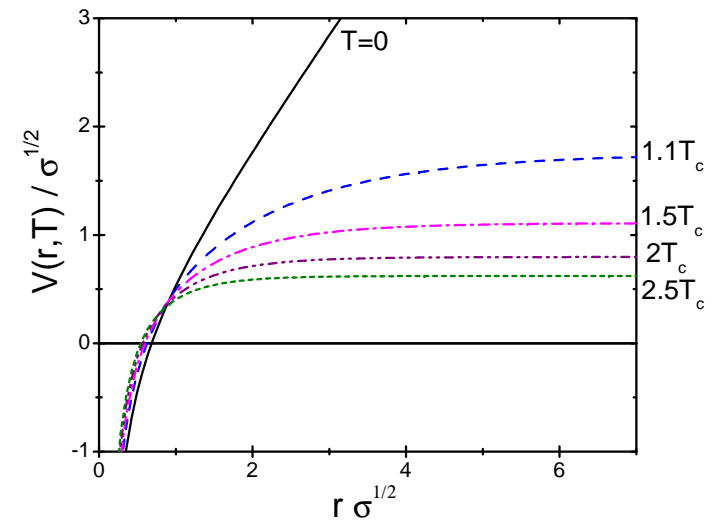
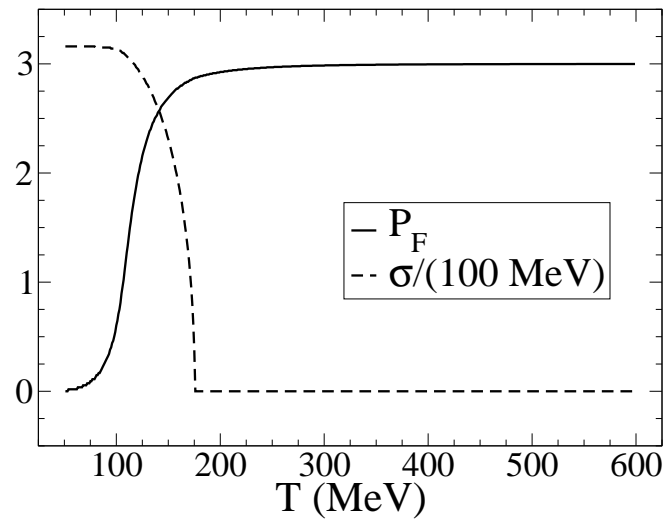
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Critical temperature T_c

→ Deconfinement

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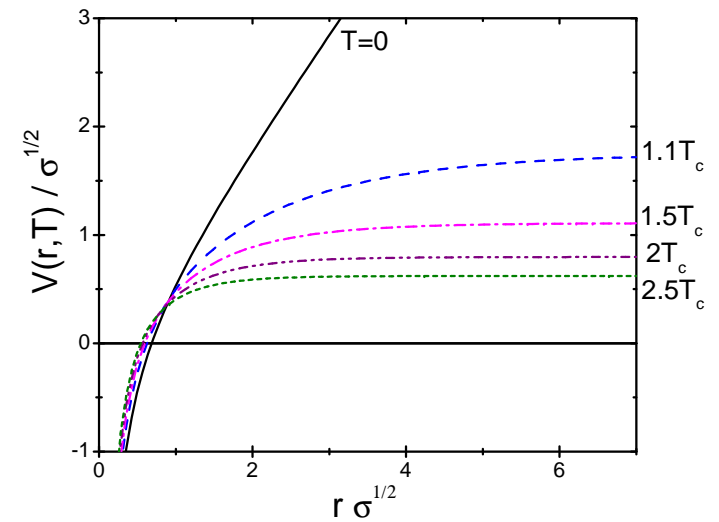
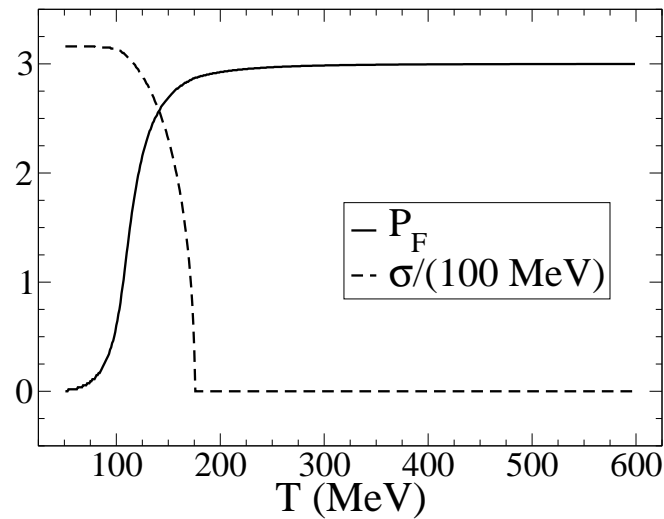
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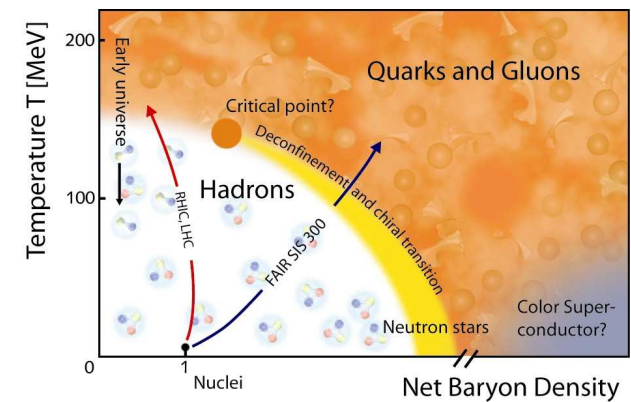
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■ Lattice QCD: phase transition



Critical temperature T_c
 \rightarrow Deconfinement



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■ J/ψ suppression

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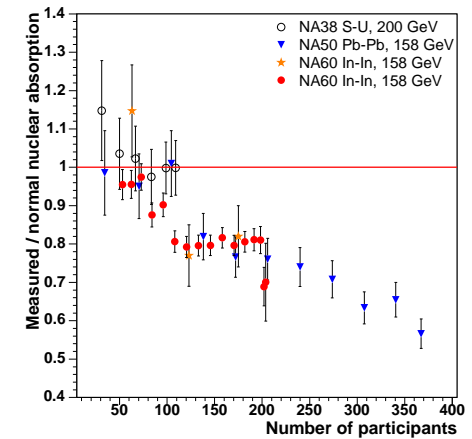
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■ J/ψ suppression

□ Observed at SPS



R.Arnaldi @ QM2005

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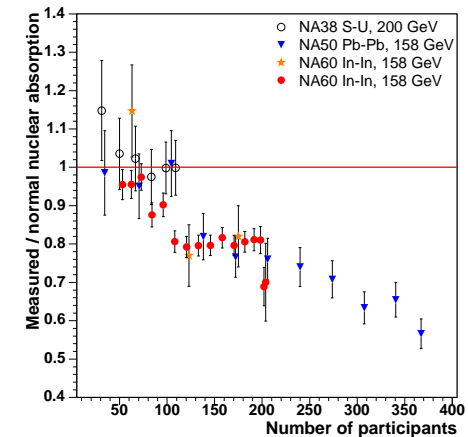
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■ J/ψ suppression

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R.Arnaldi @ QM2005

■ Why the suppression ?

- Color screening prevents formation of bound states (Matsui and Satz, Phys. Lett. B 178, 416)
- Dynamical counterpart: Scattering with hard deconfined gluons in the plasma breaks up bound states (Kharzeev and Satz, Phys. Lett. B 334, 155)

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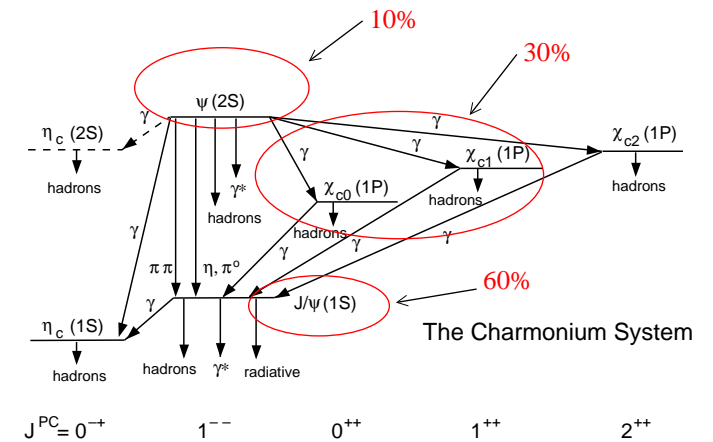
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- Around 40% of the observed J/ψ are generated by feed-down from heavier charmonia
- Same rule applies to other quarkonia



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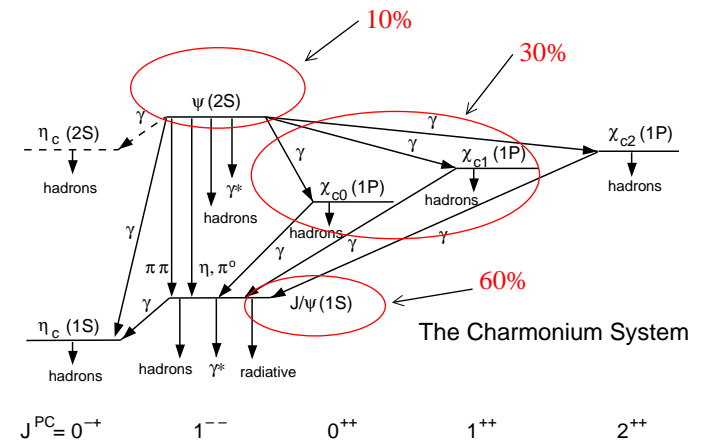
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- Need for a "sequential suppression" model.



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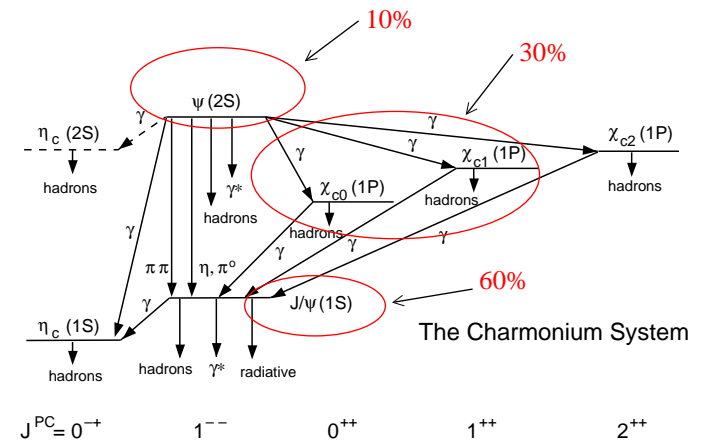
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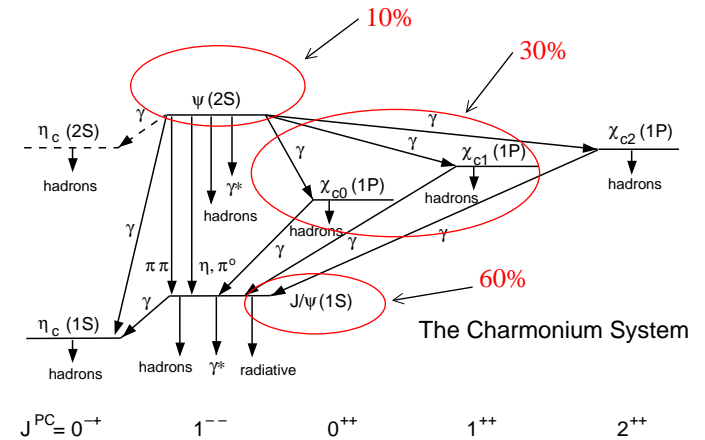
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$$\square c\bar{c} \quad (J/\psi, \psi', \chi_c \dots)$$



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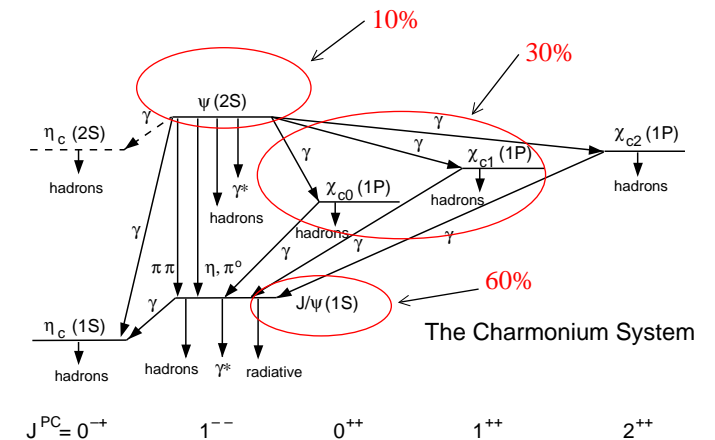
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□ $c\bar{c}$ ($J/\psi, \psi', \chi_c \dots$)

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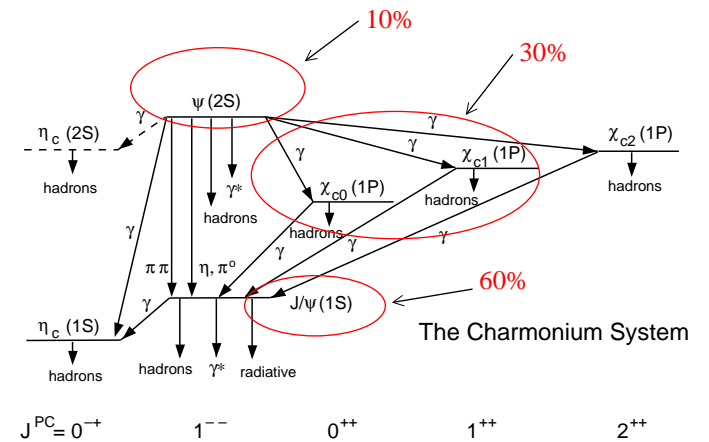
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- Study of heavy mesons:
 - $c\bar{c}$ ($J/\psi, \psi', \chi_c \dots$)
 - $b\bar{b}$ ($\Upsilon, \Upsilon', \chi_b \dots$)
 - $b\bar{c}, c\bar{b}$ ($B_c, B_c', \chi_{B_c} \dots$)

