How well can we describe the quark-gluon plasma?

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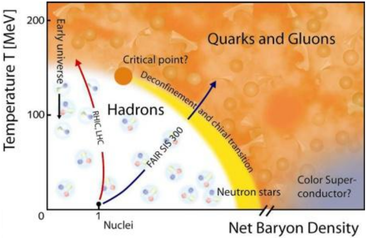
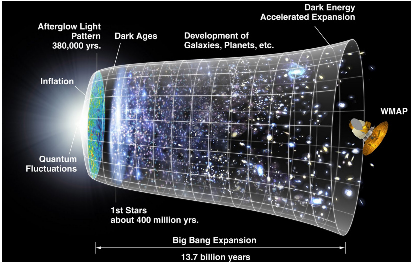
Torino, March 27, 2015

- quantum chromodynamics (QCD) - fundamental theory of strong interactions
  $\Rightarrow$ hadrons are bound states of quarks and gluons
- color confinement - at lower energies quarks form color neutral hadrons
  $\Rightarrow$ interacting gas of hadrons
- asymptotic freedom - at high energies quarks interact weakly
  $\Rightarrow$ non-interacting gas of quarks and gluons - quark-gluon plasma (wQGP) (1978, E. Shuryak)
- phase transition
- QGP - existed in Early Universe ($10^6 \times 1 \cdot 10^7\mathrm{K}$) or, possibly, in cores of neutral stars ($10 \times 2 \cdot 10^{17}\mathrm{kg\,m^{-3}}$)

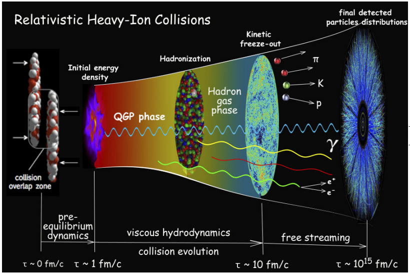


University of Jyväskylä (www.jyu.fi)

ultra relativistic heavy-ion collisions - a controlled way to produce QGP in laboratory

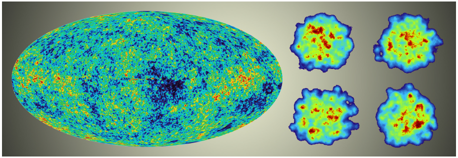


NASA



P. Sorensen, C. Shen

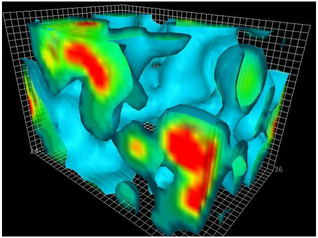
similarities:
- explosive character
- Hubble expansion
- chemical freeze-out (nucleosynthesis (180 s) - hadronization (30×10^{-24} s))
- kinetic freeze-out (charge recombination - microwave radiation, 380000 years - freeze-out of hadron momenta, 45×10^{-24} s)
- fluctuations in the initial state



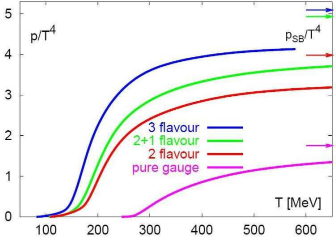
R. Tribble, A. Burrows et al., Implementing the 2007 Long Range Plan

differences:
- slower expansion rate by 18 orders of magnitude
- expansion controlled by gravity vs. expansion controlled by pressure gradients
- larger timescale (10^9 years - 10 ys)
- larger distances (ly - 10 fm)

- description of non-perturbative phenomena in QCD is very complicated
  $\Rightarrow$ numerical simulations are performed on discretised space-time lattices: lattice QCD (lQCD)
- precise results for vanishing baryon chemical potential
- problems to include the finite baryon chemical potential
- QGP properties:
  - "cross-over" phase transition at $T = T_C \approx 160\,\mathrm{MeV}$
  - hypothesis of critical point for finite baryon chemical potential
  - in the energy range available at RHIC and LHC departures from the Stefan-Boltzmann law can be seen (non-negligible interactions in the plasma)



