Computing jet quenching and the transport coefficients of the Quark-Gluon Plasma

Jacopo Ghiglieri, CERN

Theory Colloquium, Torino, June 9 2017

Outline

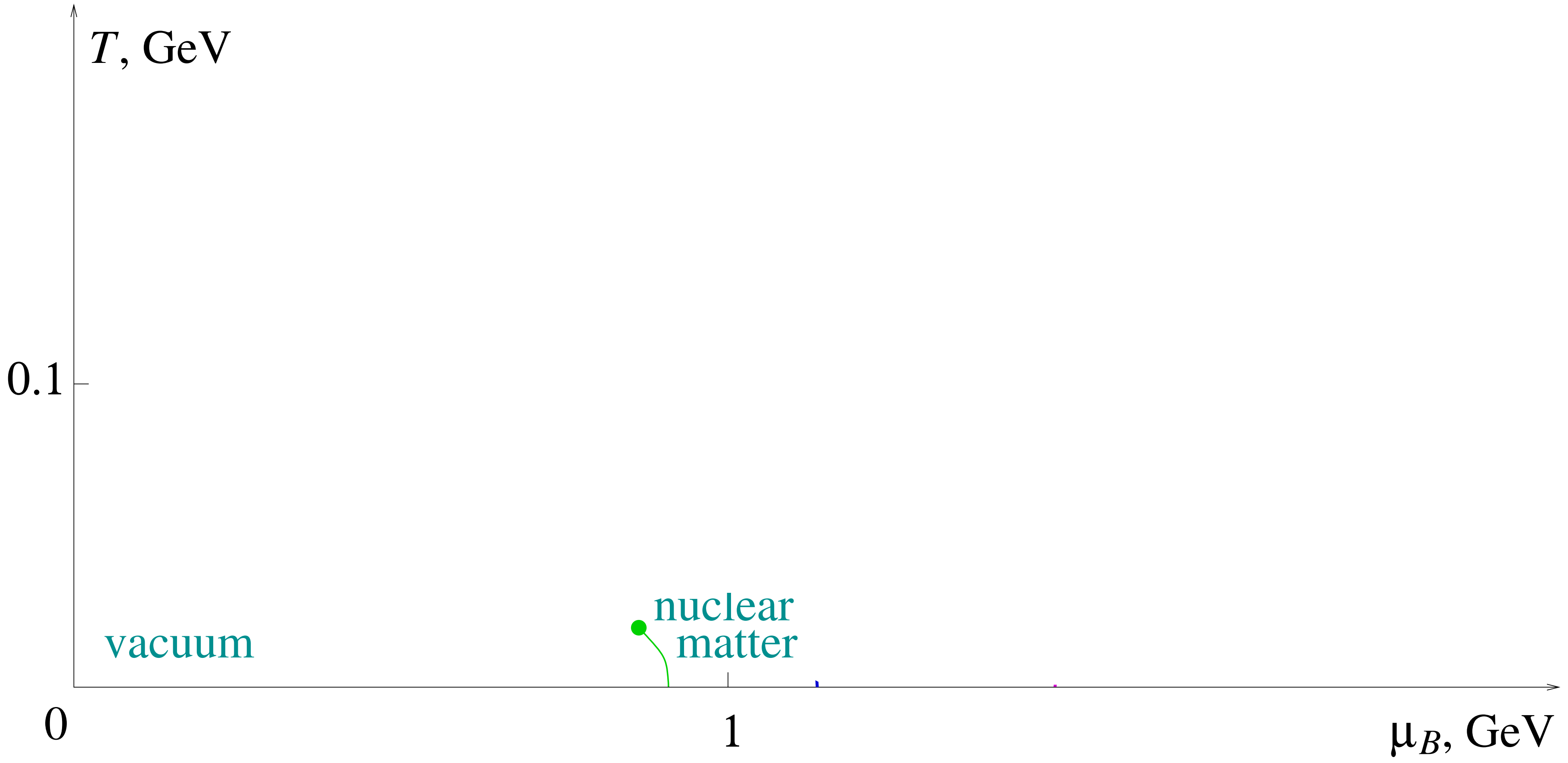
- Jets and transport in heavy ion collisions
- A modern approach to an effective kinetic theory for jets and transport
- Incorporating NLO (O(g)) and non-perturbative effects: testing the stability of these perturbative results

Pedagogical review in JG Teaney **1502.03730** (in QGP5)
Gritty details for jets in JG Moore Teaney **1509.07773**
NLO transport JG Moore Teaney, in preparation

Overview

The phase diagram of QCD

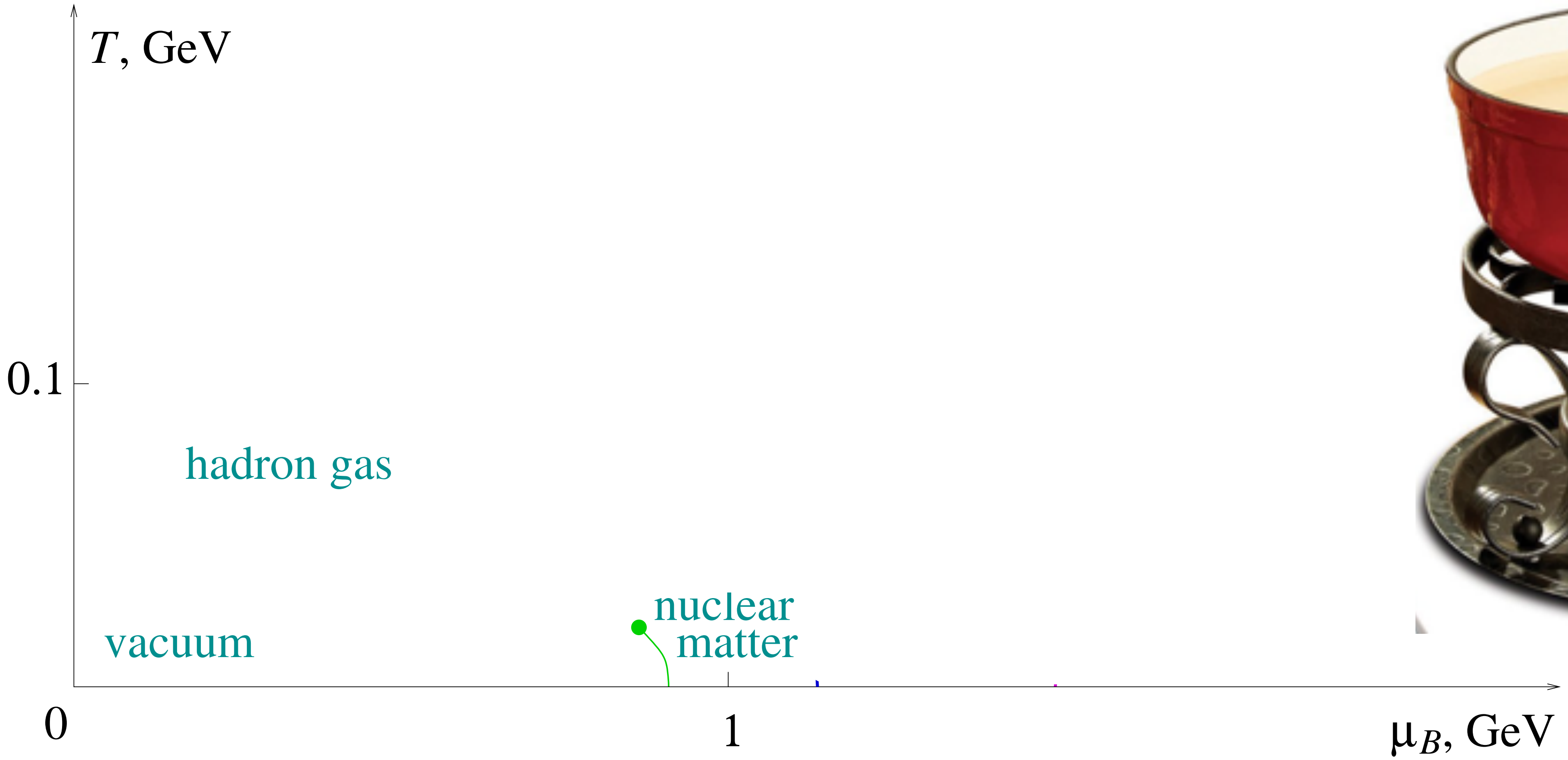
- In the temperature/baryon chemical potential plane:



- At low temperature and moderate densities: ordinary hadrons and nuclear matter.
- Colour confinement

The quark-gluon plasma

- As the temperature is increased:



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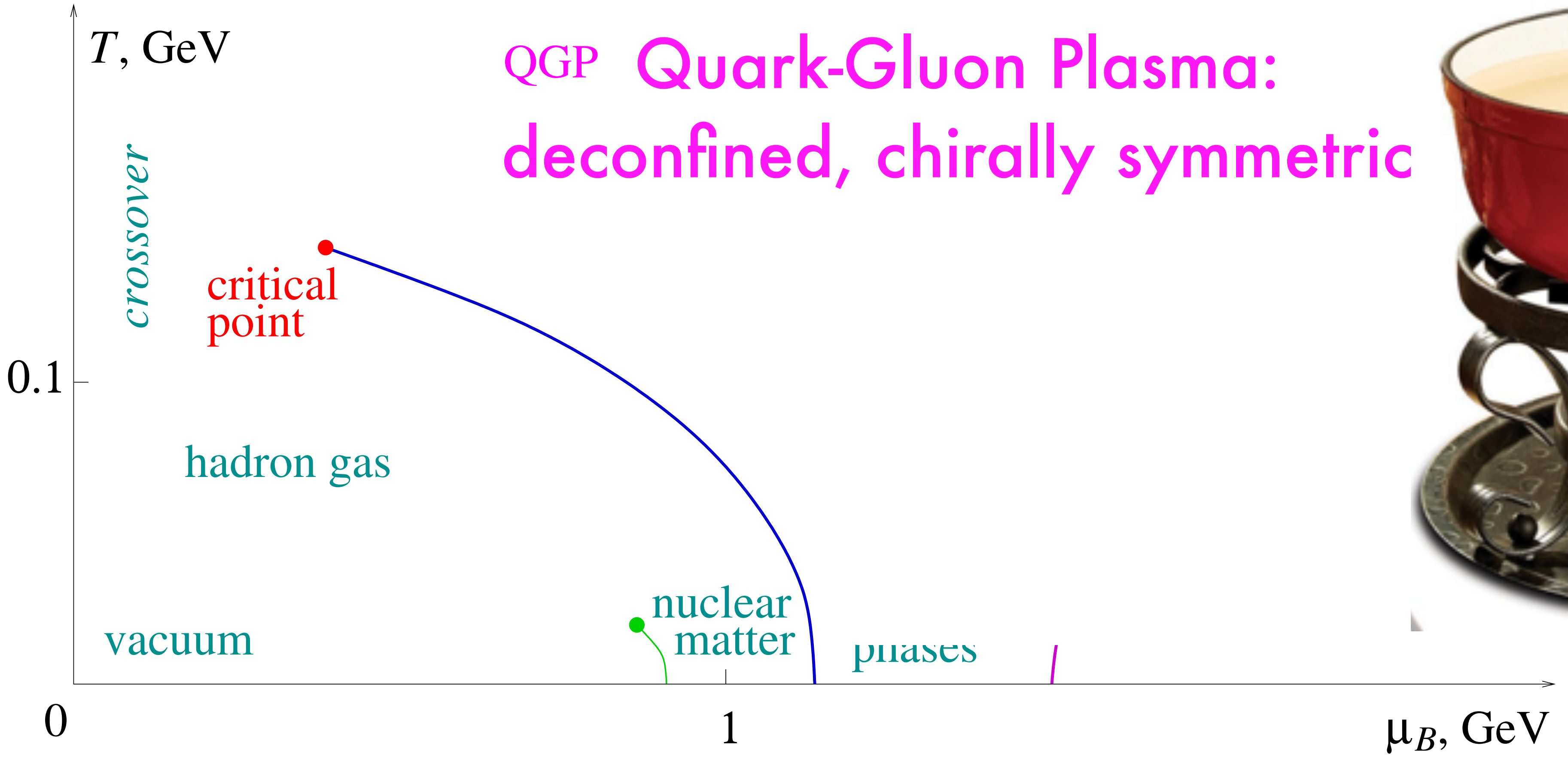

- In the upper-left region, lattice QCD indicates a (pseudo)critical temperature T_c~160 MeV ~2x10^{12} K
- For comparison, sun's core: T~1.5x10^7 K

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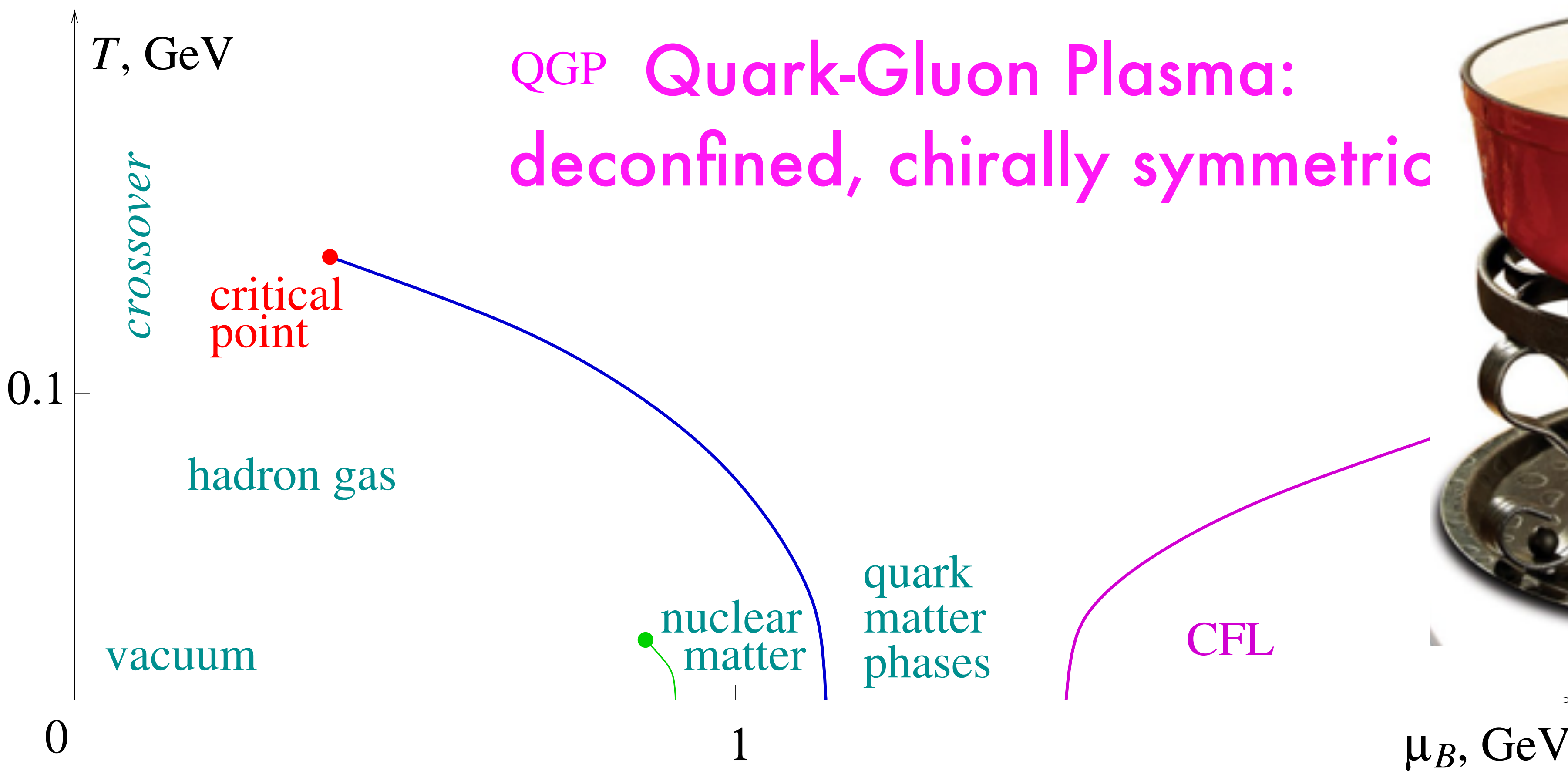

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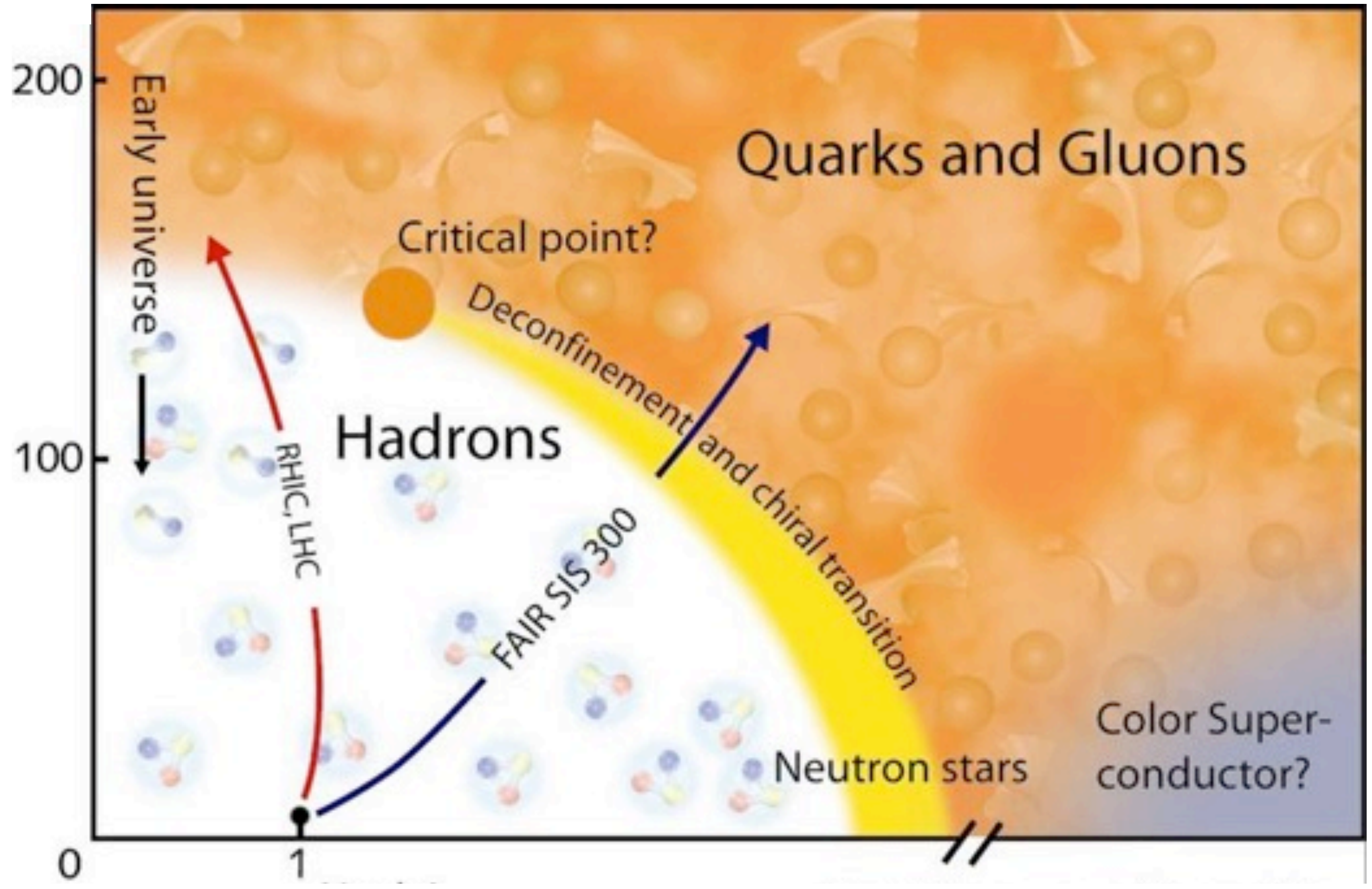
Heavy-ion collisions

- A (transient) QGP can be formed in heavy ion collision experiments. RHIC (@BNL), up to $\sqrt{s_{NN}}$=200GeV. LHC up to $\sqrt{s_{NN}}$=5.5 TeV (5 so far).



