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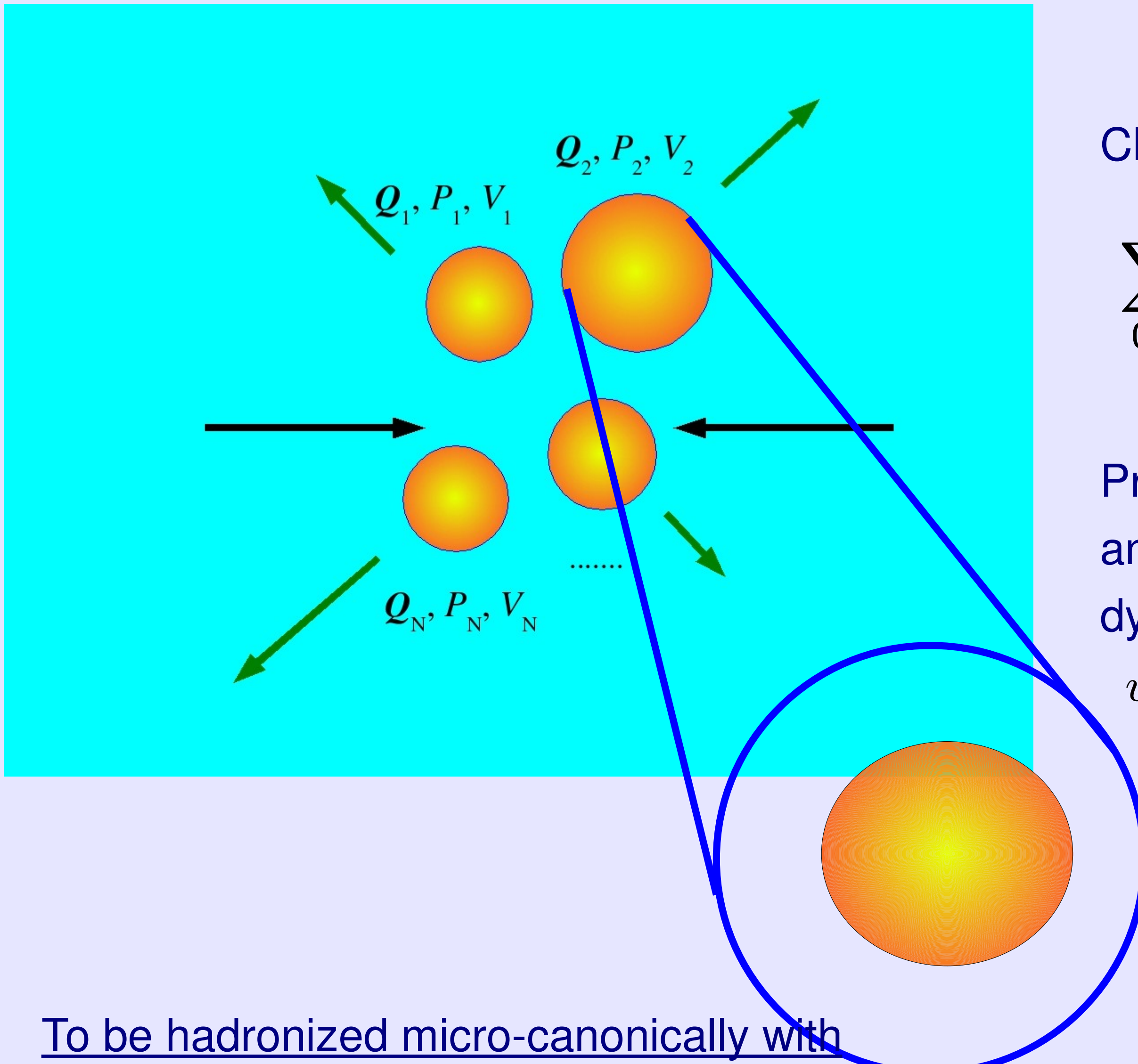
"An introduction to the Statistical Hadronization Model"
Lecture 3

OUTLINE

- Reduction to one cluster
- Results in elementary collisions
- Heavy ion collisions
- Speculative thoughts about thermalization

High energy collisions: multiple cluster production



Clusters are charge-entangled

$$\sum_{\mathsf{Q}_i} c(\mathsf{Q}_1, \mathsf{Q}_2, \ldots) |\mathsf{Q}_1, \mathsf{Q}_2, \ldots\rangle$$

Probabilities are unknown to the SHM and are determined by previous dynamics

$$w(\mathsf{Q}_1, \mathsf{Q}_2, \ldots) = |c(\mathsf{Q}_1, \mathsf{Q}_2, \ldots)|^2 = ?$$

To be hadronized micro-canonically with

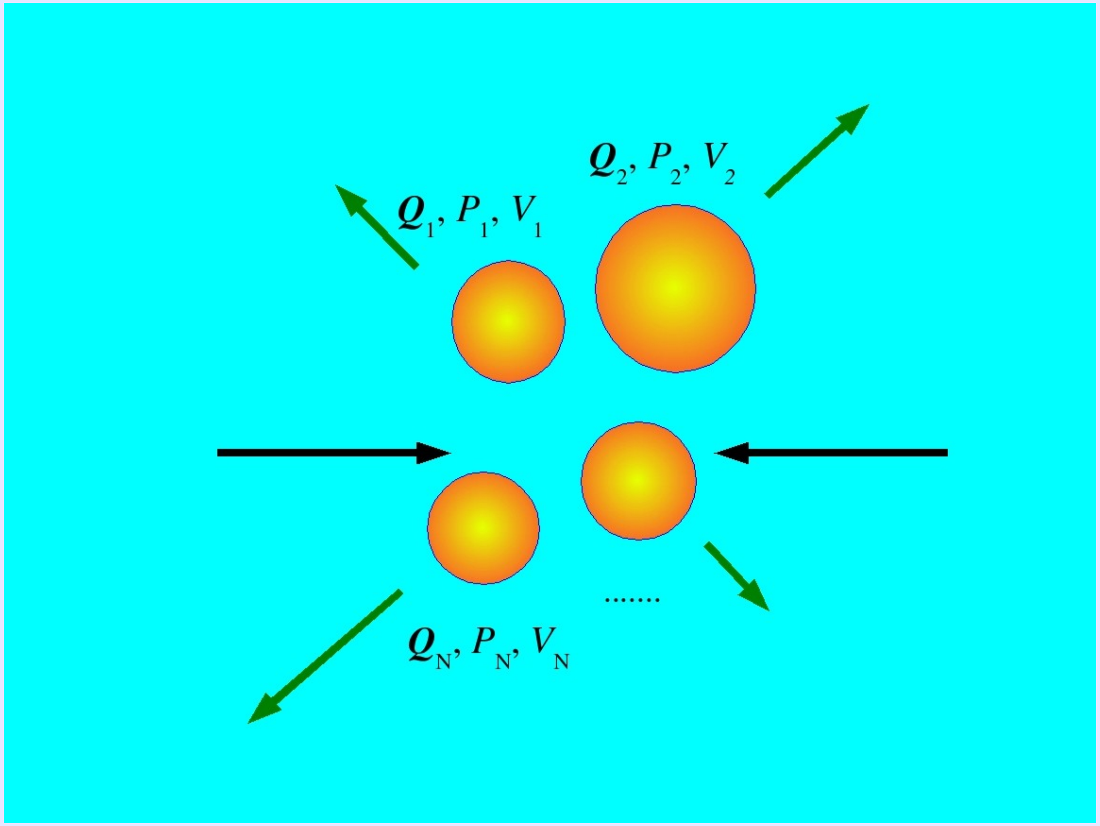
$$\Gamma_{\{N_j\}} \propto \frac{V^N}{(2\pi)^{3N}} \left(\prod_j \frac{1}{N_j!} \right) \int d^3p_1 \ldots \int d^3p_N \; \delta^4(P_0 - \sum_i p_i)$$

Momentum spectra vs multiplicity

- In high energy collisions, especially e+e-, momentum spectra of hadrons are determined by the total momentum of the clusters, hence by early perturbative dynamics (parton shower etc.)

- Conversely, the relative multiplicities are Lorentz invariants with respect a boost of the cluster and, THEREFORE, they have to be determined by non-perturbative dynamics, i.e. they just have to do with hadronization.

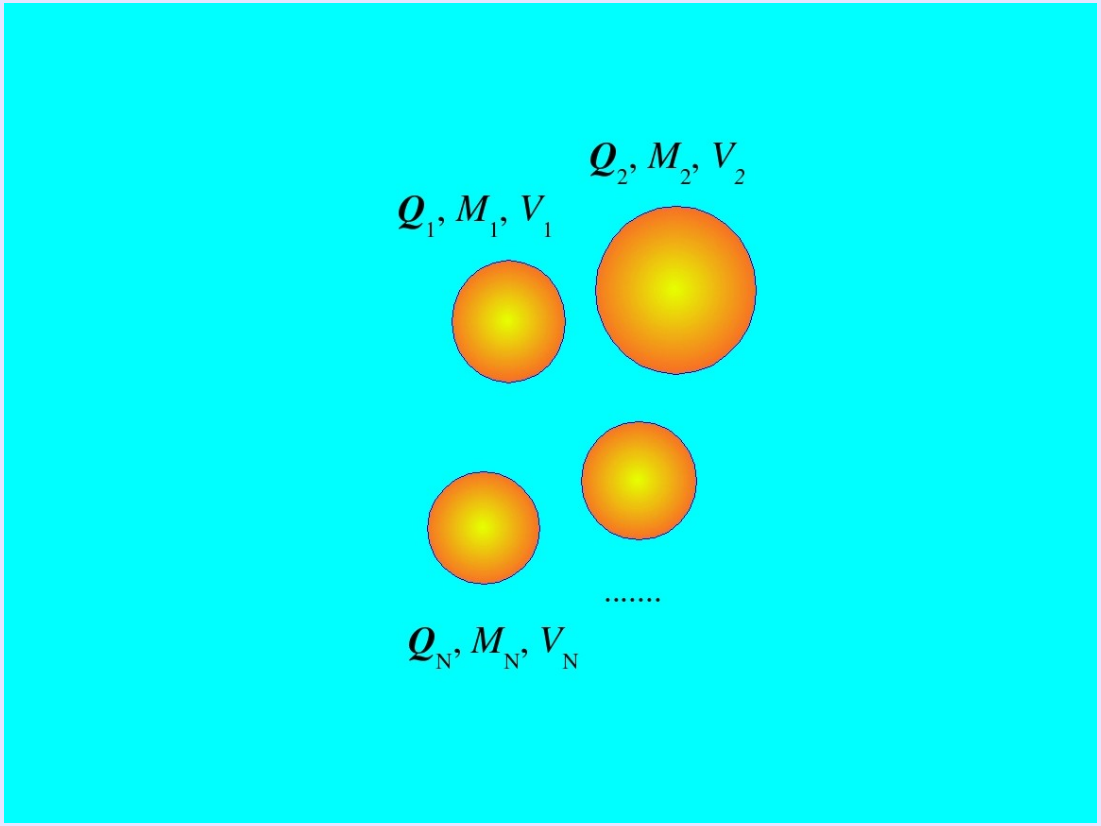
We need to know the charge-momentum distribution probabilities $w(P_1, \mathsf{Q}_1; P_2, \mathsf{Q}_2, \ldots)$



We need to know the charge-momentum distribution probabilities $w(P_1, \mathsf{Q}_1; P_2, \mathsf{Q}_2, \ldots)$