D. Leinweber (www.physics.adelaide.edu.au)



F. Karsch, E. Laermann, A. Peikert, Phys. Lett. B478, 447 (2000)

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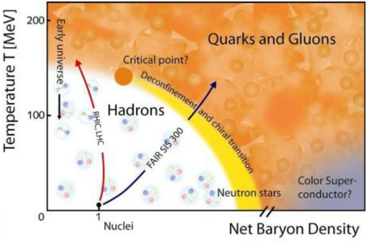

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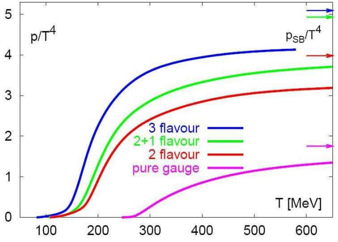


F. Karsch, E. Laermann, A. Peikert, Phys. Lett. B478, 447 (2000)

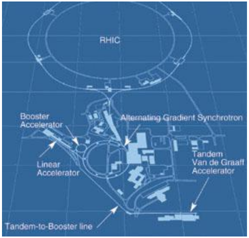
- relatively large volumes ($10^3\times$ proton's volume)
- relatively large energy densities ($10^3\times$ proton's energy density)

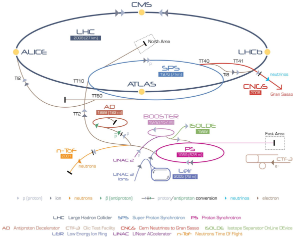
<u>short history of the experimental program:</u>

- 1980 - Bevalac at LBNL
- 1992 - AGS at BNL, $\sqrt{s} = 5$ GeV
- 1995 - SPS at CERN, $\sqrt{s} = 17$ GeV
- 2001 - Relativistic Heavy-Ion Collider (RHIC) at BNL - BRAHMS, PHENIX, PHOBOS, STAR, $\sqrt{s} = 200$ GeV
- 2011 - Large Hadron Collider (LHC) at CERN - ALICE, ATLAS, CMS, LHCb, $\sqrt{s} = 2760$ GeV

<u>future:</u>

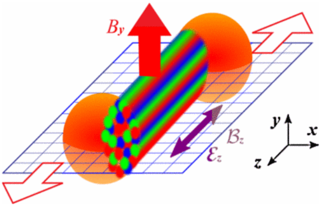
- CBM at FAIR in GSI



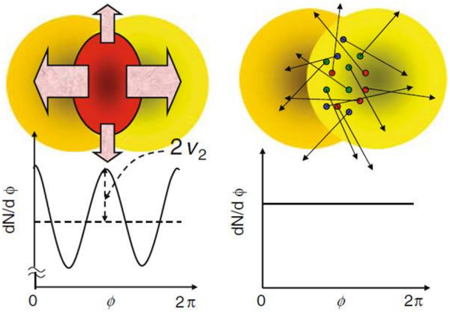


experimental data suggest formation of a new state of matter – almost perfect fluid

QGP as a perfect fluid



K. Fukushima, D. E. Kharzeev, H. J. Warringa, Phys. Rev. Lett. 104, 212001



T. Hirano, N. van der Kolk, A.Bilandzic, Lect.Notes Phys. 785 (2010) 139-178

only indirect studies of QGP are possible
⇒ measurements of the momenta and energies of emitted particles

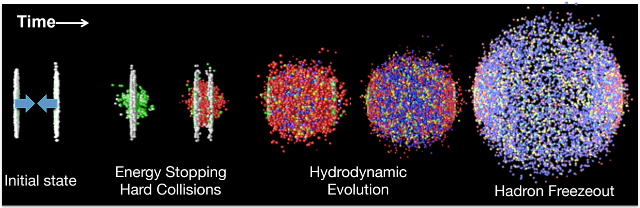
- strong elliptic flow (v_2) at RHIC and the LHC
  ⇒ collective behavior
  ⇒ strongly-interacting system
- good description in terms of perfect fluid hydrodynamics
  ⇒ *"RHIC serves the perfect fluid" (U.Heinz)*