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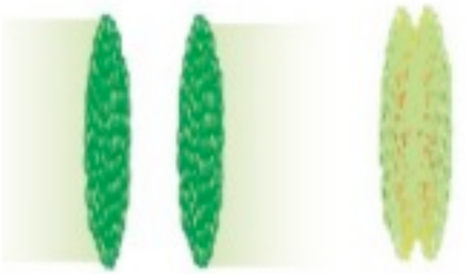

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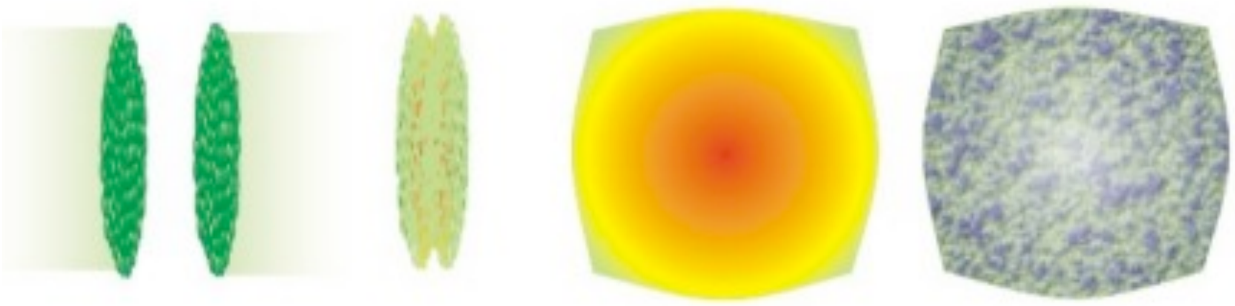

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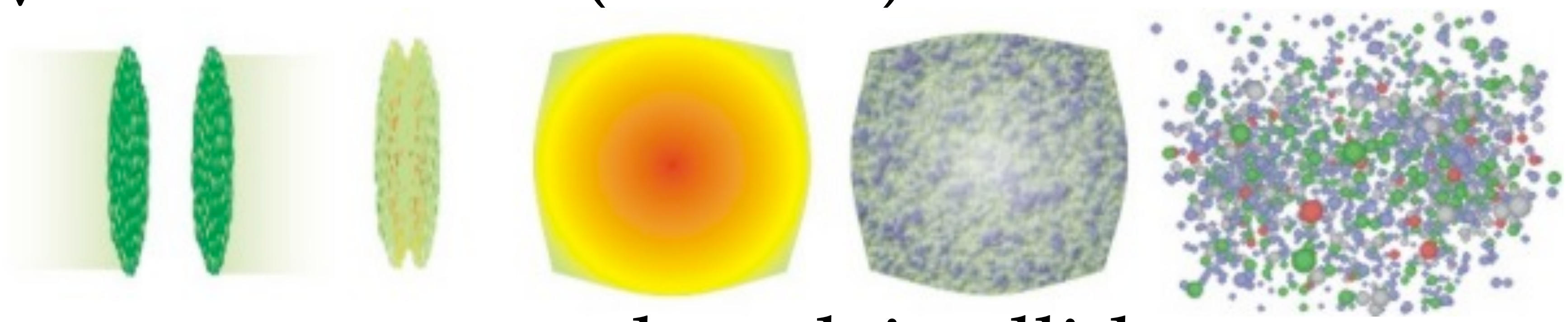

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- Expansion and cooling for up to 5-10 fm/c, then
- Hadronizazion
- Lots of particles ($dN_{\rm ch}/dy$ $O(1000)$) stream to the detectors

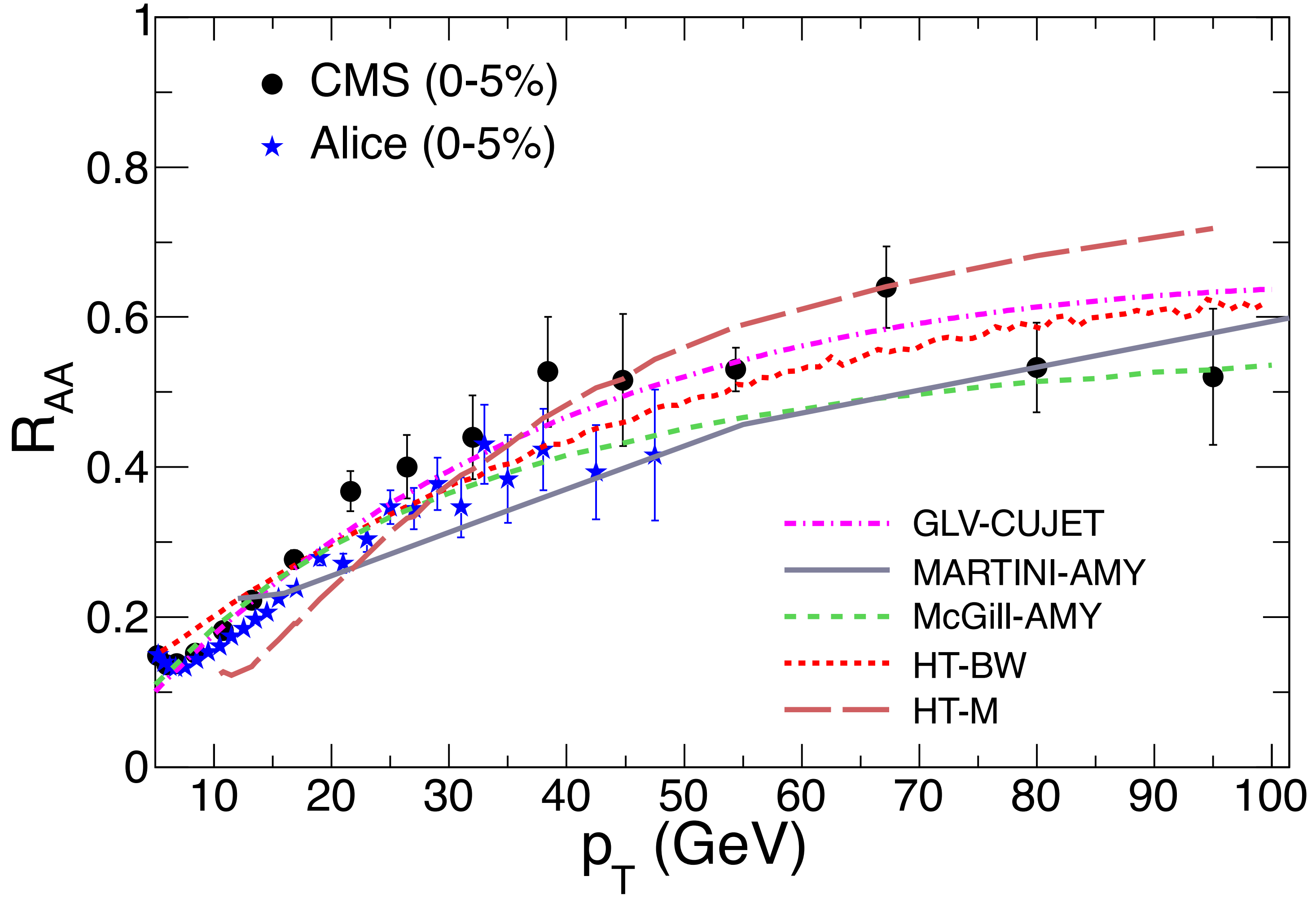
Characterizing the QGP

- Characterization of the medium through **two** classes of observables
  - **Bulk** properties: "macroscopic" evolution of the fireball effectively described by hydrodynamics. The QGP behaves as a strongly coupled, almost ideal fluid
  - **Hard probes:** *high-energy* particles *not in equilibrium* with the medium (jets, e/m probes, quarkonia...).
- Medium *tomography* and characterization of its properties, such as temperature, deconfinement, χ-sym restoration...

Jet quenching

- One of the main results of the HIC program: jets are suppressed with respect to proton-proton collisions
- Quantitatively: look at deviations from binary scaling

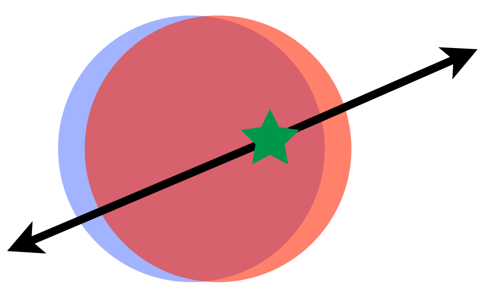
$$R_{AA} = \frac{\text{Yield}_{AA}}{\text{Yield}_{pp} \times N_{bin}}$$

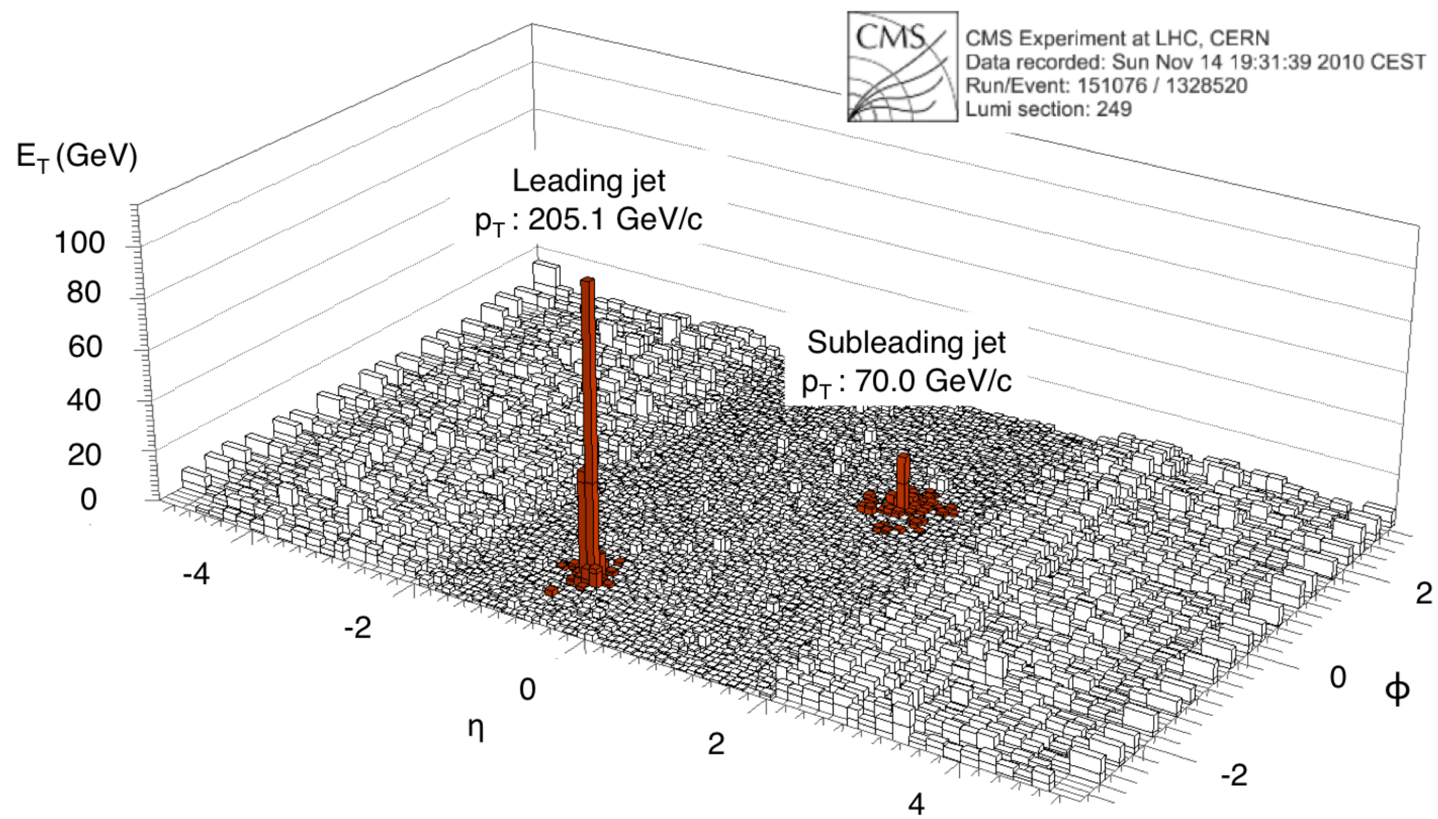


JET Collaboration

Jets quenching

- Qualitatively striking aspect: the dijet asymmetry





CMS **PRC84** (2011)

Flow: a bulk property

- Initial asymmetries in position space are converted by collective, macroscopic (many body) processes into final state momentum space asymmetries

- Quantitatively: azimuthal Fourier decomposition of the final state particle spectra

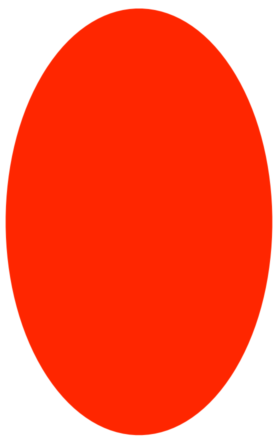
$$\frac{dN_i}{dy\, d^2p_T} = \frac{dN_i}{2\pi p_T dP_T dy}\left(1 + \sum_{n=1}^{\infty} 2v_{i,n}(p_T, y)\cos(n\phi)\right)$$

vzero amplitude + v_n coefficients

- 2D analogue of the multipole expansion of the CMB

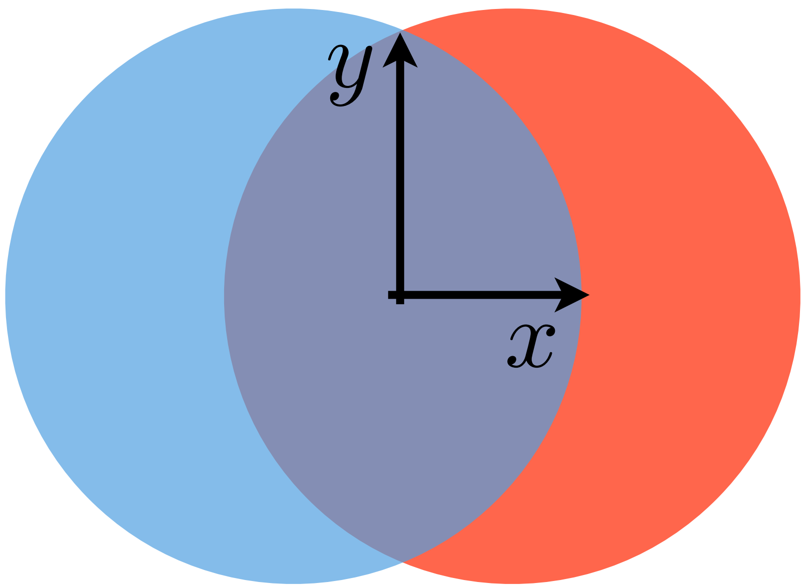
A famous example:elliptic flow

Position space



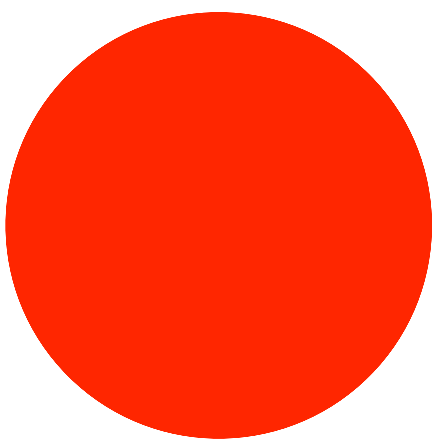
Large pressure gradients

Initial asymmetry



Beam along z

Momentum space



From initial symmetry

A famous example:elliptic flow

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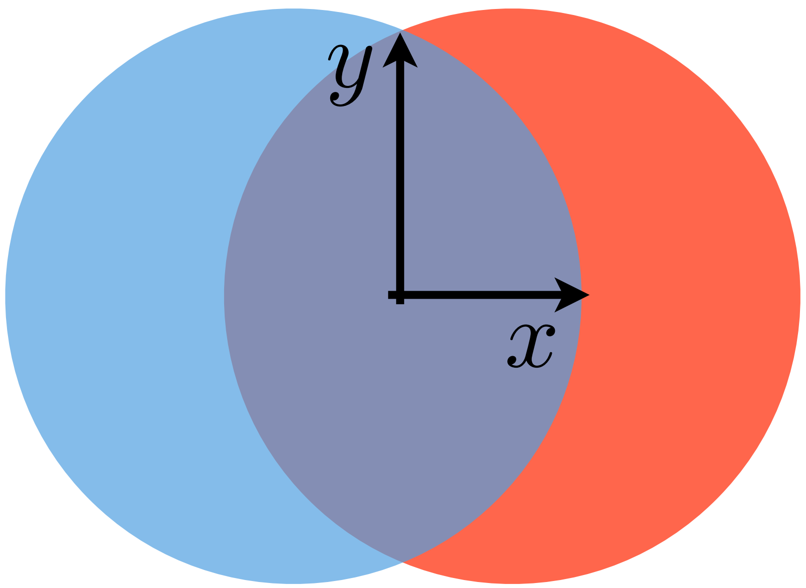