However, for Lorentz scalars, such as multiplicities, only mass distribution matters

$$w(P_1^2, \mathsf{Q}_1; P_2^2, \mathsf{Q}_2, \ldots)$$



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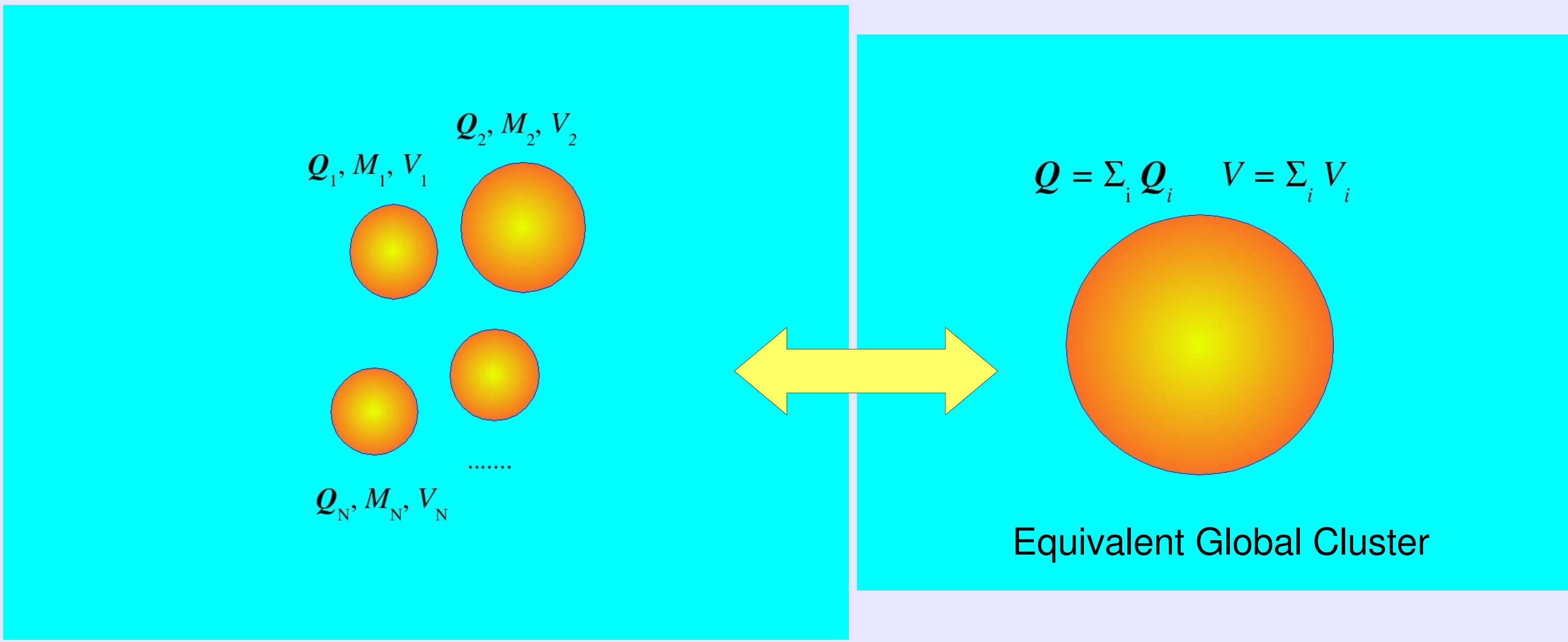


The equivalence applies provided that the probabilities *w* are “statistical”, i.e. they are obtained from maximizing the splitting entropy

For a detailed derivation see F. B., G. Passaleva, Eur. Phys. J. C 23 (2002) 551

If the EGC is large enough, one can go from a micro-canonical to a canonical calculation where (M,V) is replaced by (T,V)

Primary multiplicities in Boltzmann approx.

$$\langle n_j \rangle = \frac{(2S+1)V}{(2\pi)^3} \gamma_S{}^{N_s} \int d^3p \, \mathrm{e}^{-\sqrt{p^2+m_j^2}/T} \, \frac{Z(\mathbf{Q} - \mathbf{q}_j)}{Z(\mathbf{Q})}$$

extra strangeness suppression

$$\mathbf{Q} = (Q, B, S, ..)$$
$$\mathbf{q}_j = (Q_j, B_j, S_j, ...)$$

REMARKS

- Note that M as well as V of the EGC can fluctuate and so can T
- In general, for individual hadronizing clusters only mass and volume are physical; temperature cannot be defined for single clusters, only for the EGC.
- Therefore, in elementary collisions a local T does not exist
- Formula can be valid only for integrated multiplicities

Multiplicity fits

- Calculate (light-flavoured) primary hadron yields according to

(simplified Boltzmann formula)

$$\langle n_j \rangle = \frac{(2S_j+1)V}{(2\pi)^3} {\gamma_S}^{N_s} \int d^3p \, \mathrm{e}^{-\sqrt{p^2+m_j^2}/T} \, \frac{Z(\mathsf{Q} - \mathsf{q}_j)}{Z(\mathsf{Q})}$$

$$\mathsf{Q} = (Q, B, S, ..) \quad \mathsf{q}_j = (Q_j, B_j, S_j, ...)$$

- Perform hadronic decays until experimental definition of multiplicity is matched

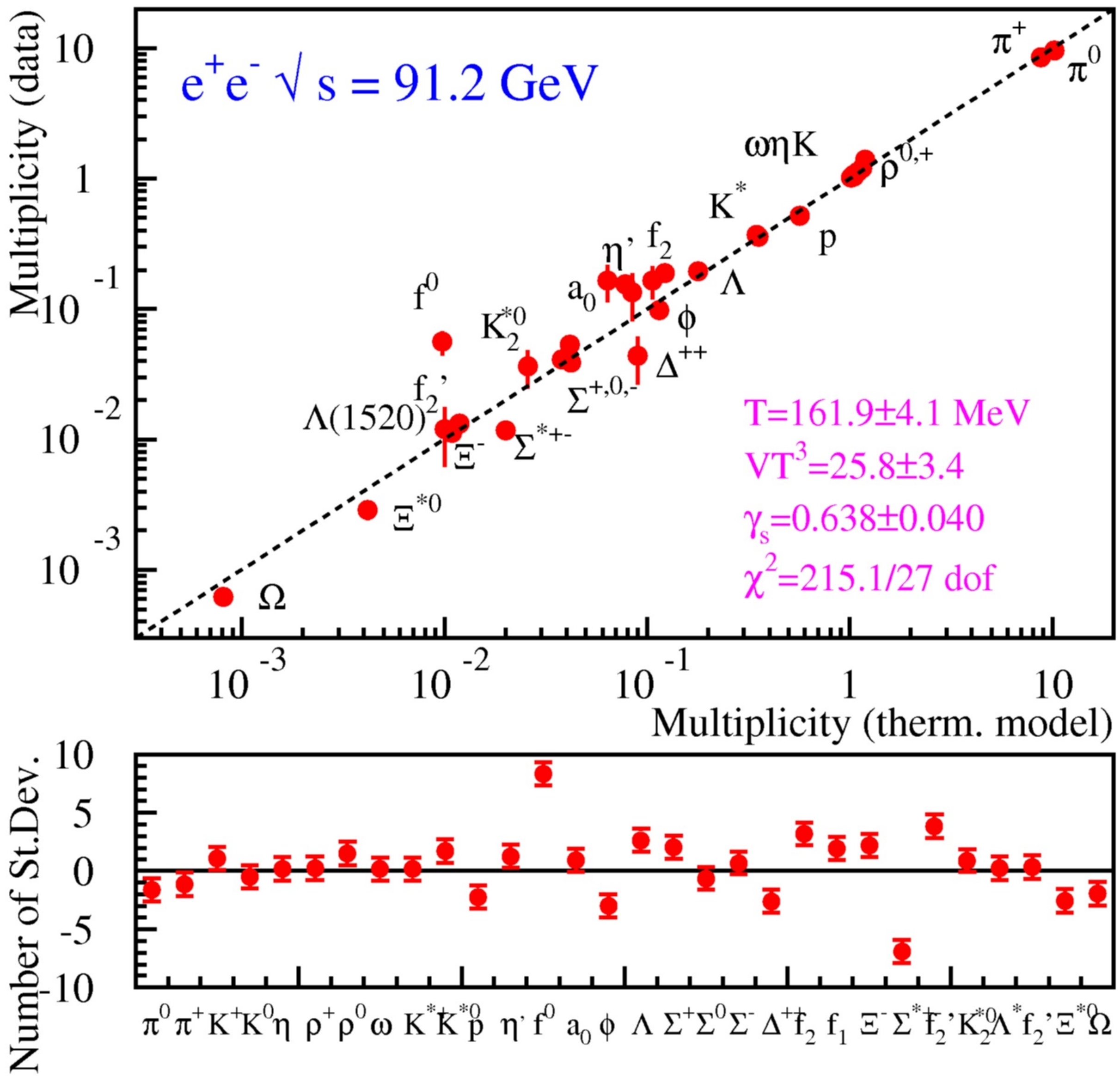
$$\langle n_j \rangle = \langle n_j \rangle_{prim} + \sum_k \langle n_k \rangle \mathrm{BR}(k \to j)$$

- Optimize parameters

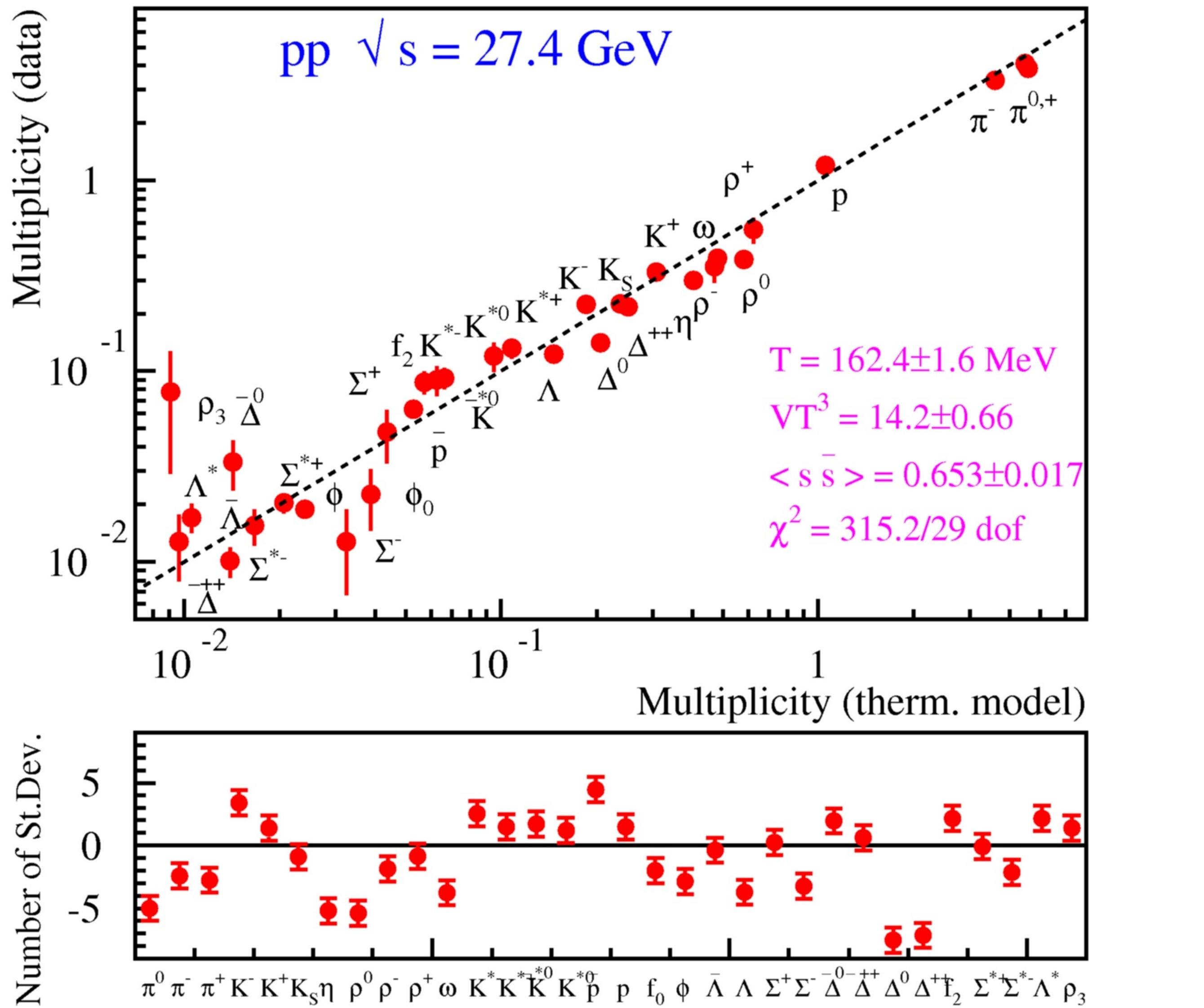
done in two steps: taking into account errors on masses, widths, BR's

iterating the fit (effective variance method)