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left[1 + 2v_1\cos(\phi) + 2v_2\cos(2\phi) + ...\right]$$

$$v_n(p_T, y) = \frac{\int d\phi \cos(n\phi)\frac{dN}{d\phi}}{\int d\phi \frac{dN}{d\phi}} \equiv \langle \cos(n\phi)\rangle$$

precise measurements need theoretical input from hydrodynamic models

"Standard model" of evolution of matter produced in heavy-ion collisions



T. K. Nayak, Lepton-Photon 2011 Conference

- initial conditions including fluctuations ($0 < \tau_0 \lesssim 1\,\mathrm{fm}$)
- non-equilibrium phase and thermalization of matter ($\tau_0 < \tau \lesssim 1\,\mathrm{fm}$)
  $\Rightarrow$ emission of hard probes: heavy quarks, photons, jets
- hydrodynamic expansion (expansion and cooling) ($1\,\mathrm{fm} \lesssim \tau \lesssim 10\,\mathrm{fm}$)

microscopic description of such a many body system is very complicated
$\Downarrow$
effective description in terms of fluid mechanics

- phase transition from QGP to hadron gas (encoded in the equation of state)
- freeze-out and free streaming ($10\,\mathrm{fm} \lesssim \tau$)

main assumption: system is in local thermal equilibrium

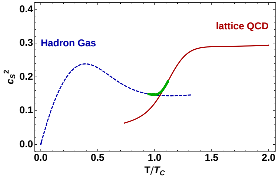
- conservation of the baryon number (and other conserved charges), energy and momentum

$$\partial_\mu N^\mu = 0 \qquad N^\mu \equiv n u^\mu$$

$$\partial_\mu T^{\mu\nu}_{id} = 0 \qquad T^{\mu\nu}_{id} \equiv \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}\mathcal{P}$$

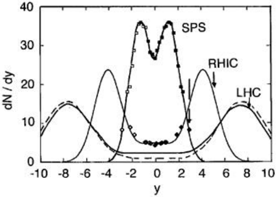
$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} \qquad \Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$$

- 6 independent variables: $(\mathcal{E}, \mathcal{P}, n, u^\mu(3))$
- 6 equations (equation of state $\mathcal{E}(n, \mathcal{P})$)



M. Chojnacki, W. Florkowski, Acta Phys.Polon. B38 (2007) 3249-3262

- in ultra relativistic collisions we may neglect the baryon number conservation



R. Kuiper and G. Wolschin, Annalen Phys. 16, 67 (2007)

main assumption: system is in local thermal equilibrium

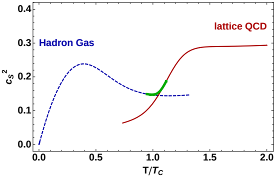
- conservation of energy and momentum

$$\cancel{\partial_\mu N^\mu = 0} \qquad \cancel{N^\mu \equiv n u^\mu}$$

$$\partial_\mu T^{\mu\nu}_{id} = 0 \qquad T^{\mu\nu}_{id} \equiv \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu} \mathcal{P}$$

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} \qquad \Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$$

- 5 independent variables $(\mathcal{E}, \mathcal{P}, \cancel{n}, u^\mu(3))$
- 5 equations (equation of state $\mathcal{E}(\cancel{n}, \mathcal{P})$)



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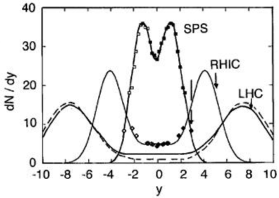
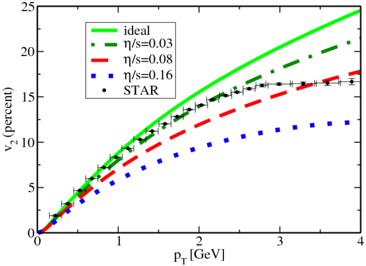


R. Kuiper and G. Wolschin, Annalen Phys. 16, 67 (2007)

- quantum mechanics (Danielewicz, Gyulassy, 1985) / AdS/CFT (Kovtun, Son, Starinets, 2005) lower bound on viscosity $\eta/\mathcal{S} > 1/4\pi$