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- Bound states of heavy quarks \rightarrow non-relativistic formalism

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- Bound states of heavy quarks → non-relativistic formalism

- Schroedinger equation:

$$\left(-\frac{\hbar^2}{2\mu} \nabla^2 + V(r, T) \right) \psi(\vec{r}, T) = \epsilon(T) \psi(\vec{r}, T)$$

$V(r, T)$ → Temperature-dependant effective potential

μ = reduced mass, ϵ = energy

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Spectral functions from lattice QCD

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- Lattice QCD \rightarrow Euclidean correlators (imaginary time)

Spectral functions from lattice QCD

- Lattice QCD → Euclidean correlators (imaginary time)

$$G_M(\tau, T) = \langle j_M(\tau) j_M^\dagger(0) \rangle_T$$

$$j_M = \bar{q} \Gamma_M q \qquad \Gamma_M = 1, \gamma_\mu, \gamma_5, \gamma_\mu \gamma_5$$

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- Lattice QCD → Euclidean correlators (imaginary time)

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- Euclidean correlators ↔ Spectral functions $\sigma_M(\omega, T)$

$$G_M(\tau, T) = \int_0^\infty d\omega \sigma_M(\omega, T) K(\tau, \omega, T)$$

$$K(\tau, \omega, T) = \frac{\cosh[\omega(\tau - \frac{T}{2})]}{\sinh(\frac{\omega T}{2})}$$

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- Integral transform \rightarrow hard to extract $\sigma_M(\omega, T) \rightarrow$ MEM

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■ From lattice QCD:

$$\square \langle L \rangle_T \rightarrow F_1 \quad \text{color singlet free energy}$$

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■ From lattice QCD:

- $\langle L \rangle_T \rightarrow F_1$ color singlet free energy
- Fit on lattice data \rightarrow functional dependence $F_1(T)$
(W.M. Alberico et al., Phys. Rev. D, 72, 114011)

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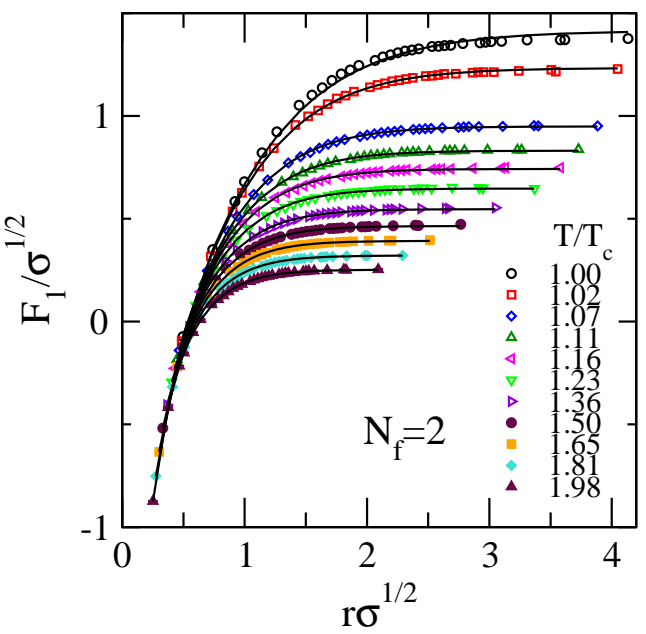
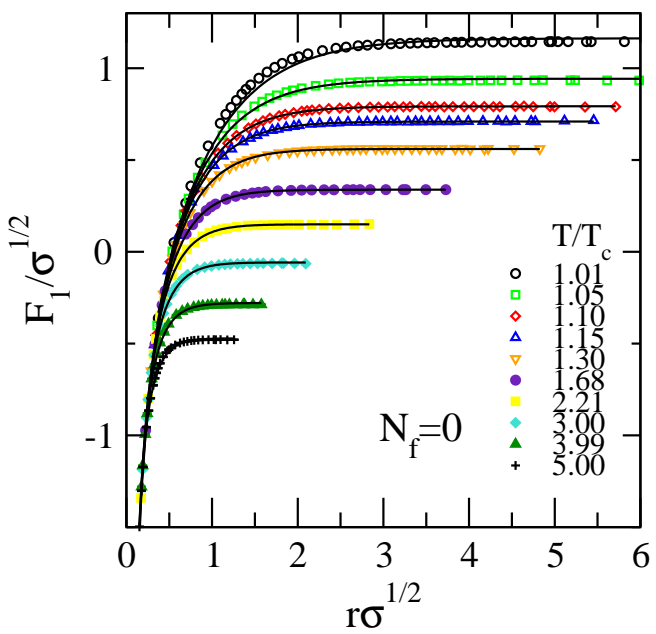
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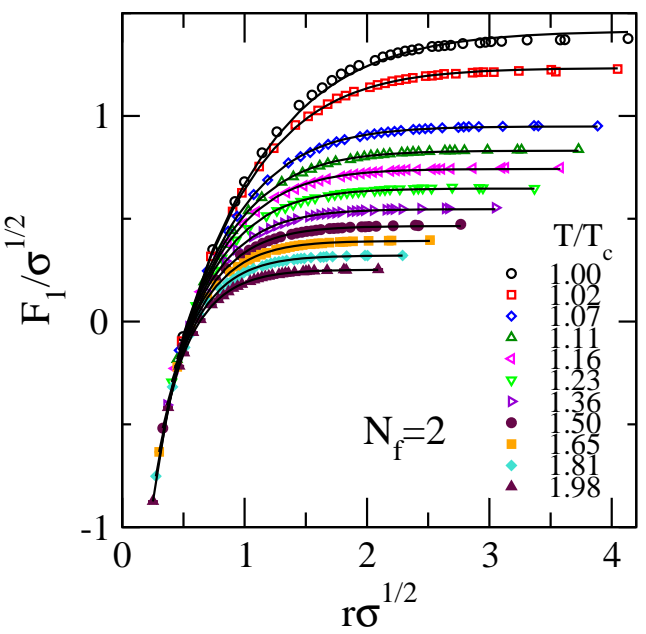
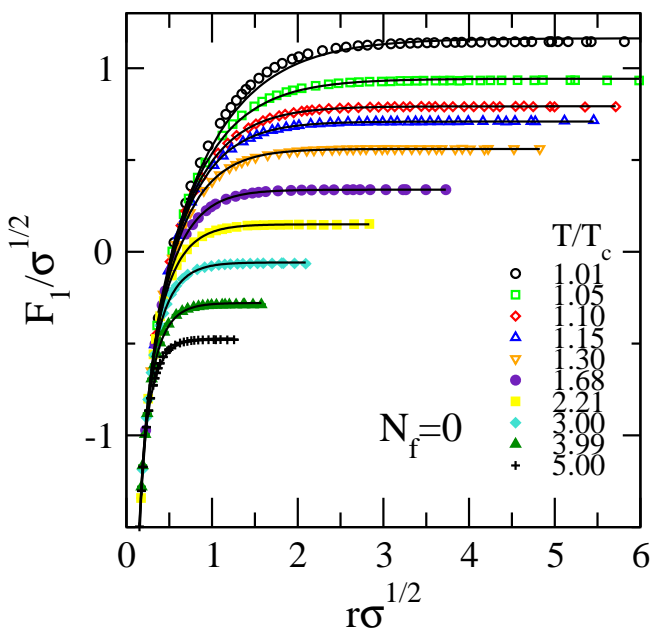
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- $F_1(T) \rightarrow U_1 = -T^2 \frac{\partial}{\partial T} \left(\frac{F_1}{T} \right)$ singlet internal energy

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■ Possibilities: $F_1, U_1 \dots$

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- Possibilities: $F_1, U_1 \dots$
- Hydrodynamical / Thermodynamical considerations \rightarrow
isolating effective $Q\bar{Q}$ potential from the plasma contribution
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- Possibilities: $F_1, U_1 \dots$
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$$U_{Q\bar{Q}}(r, T) = f_F(T)F_1(r, T) + f_U(T)U_1(r, T)$$

$$f_F = \frac{3}{3+a(T)}$$

$$f_U = \frac{a(T)}{3+a(T)}$$

$$a(T) = \frac{3p}{\epsilon}$$

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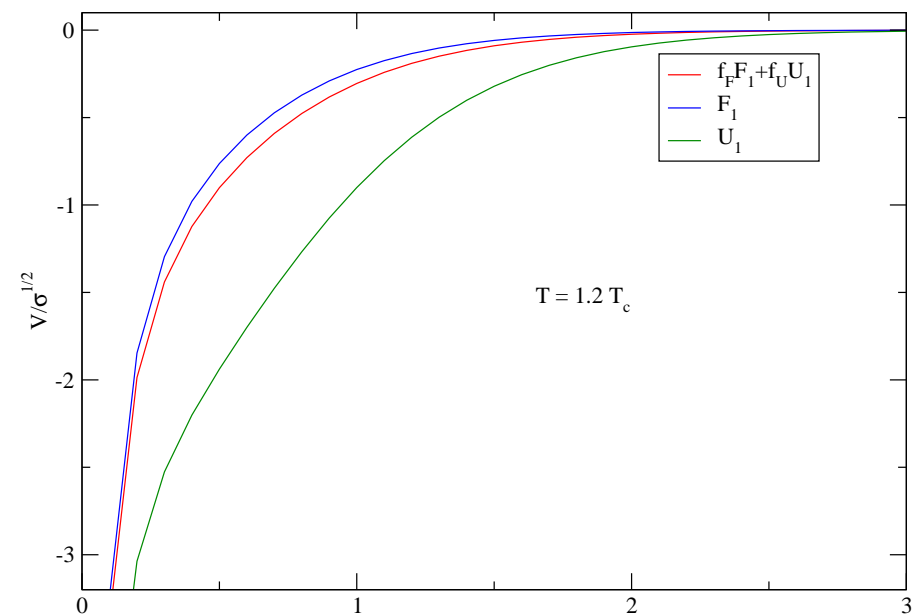
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- Quark charm: $m_c = 1.4 - 1.6 \text{ GeV}$
- Quark bottom: $m_b = 4.3 - 4.7 \text{ GeV}$

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 - Interpreted as 'thermal' mass
 - Improves binding of the states.
 - Continuum threshold:
$$s_0 = m_{Q_1} + m_{Q_2} + U_{Q\bar{Q}}(r \rightarrow \infty, T) = \tilde{m}_{Q_1}(T) + \tilde{m}_{Q_2}(T)$$
- Does the model strongly depend on the chosen values for the quark masses ?

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The analyzed states

Masses $M(T) =$
 $s_0(T) + \epsilon(T)$

Dissociation
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More masses: B_c

Even more masses: P
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■ States $c\bar{c}, b\bar{b}, b\bar{c} (c\bar{b})$

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 $s_0(T) + \epsilon(T)$

Dissociation
temperatures

More masses: B_c

Even more masses: P
wave states

Spectral functions

■ States $c\bar{c}, b\bar{b}, b\bar{c} (c\bar{b})$

■ S Wave (L=0)

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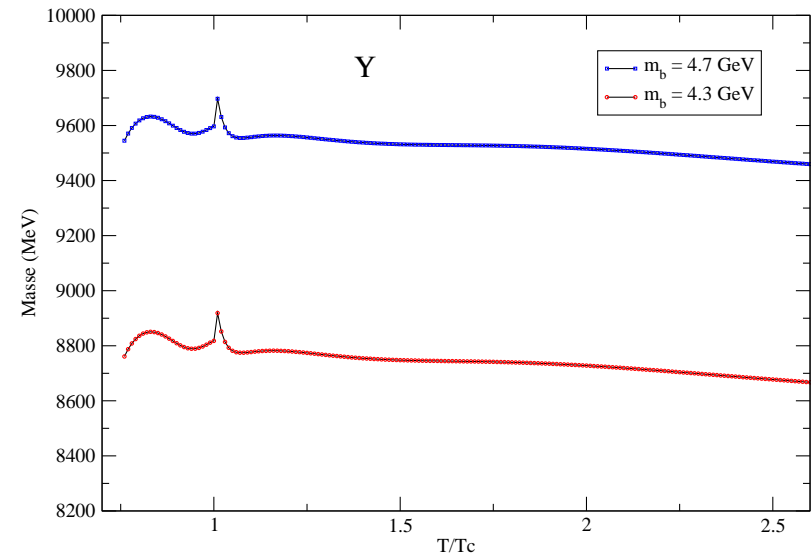
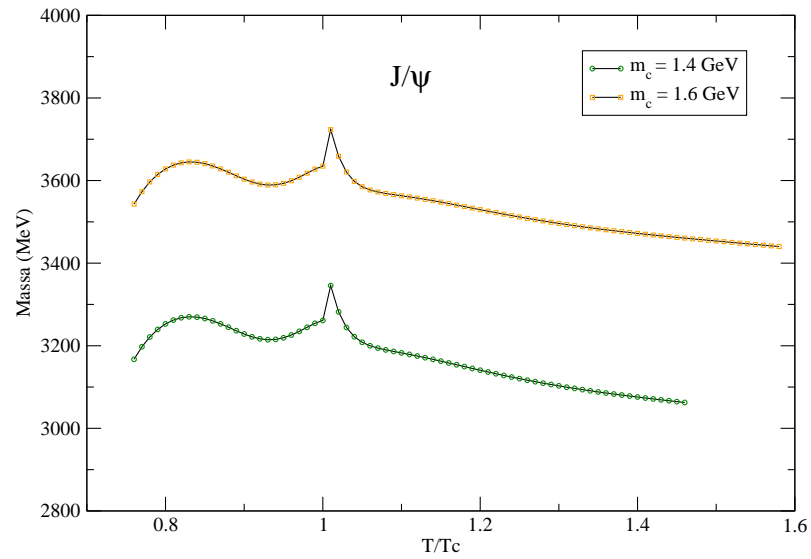
Fundamental bound states