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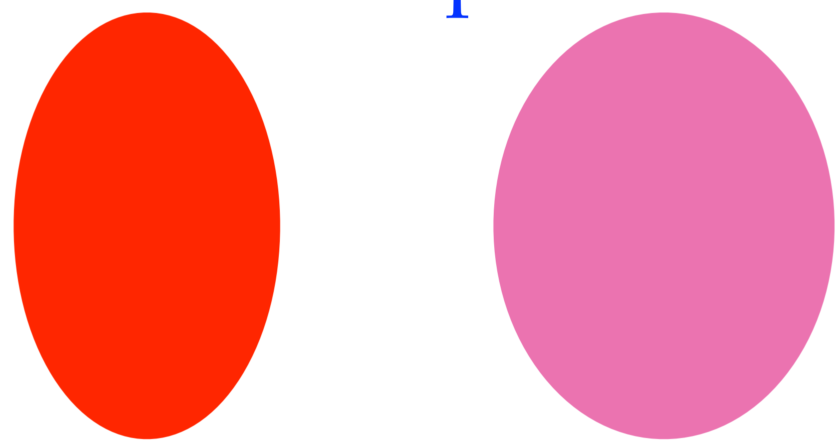

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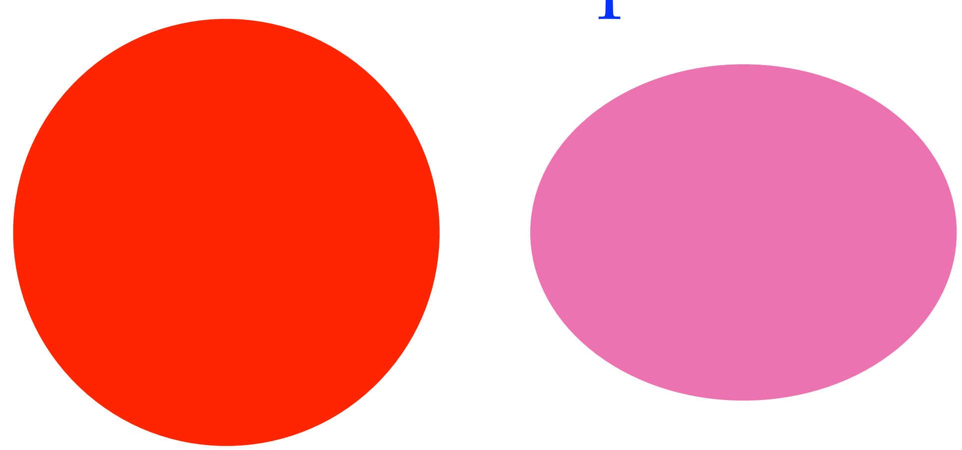

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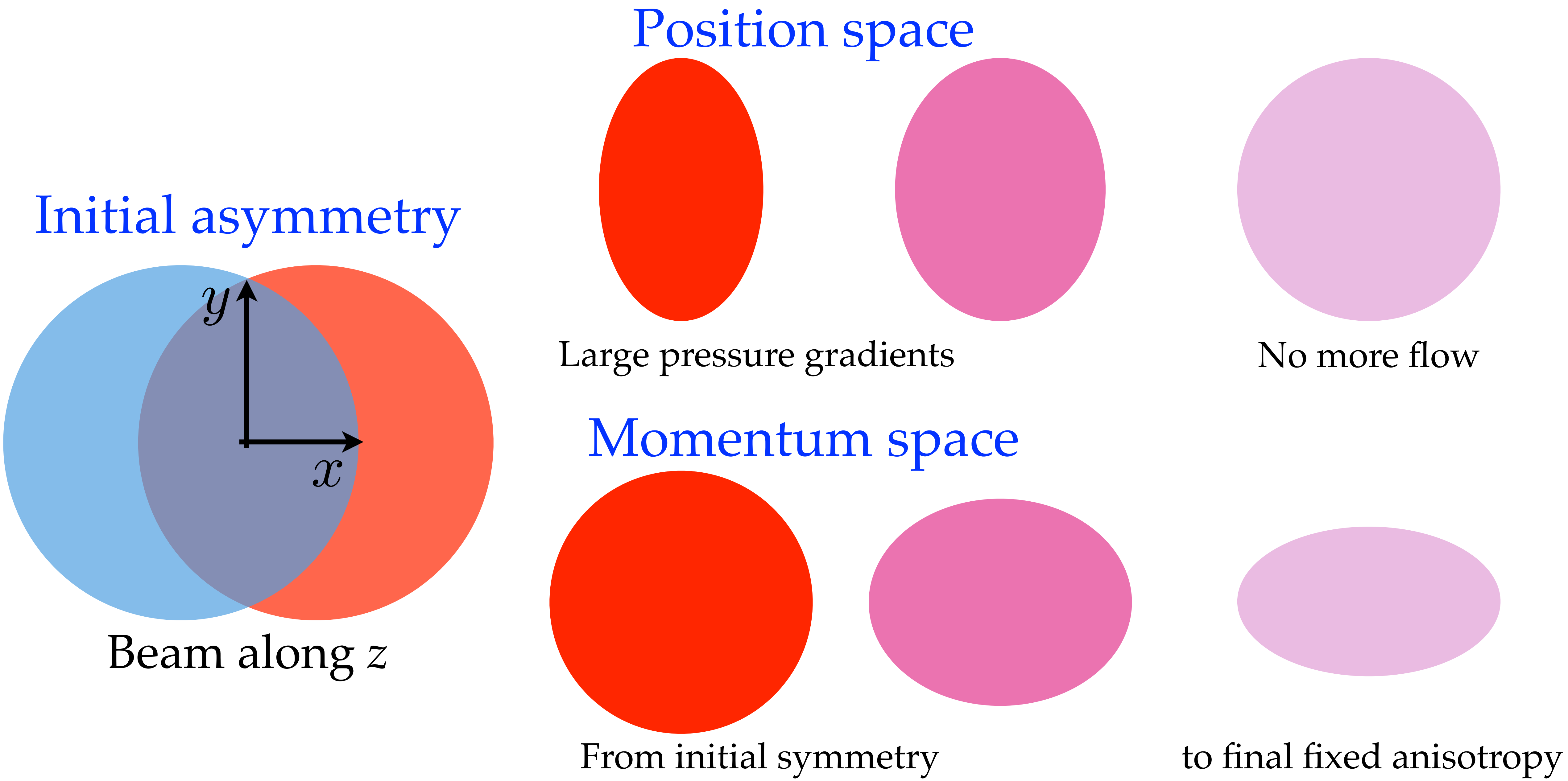

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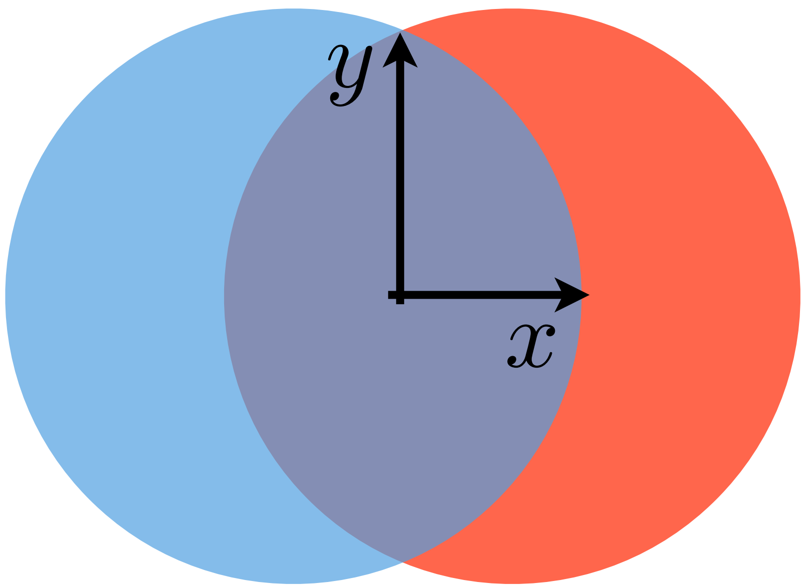

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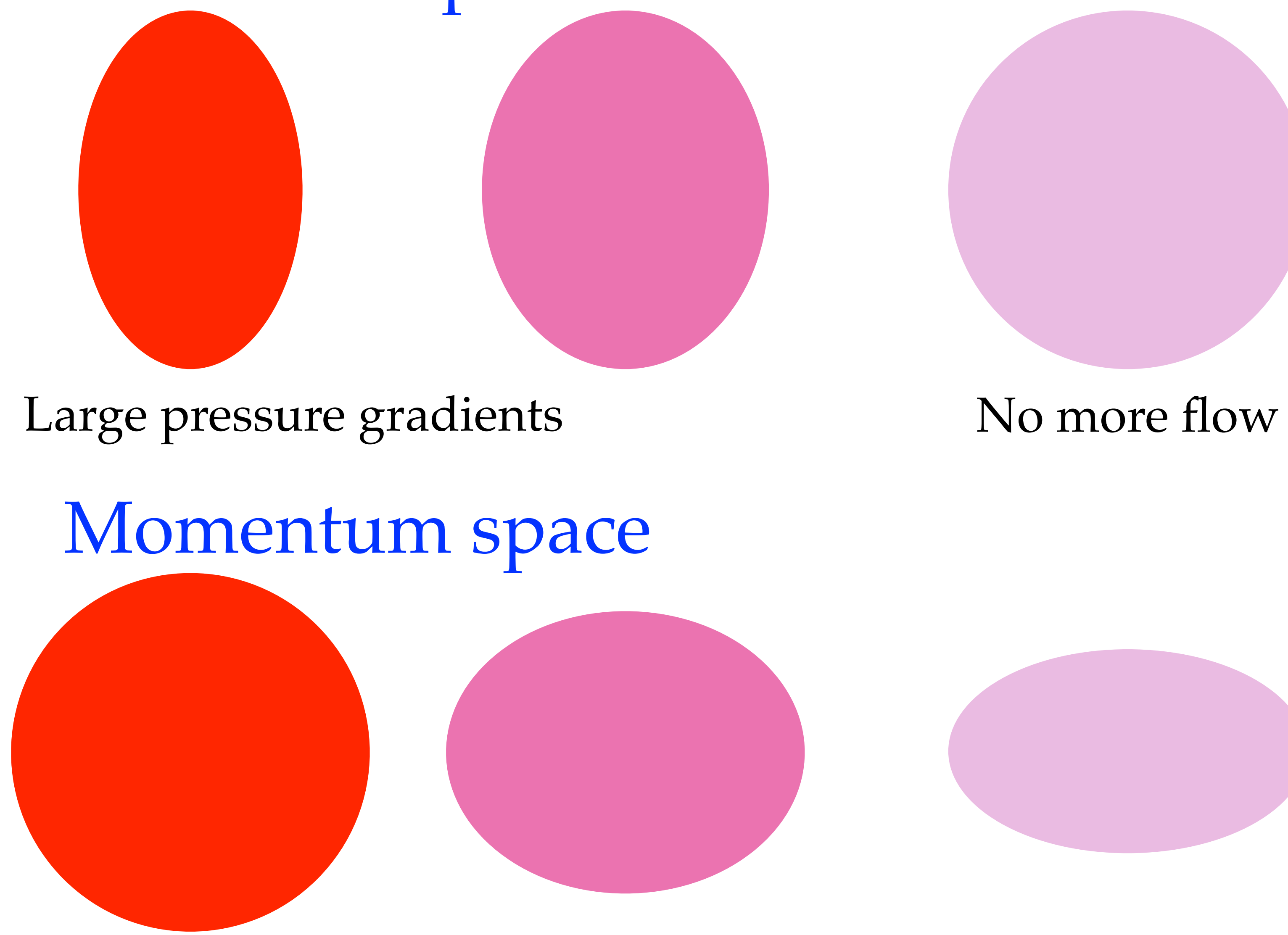


From initial symmetry

to final fixed anisotropy

- Hydrodynamics describes the buildup of flow. The shear viscosity parametrizes the efficiency of the conversion

Hydrodynamics

- Field theories admit a long-wavelength hydrodynamical limit. Hydrodynamics: Effective Theory based on a gradient expansion of the flow velocity

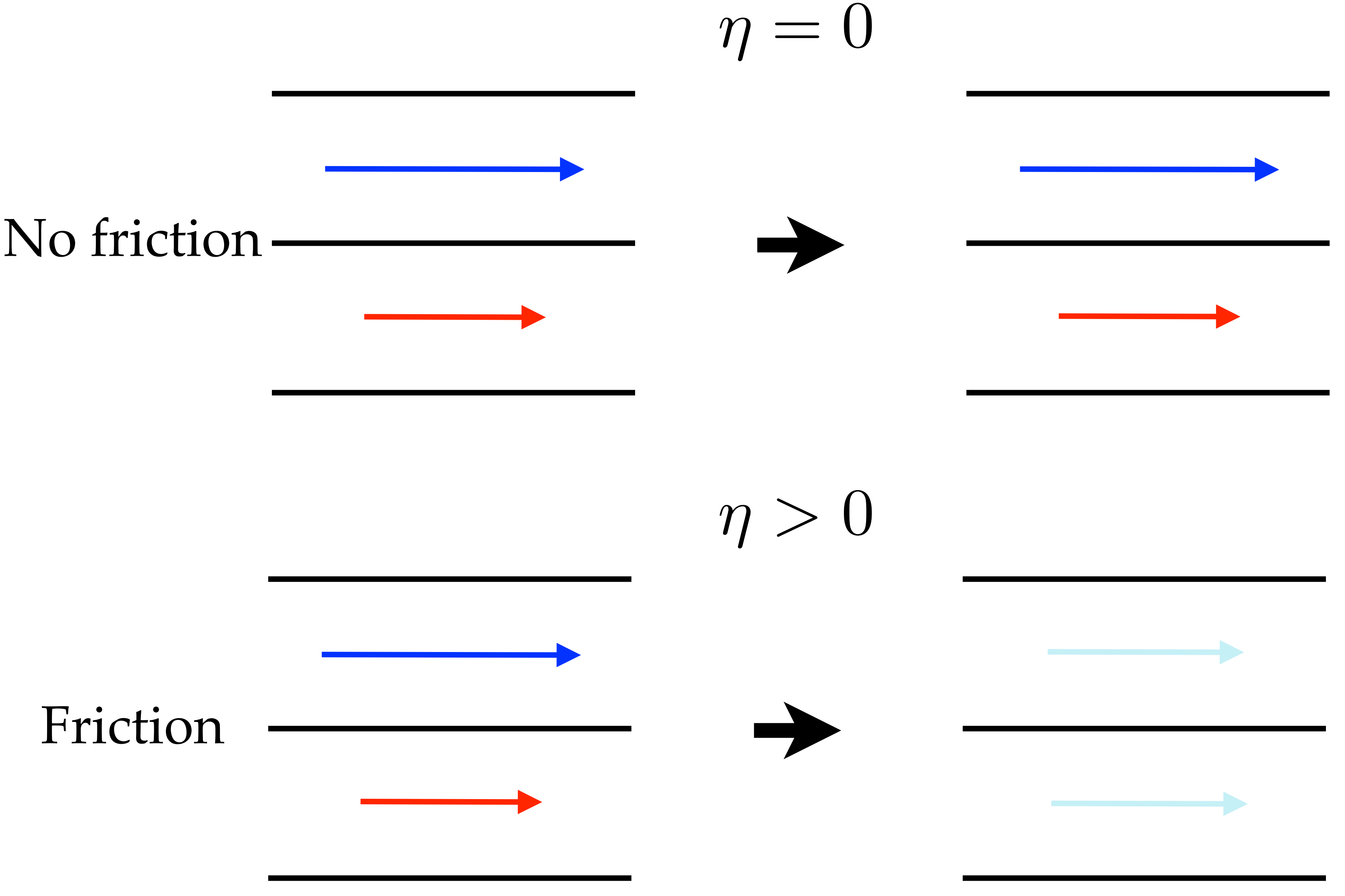
- For hydro fluctuations with local flow velocity **v** around an equilibrium state (with temp. *T*), at first order in the gradients and in **v**

$$T^{00} = e, \qquad T^{0i} = (e+p)v^i$$

$$T^{ij} = (p - \zeta \nabla \cdot \mathbf{v})\delta^{ij} - \eta \left(\partial_i v^j + \partial_j v^i - \frac{2}{3}\delta^{ij} \nabla \cdot \mathbf{v} \right)$$

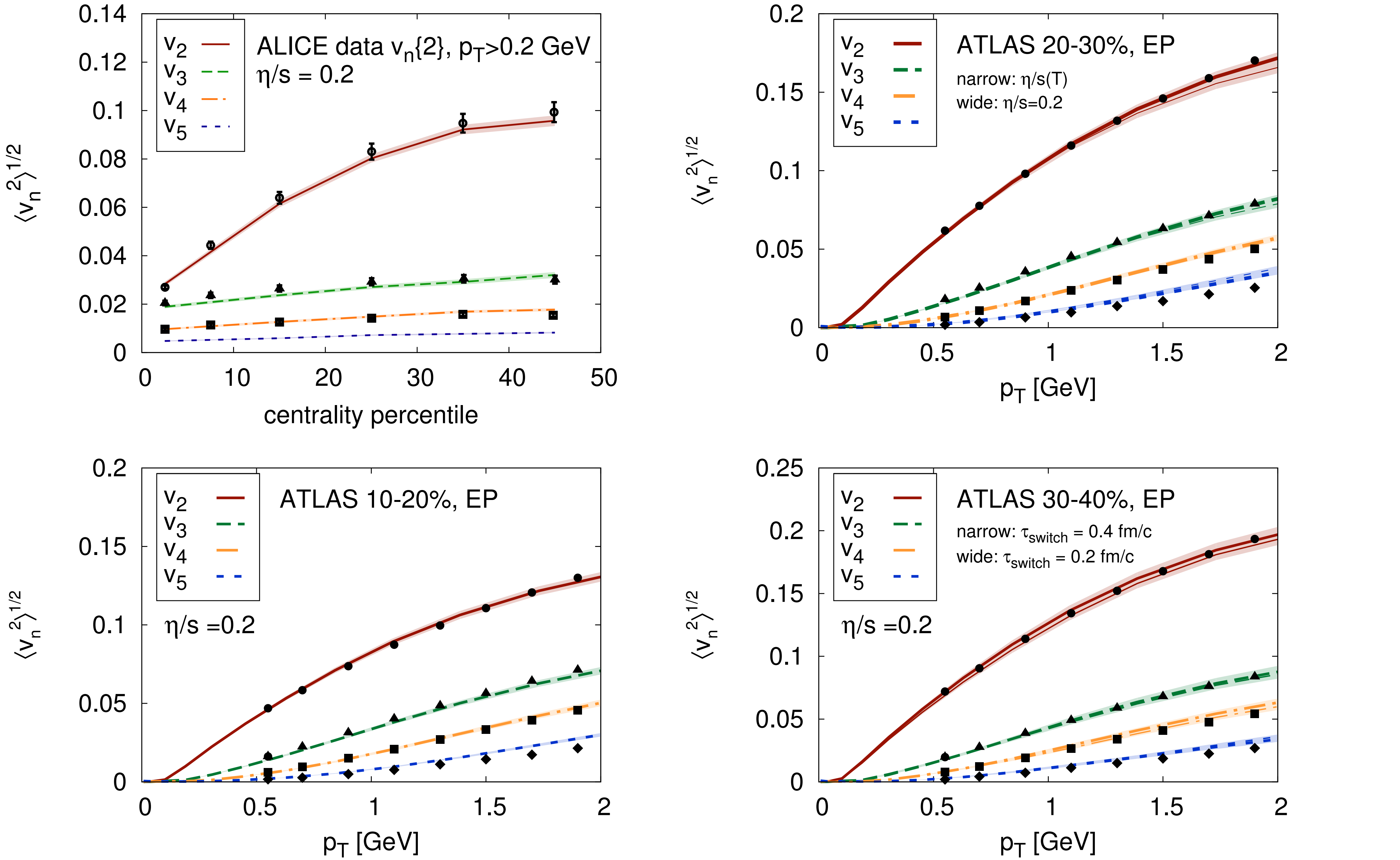
Navier-Stokes hydro, two *transport coefficients*: bulk and shear viscosity