  $\Rightarrow$ one should use relativistic dissipative hydrodynamics

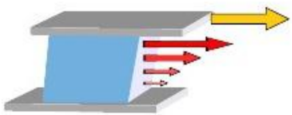
  $\Rightarrow$ better description (assuming small $\eta/\mathcal{S} = \text{const}$)



P. Romatschke and U. Romatschke, Phys. Rev. Lett. 99, 172301 (2007) shear viscosity η

$\Downarrow$

reaction to a change of shape

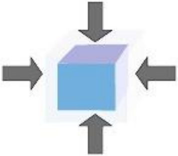


$$\pi^{\mu\nu}_{Navier-Stokes} \propto \eta \, (\partial^{\langle\mu} u^{\nu\rangle})$$

bulk viscosity ζ

$\Downarrow$

reaction to a change of volume



$$\Pi_{Navier-Stokes} \propto \zeta \, (\partial_\mu u^\mu)$$

Comparison of shear viscosity for different substances

płyn	η [Pa s]
woda	2.9 10^{-4}
miód	10000
^{4}He	10^{-6}
^{6}Li	$<10^{-15}$
plazma kw.-gl.	$<2\ 10^{11}$

płyn	η / s [$\hbar$]
woda	8.2
miód	5 10^7
^{4}He	1.9
^{6}Li	<0.5
plazma kw.-gl.	<0.3

Prof. John Mainstone (Wikipedia)

Pitch

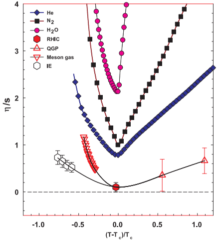
Start 1927

1st drop 1938

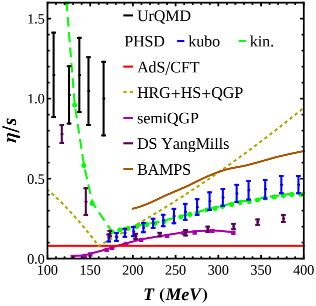
8th drop 2000

$\eta \sim 2\ 10^8$ Pa s

$\sim 10^{11} \eta_{H_2O}$

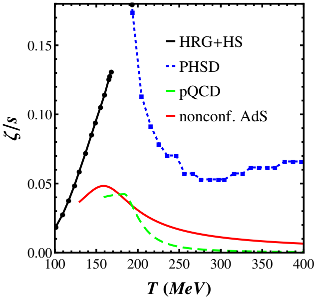


R. A. Lacey et al., Phys.Rev.Lett. 98 (2007) 092301



S. I. Finazzo et al., arXiv:1412.2968

- $\eta/\mathcal{S}$ depends on temperature
  $\Rightarrow$ first estimate:
  P. Bozek, Phys.Rev. C81 (2010) 034909
- $\eta/\mathcal{S}$ reaches minimum in the region of the phase transition
- $\zeta/\mathcal{S}$ reaches maximum in the region of the phase transition



J. Noronha-Hostler

the system is never in local equilibrium (diffusion, friction, heat conduction,...)

- energy-momentum conservation

$$\partial_\mu T^{\mu\nu}_{vis} = 0 \qquad T^{\mu\nu}_{vis} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P} + \Pi) + \pi^{\mu\nu}$$

- number of variables: 5 + 6 ($\mathcal{E}, \mathcal{P}, u^\mu(3), \Pi, \pi^{\mu\nu}(5)$)
- number of equations: 4 + 1 (equation of state $\mathcal{E}(\mathcal{P})$)
- we need 6 extra equations - different methods possible

$$\frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta$$

$$\frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu}$$

Navier-Stokes equations

$\tau_\Pi \beta_\Pi = \zeta \rightarrow$ bulk viscosity, $\tau_\pi \beta_\pi = \eta \rightarrow$ shear viscosity

the system is never in local equilibrium (diffusion, friction, heat conduction,...)