Scattering states

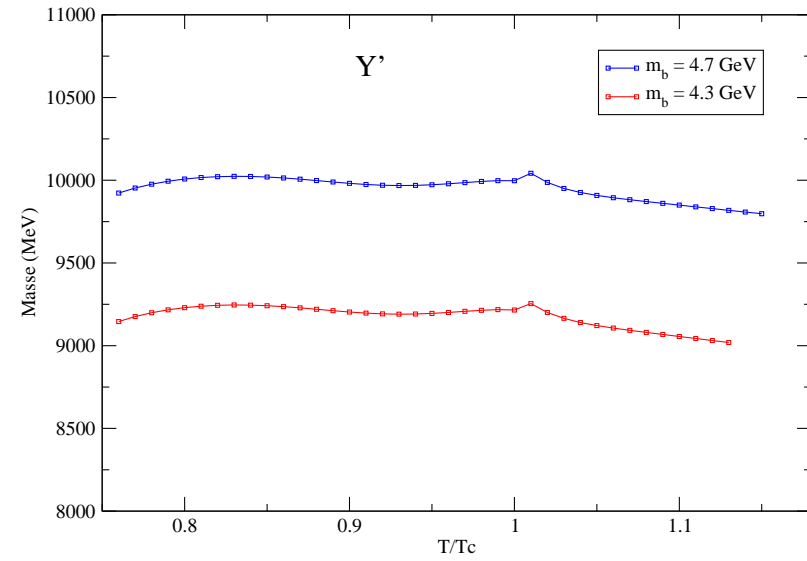
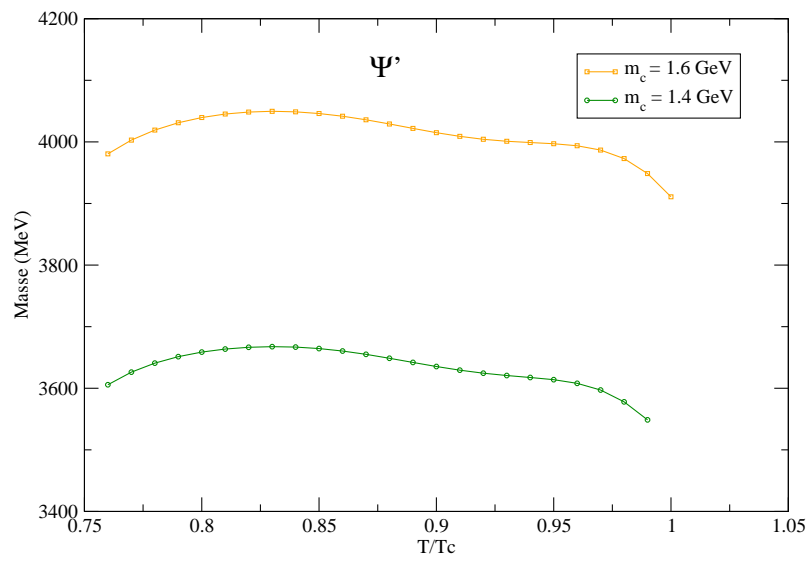
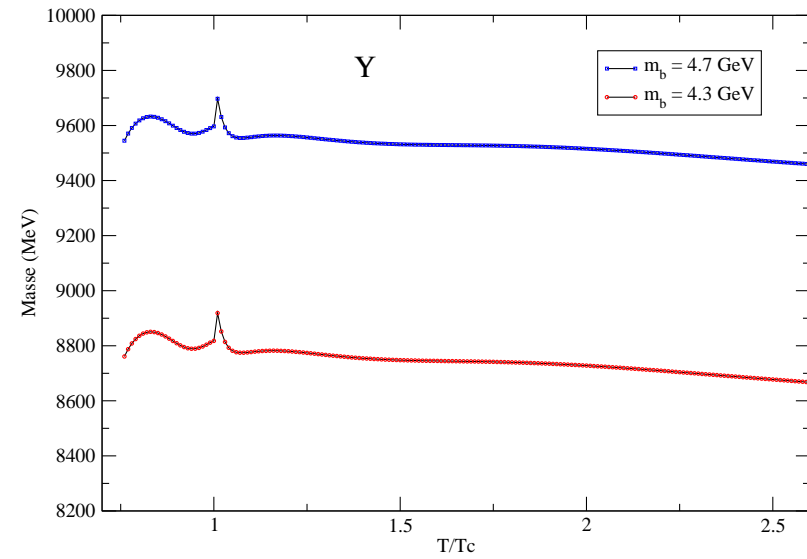
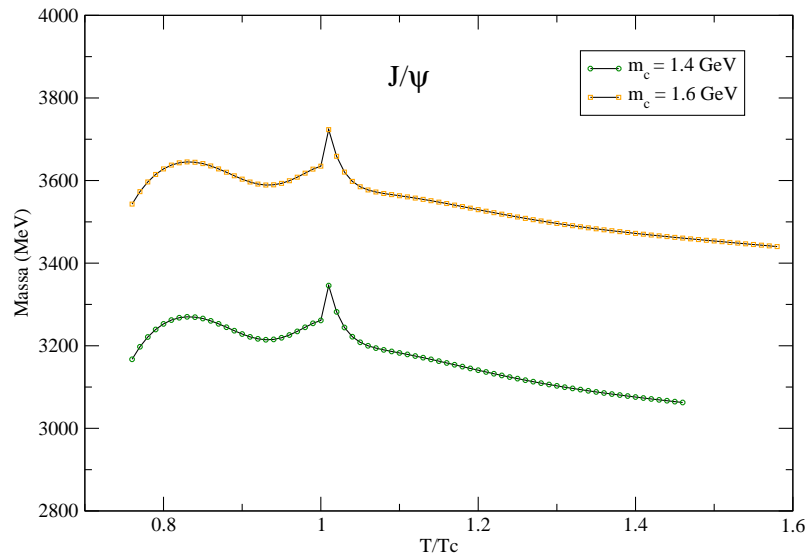
■ Lattice data for F_1 obtained using $N_f = 2$
(F. Karsch et al., Phys. Lett. B 478, 447) (2000)

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Dissociation temperatures

Dissociation temperatures (T/T_c)

$c\bar{c}$	$m_c=1.4$ GeV	$m_c=1.6$ GeV	$b\bar{b}$	$m_b=4.3$ GeV	$m_b=4.7$ GeV
$J/\psi, \eta_c$	1.45	1.57	Υ, η_b	3.9	4.4
χ_c	0.99	1.00	χ_b	1.14	1.16
ψ'	0.98	1.00	Υ'	1.14	1.15

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- Dissociation temperatures for charmonium in good agreement with lattice QCD
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Dissociation temperatures

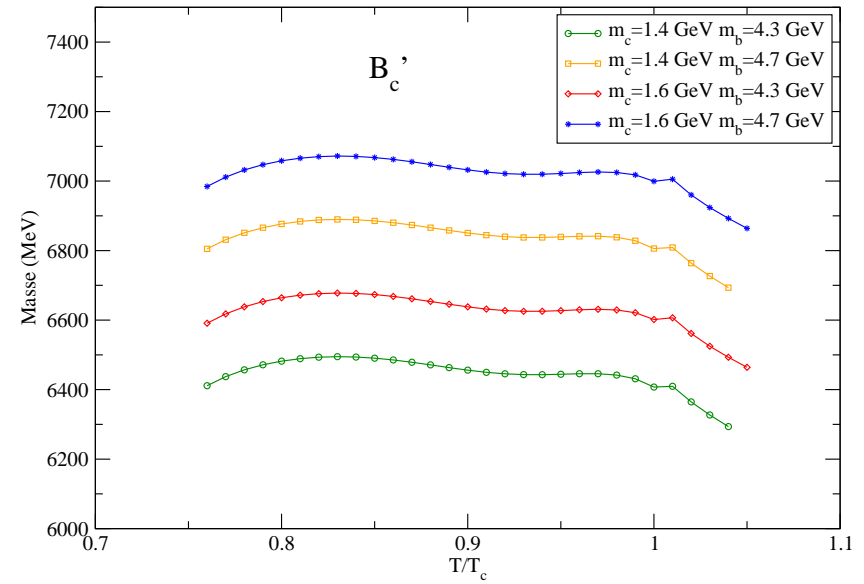
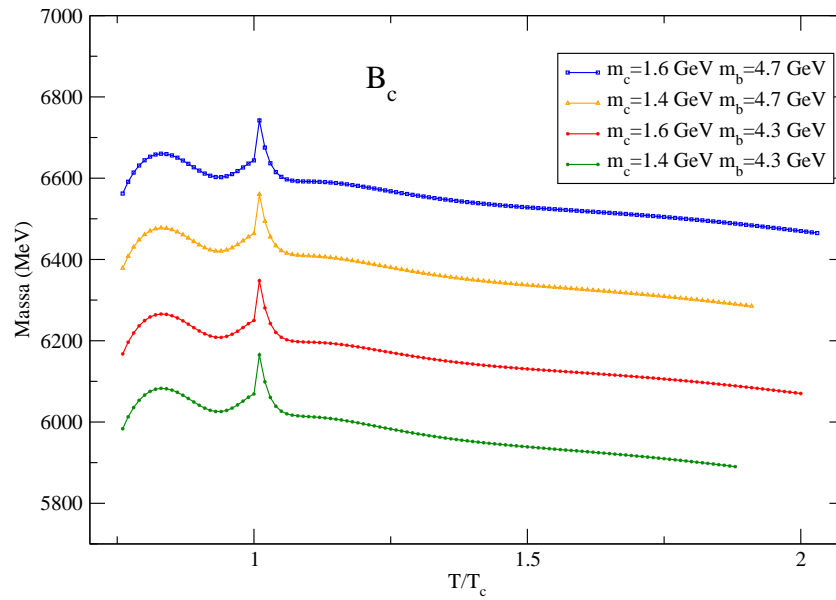
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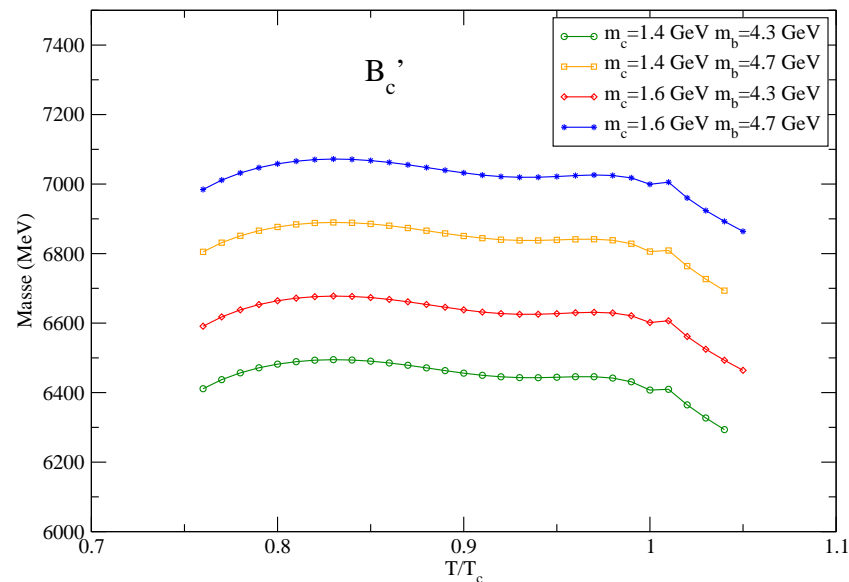
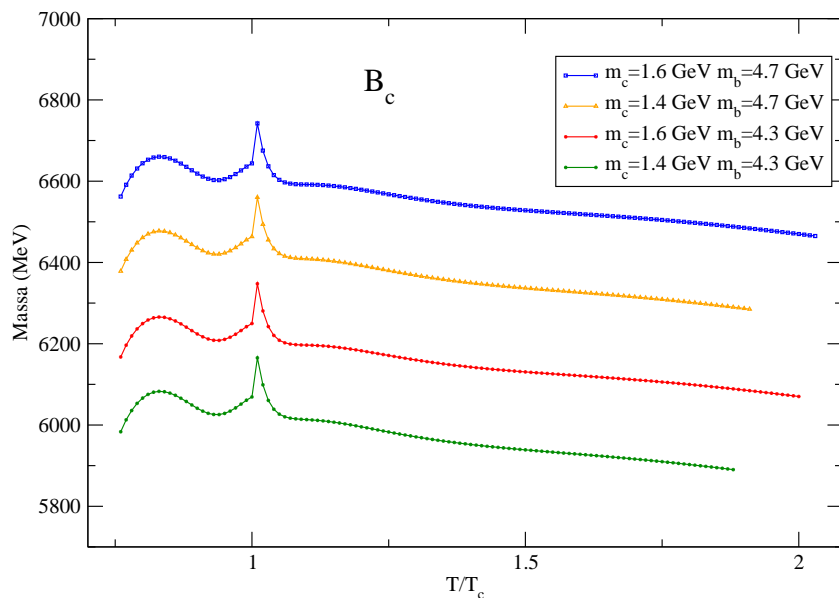
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- Dissociation temperatures for charmonium in good agreement with lattice QCD
- Mass variation \rightarrow Difficult comparison with lattice QCD
- Dissociation temperatures do not vary significantly with different quark masses.