F. B., P. Castorina, J. Manninen, H. Satz, arXiv:0805.0964, Eur. Phys. J. C in press



F. B., G. Passaleva, Eur. Phys. J. C 23 (2002) 551



NOTE: no systematic errors were quoted in the paper where most data comes from

Free parameters in PYTHIA

Specific
for light-flavoured
abundances

Parameter	Name	Default	Range gen.	Fit Result Val.	stat.	sys.
λ_{QCD}	PARJ(81)	0.4	0.25 - 0.35	0.297	± 0.004	$^{+0.007}_{-0.008}$
Q_0	PARJ(82)	1.0	1.0 - 2.0	1.56	± 0.11	$^{+0.21}_{-0.15}$
a	PARJ(41)	0.5	0.1 - 0.5	0.417	± 0.022	$^{+0.011}_{-0.015}$
b	PARJ(42)	0.9	0.850		optimized	
σ_q	PARJ(21)	0.35	0.36 - 0.44	0.408	± 0.005	$^{+0.004}_{-0.004}$
$P(^1S_0)_{ud}$	-	0.5	0.3 - 0.5	0.297	± 0.021	$^{+0.102}_{-0.011}$
$P(^3S_1)_{ud}$	-	0.5	0.2 - 0.4	0.289	± 0.038	$^{+0.004}_{-0.026}$
$P(^1P_1)_{ud}$	-	0.	see text		0.096	
$P(oth.P-states)_{ud}$	-	0.	see text		0.318	
γ_s	PARJ(2)	0.30	0.27 - 0.31	0.308	± 0.007	$^{+0.004}_{-0.036}$
$P(^1S_0)_s$	-	0.4	0.3 - 0.5	0.410	± 0.038	$^{+0.026}_{-0.013}$
$P(^3S_1)_s$	-	0.6	0.2 - 0.4	0.297	± 0.021	$^{+0.020}_{-0.004}$
$P(P-states)_s$	-	0.	see text		0.293	
ϵ_c	PARJ(54)	-	variable	-0.0372	± 0.0007	$^{+0.0011}_{-0.0012}$
$P(^1S_0)_c$	-	0.25	0.26			
$P(^3S_1)_c$	-	0.75	0.44		adj. to data	
$P(P-states)_c$	-	0.	0.3			
ϵ_b	PARJ(55)	-	variable	-0.00284	± 0.00005	$^{+0.00012}_{-0.00010}$
$P(^1S_0)_b$	-	0.25	0.175			
$P(^3S_1)_b$	-	0.75	0.525		adj. to data	
$P(P-states)_b$	-	0.	0.3			
$P(qq)/P(q)$	PARJ(1)	0.1	0.08 - 0.11	0.099	± 0.001	$^{+0.005}_{-0.002}$
$P(us)/P(ud)/\gamma_s$	PARJ(3)	0.4	0.65		adj. to data	
$P(ud1)/P(ud0)$	PARJ(4)	0.05	0.07		adj. to data	
extra baryon supp.	PARJ(19)	0.	0.5		adj. to data only uds	
extra η supp.	PARJ(25)	1.0	0.65		0.65 ± 0.06	
extra η' supp.	PARJ(26)	1.0	0.23		0.23 ± 0.05	

Table 49: Parameter setting and fit results for JETSET 7.4 PS with default decays

From DELPHI coll., *"The next round of....identified particles"*, hep-ex 9511011

What if using incorrect framework?

If this result was accidental, one would expect random χ^2 fluctuations

FIT TO LONG-LIVED PARTICLES AT 91.2 GeV

- **No resonances (with Γ > 9 MeV)**

χ^2 = 391/12

- **Resonances, but no exact conservation of B, S, Q**

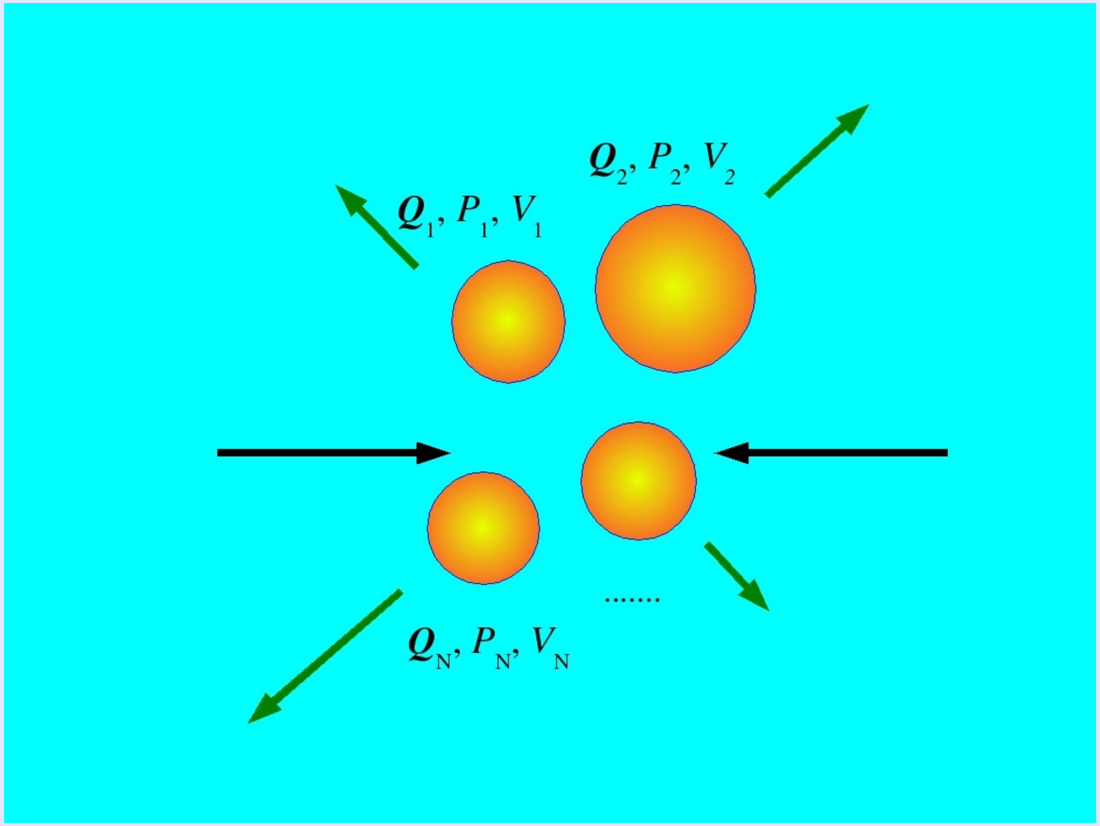
$$\langle n_j \rangle = \frac{(2S_j+1)V}{(2\pi)^3} {\gamma_S}^{N_s} \int d^3p \, \mathrm{e}^{-\sqrt{p^2+m_j^2}/T} \cancel{\frac{Z(\mathsf{Q}-\mathsf{q}_j)}{Z(\mathsf{Q})}}$$

χ^2 = 103/12

- **Correct implementation**

χ^2 = 39.3/12

Heavy flavoured hadron production



In $e^+e^- \to Q\bar{Q}$ where Q is a heavy quark, enforce two clusters with non-vanishing numbers of heavy quarks

Primary multiplicity of the jth heavy flavoured species is predicted to be

$$\langle n_j \rangle = \gamma_s^{N_{Sj}} z_j \, \frac{\sum_i \gamma_s^{N_{si}} z_i \zeta(\mathbf{Q} - \mathbf{q}_j - \mathbf{q}_i)}{\sum_{i,k} \gamma_s^{N_{si}} \gamma_s^{N_{sk}} z_i z_k \zeta(\mathbf{Q} - \mathbf{q}_i - \mathbf{q}_k)} \, ,$$

$$z_j = \frac{(2S_j+1)V}{(2\pi)^3} \int d^3p \; \mathrm{e}^{-\sqrt{p^2+m_j^2}/T}$$