- energy-momentum conservation

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$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu}$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu} \omega^{\nu\rangle\gamma} - \delta_{\pi\pi} \pi^{\mu\nu} \theta - \tau_{\pi\pi} \pi_\gamma^{\langle\mu} \sigma^{\nu\rangle\gamma} + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu}$$

Israel-Stewart equations

$\tau_\Pi \beta_\Pi = \zeta \to$ bulk viscosity, $\tau_\pi \beta_\pi = \eta \to$ shear viscosity

the system is never in local equilibrium (diffusion, friction, heat conduction,...)

- energy-momentum conservation

$$\partial_\mu T^{\mu\nu}_{vis} \quad = \quad 0 \qquad T^{\mu\nu}_{vis} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P} + \Pi) + \pi^{\mu\nu}$$

- number of equations: 5 + 6 ($\mathcal{E}, \mathcal{P}, u^\mu(3), \Pi, \pi^{\mu\nu}(5)$)
- number of equations: 4 + 1 (equations of state $\mathcal{E}(\mathcal{P})$)
- we need 6 extra equations - different methods possible

$$\begin{aligned} \dot{\Pi} + \frac{\Pi}{\tau_\Pi} &= -\beta_\Pi \theta - \delta_{\Pi\Pi}\Pi\theta \underline{+ \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}} \\ \dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \delta_{\pi\pi}\pi^{\mu\nu}\theta \underline{- \tau_{\pi\pi}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}} \end{aligned}$$

New approaches (shear-bulk coupling $\eta - \zeta$)

$\tau_\Pi \beta_\Pi = \zeta \to$ bulk viscosity, $\tau_\pi \beta_\pi = \eta \to$ shear viscosity

- Eckart (1940), Landau-Lifshitz (1959)
  → relativistic version of Navier-Stokes equations
  breaking of causality → unstable equations
- Israel-Stewart (1979) (most commonly used nowadays)
  → no problems with causality, relaxation times introduced
  distribution function - 14-moment approximation
  evolution equations - 2nd moment of the Boltzmann equation
- Denicol-Koide-Rischke (2010)
  distribution function - 14-moment approximation
  evolution equations - derived directly for the viscous correction terms
- Denicol-Niemi-Molnar-Rischke (2012)
  distribution function - systematic developments in the Knudsen and Reynolds numbers
  evolution equations - derived directly for the viscous correction terms
- Jaiswal (2013)
  distribution function - modified Chapman-Enskog method
  evolution equations - derived directly for the viscous correction terms

different approaches lead to different form of the kinetic coefficients

most of the approaches rely on the kinetic theory

in the kinetic theory the basic quantity is the one-particle phase-space distribution function

- perfect-fluid hydrodynamics: local thermal equilibrium

$$f(x,p) = f_{\rm iso}\left(\frac{p^\mu u_\mu}{T(x)}\right) \qquad \Rightarrow \qquad T^{\mu\nu}_{id} = \int dP\, p^\mu p^\nu f(x,p)$$

- dissipative hydrodynamics: linear deviations from local equilibrium

$$f(x,p) = \underbrace{f_{\rm iso}\left(\frac{p^\mu u_\mu}{T(x)}\right)}_{\rm LO} + \underbrace{\delta f(x,p)}_{\rm NLO} \qquad \Rightarrow \qquad T^{\mu\nu}_{vis} = T^{\mu\nu}_{id} + \delta T^{\mu\nu}_{id}$$

URHIC → EXTREME SPACETIME SCALES
very small systems
very large gradients
very fast expansion
⇓
dissipative corrections are substantial

standard dissipative hydrodynamics assumes that the system is always close to local equilibrium, this is in contrast with microscopic calculations showing that plasma is highly anisotropic in the momentum space (at the early stages of the evolution)

NEW DEVELOPMENTS

W. Florkowski, R. Ryblewski, Phys. Rev. **C 83**, 034907 (2011)
W. Florkowski, R. Ryblewski, Phys. Rev. **C 85**, 044902 (2012)
M. Martinez, R. Ryblewski, M. Strickland, Phys. Rev. **C 85**, 064913 (2012)
W. Florkowski, R. Ryblewski, Phys. Rev. **C 85**, 064901 (2012)
M. Nopoush, R. Ryblewski, M. Strickland, Phys. Rev. **C 90**, 014908 (2014)