More masses: B_c

More masses: B_c

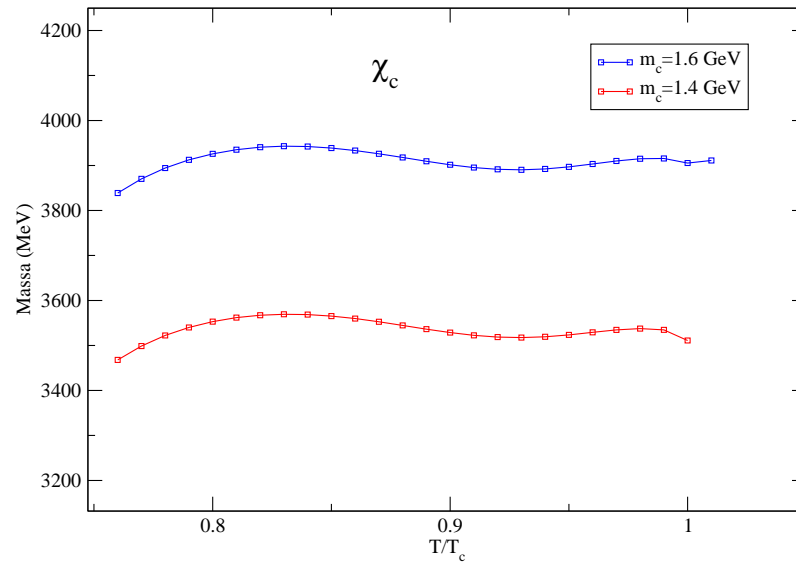




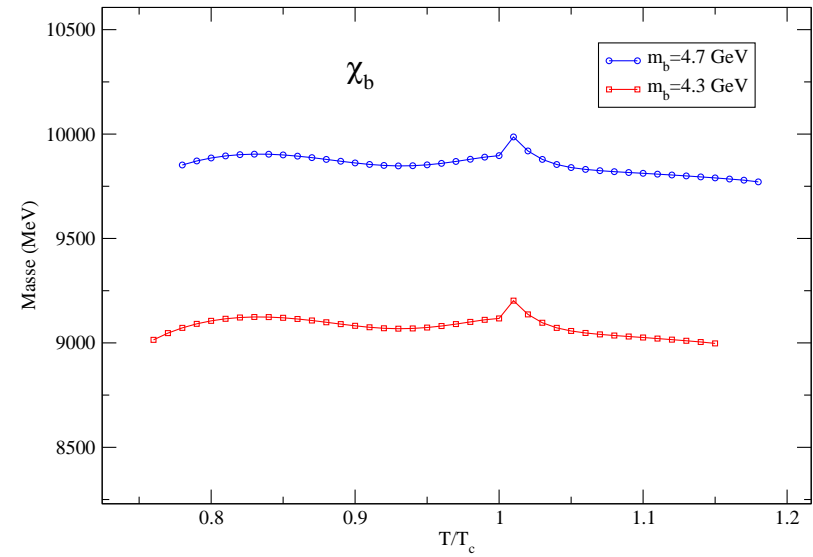
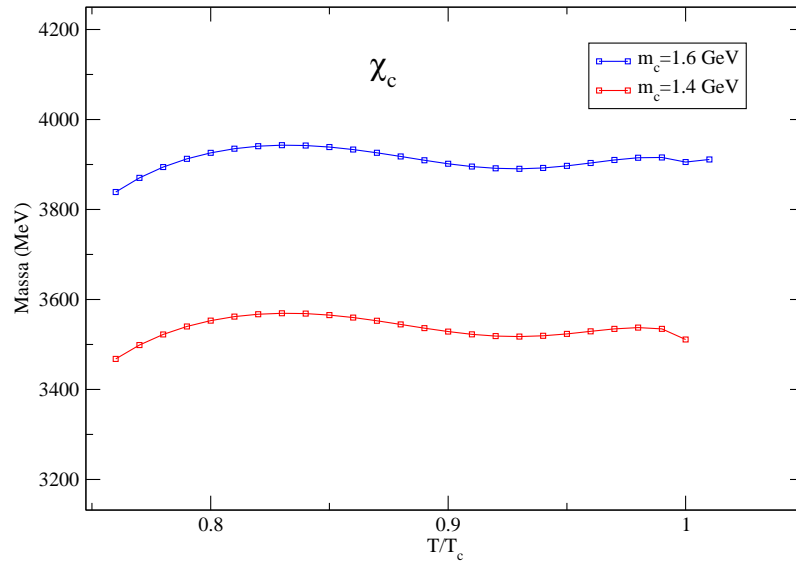
Dissociation temperatures (T/T_c)

$\bar{c}b\bar{b}\bar{c}$	$m_c=1.4$ GeV $m_b=4.3$ GeV	$m_c=1.4$ GeV $m_b=4.7$ GeV	$m_c=1.6$ GeV $m_b=4.3$ GeV	$m_c=1.6$ GeV $m_b=4.7$ GeV
B_c	1.87	1.90	1.99	2.02
χ_{B_c}	1.05	1.05	1.06	1.06
B'_c	1.03	1.04	1.04	1.05

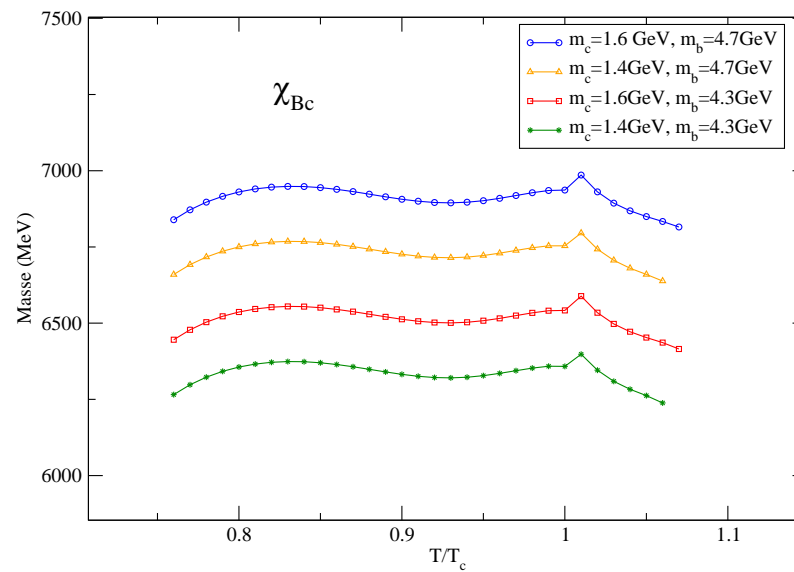
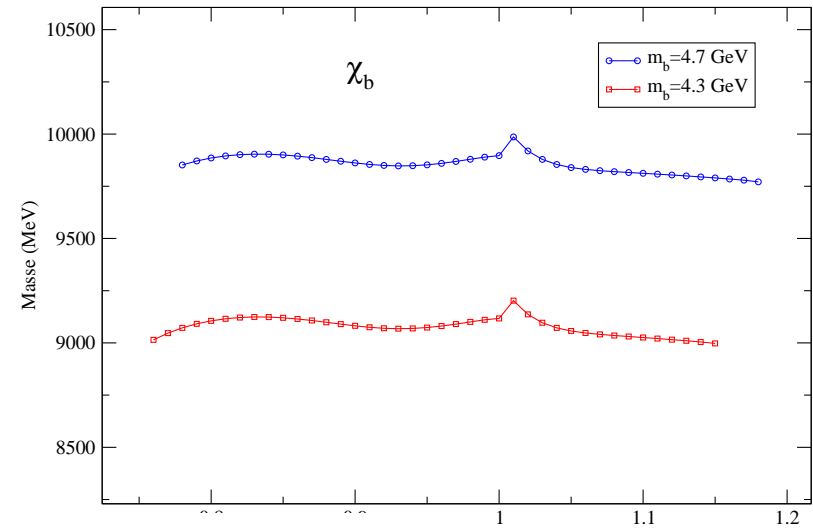
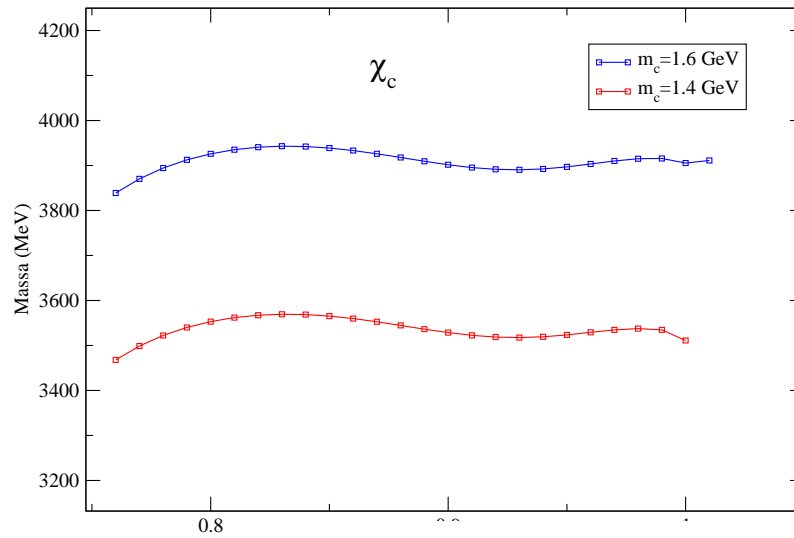
Even more masses: P wave states



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Summing up...

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$$\sigma_M(\omega, T) = \frac{1}{\pi} \text{Im} G_M(\omega) = \sum_n |\langle 0 | j_M | n \rangle|^2 \delta(\omega - E_n) =$$

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■ Non-relativistic QCD:

$$F_{M,n}^2 = |\langle 0 | j_M | n \rangle|^2 = f(|R(0)|^2, |R'(0)|^2)$$

$R(0) \rightarrow$ Value of the radial wave function $R(r)$ in the origin

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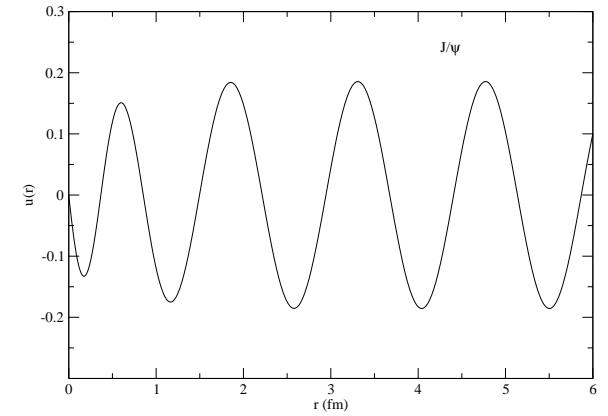
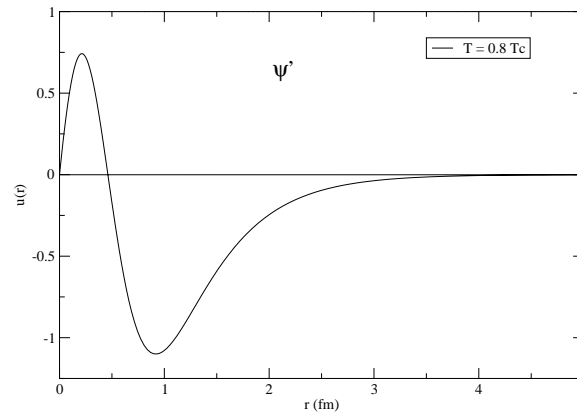
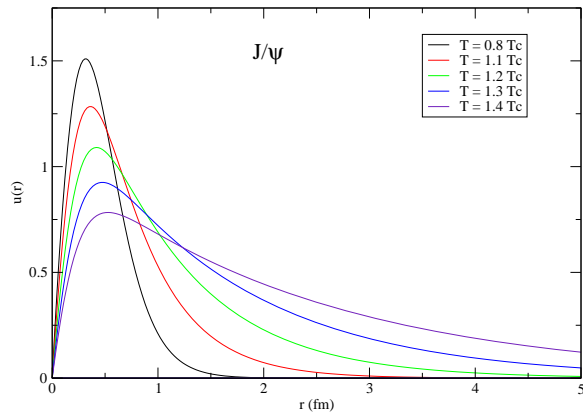
□ Vector (S wave): $F_V^2 = \frac{3N_c}{2\pi} |R(0)|^2 ;$

□ Scalar (P wave): $F_S^2 = \frac{9N_c}{2\pi m^2} |R'(0)|^2 ;$

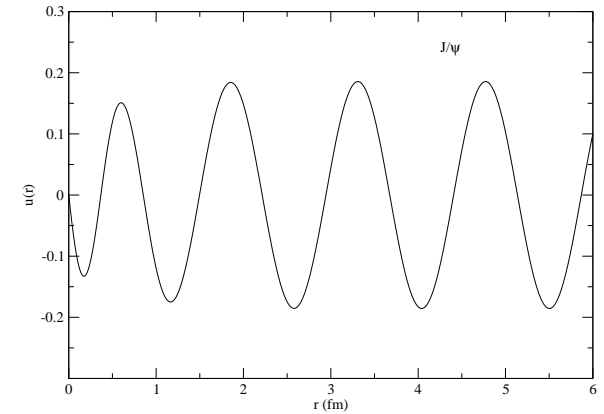
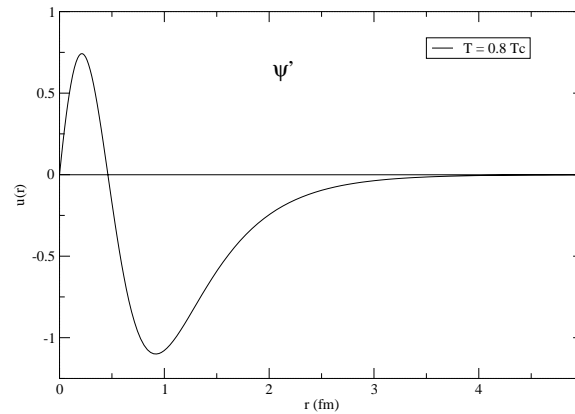
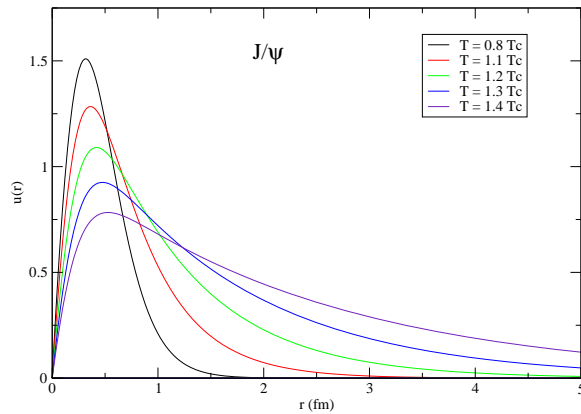
- By integrating Schroedinger's equation \rightarrow Radial wave functions