The shear viscosity



- Finite shear viscosity smears out flow differences (diffusion)

Hydro meets data



- Description of initial state also very important
  Gale Jeon Schenke Tribedy Venugopalan **PRL110** (2013)

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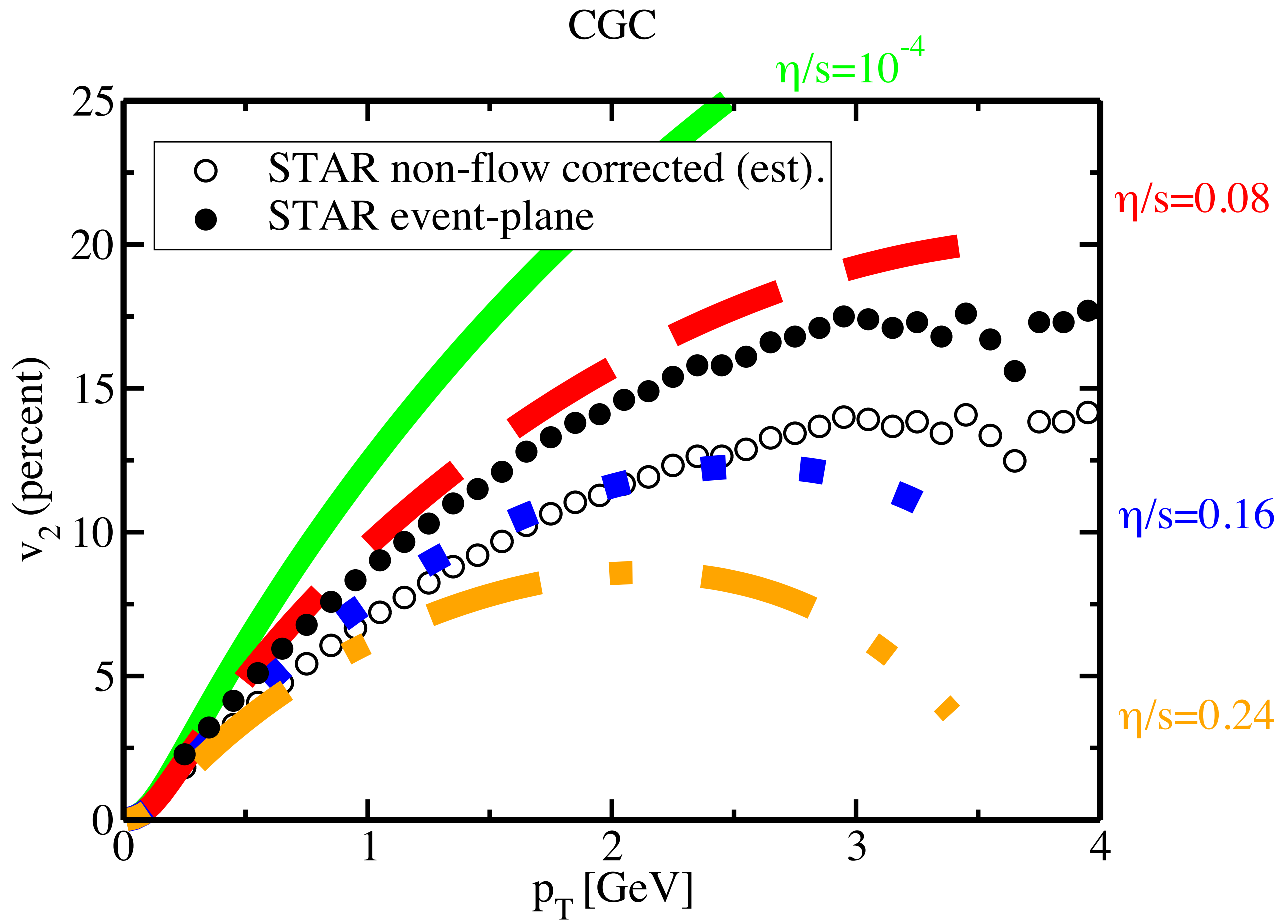
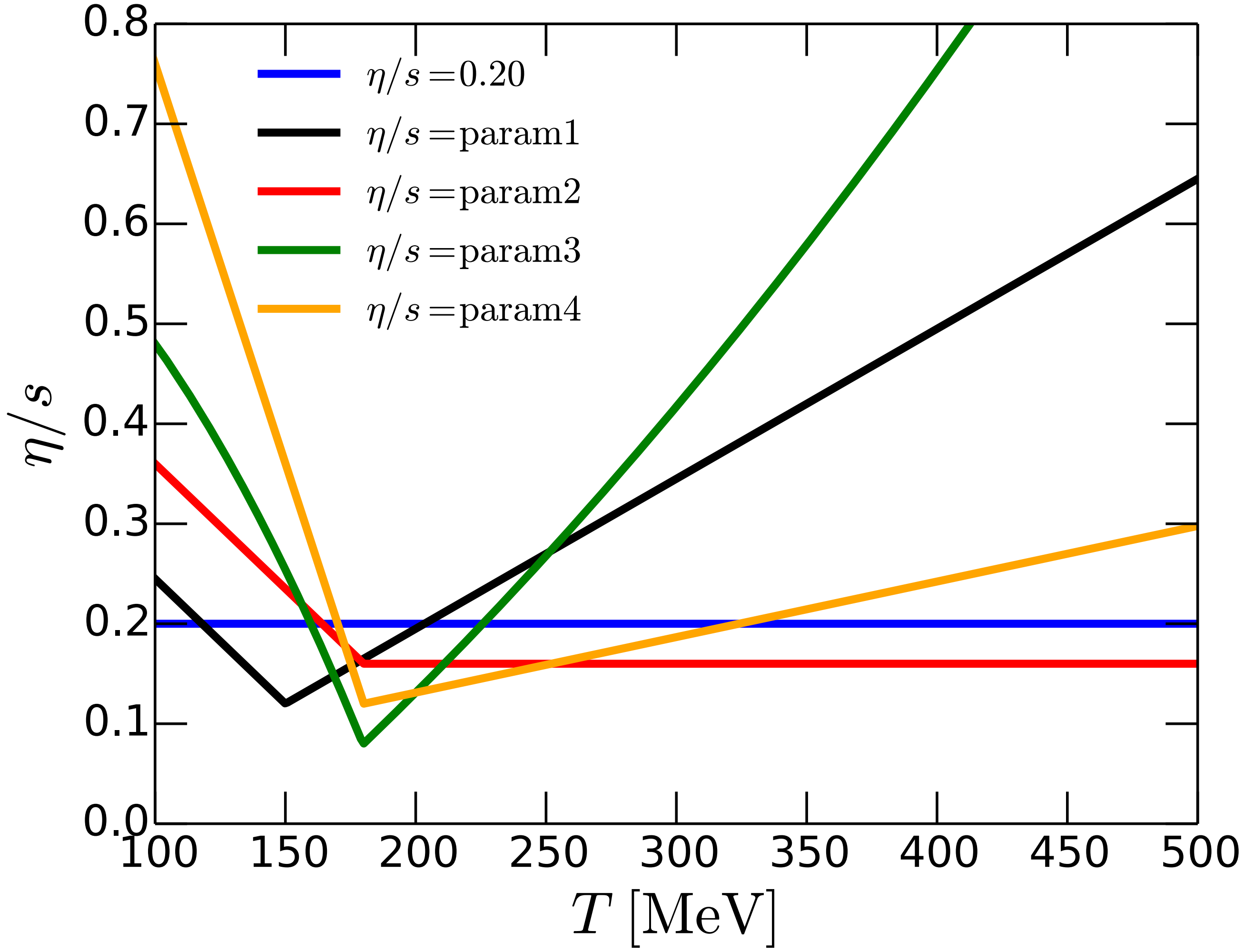
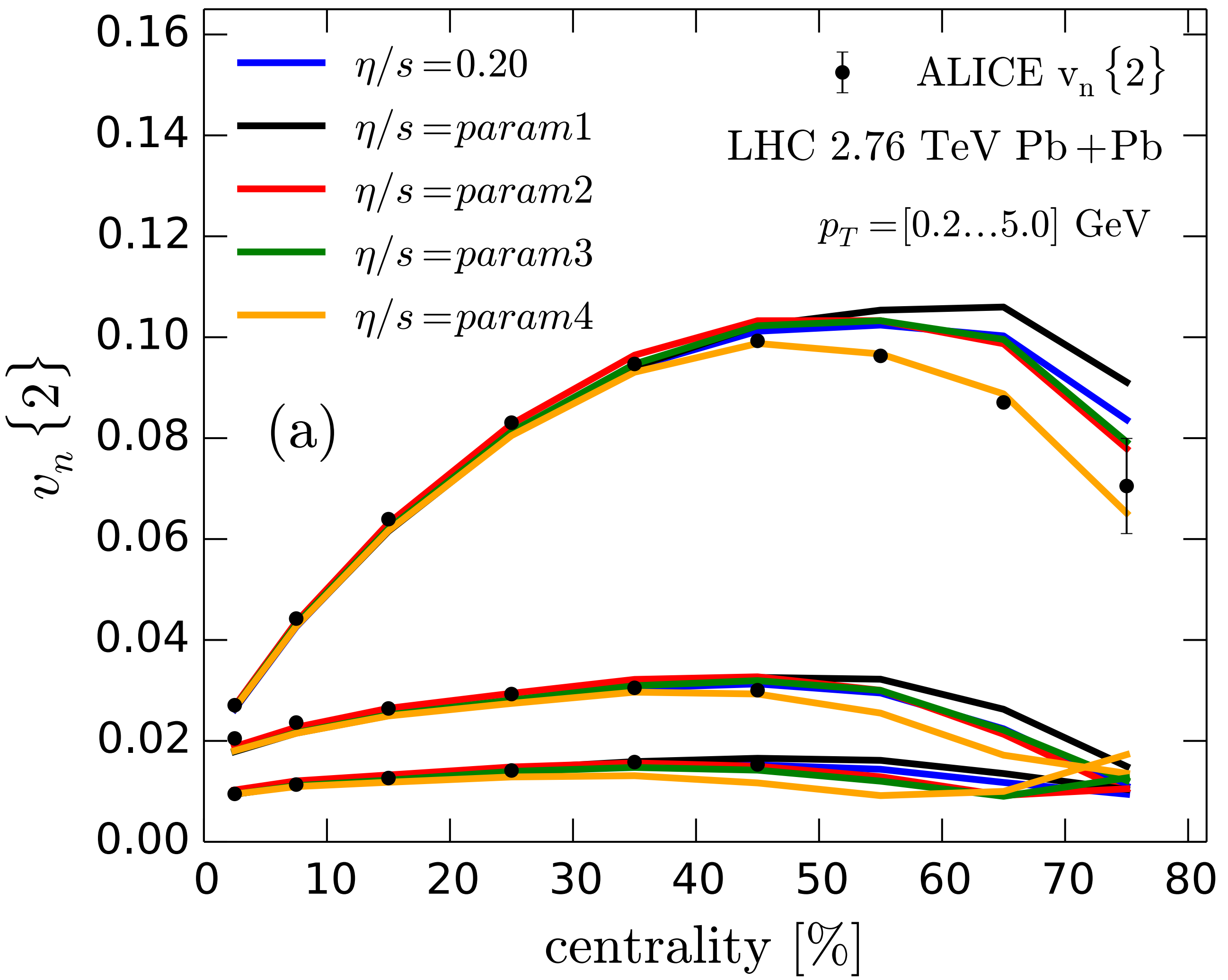


- The shear viscosity, being dissipative, smears out flow differences and makes the position→momentum conversion less efficient

  Plot from Luzum Romatschke **PRC78** (2008)

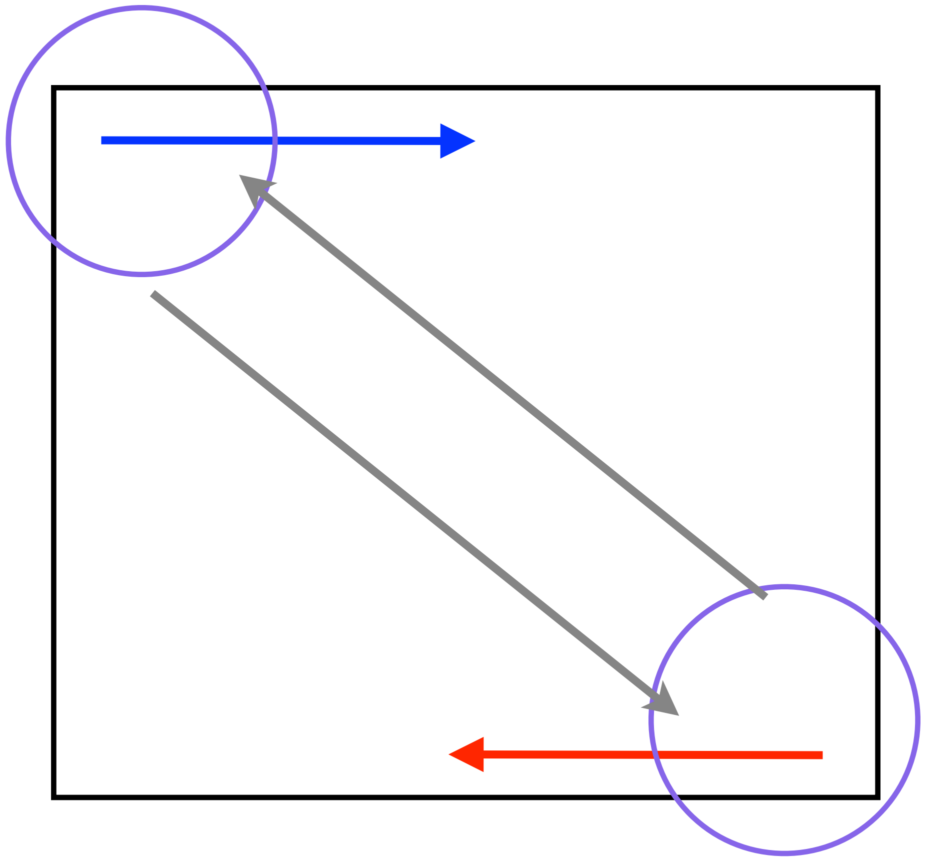
Hydro meets data

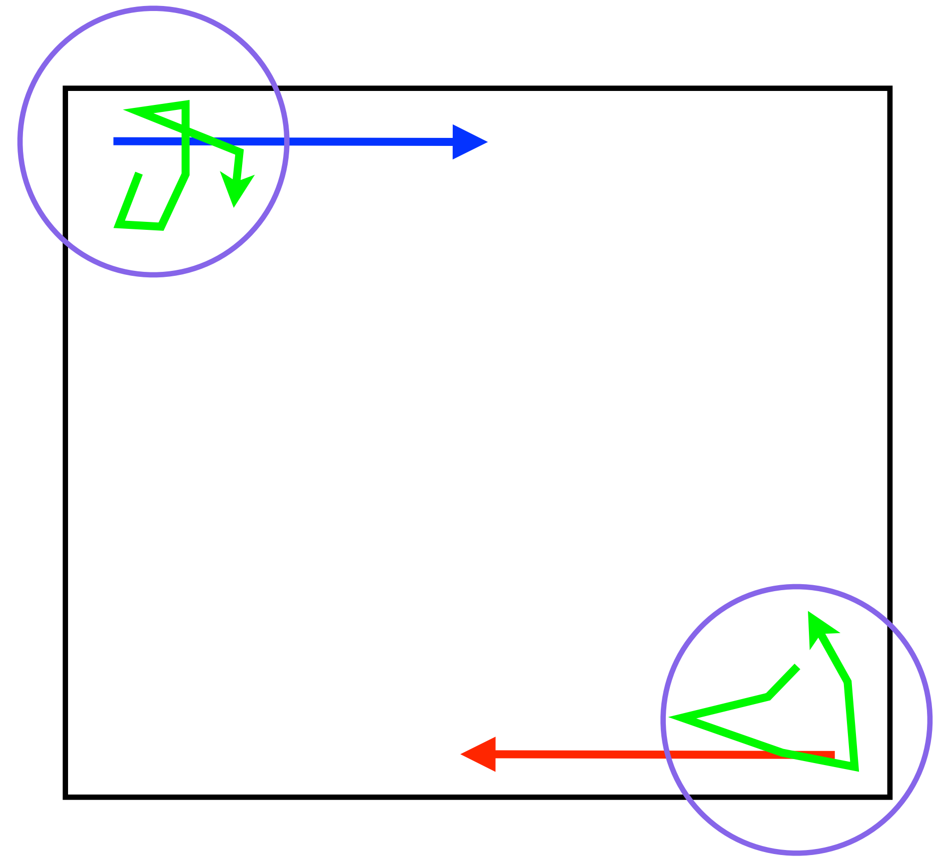




- Current hydro analyses now sensitive to the temperature dependence of the shear viscosity Niemi Eskola Paatelainen **1505.02677**

Estimating η: counterintuitive?