- anisotropic hydrodynamics: reorganization of the hydro expansion, anisotropy included in the leading order

$$f(x,p) = \underbrace{f_{\rm iso}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu} p^\nu}}{\Lambda(x)}\right)}_{\rm LO} + \cancel{\underbrace{\delta\tilde{f}(x,p)}_{\rm NLO}} \qquad \Rightarrow \qquad \delta\tilde{f}(x,p) << \delta f(x,p)$$

- evolution equations obtianed from relativistic Boltzmann equation (similarly to standard approaches)

$$p^\mu \partial_\mu f = p^\mu \frac{u_\mu}{\tau_{\rm eq}}(f^{\rm eq} - f) \qquad \rightarrow \qquad \partial_{\mu_1} \int dP\, p^{\mu_1} ... p^{\mu_n} f = u_{\mu_1} \int dP\, p^{\mu_1} ... p^{\mu_n} \frac{1}{\tau_{\rm eq}}(f^{\rm eq} - f)$$

problem of large linear corrections is solved

W. Florkowski, R. Ryblewski, M. Strickland, Nucl. Phys. **A916**, 249 (2013)
W. Florkowski, R. Ryblewski, M. Strickland, Phys. Rev. **C 88**, 024903 (2013)
W. Florkowski, E. Maksymiuk, R. Ryblewski, M. Strickland, Phys. Rev. **C 89**, 054908 (2014)

- Boltzmann equation in the relaxation time approximation

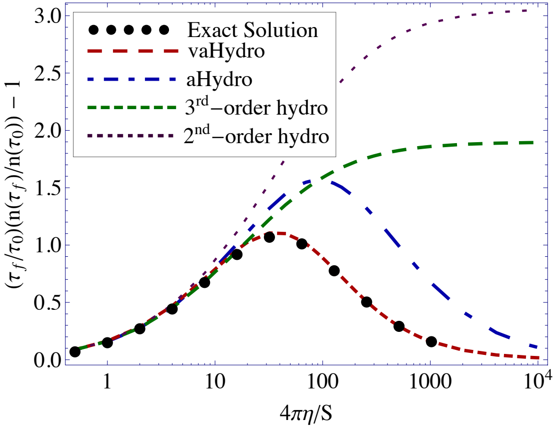
$$\boxed{p^\mu \partial_\mu f = p^\mu \frac{u_\mu}{\tau_{\rm eq}} \left(f^{\rm eq} - f \right)}$$

- analytic solutions are possible in the cases of simple geometry - e.g., for boost-invariant and transversally homogenous systems, denoted as (0+1)D fluids
- formal solution

$$f(\tau, w, p_\perp) \quad = \quad D(\tau,\tau_0) f_0(w,p_\perp) + \int\limits_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\rm eq}(\tau')} \, D(\tau,\tau') \, f^{\rm eq}(\tau', w, p_\perp)$$

- knowing the distribution function we determine dissipative currents (for (0+1)D case only Π and $\pi^{\eta\eta}$ (connected with the ratio $\mathcal{P}_L/\mathcal{P}_T$))

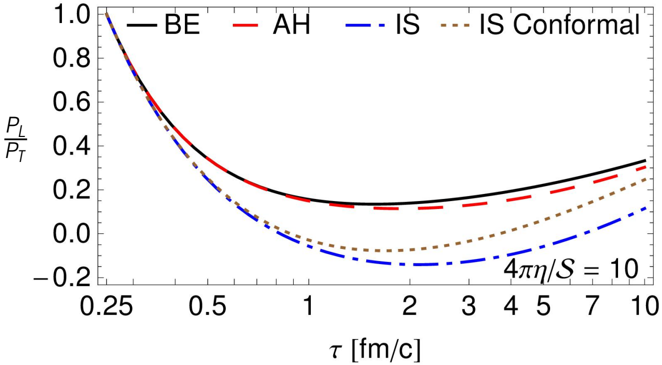
analytic solutions serve to check different hydro approaches