Wave functions

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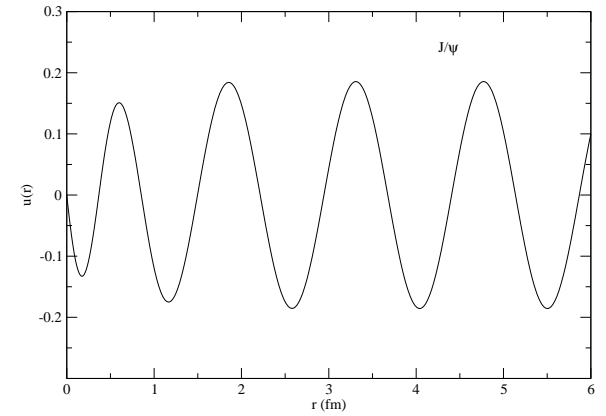
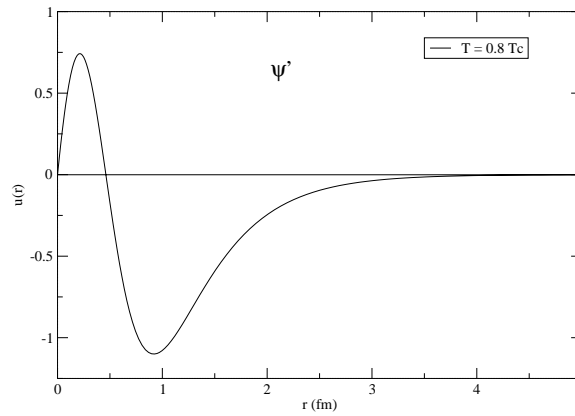
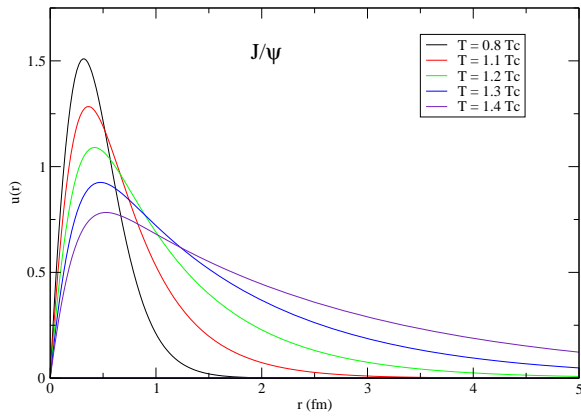
- By integrating Schroedinger's equation \rightarrow Radial wave functions



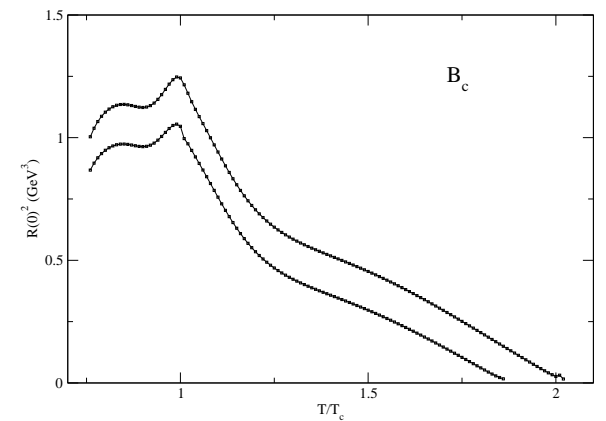
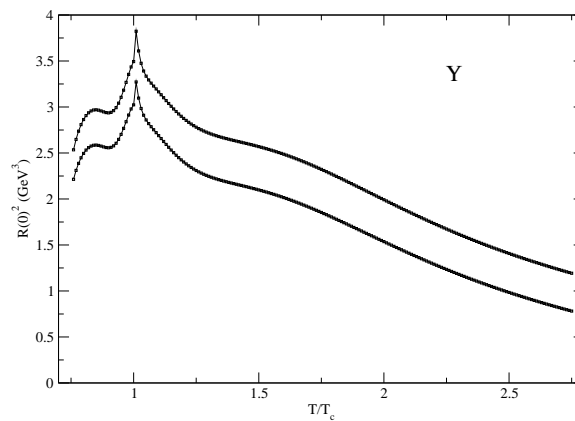
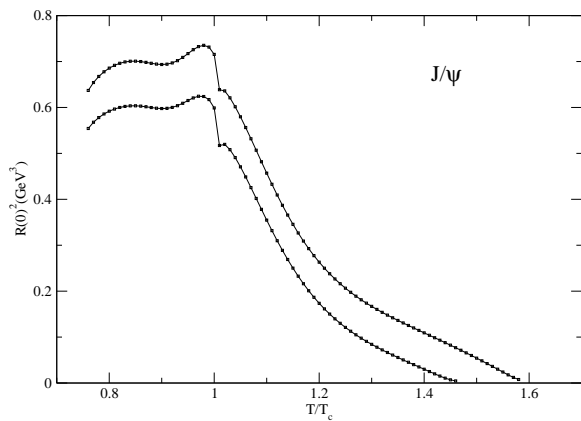
- From the radial wave functions $R(r) \rightarrow R(0), R'(0)$..

Wave functions

- By integrating Schroedinger's equation \rightarrow Radial wave functions

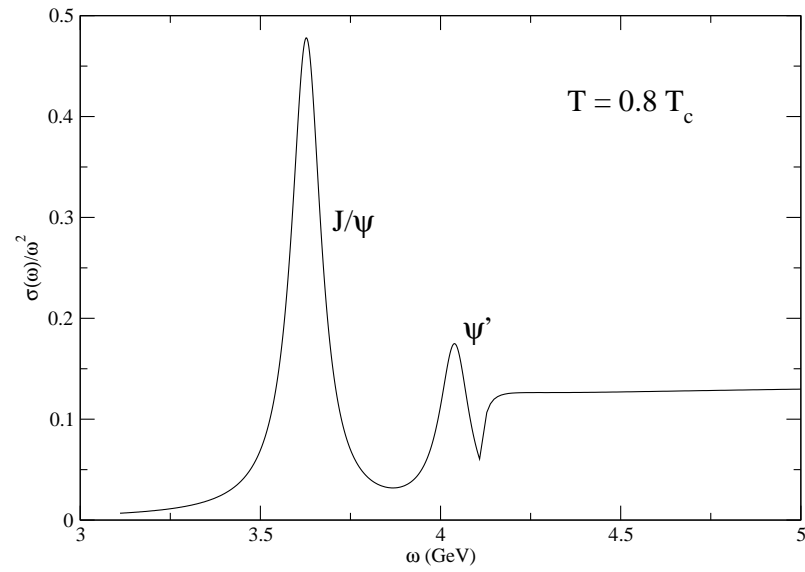


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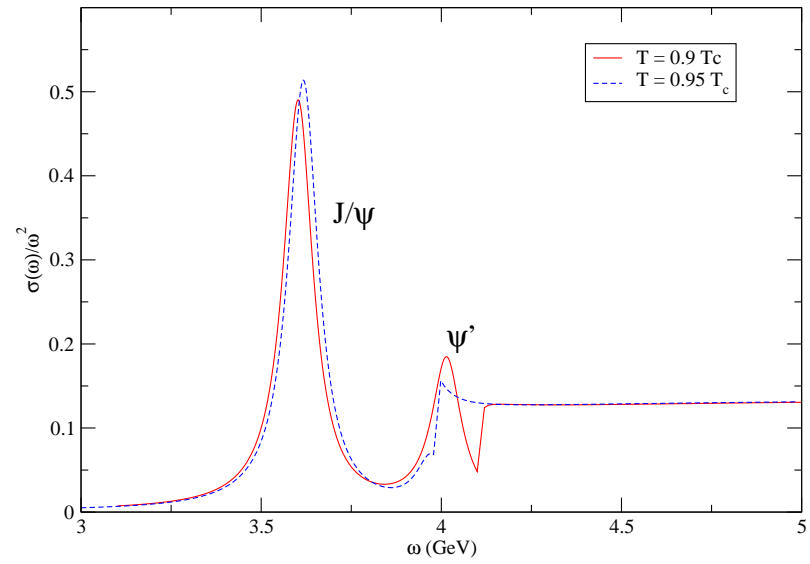
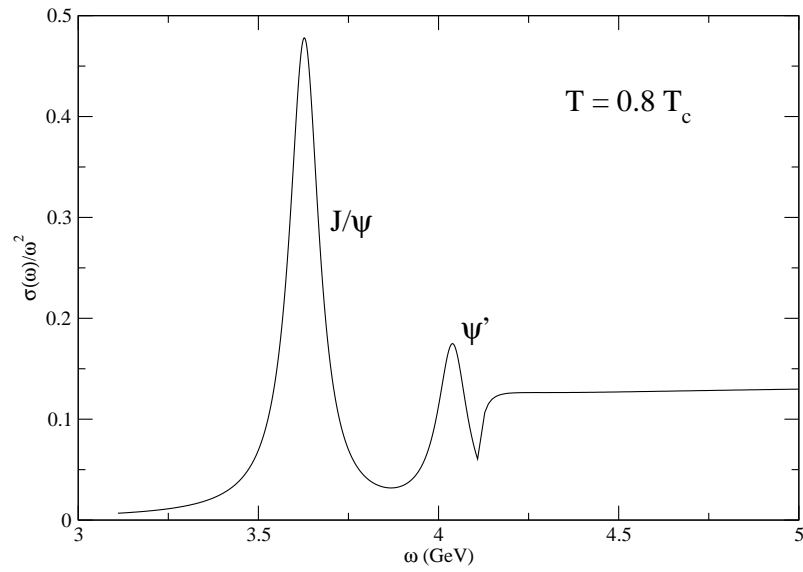


Spectral Functions: J/ψ

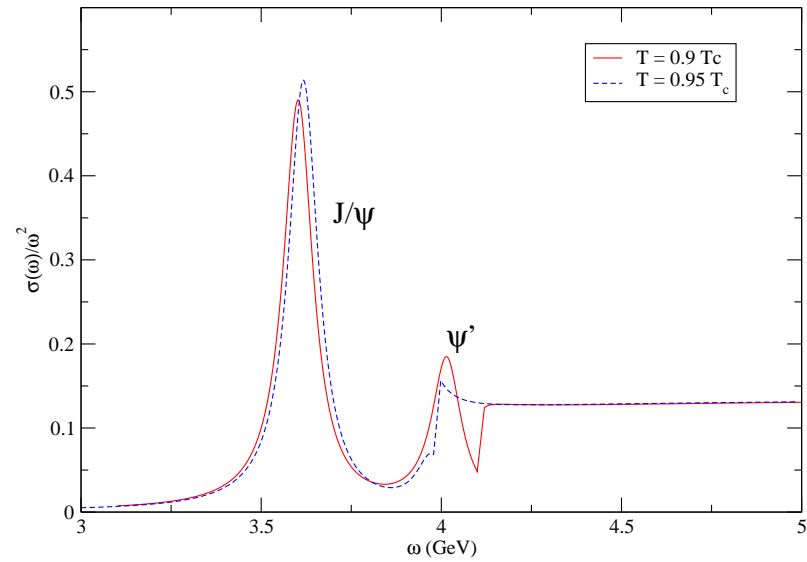
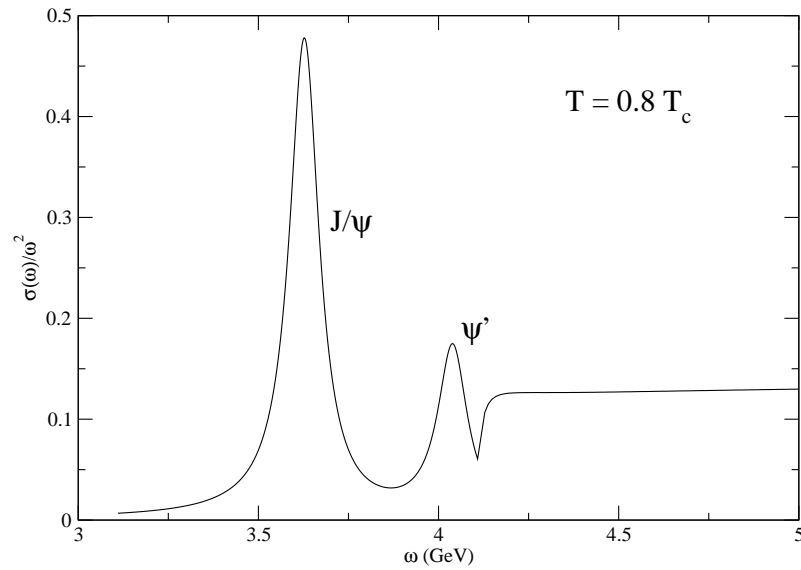
Spectral Functions: J/ψ