- Weak coupling: long distances between collisions, easy diffusion. **Large** η
- Strong coupling: short distances between collisions, little diffusion. **Small** η

Estimating η
(or why is η/s natural)

- u flow velocity, v_x microscopical velocity of particles

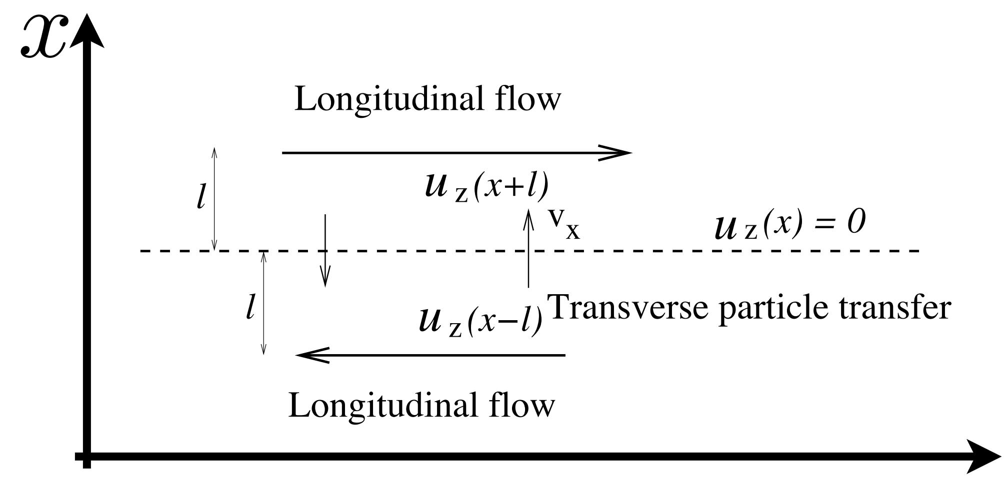


Figure by S. Jeon

- $T^{0z}=(e+P)u^0u^z$ diffuses along x with $v^x=u^x/u^0$. Net change

$$(e+p)v^x u^0(u^z(x-l_{\rm mfp})-u^z(x+l_{\rm mfp})\approx -2(e+p)v^x u^0 l_{\rm mfp}\partial_x u^z(x)\sim -\eta u^0\partial_x u^z(x)$$

- Using $e+p=sT$ and in the high-T limit (v^x~1)

$$\frac{\eta}{s}\sim T l_{\rm mfp}$$

Estimating η
(or why is η/s natural)

- (Mean free path)$^{-1}$~ cross section x density

$$\frac{\eta}{s} \sim T l_{\rm mfp} \sim \frac{T}{n\sigma} \sim \frac{1}{T^2\sigma}$$

- Cross section in a perturbative gauge theory (T only scale*)

$$\sigma \sim \frac{g^4}{T^2} \qquad \frac{\eta}{s} \sim \frac{1}{g^4}$$

* Coulomb divergences and screening scales (m_D~gT) in gauge theories

$$\sigma \sim \frac{g^4}{T^2}\ln(1/g) \qquad \frac{\eta}{s} \sim \frac{1}{g^4\ln(1/g)}$$

- From holography one instead has η/s=1/(4π) (for $\mathcal{N}=4$ SYM) and a conjectured lower limit

  Kovtun Son Starinets Policastro **PRL87** (2001) **PLR94** (2004)

The effective kinetic theory

Theory approaches to transport coefficients and jets

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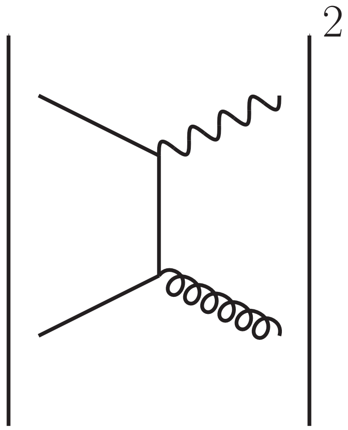

pQCD: QCD action (and EFTs thereof). Can be done both in and out of equilibrium. Real world: extrapolate from $g \ll 1$ to $\alpha_s \sim 0.3$

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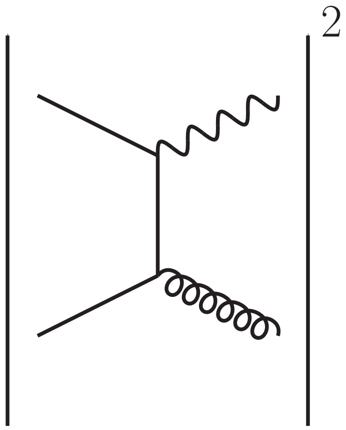

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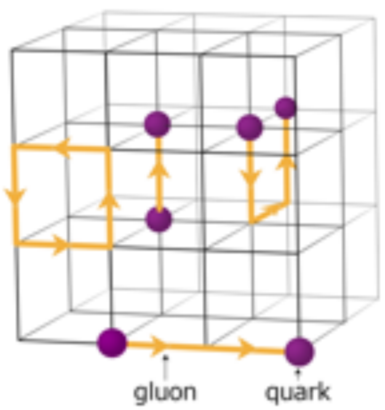

lattice QCD: Euclidean QCD action, equilibrium only. Real world: analytically continue to Minkowskian domain

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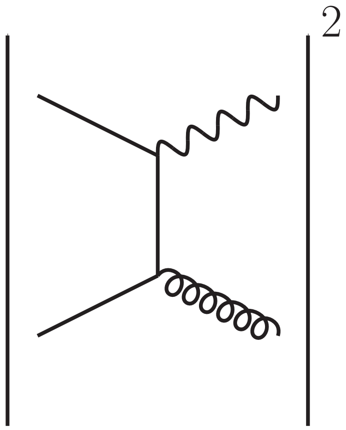

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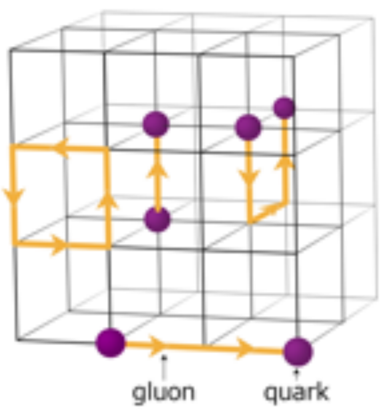

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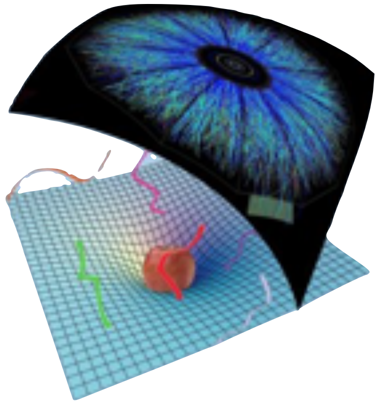


AdS/CFT: $\mathcal{N}=4$ action, in and out of equilibrium, weak and strong coupling. Real world: extrapolate to QCD

The weak-coupling picture

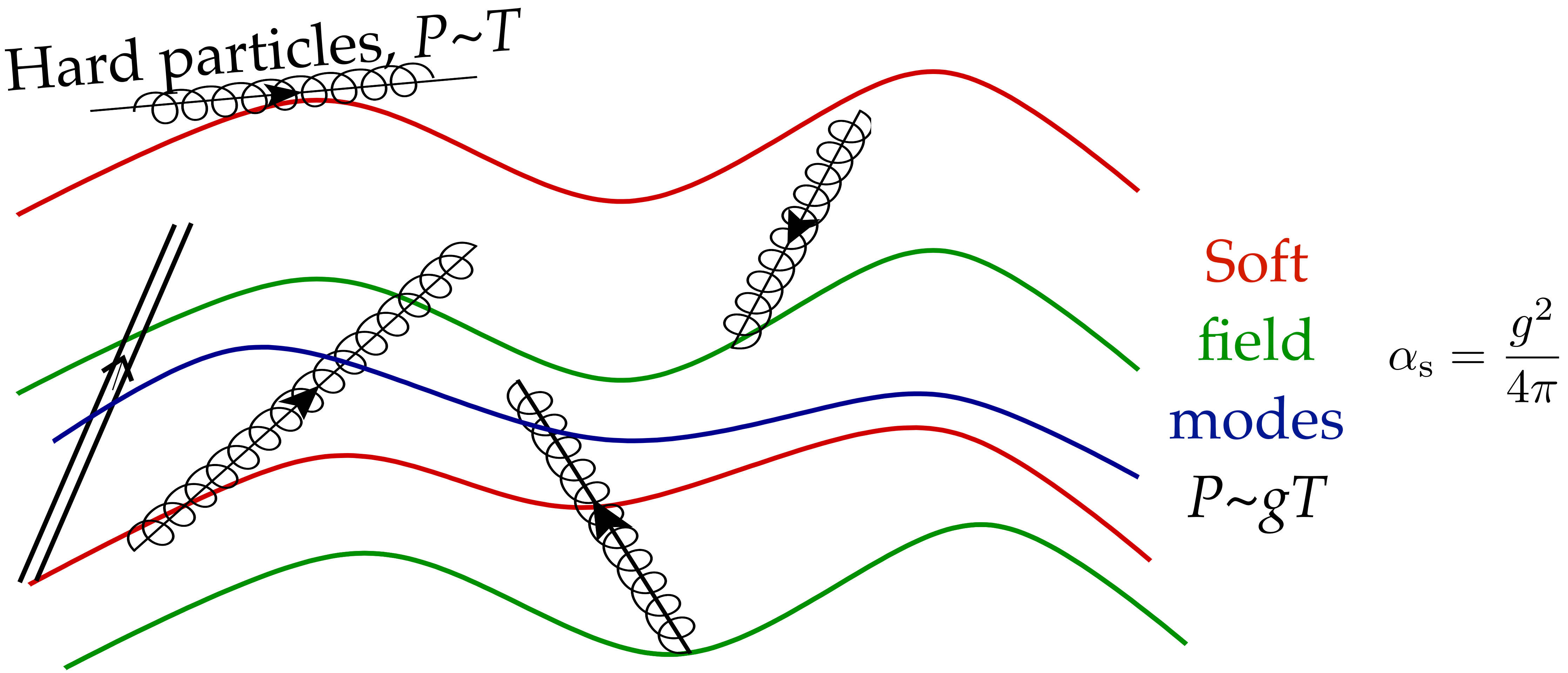


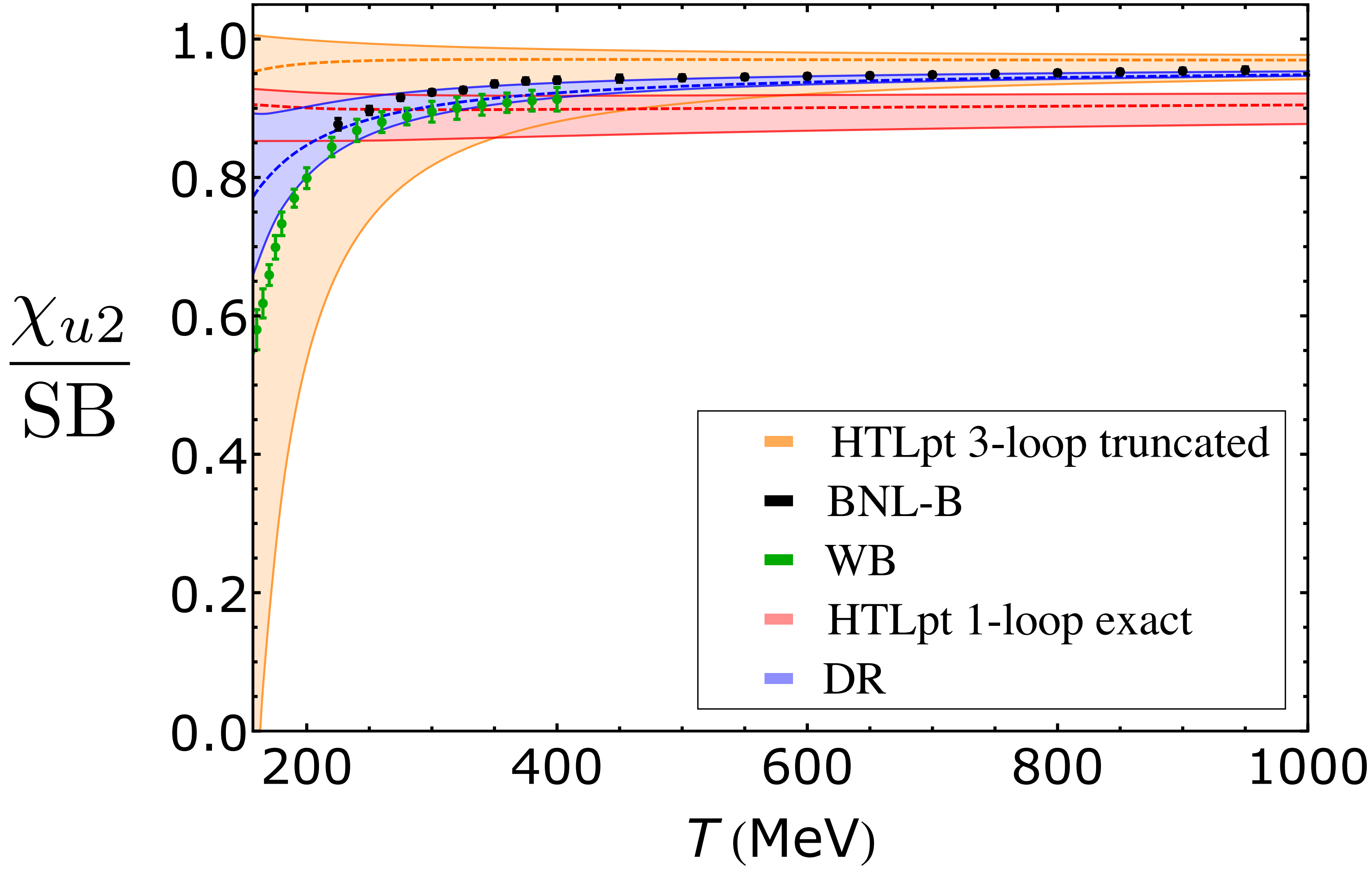
Figure by D. Teaney

- The gluonic soft fields have large occupation numbers ⇒ they can be treated classically

$$n_B(\omega) = \frac{1}{e^{\omega/T} - 1} \overset{\omega \sim gT}{\simeq} \frac{T}{\omega} \sim \frac{1}{g}$$

Weak-coupling thermodynamics

$$\chi_{u2} = \frac{\partial^2 p(T,\mu)}{\partial \mu_u^2}$$



Mogliacci Andersen Strickland Su Vuorinen **JHEP1312** (2013)

- Successful for static (thermodynamical) quantities. Possibility of solving the soft sector non-perturbatively (dimensionally-reduced theory on the lattice)

The effective kinetic theory

Baym Braaten Pisarski Arnold Moore Yaffe Baier Dokshitzer Mueller Schiff Son Peigné Wiedemann Gyulassy Wang Aurenche Gelis Zaraket Blaizot Iancu . . .

The effective kinetic theory

- Justified at weak coupling, but can be extended to factor in non-perturbative contributions (in progress, more later)

- The effective theory is obtained by integrating out (off-shell) quantum fluctuations (for instance from Kadanoff-Baym equations). Appropriate for describing the dynamics of excitations on scales large compared to *1/T*, which is the size of the typical de Broglie wavelength of an excitation.

- Boltzmann equation for the single-particle phase space-distribution: its convective derivative equals a collision operator

$$(\partial_t + \mathbf{v_p} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C[f]$$

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- In other words at weak coupling the underlying QFT has well-defined quasi-particles. These are weakly interacting with a *mean free time ($1/g^4T$) large compared to the actual duration of an individual collision ($1/T$)*

The collision operator

- A modern approach to the (LO) collision operator

$$(\partial_t + \mathbf{v}_\mathbf{p} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C^{\text{large}}[\mu_\perp] + C^{\text{diff}}[\mu_\perp] + C^{\text{coll}}$$

- For illustration purposes, quarks are omitted from the plasma in this talk

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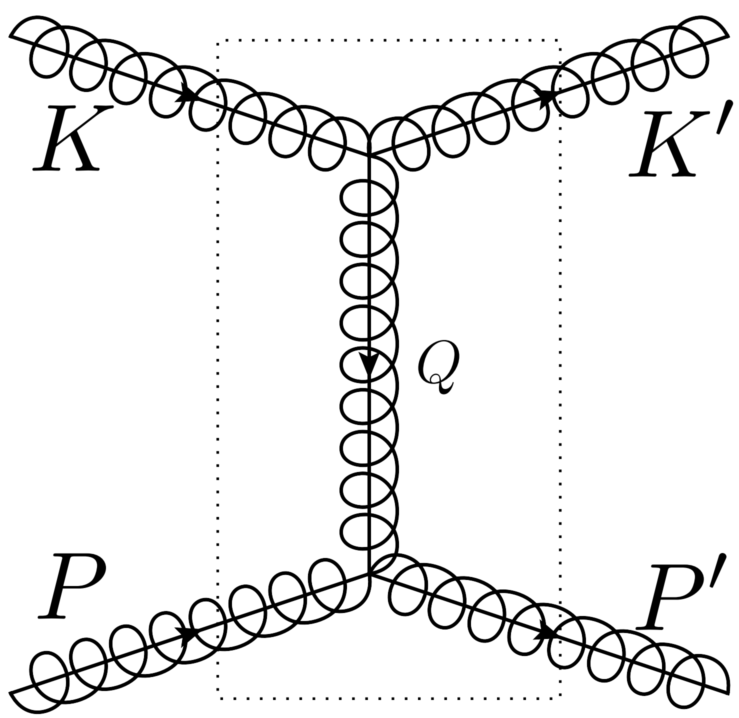
$$C^{\text{large}}[\mu_\perp] = \frac{1}{4p\nu_g}\sum_{bcd}\int_{\mathbf{kp}'\mathbf{k}'}\left|\mathcal{M}^{ab}_{cd}\right|^2(2\pi)^4\delta^{(4)}(P+K-P'-K')\theta(q_\perp-\mu_\perp)$$
$$\times\left\{f_\mathbf{p}f_\mathbf{k}[1+f'_\mathbf{p}][1+f'_\mathbf{k}] - f'_\mathbf{p}f'_\mathbf{k}[1+f_\mathbf{p}][1+f_\mathbf{k}]\right\}$$

- 2↔2 processes with large momentum transfer
- Loss - gain structure
- $Q>gT$, $O(1)$ deflection angles
- Need to exclude the IR with a cutoff $\mu_\perp$
- Logarithmic sensitivity to the cutoff⇒ Can use bare matrix elements



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- How to deal with the soft Q region?
- Older approach: dressing the intermediate propagator with Hard Thermal Loops for IR finiteness
Braaten Pisarski, Arnold Moore Yaffe (AMY)
- Hard Thermal Loops: resummation of 1-loop hard off-shell loops into soft propagators (and vertices). Rich structure

$$m_D^2 = g^2 T^2 (N_c/3 + n_f/6)$$

$$G_R^{00}(\omega, \mathbf{q}) = \frac{i\eta^{00}}{q^2 + \Pi_L(\omega/q)}$$

$$G_R^{ij}(\omega, \mathbf{q}) = \frac{-i(\delta^{ij} - \hat{q}^i \hat{q}^j)}{-(q^0)^2 + q^2 + \Pi_T(\omega/q)}$$

$$\Pi_L = m_D^2 \left(1 - \frac{\omega}{2q} \log\left(\frac{\omega+q}{\omega-q}\right)\right)$$

$$\Pi_T = \frac{m_D^2}{2}\left(\left(\frac{\omega}{q}\right)^2 - \frac{(\omega^2 - q^2)\omega}{2q^3} \log\left(\frac{\omega+q}{\omega-q}\right)\right)$$

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$$(\partial_t + \mathbf{v_p} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C^{\text{large}}[\mu_\perp] + C^{\text{diff}}[\mu_\perp] + C^{\text{coll}}$$

- New approach: **diffusion.** Fokker-Planck drag limit for small Q, with the soft background factored into Wilson-line operators

$$C^{\text{diff}}[\mu_\perp] = \frac{\partial}{\partial p^i}\left[\eta_D(p) p^i f(\mathbf{p})\right] + \frac{1}{2}\frac{\partial^2}{\partial p^i \partial p^j}\left[\left(\hat{p}^i\hat{p}^j \hat{q}_L(\mu_\perp) + \frac{1}{2}(\delta^{ij} - \hat{p}^i\hat{p}^j)\hat{q}(\mu_\perp)\right) f(\mathbf{p})\right]$$

- Three operators:
  - Transverse momentum broadening
  - Longitudinal momentum broadening
  - Drag

Momentum broadening

- In this soft background the lightlike particle experiences a "force"

$$\mathcal{F}^i(x^+) \equiv U^\dagger(x^+, -\infty)\, gF^{i\mu}(x^+)v_\mu\, U(x^+, -\infty)$$

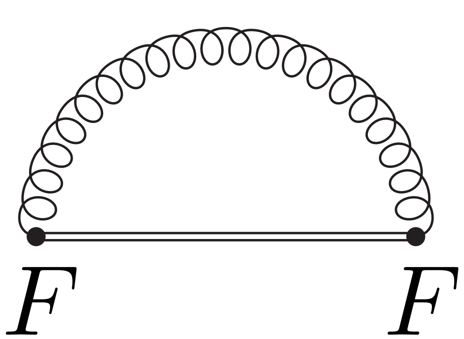
field strength dressed by Wilson lines on the light cone

- Momentum broadening is then given by

$$\hat{q}^{ij} \equiv \frac{1}{d_R}\int_{-\infty}^{+\infty} dt' \,\left\langle \mathcal{F}^i(t)\mathcal{F}^j(0)\right\rangle$$

- Rigorous formulation from SCET possible
Benzke Brambilla Escobedo Vairo **JHEP1302** (2013)

- At leading order: integrals over HTL propagator?