Heavy flavoured hadron yields measured at √s = 91.2 GeV

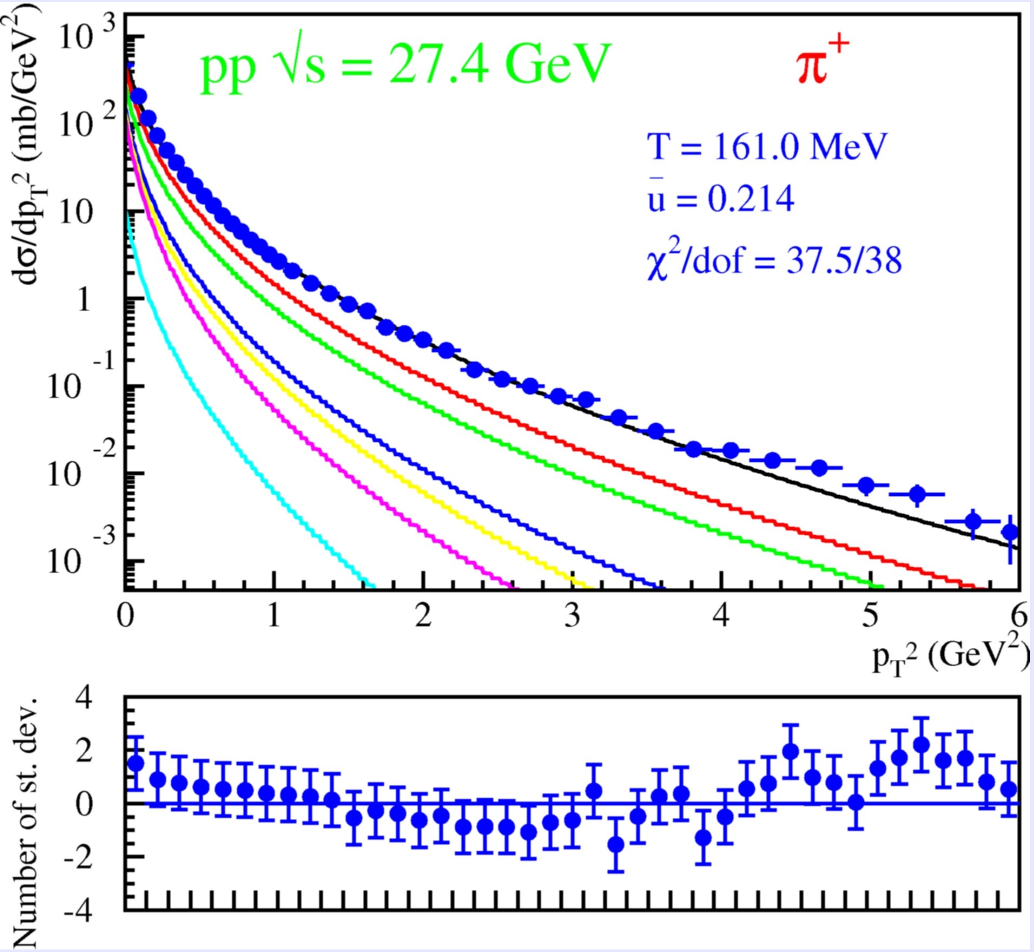
14

Particle		Experiment (E)	Model (M)	Residual	$(M-E)/E$ [%]
D^0	[28]	0.559 ± 0.022	0.5406	-0.83	-3.2
D^+	[28]	0.238 ± 0.024	0.2235	-0.60	-6.1
D^{*+}	[28–30]	0.2377 ± 0.0098	0.2279	-1.00	-4.1
D^{*0}	[31]	0.218 ± 0.071	0.2311	0.18	6.0
D_1^0	[32, 33]	0.0173 ± 0.0039	0.01830	0.26	5.8
D_2^{*0}	[32, 33]	0.0484 ± 0.0080	0.02489	-2.94	-48.6
D_s	[28]	0.116 ± 0.036	0.1162	0.006	0.19
D_s^*	[28]	0.069 ± 0.026	0.0674	-0.06	-2.4
D_{s1}	[33, 34]	0.0106 ± 0.0025	0.00575	-1.94	-45.7
D_{s2}^*	[34]	0.0140 ± 0.0062	0.00778	-1.00	-44.5
Λ_c	[28]	0.079 ± 0.022	0.0966	0.80	22.2
$(B^0+B^+)/2$	[35]	0.399 ± 0.011	0.3971	-0.18	-0.49
B_s	[35]	0.098 ± 0.012	0.1084	0.87	10.6
$B^*/B(\mathrm{uds})$	[36–39]	0.749 ± 0.040	0.6943	-1.37	-7.3
$B^{**} \times BR(B(^*)\pi)$	[40–42]	0.180 ± 0.025	0.1319	-1.92	-26.7
$(B_2^* + B_1) \times BR(B(^*)\pi)$	[41]	0.090 ± 0.018	0.0800	-0.57	-11.4
$B_{s2}^* \times BR(BK)$	[41]	0.0093 ± 0.0024	0.00631	-1.24	-32.1
b-baryon	[35]	0.103 ± 0.018	0.09751	-0.30	-5.3
Ξ_b^-	[35]	0.011 ± 0.006	0.00944	-0.26	-14.2

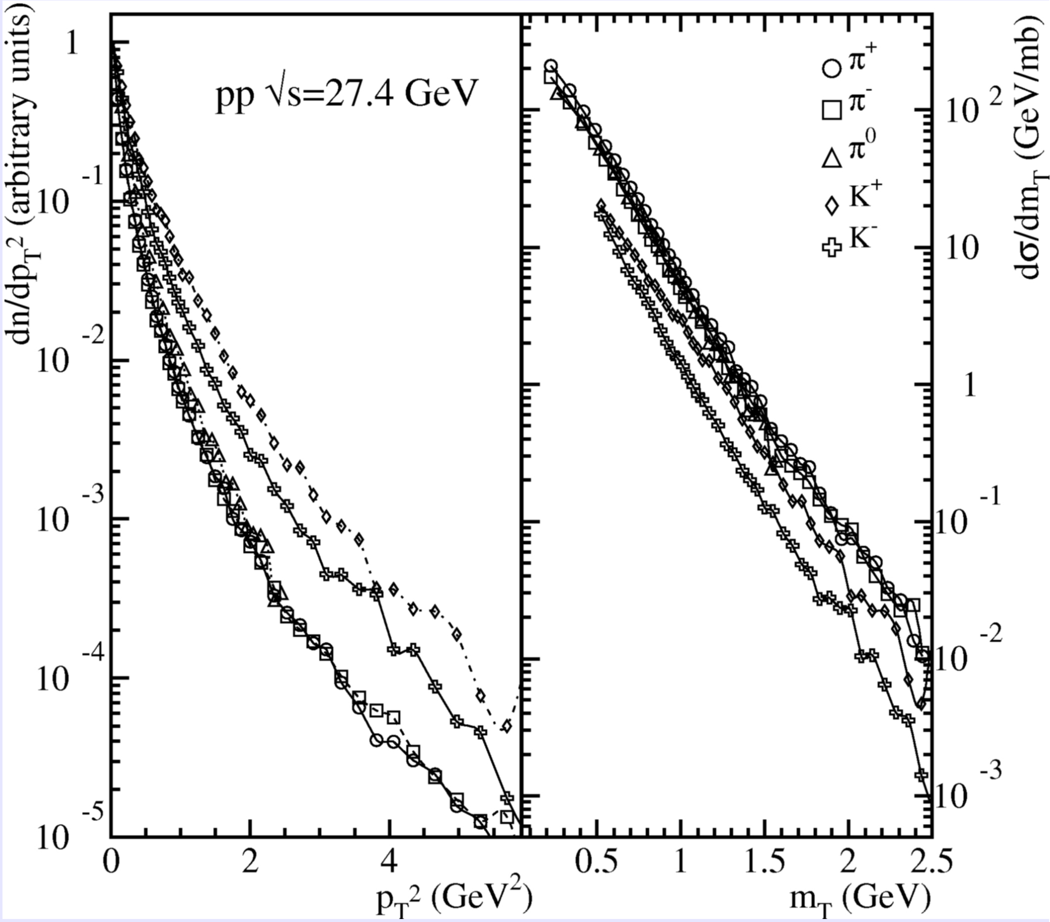
χ^2/dof = 24.6/19 *WITHOUT ANY NEW FREE PARAMETER*

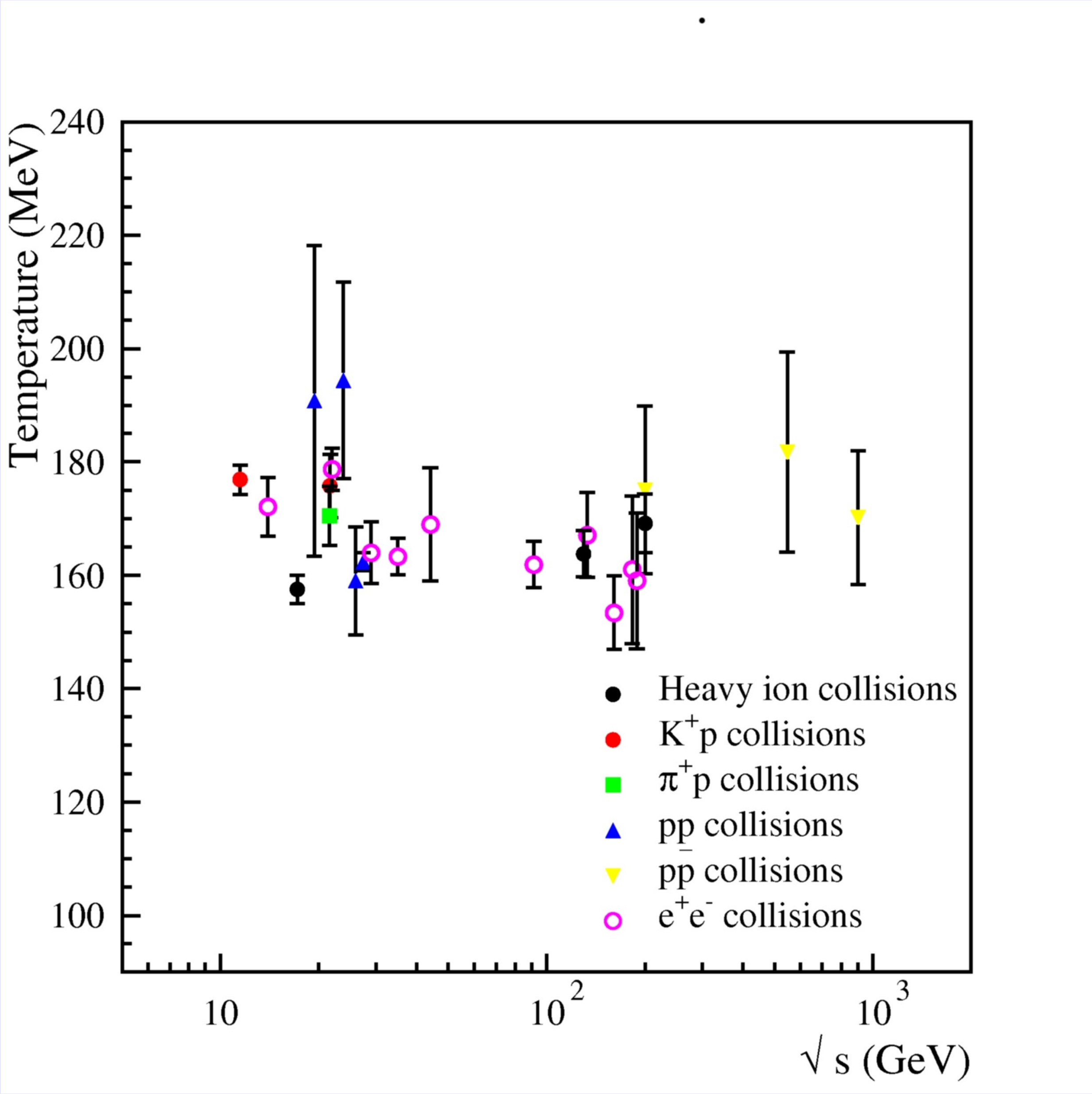
F. B., P. Castorina, J. Manninen, H. Satz, arXiv:0805.0964, Eur. Phys. J. C in press

Transverse momentum spectra in hadronic collisions



F. B., G. Passaleva, Eur. Phys. J. C 23 (2002) 551





Critical energy density at the hadronization ?