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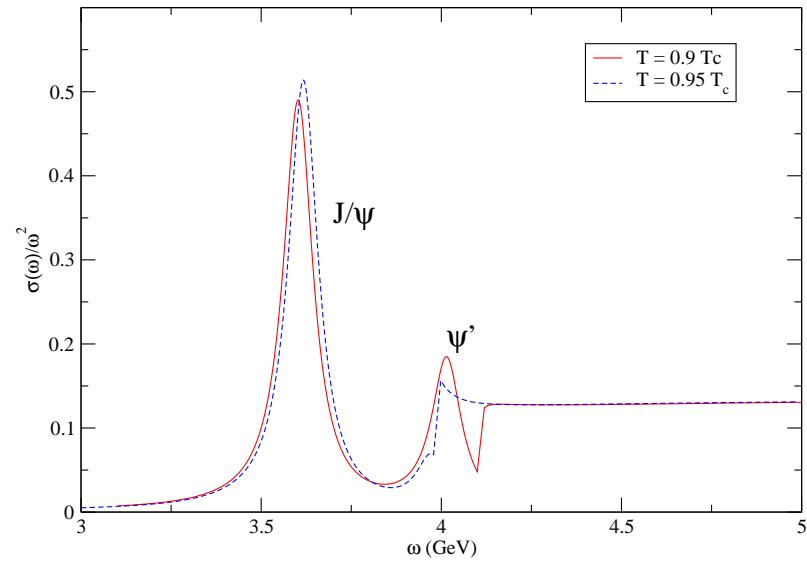
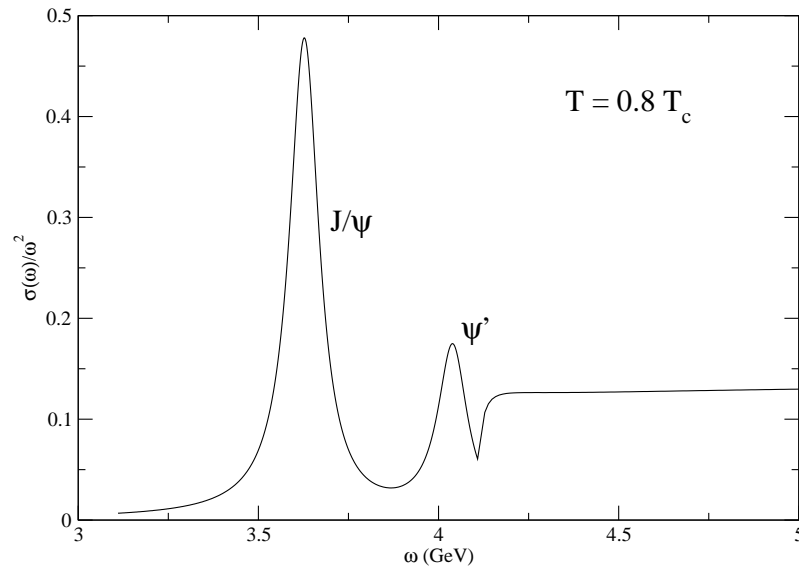


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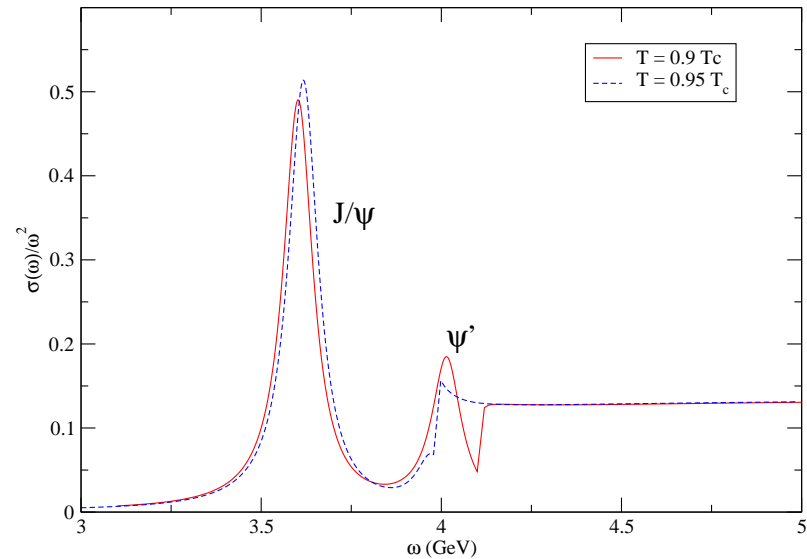
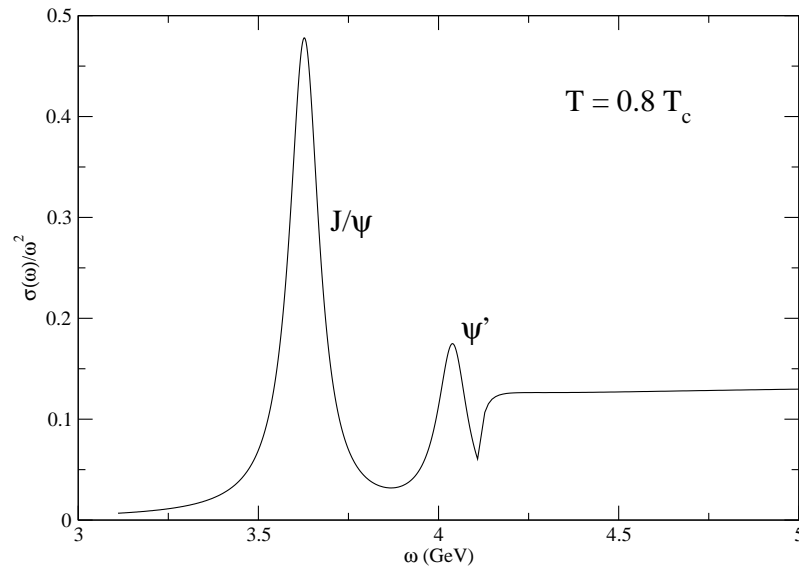
- Broadening of the bound state peaks added 'ad hoc' ($\Gamma \approx 100 MeV$)

Spectral Functions: J/ψ



- Broadening of the bound state peaks added 'ad hoc' ($\Gamma \approx 100 MeV$)
- Kinematic corrections to the scattering contribution \rightarrow reproduce correct $\propto \omega^2$ asymptotic behaviour of perturbative QCD

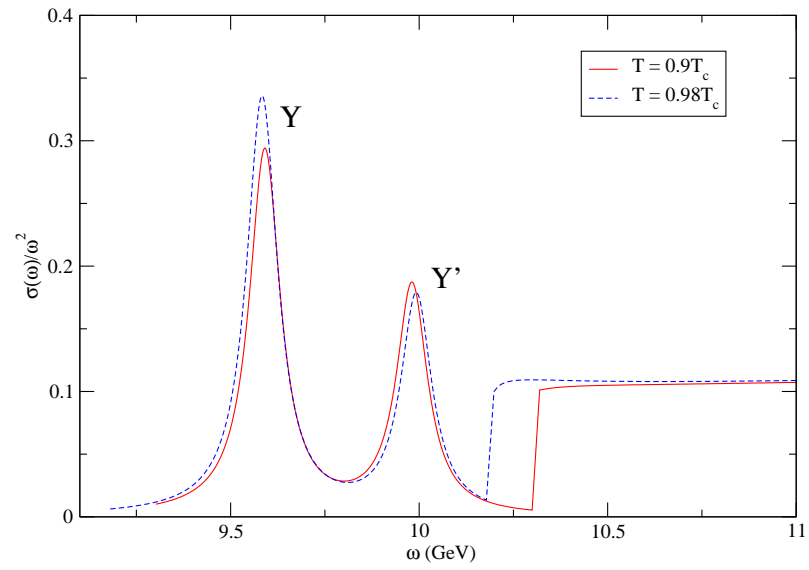
Spectral Functions: J/ψ



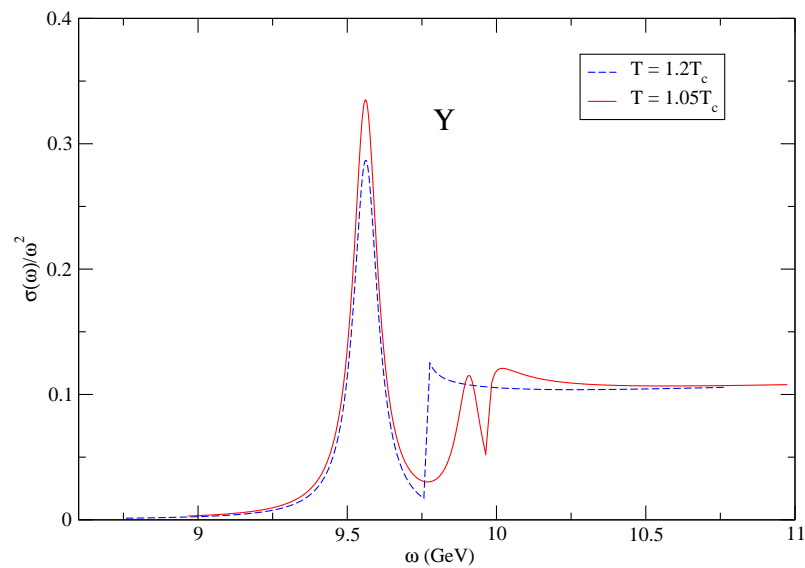
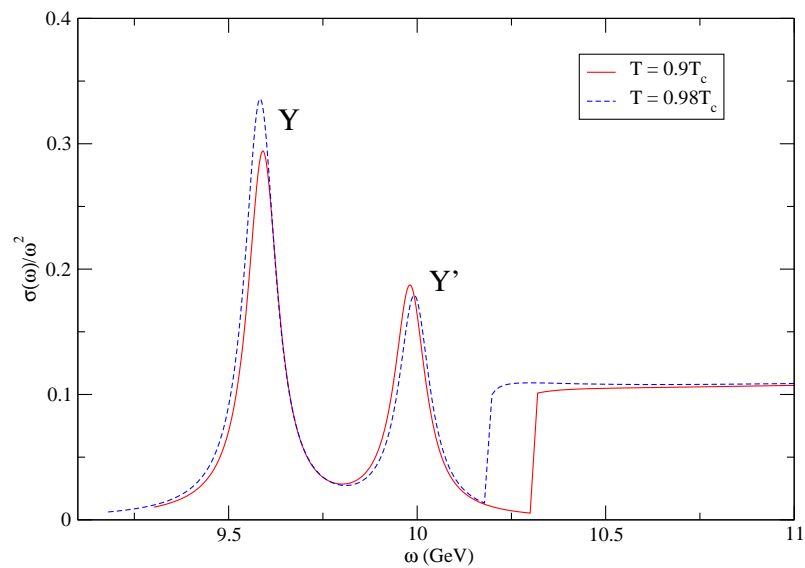
- Broadening of the bound state peaks added 'ad hoc' ($\Gamma \approx 100 MeV$)
- Kinematic corrections to the scattering contribution \rightarrow reproduce correct $\propto \omega^2$ asymptotic behaviour of perturbative QCD
- A direct comparison with lattice QCD results is very hard to realize.

Spectral functions: Υ

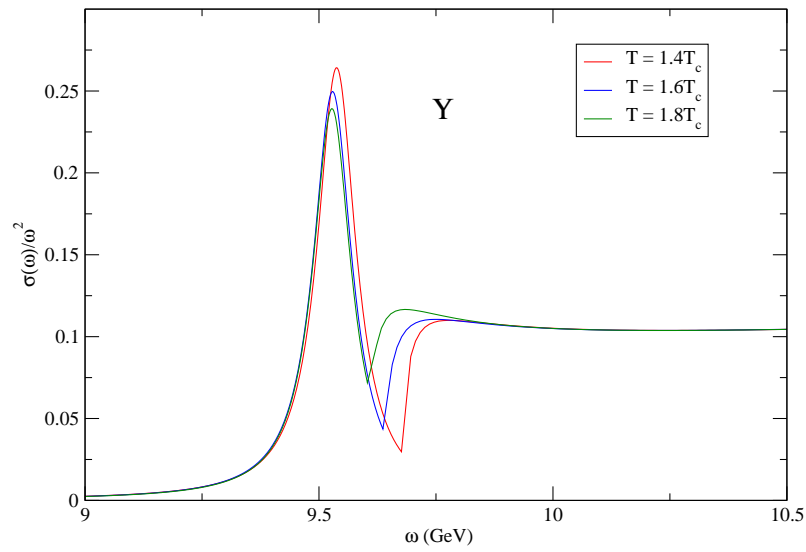
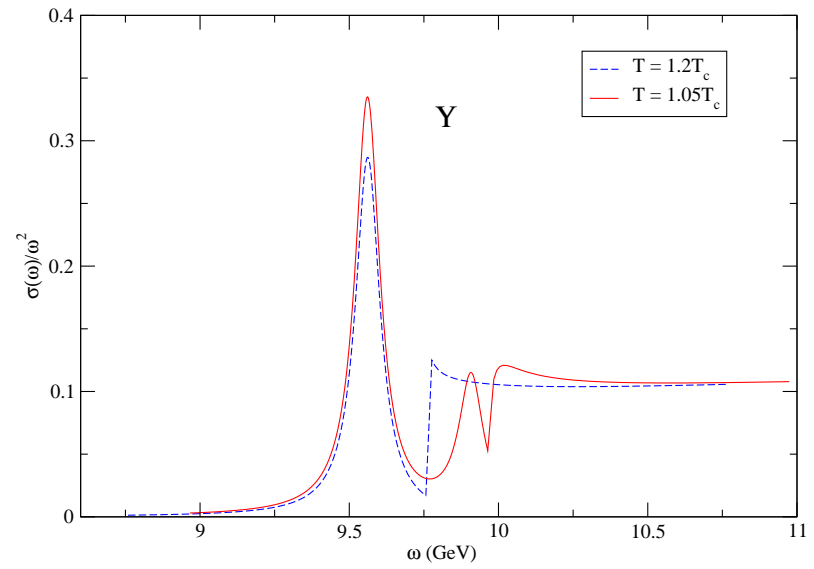
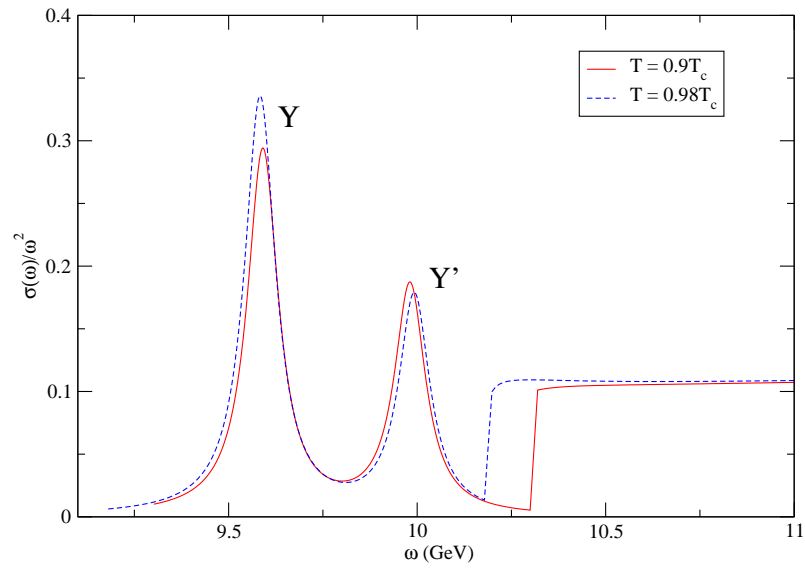
Spectral functions: Υ