Momentum broadening

$$\mathcal{F}^i(x^+) \equiv U^\dagger(x^+,-\infty)\, gF^{i\mu}(x^+)v_\mu\, U(x^+,-\infty) \qquad \hat{q}^{ij} \equiv \frac{1}{d_R}\int_{-\infty}^{+\infty} dt' \,\langle \mathcal{F}^i(t)\mathcal{F}^j(0)\rangle$$

- Breakthrough over the past ~10 years. Heuristically, the hard, light-like parton sees undisturbed soft modes, which *"can't keep up"* with it (up to $O(g^2)$ suppressed collinear effects)
- Mathematically, this translates into analytical properties of retarded and advanced correlators at light-like momenta
- In transverse diffusion: dimensional reduction becomes applicable

$$\hat{q}(\mu_\perp) = g^2 C_A \int^{\mu_\perp} \frac{d^2q_\perp}{(2\pi)^2}\int \frac{dq^+}{2\pi}\langle F^{-\perp}(Q)F^{-}{}_{\perp}\rangle_{q^-=0}$$
$$= g^2 C_A T \int^{\mu_\perp} \frac{d^2q_\perp}{(2\pi)^2} q_\perp^2 \left(\frac{1}{q_\perp^2} - \frac{1}{q_\perp^2 + m_D^2}\right) = \frac{g^2 C_A T m_D^2}{2\pi}\ln\frac{\mu_\perp}{m_D}$$

Caron-Huot **PRD79** (2008)

Momentum broadening

$$\mathcal{F}^i(x^+) \equiv U^\dagger(x^+,-\infty)\, gF^{i\mu}(x^+)v_\mu\, U(x^+,-\infty) \qquad \hat{q}^{ij} \equiv \frac{1}{d_R}\int_{-\infty}^{+\infty} dt' \,\langle \mathcal{F}^i(t)\mathcal{F}^j(0)\rangle$$

- Breakthrough over the past ~10 years. Heuristically, the hard, light-like parton sees undisturbed soft modes, which *"can't keep up"* with it (up to $O(g^2)$ suppressed collinear effects)
- Mathematically, this translates into analytical properties of retarded and advanced correlators at light-like momenta
- In longitudinal diffusion: sensitive only to $\omega \approx q \gg gT$ dispersion relation $\omega^2 - q^2 - m_\infty^2 = 0, \quad m_\infty^2 = m_D^2/2$

$$\hat{q}_L(\mu_\perp) = g^2 C_A \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-z}(Q)F^{-z}\rangle_{q^-=0}$$

$$= g^2 C_A T \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \left(1 - \frac{q_\perp^2}{q_\perp^2 + m_\infty^2}\right) = \frac{g^2 C_A T m_\infty^2}{2\pi} \ln\frac{\mu_\perp}{m_D}$$

JG Moore Teaney

The collision operator

- A modern approach to the (LO) collision operator

$$(\partial_t + \mathbf{v_p} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C^{\text{large}}[\mu_\perp] + C^{\text{diff}}[\mu_\perp] + C^{\text{coll}}$$

$$C^{\text{diff}}[\mu_\perp] = \frac{\partial}{\partial p^i}\left[\eta_D(p) p^i f(\mathbf{p})\right] + \frac{1}{2}\frac{\partial^2}{\partial p^i \partial p^j}\left[\left(\hat{p}^i\hat{p}^j \hat{q}_L(\mu_\perp) + \frac{1}{2}(\delta^{ij} - \hat{p}^i\hat{p}^j)\hat{q}(\mu_\perp)\right) f(\mathbf{p})\right]$$

- Drag: related by Einstein-like relation to momentum broadening

$$\eta_D(p) = \frac{\hat{q}_L}{2Tp} + \mathcal{O}\left(\frac{1}{p^2}\right)$$

- In the end, cutoff dependence vanishes between diffusion and large-angle scatterings

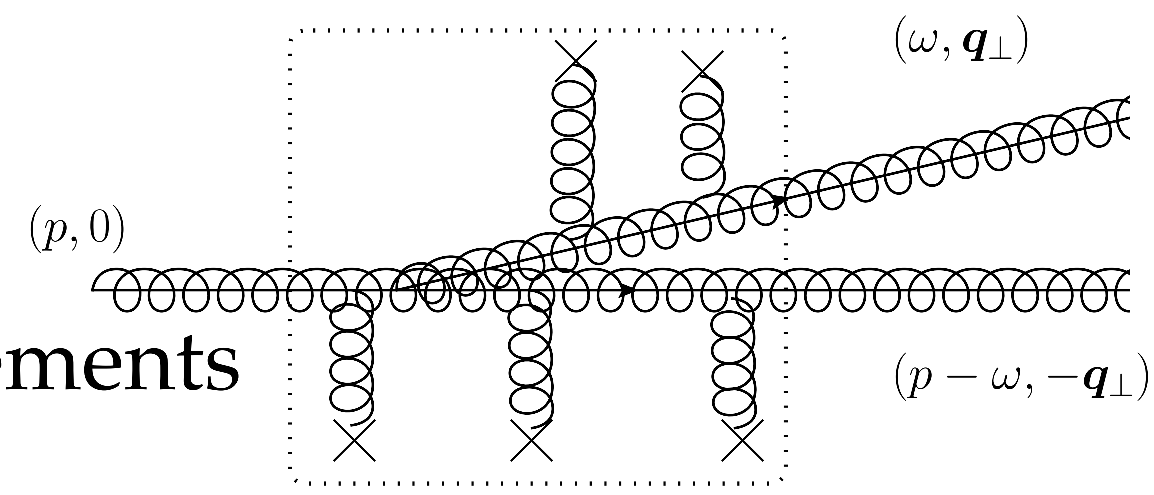
The collision operator

- A modern approach to the (LO) collision operator

$$(\partial_t + \mathbf{v_p} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C^{\rm large}[\mu_\perp] + C^{\rm diff}[\mu_\perp] + C^{\rm coll}$$

- Collinear splitting/joining induced by soft scatterings with the medium constituents

- Apparently suppressed by powers of g but
    - Soft and collinear enhancements cancel the suppression



    - Mean free time between soft collisions ($1/g^2T$) of the same order of formation time ⇒ interference of many such scatterings (Landau-Pomeranchuk-Migdal effect)
      Baier Dokshitzer Mueller Schiff Son Zakharov Arnold Moore Yaffe

The EKT and jets

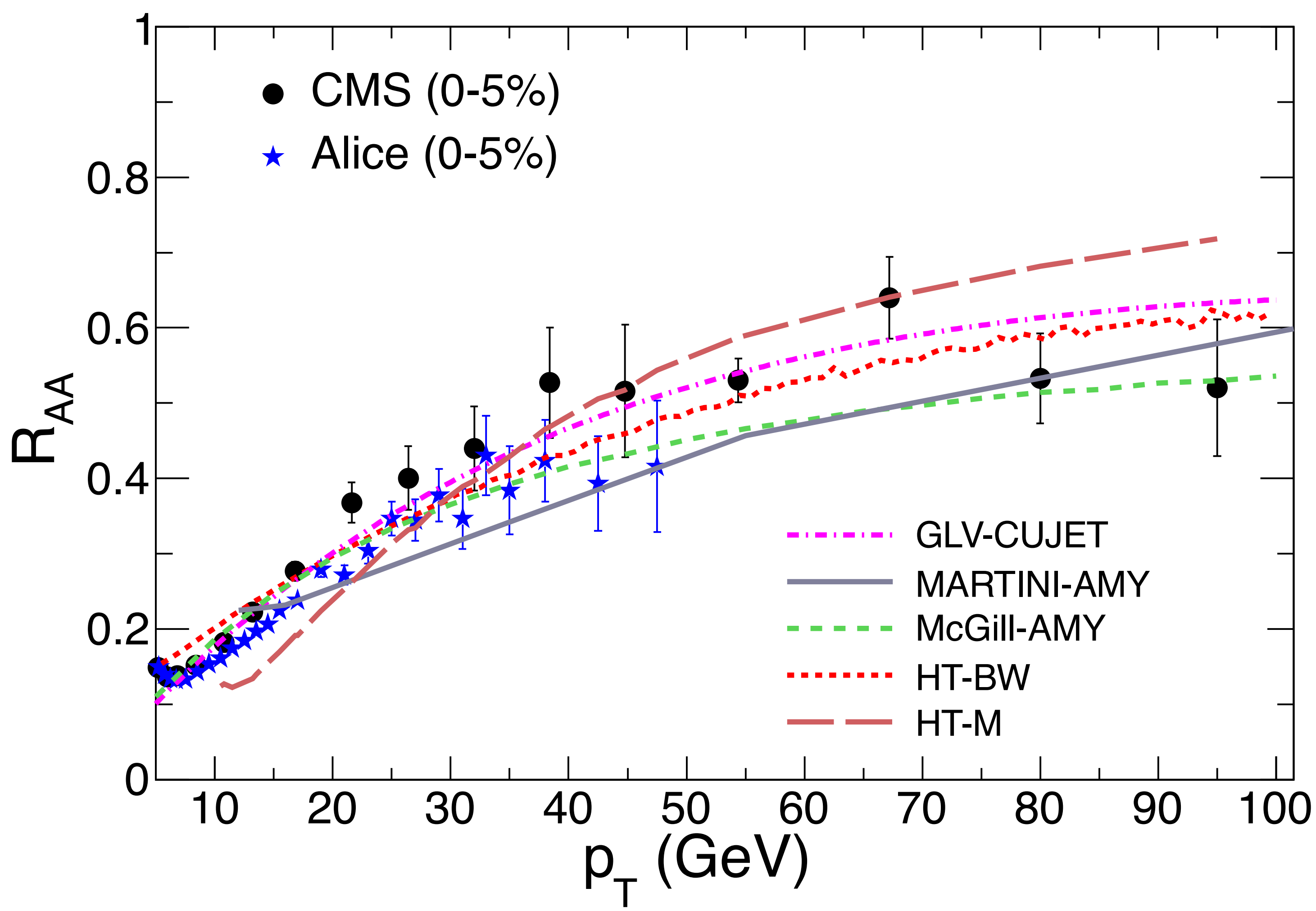
$$(\partial_t + \mathbf{v}_\mathbf{p} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C^{\mathrm{large}}[\mu_\perp] + C^{\mathrm{diff}}[\mu_\perp] + C^{\mathrm{coll}}$$

- Study how the distribution of high-energy partons $f(\mathbf{p})$ evolves by interacting with a (locally) equilibrated medium
- Leading order implemented in MARTINI
  Schenke Gale Jeon (2009)
- Kinetic picture applicable at later stages of the HIC, when the virtuality of the jet has been reduced by vacuum-like radiation. Higher twist formalism used in the community to deal with earlier stages under the influence of a medium
- Future plans: extend the kinetic picture in that direction

Jet quenching

- One of the main results of the HIC program: jets are suppressed with respect to proton-proton collisions
- Quantitatively: look at deviations from binary scaling

$$R_{AA} = \frac{\text{Yield}_{AA}}{\text{Yield}_{pp} \times N_{bin}}$$



JET Collaboration

The EKT and transport

$$(\partial_t + \mathbf{v_p} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C^{\rm large}[\mu_\perp] + C^{\rm diff}[\mu_\perp] + C^{\rm coll}$$

- The stress-energy tensor the hydrodynamic limit and in the kinetic theory is

$$T^{ij} = (p - \zeta \nabla \cdot \mathbf{v})\delta^{ij} - \eta \left(\partial_i v^j + \partial_j v^i - \frac{2}{3}\delta^{ij} \nabla \cdot \mathbf{v} \right) \qquad T^{ij} = \int \frac{d^3p}{(2\pi)^3} \frac{p^i p^j}{p} f(\mathbf{p})$$

- Linearize the EKT around local equilibrium and solve for the non-eq. part under the source given by the perturbed local equilibrium ⇒ numerical inversion of the collision operator

$$f(\mathbf{p}, \mathbf{x}, t) = f_{\rm eq}(p, \mathbf{x}, t) + f^{(1)}(\mathbf{p}, \mathbf{x}, t)$$

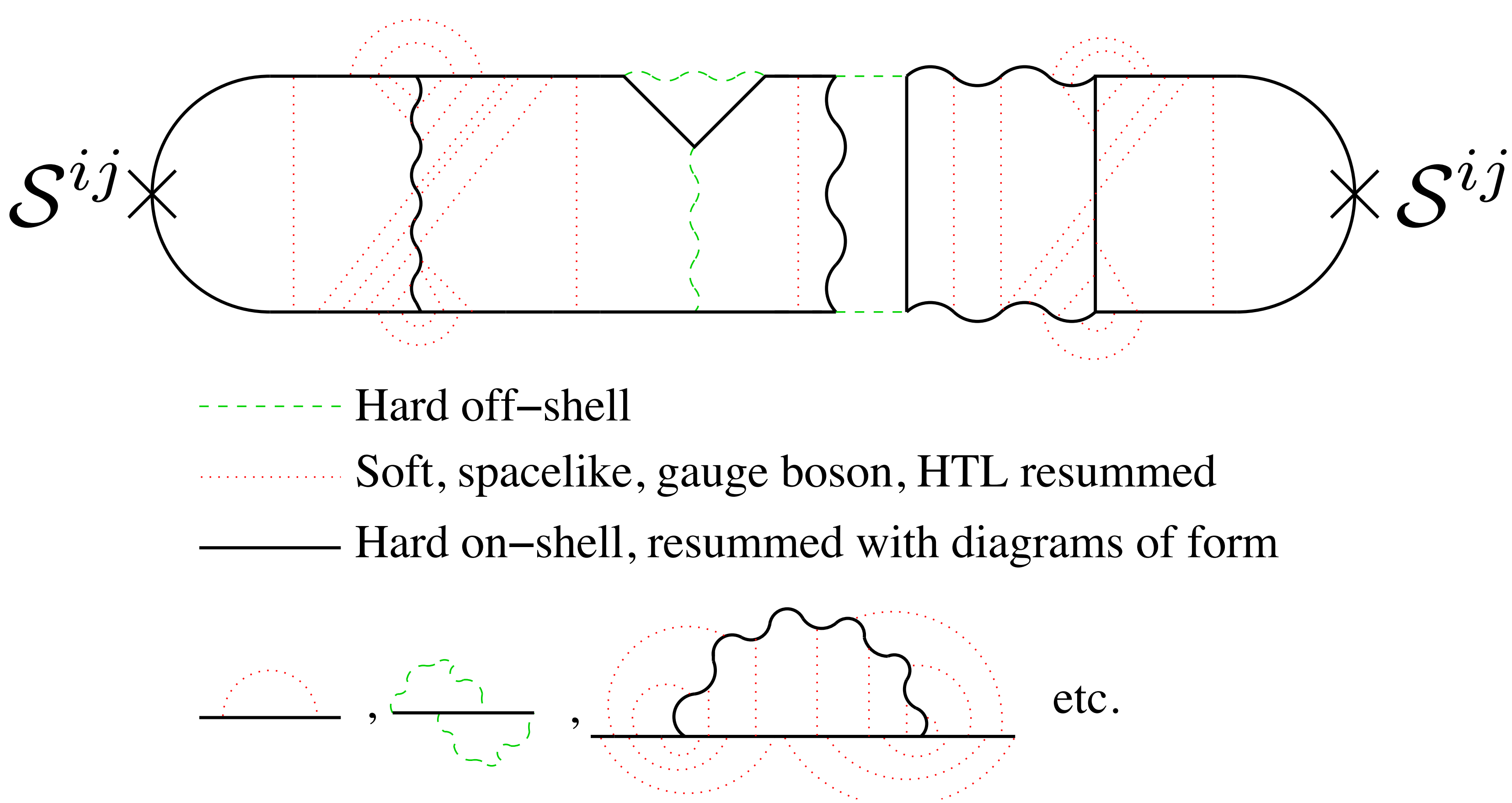
LO results (shown later) in Arnold Moore Yaffe (AMY) 2000-2003

The EKT and transport

- Linearized EKT equivalent to Kubo formula ($\mathcal{S}$ TT part of T)

$$\eta = \frac{1}{20}\lim_{\omega\to 0}\frac{1}{\omega}\int d^4x\, e^{i\omega t}\left\langle\left[\mathcal{S}^{ij}(t,\mathbf{x}),\, \mathcal{S}^{ij}(0,\mathbf{0})\right]\right\rangle\theta(t)$$