Moltepliicità per le specie adroniche			
Adroni	Misura	Cariche libere	Cariche fissate
π^0	9.61 ± 0.29	10.27 ± 0.01	10.42 ± 0.04
$\pi^\pm$	8.50 ± 0.10	9.08 ± 0.01	8.81 ± 0.03
$K^\pm$	1.127 ± 0.026	1.098 ± 0.003	1.16 ± 0.01
K^0_S	1.0376 ± 0.0096	0.999 ± 0.003	1.09 ± 0.01
η	1.059 ± 0.086	0.993 ± 0.003	1.14 ± 0.01
ρ^0	1.40 ± 0.13	1.220 ± 0.004	1.25 ± 0.01
$\rho^\pm$	1.20 ± 0.22	1.145 ± 0.003	1.06 ± 0.01
ω	1.024 ± 0.059	0.993 ± 0.003	0.97 ± 0.01
K^{*0}	0.357 ± 0.022	0.35 ± 0.002	0.352 ± 0.006
$K^{*\pm}$	0.370 ± 0.012	0.359 ± 0.002	0.0356 ± 0.006
η'	0.166 ± 0.047	0.095 ± 0.001	0.107 ± 0.003
ϕ	0.0977 ± 0.0058	0.108 ± 0.001	0.205 ± 0.005
$f_2(1270)$	0.188 ± 0.020	0.124 ± 0.001	0.127 ± 0.003
K^*_2	0.036 ± 0.011	0.0217 ± 0.0005	0.020 ± 0.001
$f'_2(1525)$	0.0120 ± 0.0058	0.0088 ± 0.0003	0.019 ± 0.001
p	0.519 ± 0.018	0.543 ± 0.002	0.506 ± 0.006
$a_0(980)$	0.135 ± 0.054	0.0890 ± 0.0009	0.071 ± 0.03
f_0	0.1555 ± 0.0085	0.0663 ± 0.0008	0.067 ± 0.03
Λ	0.1943 ± 0.0038	0.196 ± 0.001	0.177 ± 0.005
Σ^+	0.0535 ± 0.0052	0.0473 ± 0.0007	0.037 ± 0.002
Σ^0	0.0389 ± 0.0041	0.0473 ± 0.0007	0.043 ± 0.002
Σ^-	0.0410 ± 0.0037	0.0436 ± 0.0004	0.046 ± 0.002
$\Sigma^\pm$	0.0868 ± 0.0087	0.091 ± 0.001	0.082 ± 0.004
Δ^{++}	0.044 ± 0.016	0.0846 ± 0.0007	0.059 ± 0.002
Ξ^-	0.01319 ± 0.00052	0.0123 ± 0.0003	0.009 ± 0.001
Σ^{*+}	0.0118 ± 0.0011	0.0206 ± 0.0005	0.015 ± 0.001
$\Lambda(1520)$	0.0112 ± 0.0014	0.0109 ± 0.0003	0.010 ± 0.001
Ξ^{*0}	0.00289 ± 0.00050	0.0041 ± 0.0002	0.0028 ± 0.0005
Ω^-	0.00062 ± 0.00012	0.00088 ± 0.00009	0.0010 ± 0.0003

Releasing the one-cluster reduction hypothesis: implement SHM in a Monte-Carlo code

- Parton shower
- HERWIG cluster formation
- Microcanonical decay of each cluster

Same quality of the agreement data-model

T. Gabbriellini, Diploma thesis, Universita' di Firenze

Ongoing project:

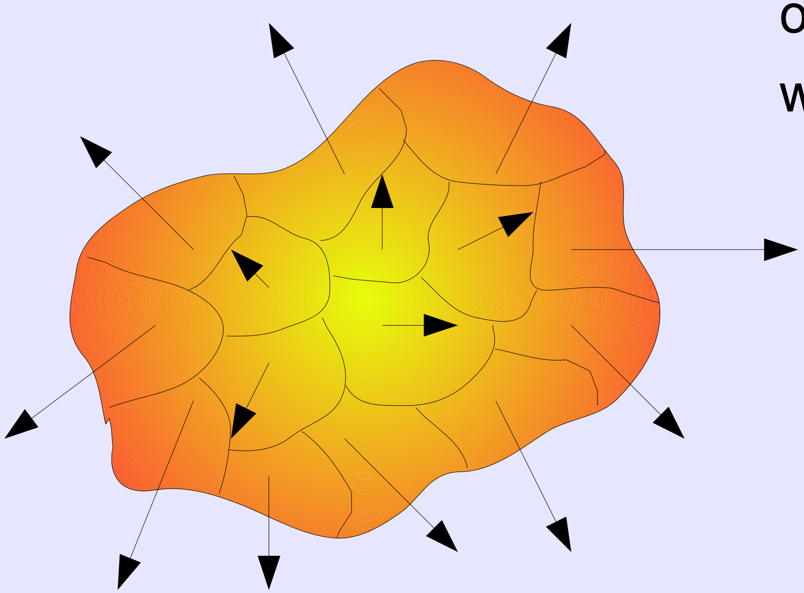
F. B., C. Bignamini, L. Ferroni, F. Piccinini

Heavy ion collisions

In heavy ion collisions:

- either clusters are individually large
- or they are hydrodynamical cells in contact with each other and the system is large overall

In both cases, a grand-canonical formalism is allowed



Hydrodynamical approach requires
a strong correlation between momentum
of clusters (velocity) and their position,
which is not the case in elementary collisions

If the hypothesis of reduction to one cluster (EGC) holds in heavy ion collisions, then:

$$\langle n_j \rangle_{\rm primary} = \frac{(2S+1)V}{(2\pi)^3} {\gamma_S}^{N_s} \int d^3p \; {\rm e}^{-\sqrt{p^2+m_j^2}/T} \; {\rm e}^{\boldsymbol{\mu}\cdot \mathsf{q}_j/T}$$

where V is the global volume sum of the proper volumes of single clusters or cells.

If we aim at determining more local quantities, then

$$\frac{d\langle n_j \rangle}{d^3x}_{\rm primary} = \frac{(2S_j+1)}{(2\pi)^3} \gamma_S(\mathsf{x})^{N_{sj}} \int d^3p \; {\rm e}^{-\beta(\mathsf{x})\cdot p} \; {\rm e}^{\boldsymbol{\mu}(\mathsf{x})\cdot \mathsf{q}_j/T(\mathsf{x})}$$

Yet, we can make a selection only in momentum space, not in space

Midrapidity densities

- Assume that thermodynamical freeze-out parameters only depend on rapidity. Then calculate the rapidity density at midrapidity by integrating over all contributing clusters (or cells)

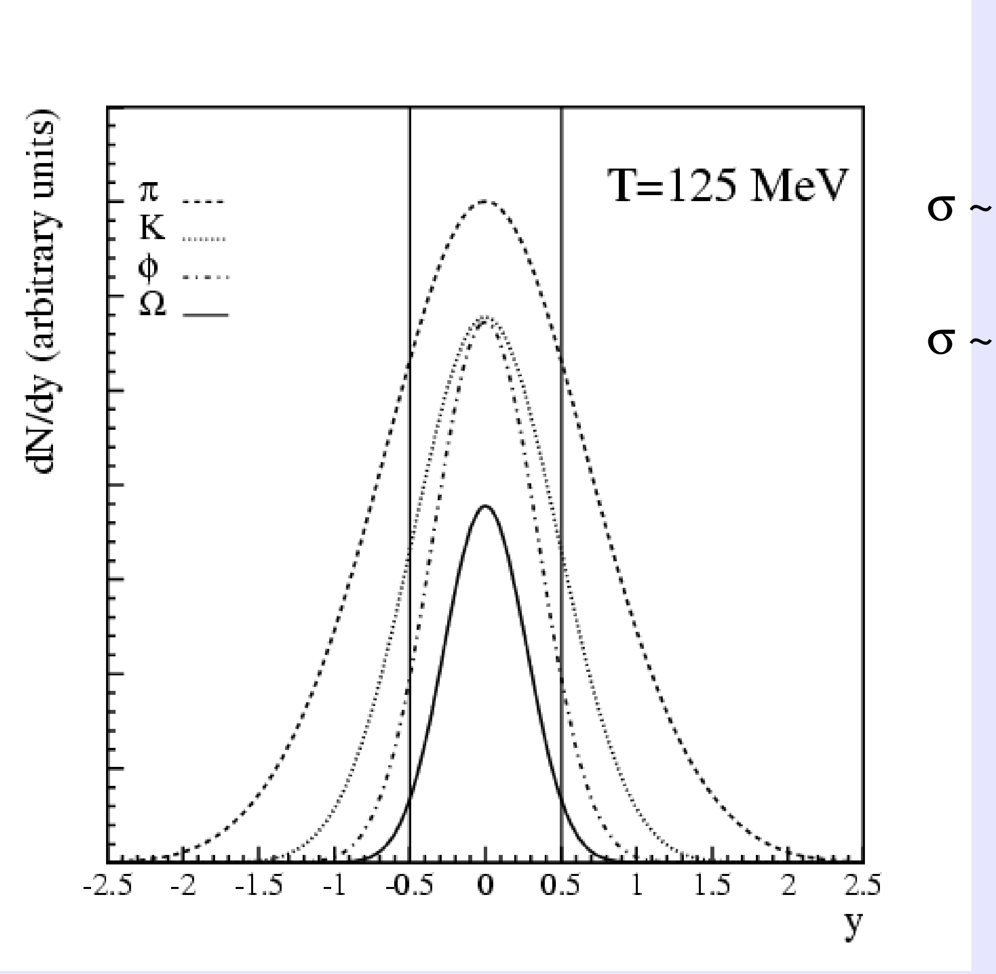
$$
\begin{aligned}
\frac{dN}{dy} &= \int_{-\infty}^{+\infty} dY\ \rho(Y)\, \frac{dn}{dy}\Big|_{\mathrm{th}} (\mu(Y), T(Y), y - Y) \\
&= \int_{-\infty}^{+\infty} dy'\ \rho(y - y')\, \frac{dn}{dy}\Big|_{\mathrm{th}} (\mu(y - y'), T(y - y'), y') \\
&\approx \rho(y) \int_{-\infty}^{+\infty} dy'\ \frac{dn}{dy}\Big|_{\mathrm{th}} (\mu(y), T(y), y') = \rho(y)\, n\Big|_{\mathrm{th}} (T(y), \mu(y))
\end{aligned}
$$

Requires the overall rapidity distribution width to be significantly larger than thermal width

$\sigma_{\mathrm{exp}} \gg \sigma_{\mathrm{fireball}}$

WARNING!

- Midrapidity densities yield exact properties of the central fireball only if there is a sufficiently large window of invariance around midrapidity (RHIC – LHC)
- If the rapidity distribution is not wide enough, the cut may artificially enhance heavier (i.e. strange) hadrons.



Rapidity widths of pions:

$\sigma \sim 1.3$ at top SPS $\sigma \sim 0.8$ for a fireball T = 125 MeV

$\sigma \sim 2.1$ at RHIC 200

EXAMPLE: fit to NA49 data at top SPS

Midrap. yields $\gamma_s = 0.95 \pm 0.06$ $\chi^2 = 37.2/8$

4π yields $\gamma_s = 0.81 \pm 0.04$ $\chi^2 = 18.8/9$

Why do we observe such thermal features?

L. McLerran, "Lectures on RHIC physics", hep-ph 0311028

This would appear to be a compelling case for the production of a Quark Gluon Plasma. The problem is that the fits done for heavy ions to particle abundances work even better in e^+e^- collisions. One definitely expects no Quark Gluon Plasma in e^+e^- collisions. There is a deep theoretical question to be understood here: How can thermal models work so well for non-thermal systems? Is there some simple saturation of phase space?

"Phase space dominance": J.Hormuzdiar, S. D. H. Hsu, G.Mahlon, Int. J. Mod. Phys. E 12, 649 (2003)

Can we derive / justify this behaviour from QCD?

In the statistical model there is essentially one scale, which is the energy density at which hadronization takes place: all the rest is conservation laws (except for γ_S, which is probably associated to the m_s).

What is the main difference between elementary and heavy ion collisions as far as thermal features are concerned?

VOLUME

Clusters decaying statistically in elementary collisions are hadronic-sized

HIC: thermalization of strongly interacting system over large region leads to collective phenomena involving correlations between position and energy like pressure, flow ecc.