D. Bazow, U. W. Heinz, and M. Strickland, Phys.Rev. C90, 044908 (2014)

anisotropic hydro reproduces two limts:
perfect fluid ($\eta/\mathcal{S} \to 0$) and free streaming ($\eta/\mathcal{S} \to \infty$)

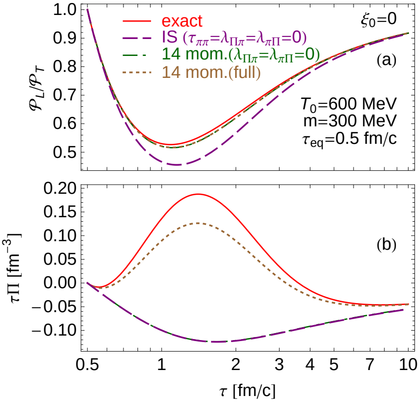
Problem of negative pressures



W. Florkowski, R. Ryblewski, M. Strickland, Nucl. Phys. **A916**, 249 (2013)

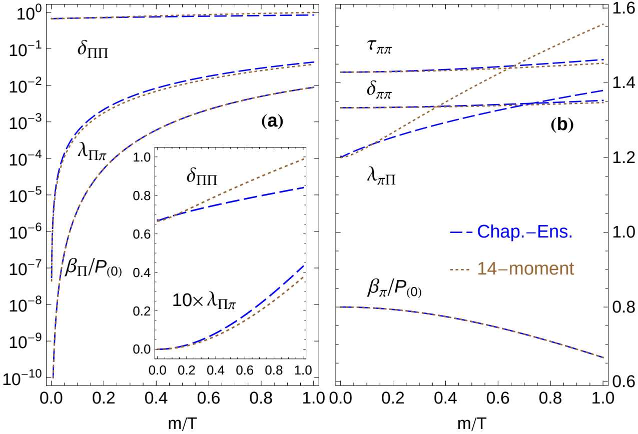
anisotropic hydrodynamics solves the problem of negative pressures present in popular hydro codes

G. Denicol, W. Florkowski, R. Ryblewski, M. Strickland, Phys. Rev. **C 90**, 044905 (2014)



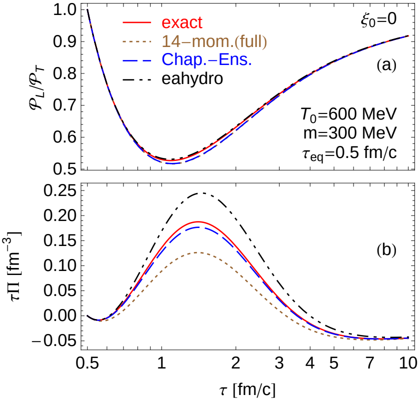
Israela-Stewart neglects $\tau_{\pi\pi}$ in the equation for η
(20% improvement)
similarly, Israel-Stewart neglects $\lambda_{\Pi\pi}$ and $\lambda_{\pi\Pi}$ ($\eta - \zeta$ couplings),
they appear in new approaches and are crucial to reproduce the bulk viscous pressure

A. Jaiswal, R. Ryblewski, M. Strickland, Phys. Rev. **C 90**, 044905 (2014)



Chapmann-Enskog method leads to different forms of the coefficients

A. Jaiswal, R. Ryblewski, M. Strickland, Phys. Rev. **C 90**, 044905 (2014)



better description with the Chapmann-Enskog method compared to 14 moment Grad method

pressure anisotropy well reproduced in anisotropic hydrodynamics

Summary

- experimental studies of QGP need a theoretical framework that can be used to ask questions about the QGP properties
- our understanding of the QGP behavior relies on the application of relativistic dissipative hydrodynamics, properties of QGP encoded in the equation of state, kinetic coefficients, etc.
- new hydro schemes: 14-moment approach, Chapman-Enskog, anisotropic hydrodynamics
- checks through analytic special solutions
- interesting phenomena, like couplings between the shear and bulk viscous pressure

Molte grazie!