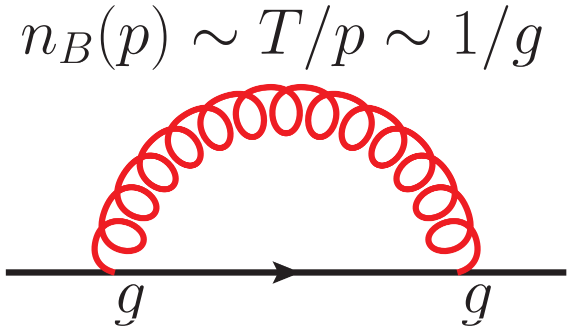
- Not practical at weak coupling: loop expansion breaks down AMY (2000-2003)



Going to NLO

Sources of NLO corrections

- As usual in thermal field theory, the soft scale gT introduces NLO $O(g)$ corrections

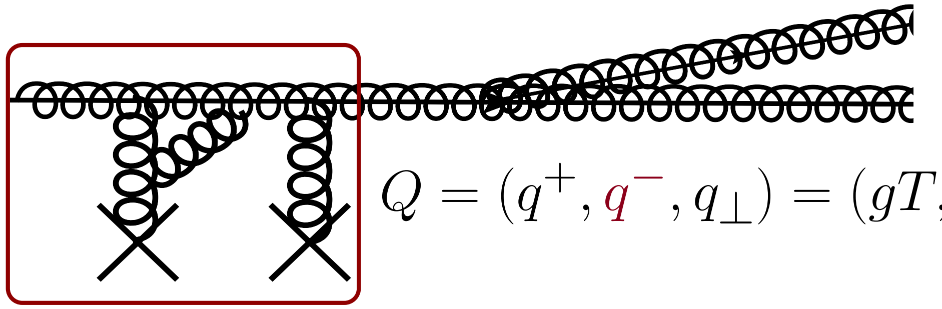


- The diffusion and the collinear regions receive $O(g)$ corrections
- There is a new semi-collinear region

Collinear corrections

- The differential eq. for LPM resummation gets correction from NLO *C($q_\perp$)* and from the thermal asymptotic mass at NLO (Caron-Huot 2009)

$$\mathcal{C}_{\rm LO}(q_\perp) = \frac{g^2 C_A T m_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)}$$



$$Q = (q^+, q^-, q_\perp) = (gT,\ g^2T,\ gT)$$

$\mathcal{C}_{\rm NLO}(q_\perp)$ complicated but analytical (Euclidean tech)
Caron-Huot **PRD79** (2009)

- Now possible to compute it on the lattice too!
Panero Rummukainen Schäfer **PRL112** (2013)

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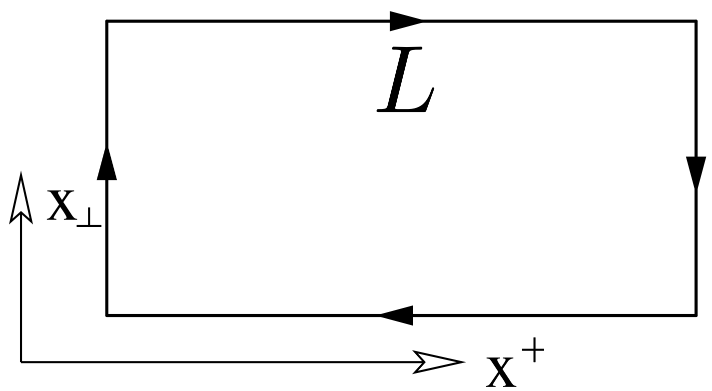
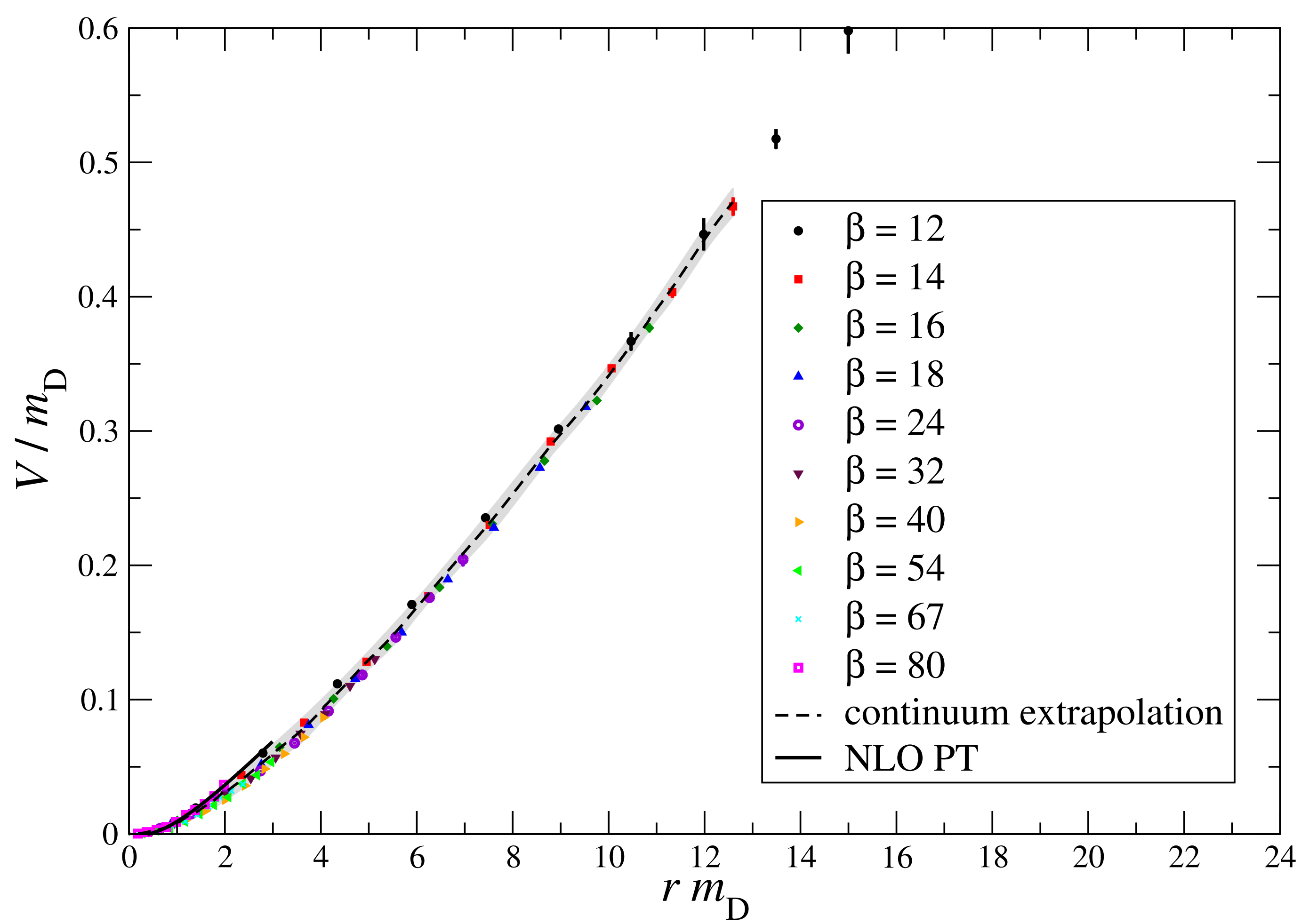


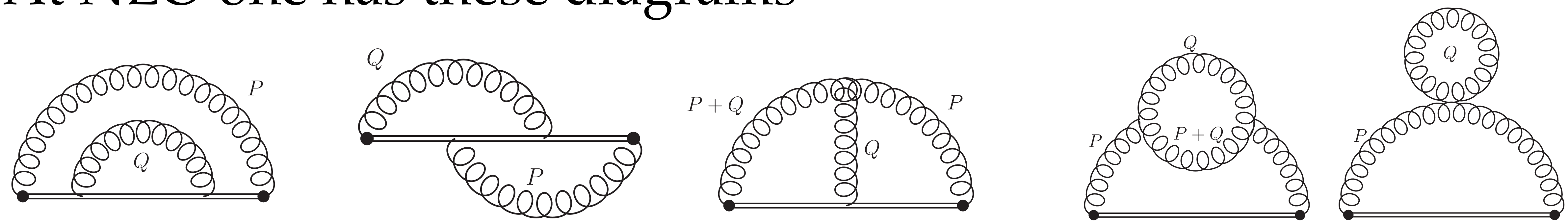
$$\propto e^{-V(r)L}$$

$$V(r) = \int \frac{d^2 q_\perp}{(2\pi)^2} \left(1 - e^{i\mathbf{q}\cdot\boldsymbol{r}}\right) C(q_\perp)$$



Diffusion corrections

- At NLO one has these diagrams



- For transverse: Euclidean calculation Caron-Huot **PRD79** (2009)

$$\hat{q}_{\rm NLO} = \hat{q}_{\rm LO} + \frac{g^4 C_A^2 T^3}{32\pi^2}\frac{m_D}{T}\left(3\pi^2 + 10 - 4\ln 2\right)$$

- For longitudinal:

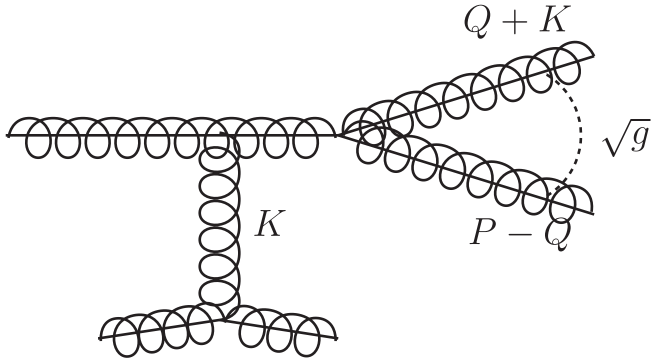
$$\hat{q}_L(\mu_\perp)_{\rm LO} = g^2 C_A T\int\frac{d^2q_\perp}{(2\pi)^2}\frac{m_\infty^2}{q_\perp^2+m_\infty^2}$$

$$\hat{q}_L(\mu_\perp)_{\rm NLO} = g^2 C_A T\int\frac{d^2q_\perp}{(2\pi)^2}\frac{m_\infty^2+\delta m_\infty^2}{q_\perp^2+m_\infty^2+\delta m_\infty^2} \approx g^2 C_A T\int\frac{d^2q_\perp}{(2\pi)^2}\left[\frac{m_\infty^2}{q_\perp^2+m_\infty^2}+\boxed{\frac{q_\perp^2\,\delta m_\infty^2}{(q_\perp^2+m_\infty^2)^2}}\right]$$

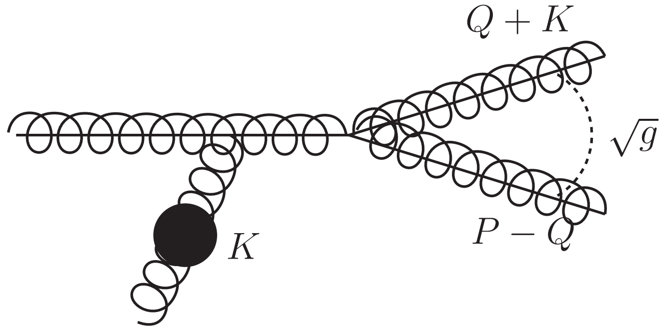
light-cone sum rule still sees only dispersion relation (with $O(g)$ correction). NLO correction UV-log sensitive

Semi-collinear processes

- Seemingly different processes boiling down to wider-angle radiation



K soft cut, spacelike



K soft plasmon, timelike

- Evaluation: introduce *"modified $\hat{q}$"* tracking the changes in the small light-cone component p^- of the gluons. Can be evaluated in EQCD

"standard" $$\hat{q} = g^2 C_A \int \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^-{}_\perp \rangle_{q^-=0}$$

"modified" $$\hat{q}(\delta E) = g^2 C_A \int \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^-{}_\perp \rangle_{q^-=\delta E}$$

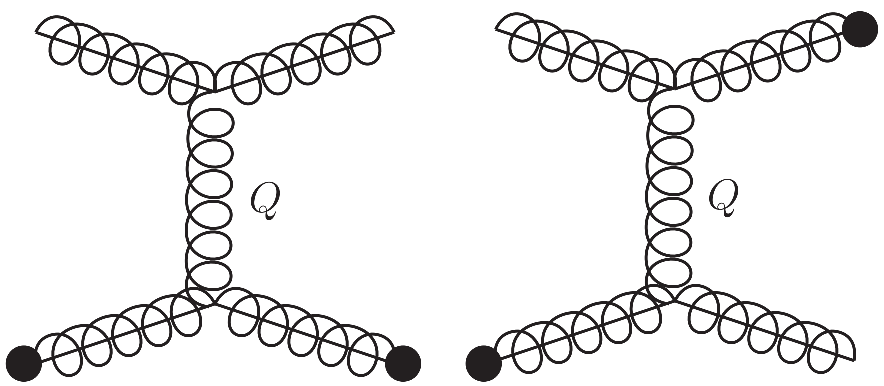
- Rate ∝ *"modified $\hat{q}$"* x DGLAP splitting. IR log divergence makes collision operator finite at NLO

A missing subtlety

- Computing transport coefficients (η) requires knowing how a T^{ij} disturbance induces a second T^{ij} disturbance

- The challenge is again in the soft regions

T^{ij} insertions on the same side, momenta correlated. **Diffusion picture applies**



T^{ij} insertions on opposite sides, momenta uncorrelated. **Diffusion picture does not apply**

- No diffusion picture = no "easy" light-cone sum rules, only bruteforce HTL. Silver lining: they're finite, so just estimate the number and vary it. NLO test ansatz: LO cross x m_D/T ($\sim g$) x arbitrary constant that we vary

$$C^{\rm cross}_{\rm NLO} = C^{\rm cross}_{\rm LO} \times \frac{m_D}{T} \times c_{\rm cross}$$

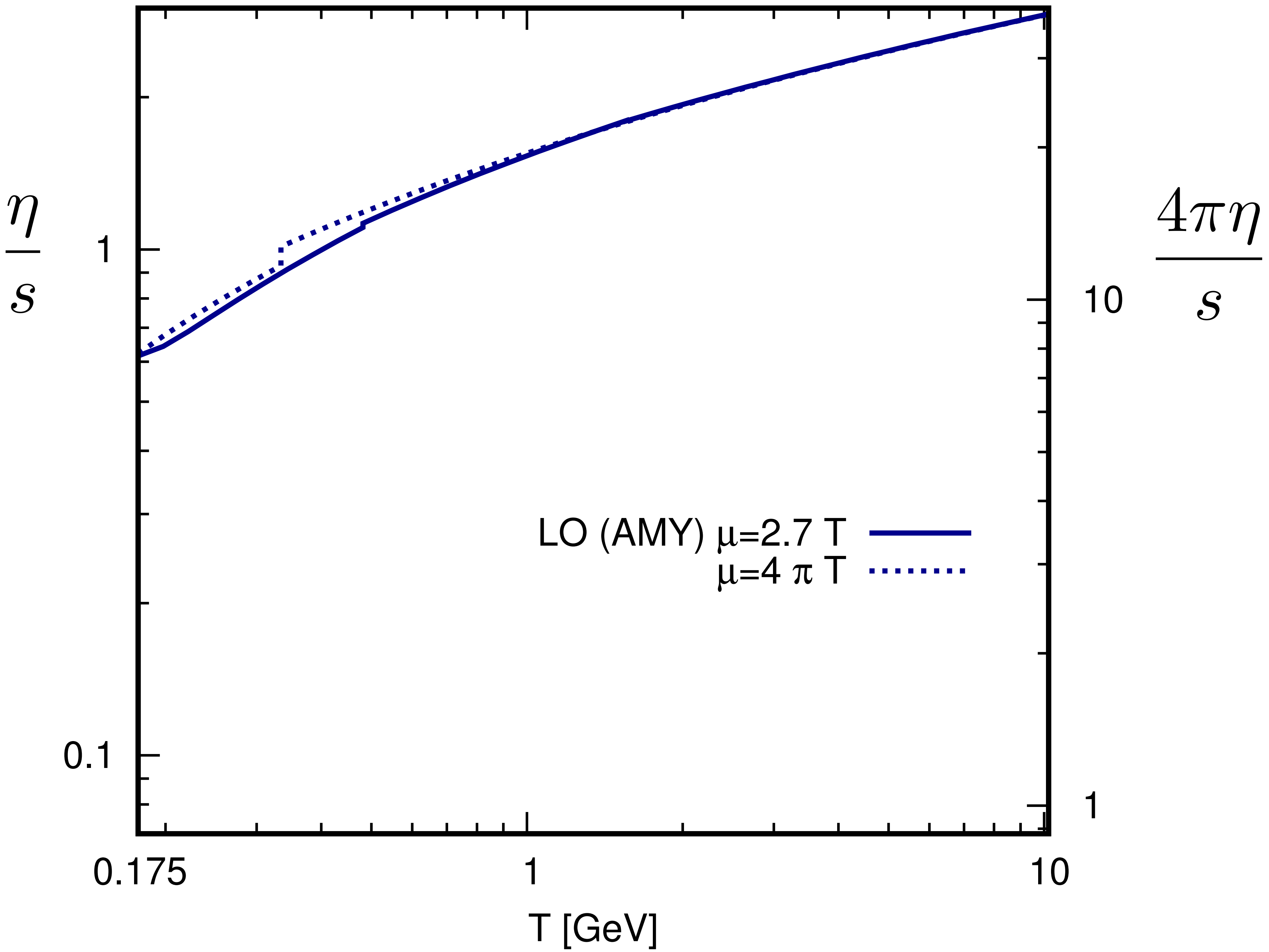
Results

Results

- Inversion of the collision operator using variational Ansatz
- At NLO just add $O(g)$ corrections to the LO collision operator, do not treat them as perturbations in the inversion
- Kinetic theory with massless quarks still conformal to NLO
- Relate parameter $m_D/T \sim g$ to temperature through two-loop $g(T)$ as in Laine Schröder **JHEP0503** (2005)
  - Degree of arbitrariness in the choice of quark mass thresholds, test several values of μ/T