Still, the ultimate reason for the apparent thermal behaviour in a rapidly evolving system might be the same
(as long as QCD is the underlying driving force)

Quantum thermalization of classically chaotic systems

M. Srednicki, Phys. Rev. E 50 (1994) 888

Based on the validity of Berry's conjecture: the high-lying energy eigenfunctions of a classically chaotic and ergodic system are Gaussian random numbers

Srednicki considers a classical gas of hard spheres (= classically chaotic, ergodic system) and shows that, provided Berry's conjecture is true, the bounded quantum gas of hard spheres must exhibit a thermal single-particle spectrum in a PURE QUANTUM STATE

Even more intriguingly, he shows that, starting from a wave function which does not have thermal spectrum, the system will become thermal within a time of the order of $1/\Delta$ where Δ is the spread in initial energy

A. Krzywicki (hep-ph 0204411) advocates this as the underlying mechanism for the apparent thermal features in hadroproduction in high energy collisions

Berry's conjecture (1)

The high-lying eigenfunctions of a quantum system whose classical counterpart is chaotic and ergodic appear to be random Gaussian numbers $\psi_\alpha(\mathsf{x})$ in configuration space, with a two-point correlation function given by

$$\langle \psi_\alpha(\mathsf{x}+\frac{\mathsf{s}}{2})\psi^*_\beta(\mathsf{x}-\frac{\mathsf{s}}{2})\rangle_{EE} = \delta_{\alpha\beta}\frac{\int d^3p\,\mathrm{e}^{i\mathsf{p}\cdot\mathsf{s}}\,\delta(E_\alpha - H(\mathsf{x},\mathsf{p}))}{\int d^3p\,d^3x\,\delta(E_\alpha - H(\mathsf{x},\mathsf{p}))}$$

EE stands for eigenstate ensemble. A fictitious ensemble whose meaning is: the typical eigenfunction behaves as if it was drawn at random from the eigenstate ensemble

The best intuitive example of such a behaviour is the superposition of many plane waves with random phases: because of the central limit theorem, the amplitudes $\psi_\alpha(\mathsf{x})$ are normally distributed

Berry's conjecture (2)

Berry's conjecture has been found to be valid numerically for many systems and it has even been suggested as a good definition of Quantum Chaos

$$\langle W_\alpha(\mathsf{x},\mathsf{p})\rangle_{EE} = \langle \frac{1}{(2\pi)^3}\int d^3s\, \mathrm{e}^{-i\mathsf{p}\cdot\mathsf{s}}\,\psi_\alpha(\mathsf{x}+\frac{\mathsf{s}}{2})\psi^*_\alpha(\mathsf{x}-\frac{\mathsf{s}}{2})\rangle_{EE} = \frac{\delta(E_\alpha - H(\mathsf{x},\mathsf{p}))}{\int d^3p\, d^3x\ \delta(E_\alpha - H(\mathsf{x},\mathsf{p}))}$$

The “typical” Wigner function becomes the statistical microcanonical distribution

It can be easily shown that, given the assumed two-point correlation function and if volume is large, the probability of a momentum state is microcanonical:

$$\langle \rho(\mathsf{p})\rangle_{EE} = \frac{\int d^3x\ \delta(E_\alpha - H(\mathsf{x},\mathsf{p}))}{\int d^3p\, d^3x\ \delta(E_\alpha - H(\mathsf{x},\mathsf{p}))}$$

QUESTIONS

Is classical QCD chaotic?

(YES: T. Biro, C. Gong, B. Muller, H. Markum, R. Pullirsch)

Does Berry's conjecture apply to quantum fields?

Can we extend Srednicki's reasoning to dynamical hadron production?

FINAL THOUGHTS

The statistical-thermal features observed in elementary as well as heavy ion collisions cannot be "accidental"

I personally believe that the reason of this behaviour is common to all collisions and must be quantum mechanical in its origin: *"hadrons are born at equilibrium"* (R. Hagedorn, 1970)

- Quantum chaos?
- Hawking-Unruh radiation?
- ...

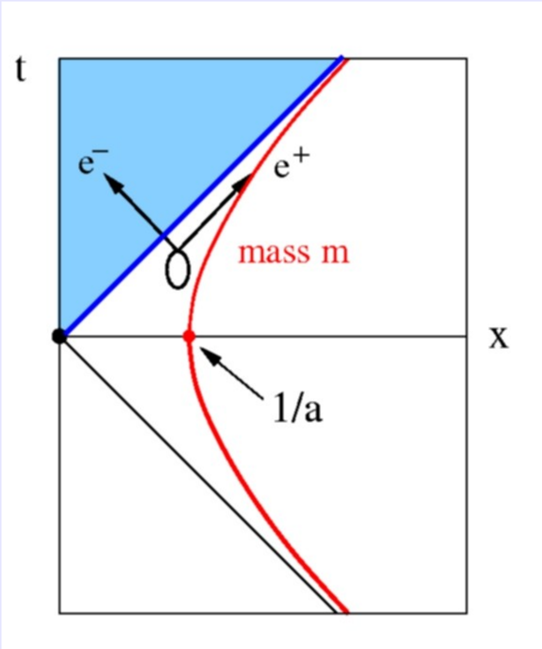
"Ai posteri l'ardua sentenza"

Hadronization as Hawking-Unruh radiation

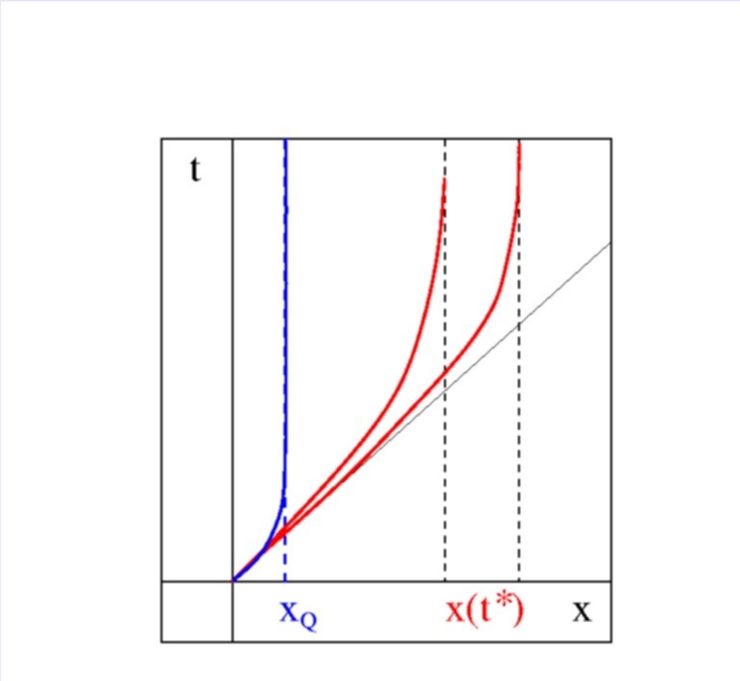
P. Castorina, D. Kharzeev, H. Satz, Eur. Phys. J. C52 (2007) 187

IDEA: analogy between black holes and QCD confinement, which is effectively seen as the formation of an event horizon for coloured signals

Hawking radiation from event horizon can only be thermal because it cannot convey any information



Unruh radiation: $T=\hbar a/2\pi c$



Event horizon in qq production

Possible deep relation between pair production in a strong field (QED = Schwinger effect) and Unruh radiation

$$R_{pair} \sim \exp[-\pi m^2/eE] = \exp[-m/T_{Unruh}]$$

$$T_{Unruh} = \frac{a}{2\pi} = \frac{2eE}{m}$$

<u>New paradigm:</u> the acceleration involved in pair-produced particles in the colour field determines the temperature of the emitted radiation.

For the colour field, inspiring of the above formula:

$$a_q = \frac{\sigma}{\sqrt{P^2}/2} = \frac{\sigma}{\sqrt{m_q^2 + k_q^2}} = \frac{\sigma}{\sqrt{m_q^2 + \frac{\sigma^2}{4m_q^2+2\pi\sigma}}} \equiv \frac{\sigma}{w_q}$$

Massless quarks

$$T = \frac{a}{2\pi} = \sqrt{\frac{\sigma}{2\pi}}$$

Massive quarks in a hadron: averaging the accelerations

$$\bar{a} = \frac{w_1 a_1 + w_2 a_2}{w_1 + w_2}$$

Temperature of hadron radiation becomes dependent on the quark masses

$\sigma = 0.2\ \mathrm{GeV}^2$

T	$m_s = 0.075$	$m_s = 0.100$	$m_s = 0.125$
$T(00)$	0.178	0.178	0.178
$T(0s)$	0.172	0.167	0.162
$T(ss)$	0.166	0.157	0.148
$T(000)$	0.178	0.178	0.178
$T(00s)$	0.174	0.171	0.167
$T(0ss)$	0.170	0.164	0.157
$T(sss)$	0.166	0.157	0.148

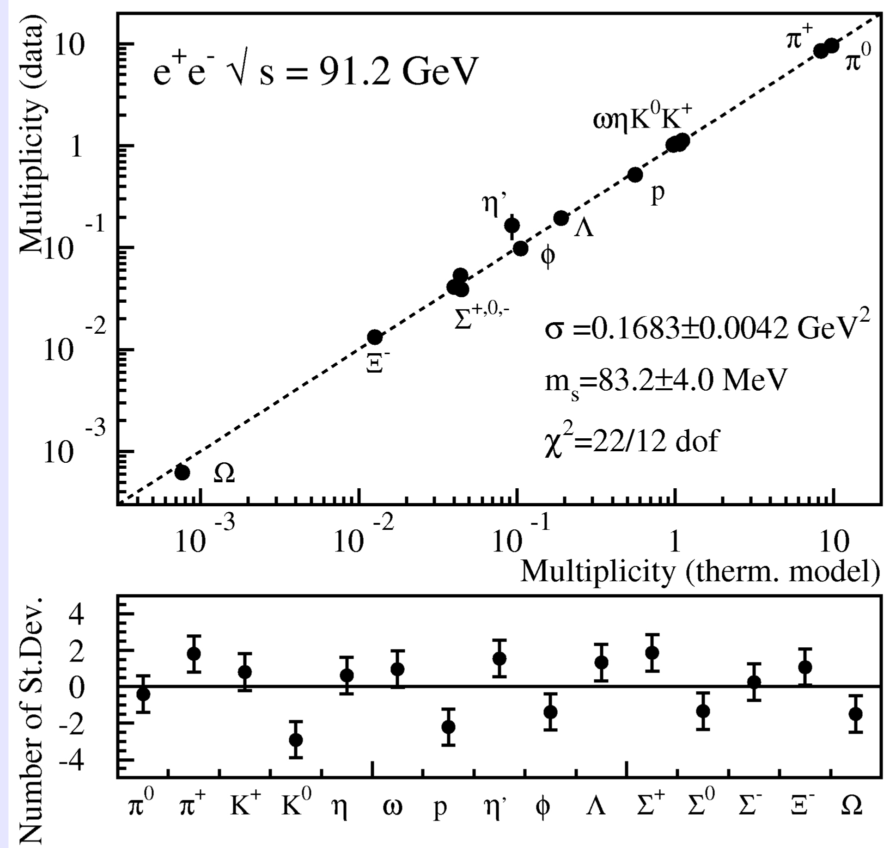
Analysis of the whole set of available data in e+e- collisions

F. B., P. Castorina, J. Manninen, H. Satz, arXiv:0805.0964 Eur. Phys. J. C in press

Primary mult's in Boltzmann approx.