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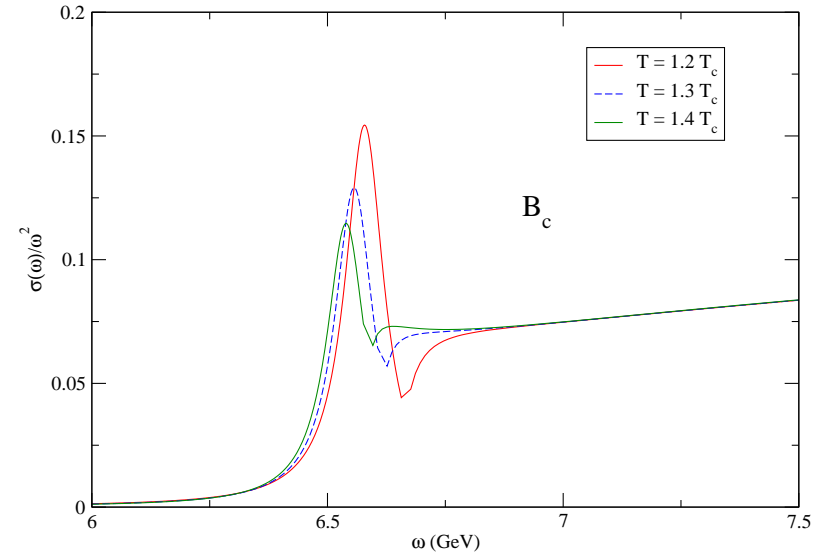
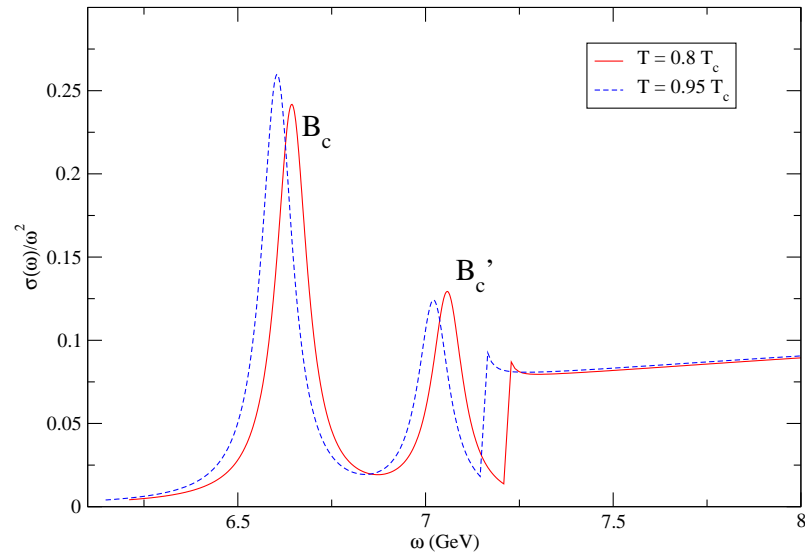


Spectral functions: Υ

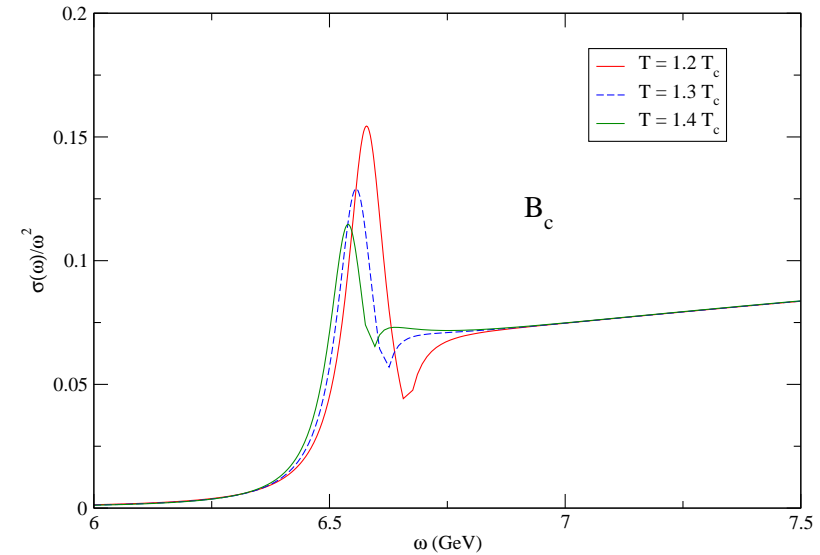
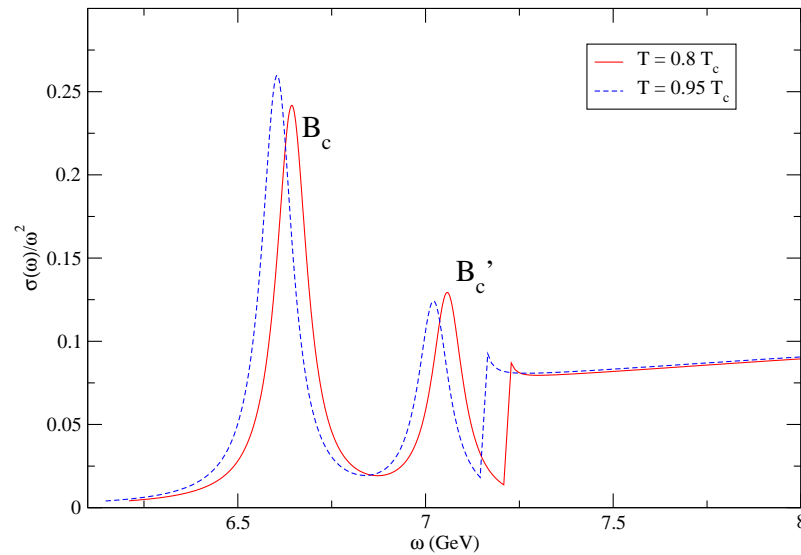


Spectral functions: B_c

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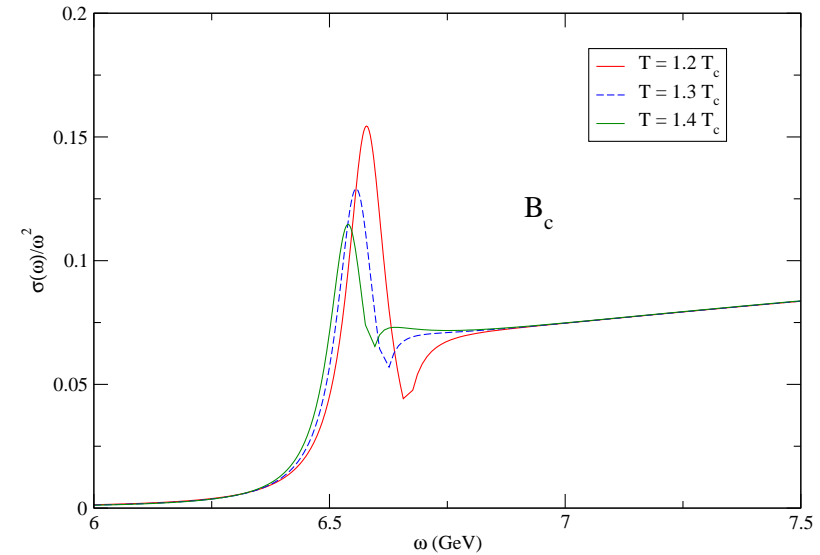
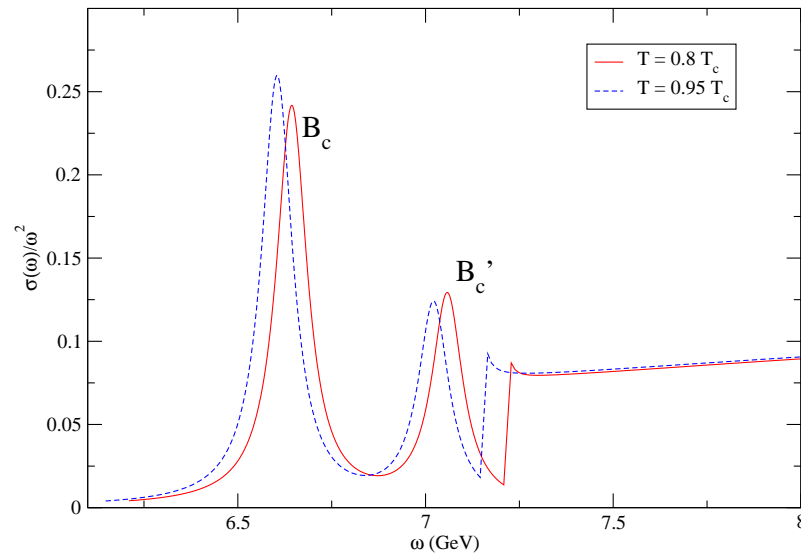