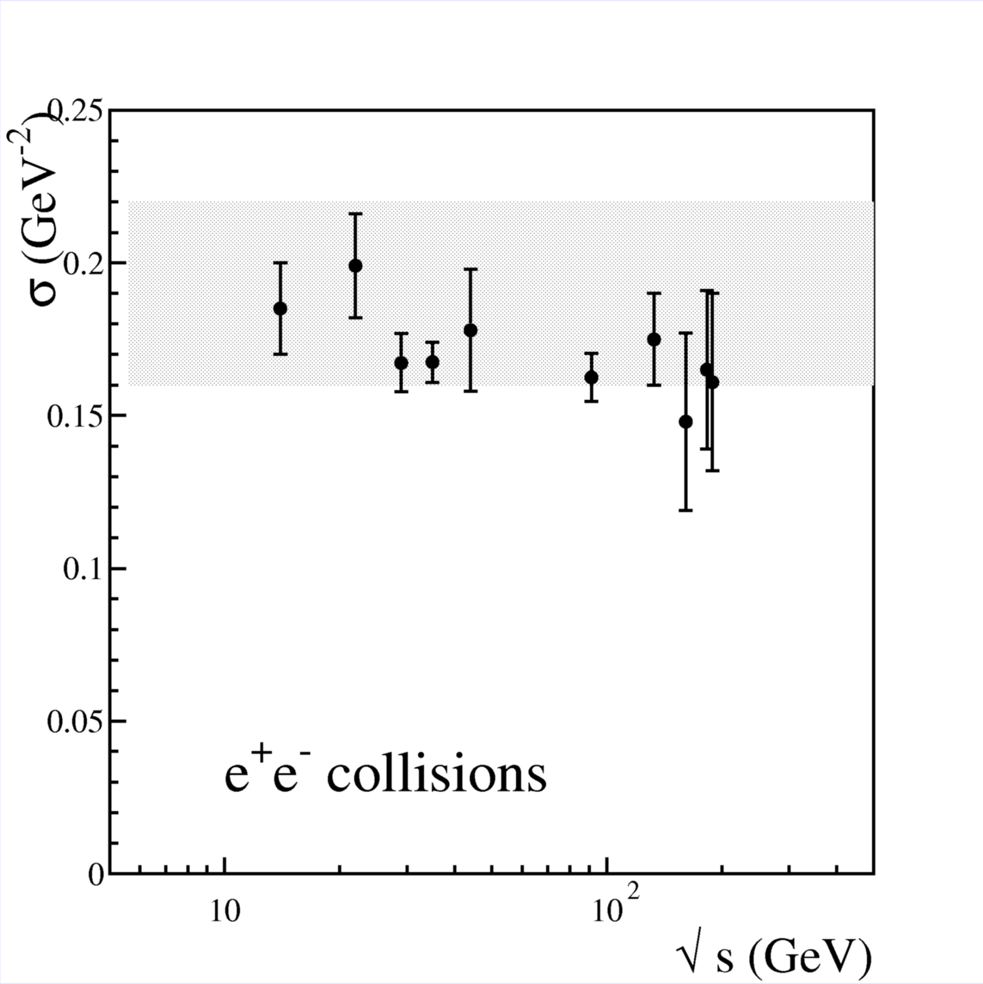
$$\langle n_j \rangle = \frac{(2S+1)V}{(2\pi)^3} \int d^3p \, e^{-\sqrt{p^2+m_j^2}/T(j)} \frac{Z(\mathbf{Q} - \mathbf{q}_j)}{Z(\mathbf{Q})} \qquad \begin{array}{l} \mathbf{Q} = (Q, B, S, ..) \\ \mathbf{q}_j = (Q_j, B_j, S_j, ...) \end{array}$$

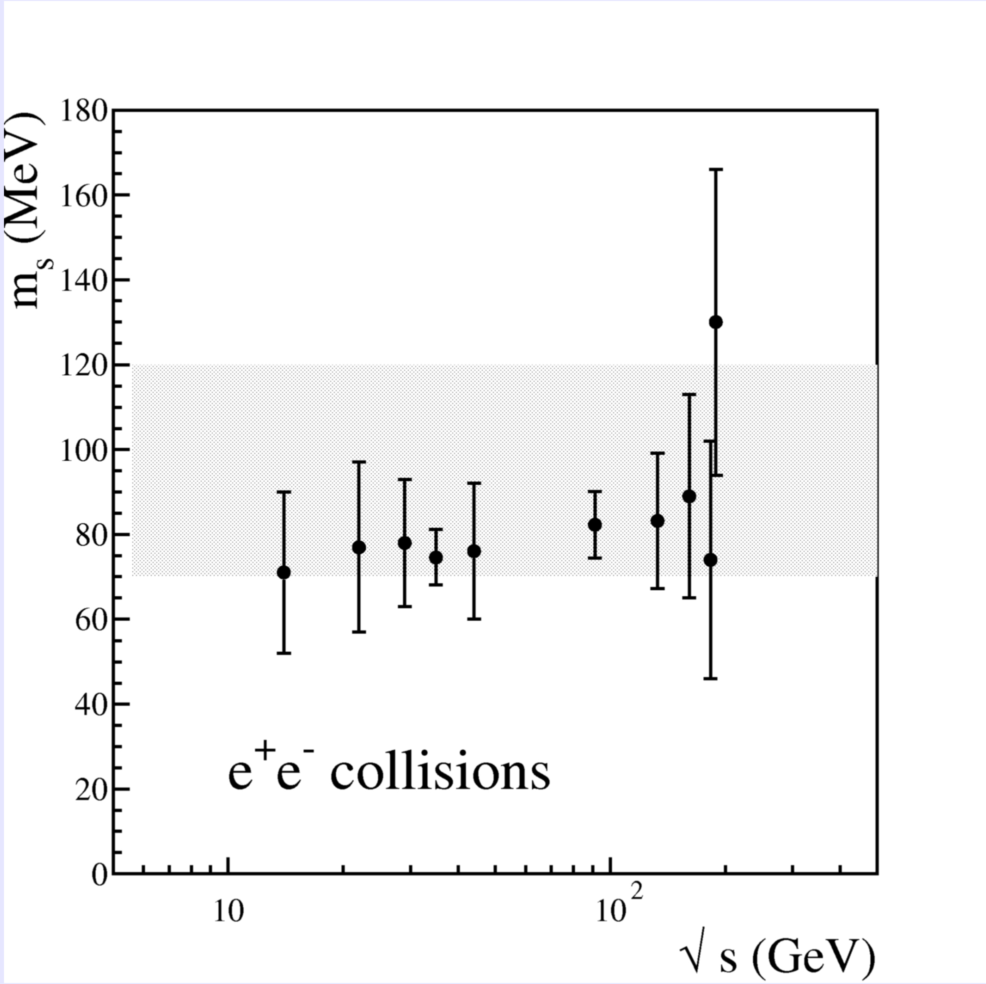
$\sqrt{s}$	$R_u + R_d$	R_s	R_c	R_b
14	0.46	0.09	0.37	0.08
22	0.46	0.09	0.36	0.09
29	0.46	0.09	0.36	0.09
35	0.46	0.09	0.36	0.09
43	0.46	0.09	0.36	0.09
91.25	0.39	0.22	0.17	0.22
133	0.41	0.18	0.23	0.18
161	0.42	0.17	0.24	0.17
183	0.42	0.16	0.26	0.16
189	0.42	0.16	0.26	0.16



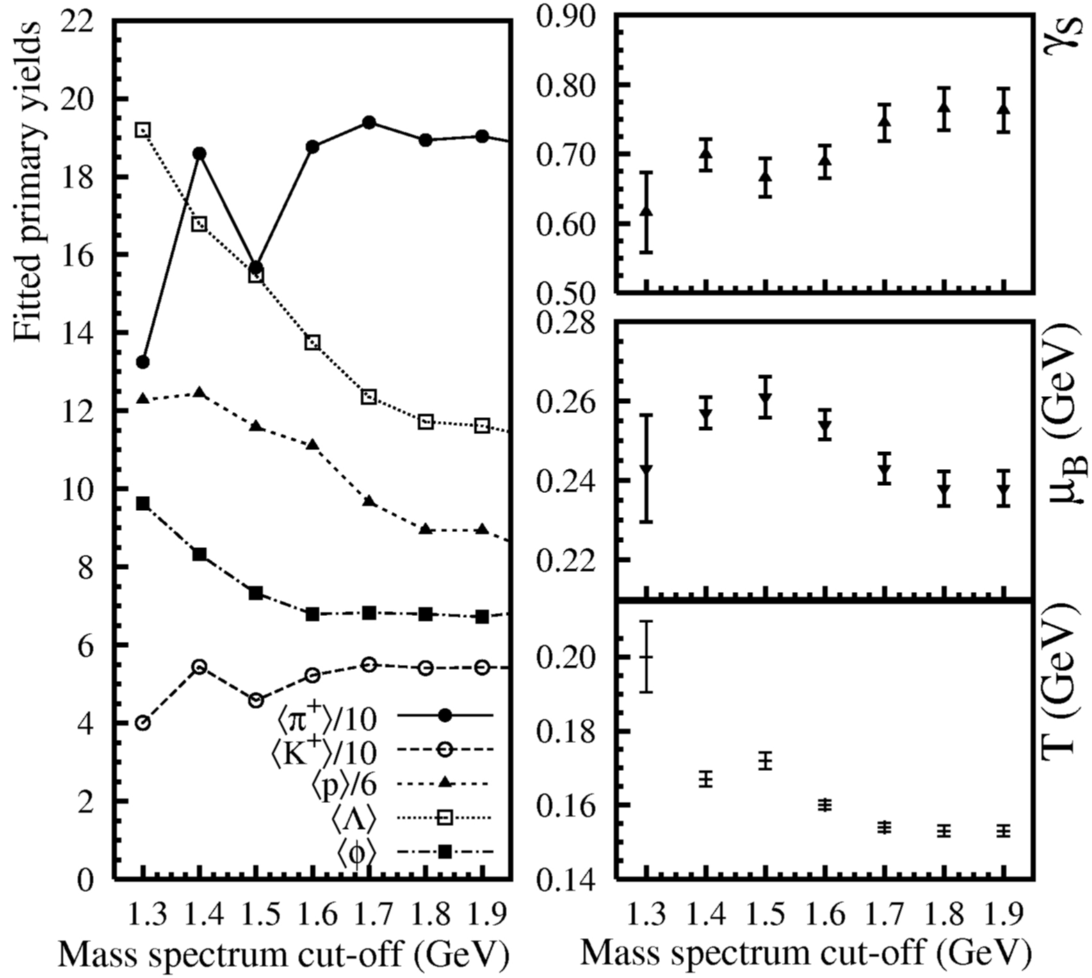
Results

Grey bands: present world best estimates





SPARE SLIDES



F. B., M. Gazdzicki, A. Keranen, J. Manninen, R. Stock, Phys. Rev. C 69 (2004) 024905

Comparison with other models

From DELPHI coll., *"The next round of....identified particles"*, hep-ex 9511011

	JETSET 7.3 PS	JETSET 7.4 PS	ARIADNE 4.06	JETSET 7.4 ME	HERWIG 5.8 C	LEP[44],[46],[47],[49]	Stat. mod.
Charged Particles							
$< N_{ch} >$	20.87	20.81	20.80	20.86	20.94	20.95 $\pm$ 0.21	
Pseudoscalar Mesons							
$\pi^{\pm}$	17.19	17.09	17.13	17.36	17.66	17.1 $\pm$ 0.4	16.93
π^0	9.85	9.83	9.82	10.03	9.81	9.9 $\pm$ 0.08	9.88
$K^{\pm}$	2.20	2.23	2.19	2.15	2.11	2.42 $\pm$ 0.13	2.29
K^0	2.13	2.17	2.12	2.10	2.08	2.12 $\pm$ 0.06	2.20
η	1.07	1.10	1.09	1.16	1.02	0.73 $\pm$ 0.07	1.00
η'(958)	0.10	0.09	0.10	0.10	0.14	0.17 $\pm$ 0.05	0.104
D^+	0.19	0.20	0.20	0.20	0.24	0.20 $\pm$ 0.03	
D^0	0.46	0.49	0.48	0.49	0.53	0.40 $\pm$ 0.06	
$B^{\pm}$,B^0	0.36	0.36	0.36	0.36	0.36	0.34 $\pm$ 0.06	
Scalar Mesons							
$f_0(980)$	0.17	0.16	0.17	0.16		0.14 $\pm$ 0.06	0.069
Vector Mesons							
$\rho^\circ(770)$	1.29	1.27	1.26	1.29	1.43	1.40 $\pm$ 0.1	1.10
$K^{*\pm}(892)$	0.78	0.77	0.79	0.77	0.74	0.78 $\pm$ 0.08	0.706
$K^{*0}(892)$	0.80	0.77	0.81	0.78	0.74	0.77 $\pm$ 0.09	0.691
$\phi(1020)$	0.109	0.107	0.107	0.102	0.099	0.086 $\pm$ 0.018	0.127
$D^{*\pm}(2010)$	0.18	0.22	0.19	0.22	0.22	0.17 $\pm$ 0.02	
$D^{*0}(2007)$	0.20	0.22	0.20	0.22	0.23	.	
Tensor Mesons							
$f_2(1270)$	0.29	0.29	0.29	0.30	0.26	0.31 $\pm$ 0.12	0.0972
Baryons							
p	0.97	0.97	0.96	0.90	0.78	0.92 $\pm$ 0.11	0.898
Λ^0	0.361	0.349	0.365	0.309	0.368	0.348 $\pm$ 0.013	0.308
Ξ^-	0.0288	0.0300	0.0300	0.0256	0.0493	0.0238 $\pm$ 0.0024	0.0227
$\Delta^{++}(1232)$	0.158	0.160	0.136	0.158	0.154	0.077 $\pm$ 0.018	0.135
$\Sigma^{\pm}(1385)$	0.037	0.036	0.032	0.033	0.065	0.0380 $\pm$ 0.0062	0.0667
$\Xi^0(1530)$	0.0073	0.0069	0.0063	0.0060	0.0249	0.0063 $\pm$ 0.0014	0.00768
Ω^-	0.0013	0.0019	0.0021	0.0010	0.0077	0.0051 $\pm$ 0.0013	0.00167
Λ_b^0	0.032	0.033	0.032	0.029	0.007	0.031 $\pm$ 0.016	

Table 8: The Production Rates for the different Generators compared to LEP data

χ^2 = 71.8/5

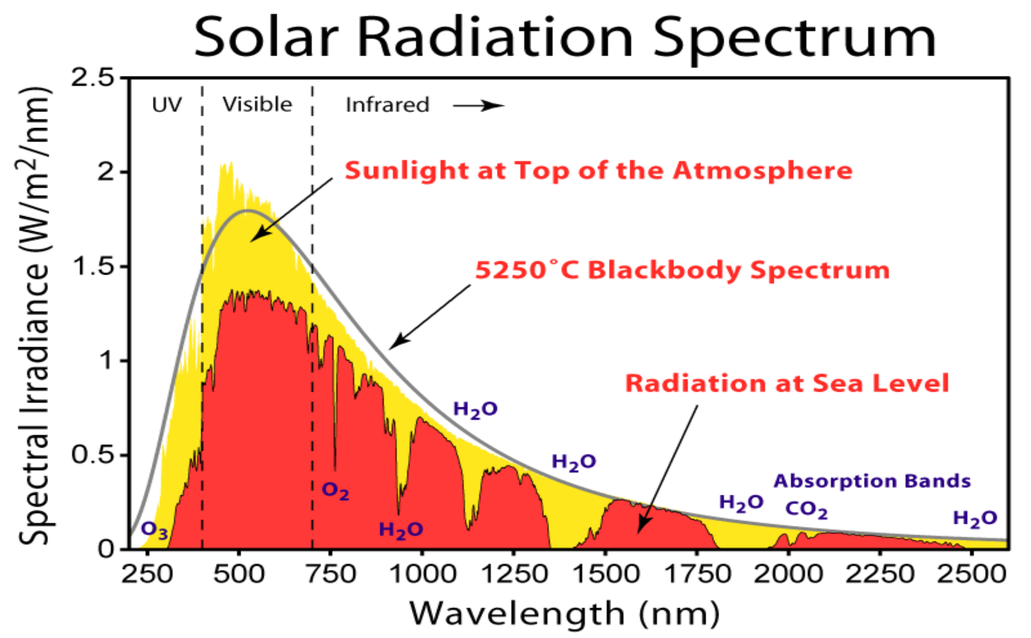
χ^2 = 90/16

The meaning of χ^2

In general: χ^2 test is used to reject a hypothesis.
This hypothesis is not “the model is good” but “the formula we are using is good”

EXAMPLE: if we were to fit electroweak measurements with tree-level Standard Model formulae, we would get a very bad χ^2 but this would not mean Standard Model is not good. Fit quality is excellent if we include radiative corrections (this is how top mass was predicted)

2nd EXAMPLE: temperature of the sun surface is inferred from a black-body fit to the light spectrum measured on top atmosphere. The fit is horrible, because the sun is not a perfect blackbody (absorption lines in the corona etc.).

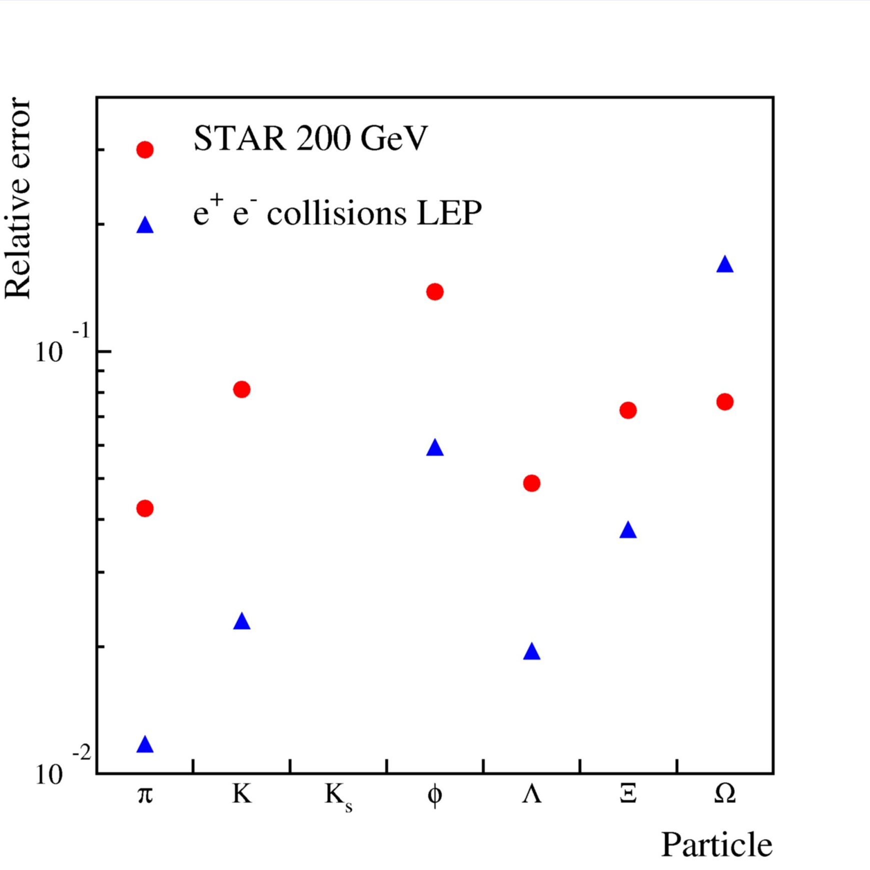


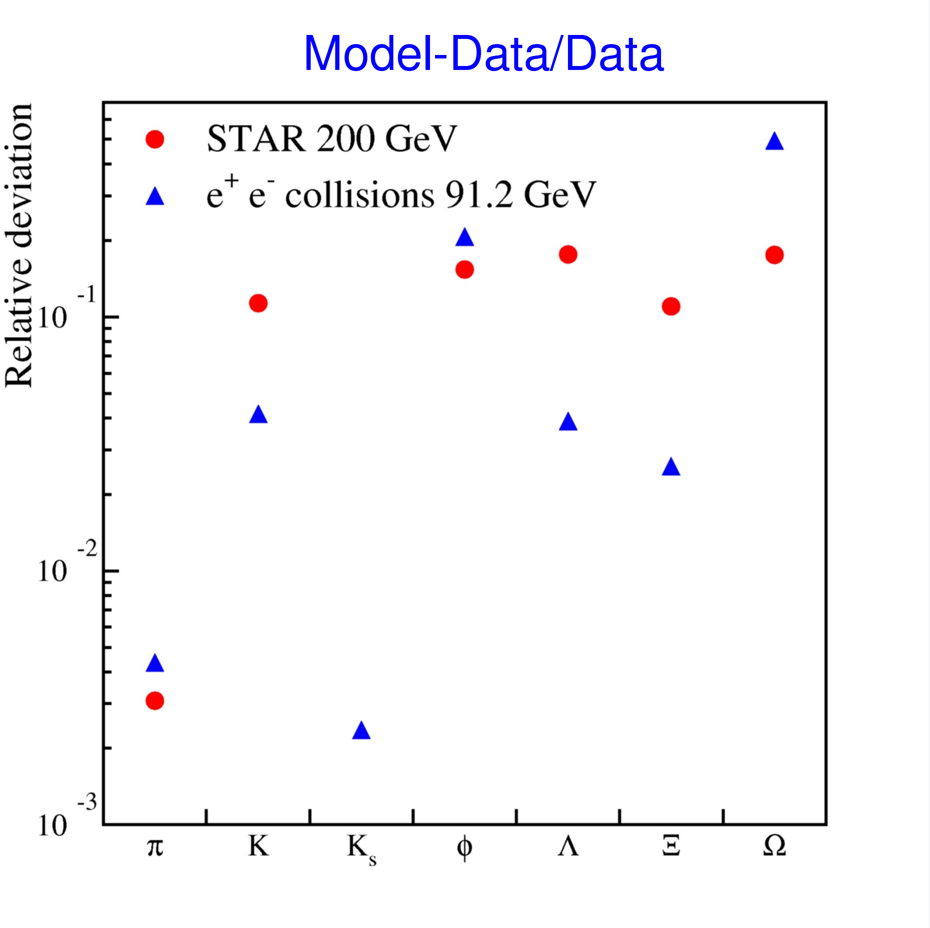
Comparison with heavy ion collisions at RHIC 200

Comparison of fits to 12 long-lived particles with the same model:

$\pi^+, \pi^-, K^+, K^-, \phi, p, \bar{p}, \Lambda, \bar{\Lambda}, \Xi^-, \bar{\Xi}^+, \Omega + \bar{\Omega}$ $\chi^2=$ [illegible] 1 more free parameter

$\pi^+, \pi^0, K^+, K^0, \phi, p, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^-, \Omega$ $\chi^2=$ [illegible]





$$\langle n_j \rangle = \frac{(2S+1)V}{(2\pi)^3} {\gamma_S}^{N_s} \int d^3p \, \mathrm{e}^{-\sqrt{p^2+m_j^2}/T} \, \frac{Z(\mathbf{Q} - \mathbf{q}_j)}{Z(\mathbf{Q})}$$

This formula used to fit multiplicities is a “tree-level” one relying on several simplifying assumptions:

- Maximal disorder in mass-charge distribution (=existence of EGC, needed to fit one T and one V)

- The use of the hadron-resonance gas model, neglecting non-resonant interaction

- Within the resonant approximation:
  - neglecting interference terms (non-symmetric diagrams in S-matrix decomposition)
  - assuming validity of DMB theorem at finite V
  - assuming validity of DMB formulae for specific channels besides full trace
  - assuming complete knowledge of the resonance spectrum up to a cut-off mass