Spectral functions: B_c



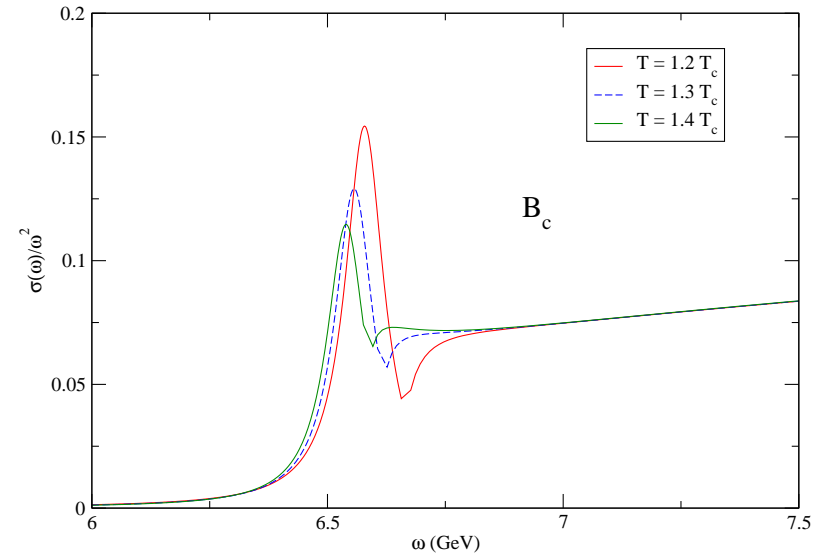
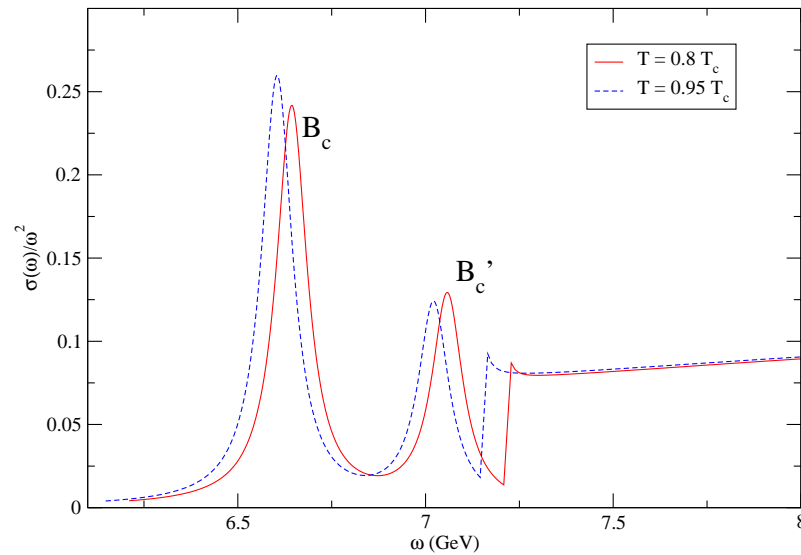
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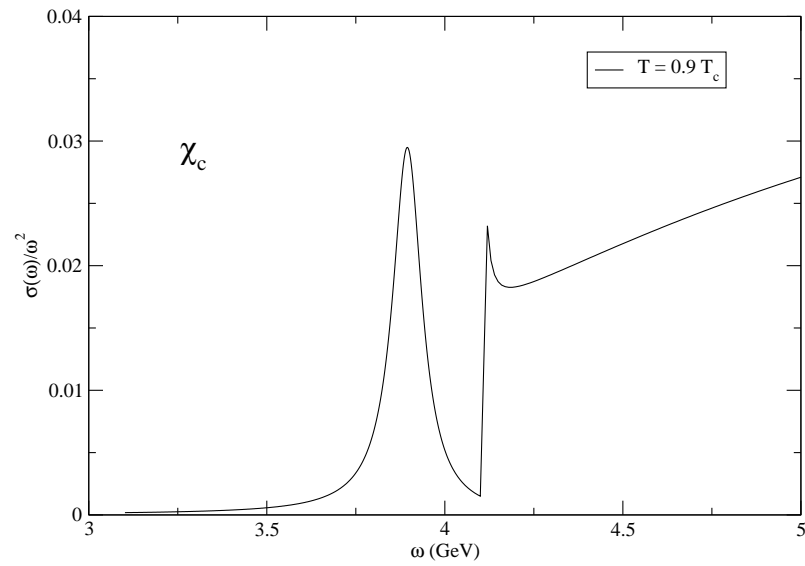
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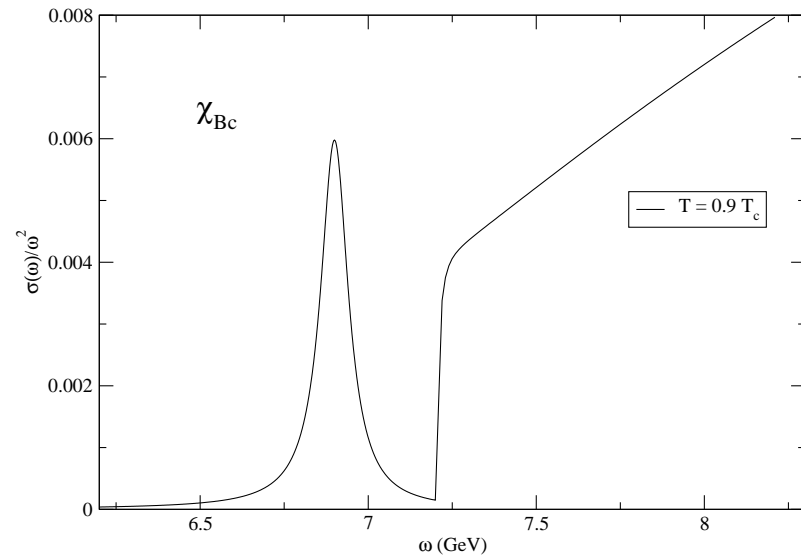
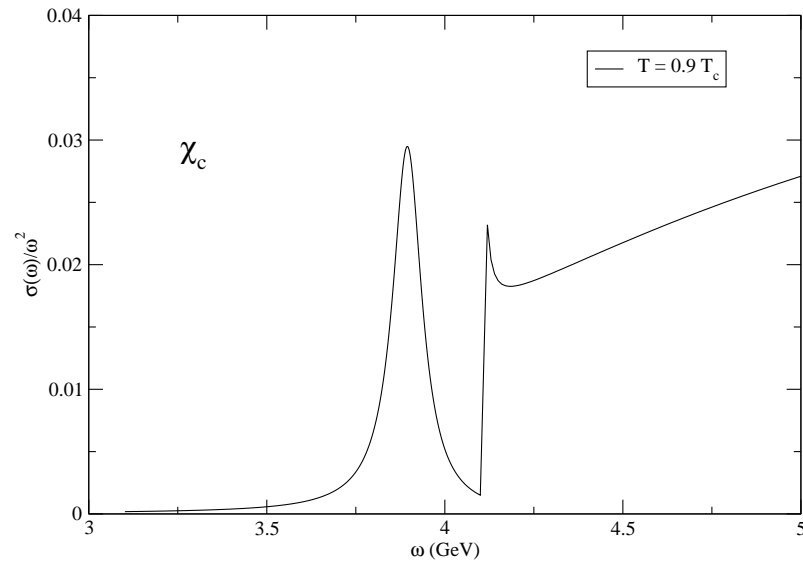
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- Radial excitation dissociates around T_c
- Fundamental state peak decreases gradually as temperature rises beyond T_c
- The peaks shift as the mass changes with temperature

Spectral functions: P wave states

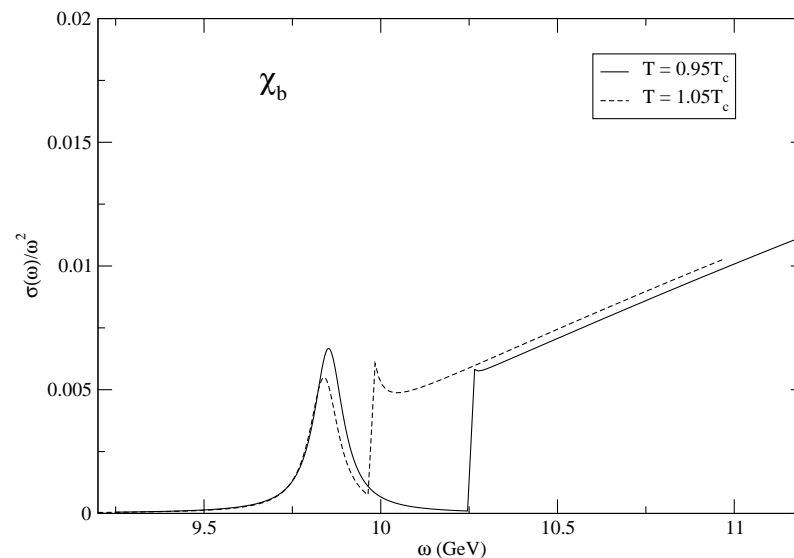
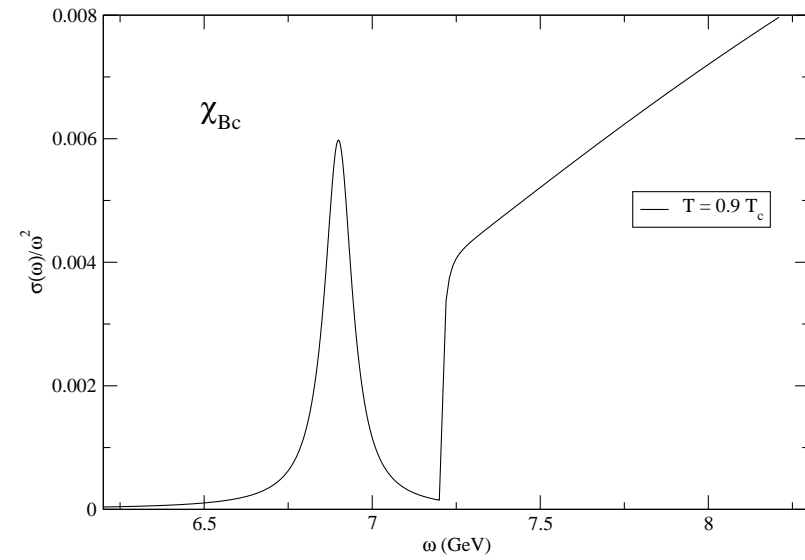
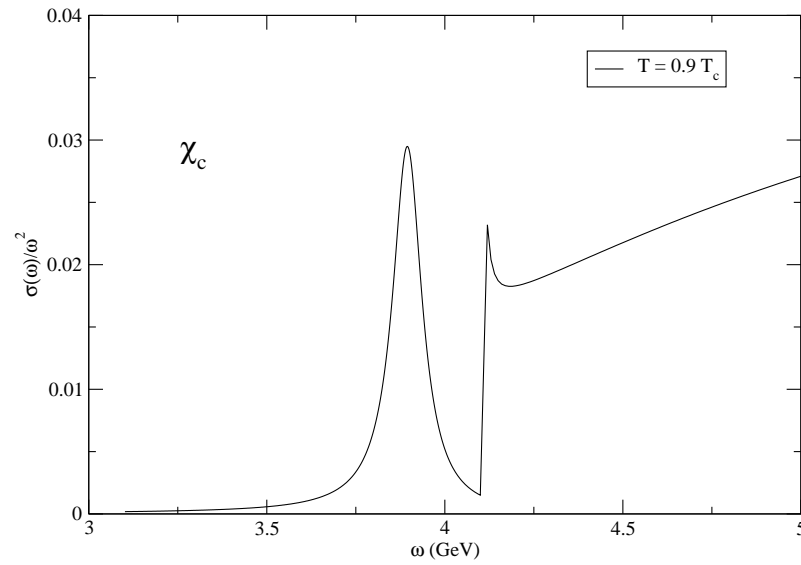
Spectral functions: P wave states



Spectral functions: P wave states



Spectral functions: P wave states



Summing up...

- Importance of the study of heavy mesons in the QGP

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- Limits of the lattice QCD approach

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 - Dissociation temperatures → Consistent with lattice QCD
 - Spectral functions → harder confrontation with lattice QCD

Intro: Quark Gluon
Plasma

Heavy mesons in the
QGP

Results

Spectral functions

-

Not enough yet?

Wong's prescription for
the potential

Wong's prescription II

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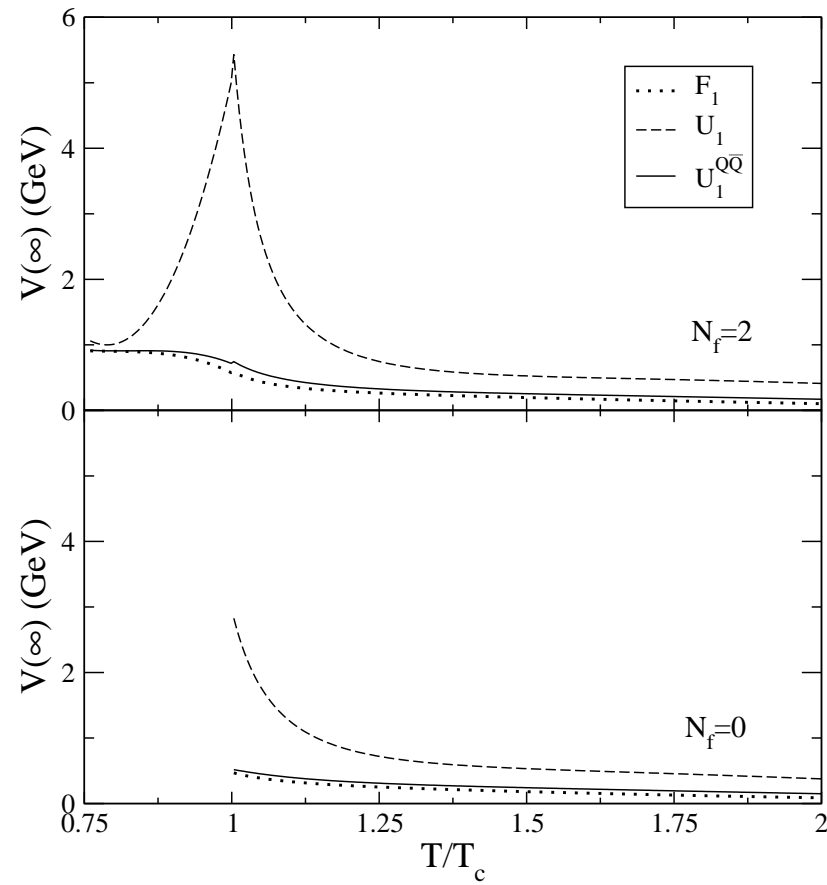
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Wong's prescription II

$$\epsilon_{\text{QGP}}^{(1)}(\vec{x}) \equiv \frac{dU_{\text{QGP}}^{(1)}}{dV}(\vec{x}) = T \frac{dS_1}{dV}(\vec{x}) - p^{(1)}(\vec{x})$$

$$\frac{p^{(1)}(\vec{x})}{\epsilon_g^{(1)}(\vec{x})/3} = a(T) \quad (\text{Local energy density})$$

$$\frac{dU_g^{(1)}}{dV}(\vec{x}) = \frac{3}{3 + a(T)} \frac{T dS_1}{dV}(\vec{x})$$

$$U_g^{(1)}(\vec{r}, T) = \int d\vec{x} \frac{dU_g^{(1)}}{dV}(\vec{x}) = \frac{3}{3 + a(T)} T S_1(\vec{r}, T)$$

$$U_g^{(1)}(\vec{r}, T) - U_{g0}(T) = \frac{3}{3 + a(T)} \{U_1(\vec{r}, T) - F_1(\vec{r}, T)\}$$

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$$U_1(\vec{r}, T) = U_{Q\bar{Q}}^{(1)}(\vec{r}, T) + U_g^{(1)}(\vec{r}, T) - U_{g0}(T)$$

From which

$$\begin{aligned} U_{Q\bar{Q}}^{(1)}(\vec{r}, T) &= \frac{3}{3 + a(T)} [F_1(\vec{r}, T) - U_1(\vec{r}, T)] + U_1(\vec{r}, T) \\ &= \frac{3}{3 + a(T)} F_1(\vec{r}, T) + \frac{a(T)}{3 + a(T)} U_1(\vec{r}, T) \end{aligned}$$

The potential then is

$$\begin{aligned} &U_{Q\bar{Q}}^{(1)}(\vec{r}, T) - U_{Q\bar{Q}}^{(1)}(\vec{r} \rightarrow \infty, T) \\ &= f_F(T) [F_1(\vec{r}, T) - F_1(\vec{r} \rightarrow \infty, T)] \\ &+ f_U(T) [U_1(\vec{r}, T) - U_1(\vec{r} \rightarrow \infty, T)] \end{aligned}$$