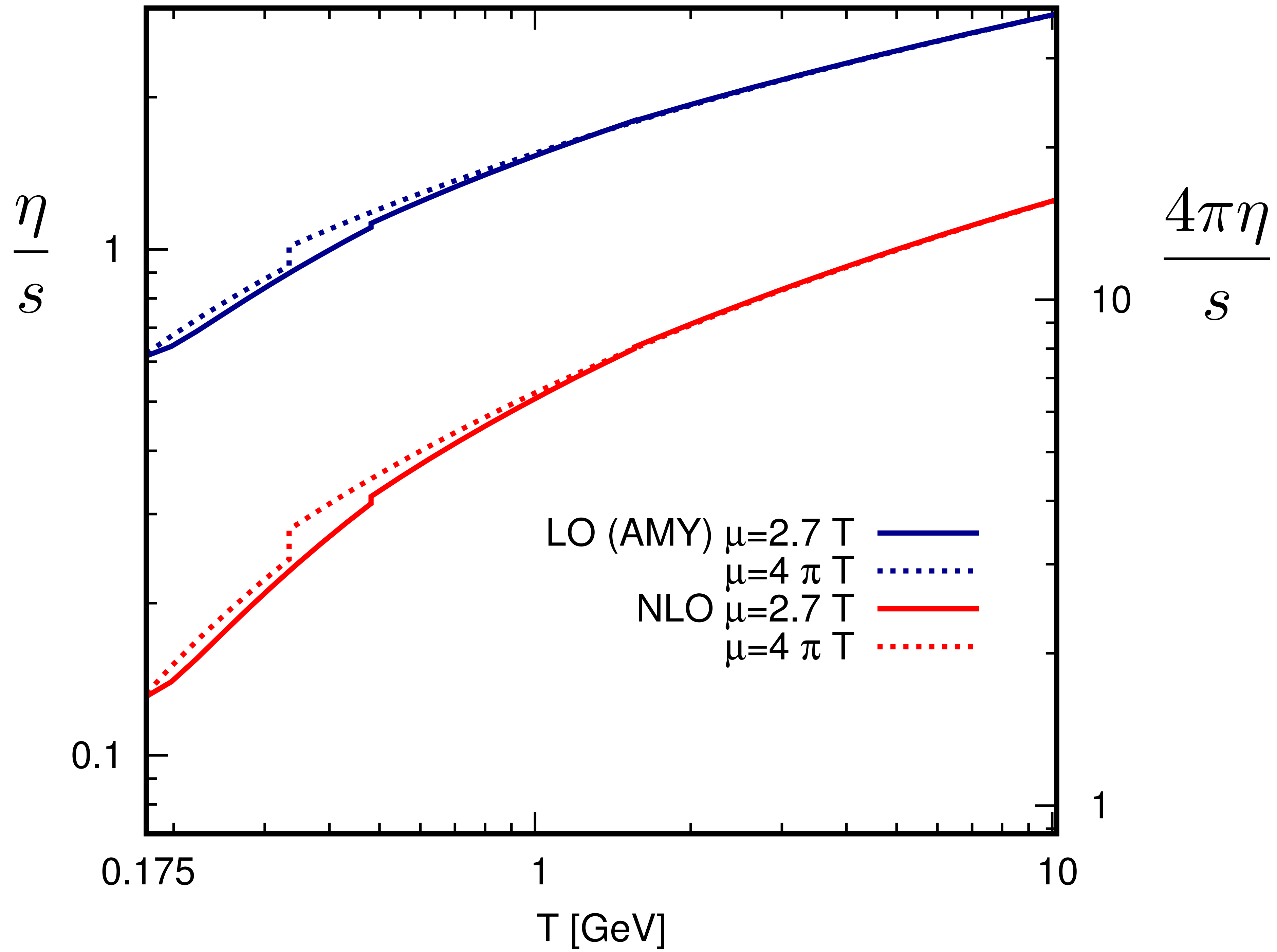
JG Moore Teaney, soon

η/s(*T*) of QCD



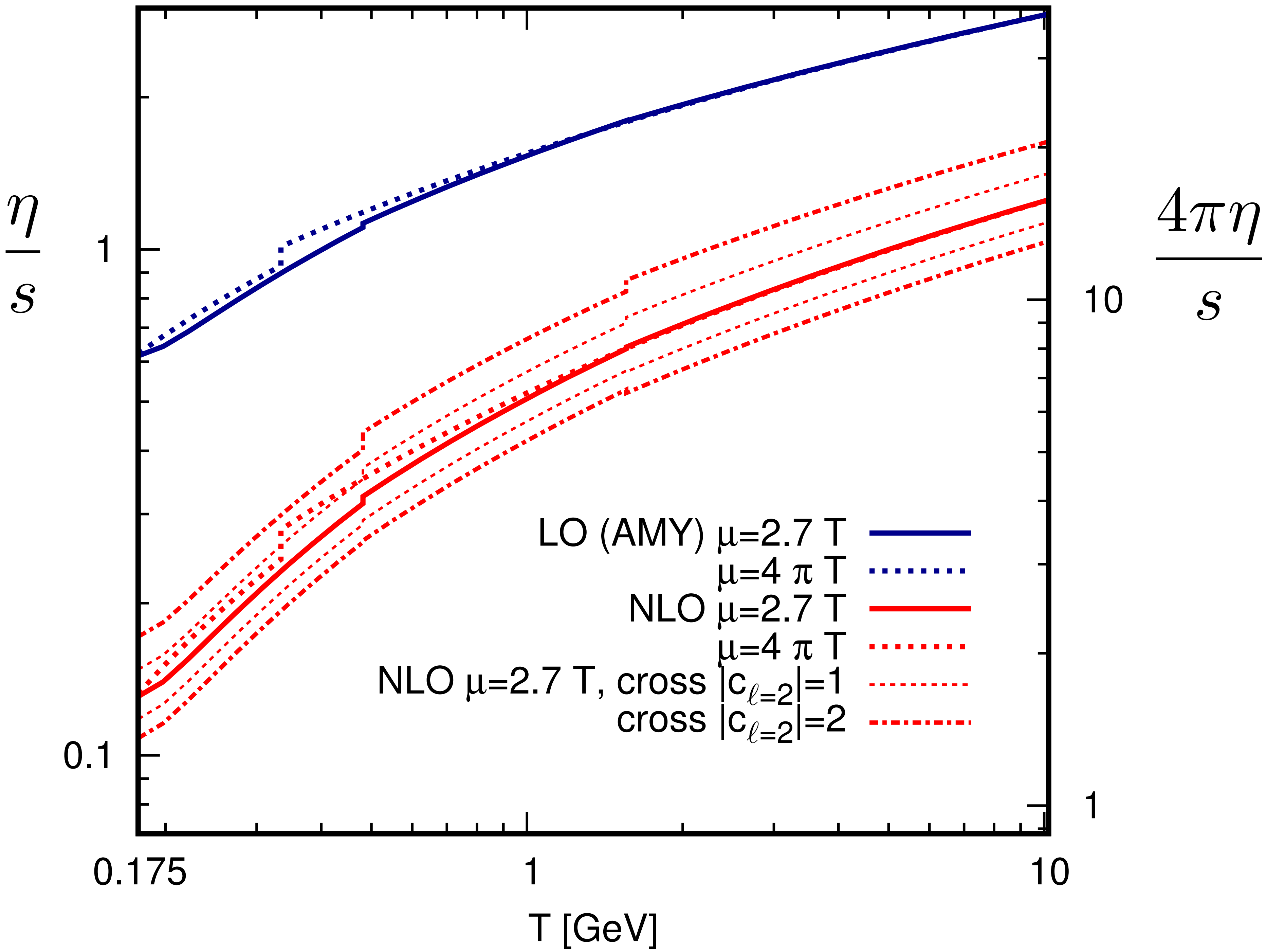
- LO results from AMY (2003)

η/s(*T*) of QCD



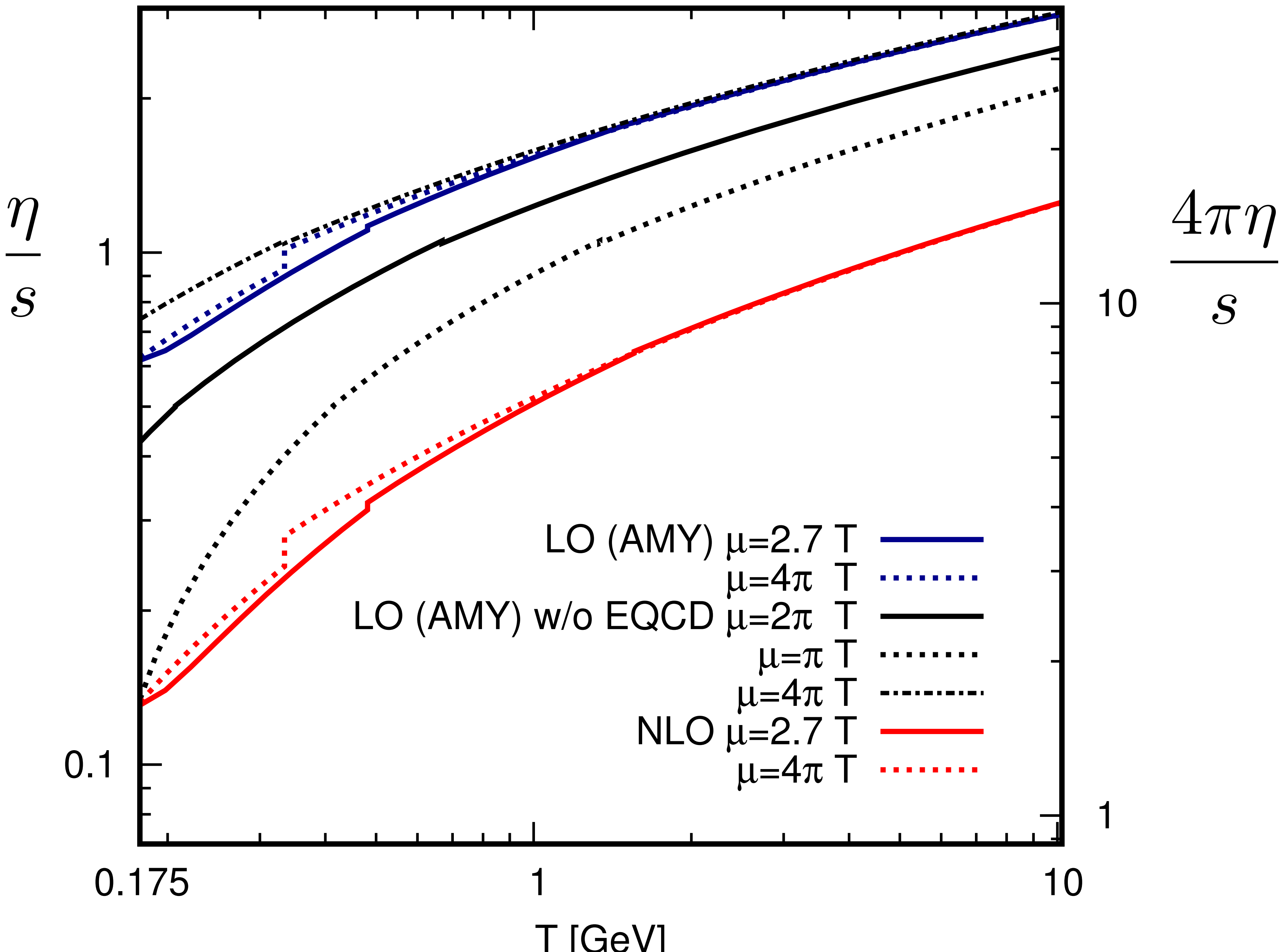
- All known NLO terms, no cross ansatz yet

η/s(T) of QCD



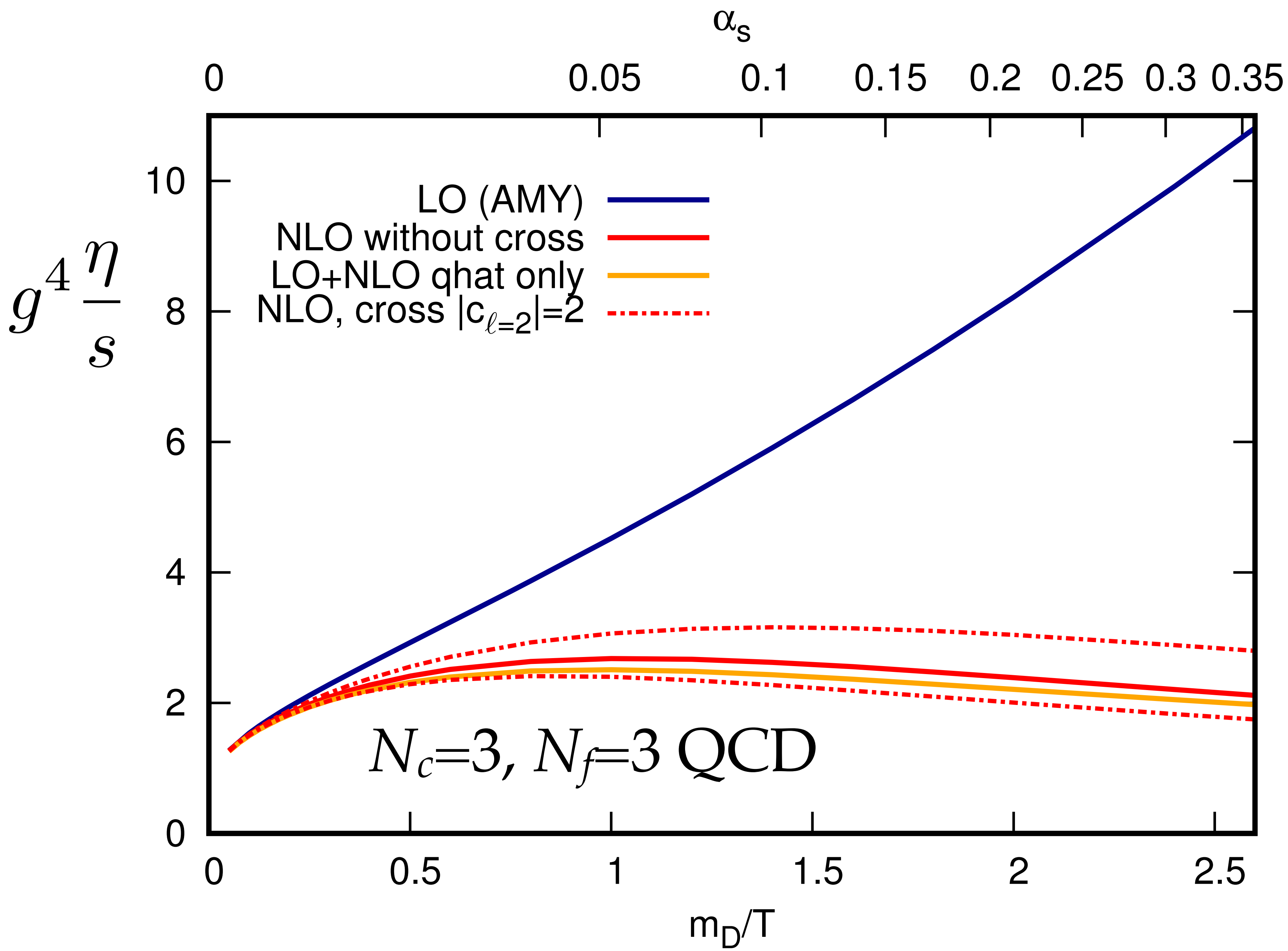
- Cross ansatz introduces $O(\pm 30\%)$ uncertainty

η/s(*T*) of QCD



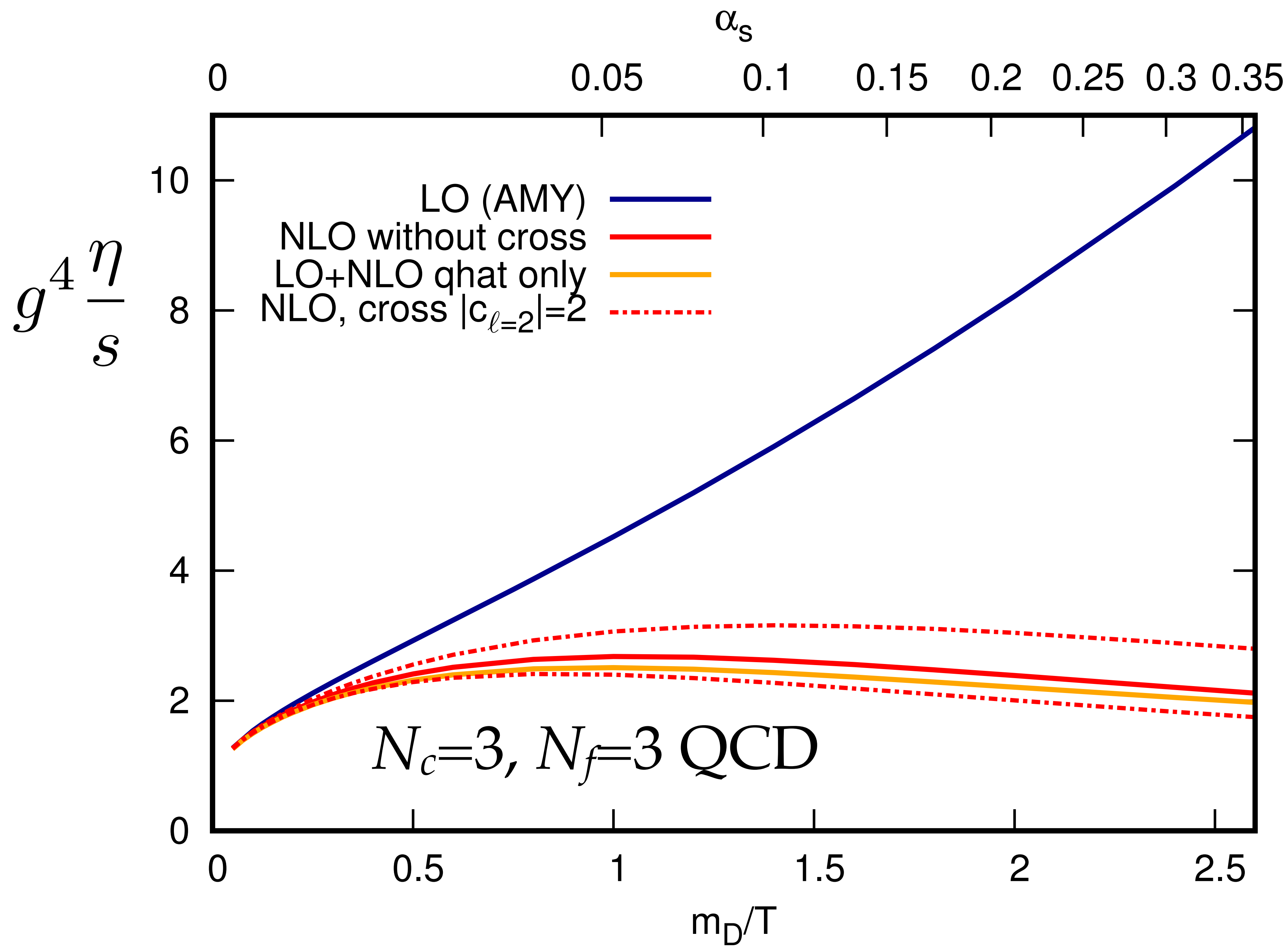
- Pure QCD running uncertainty band at LO (NNLO) smaller than NLO deviation from LO

η/s convergence



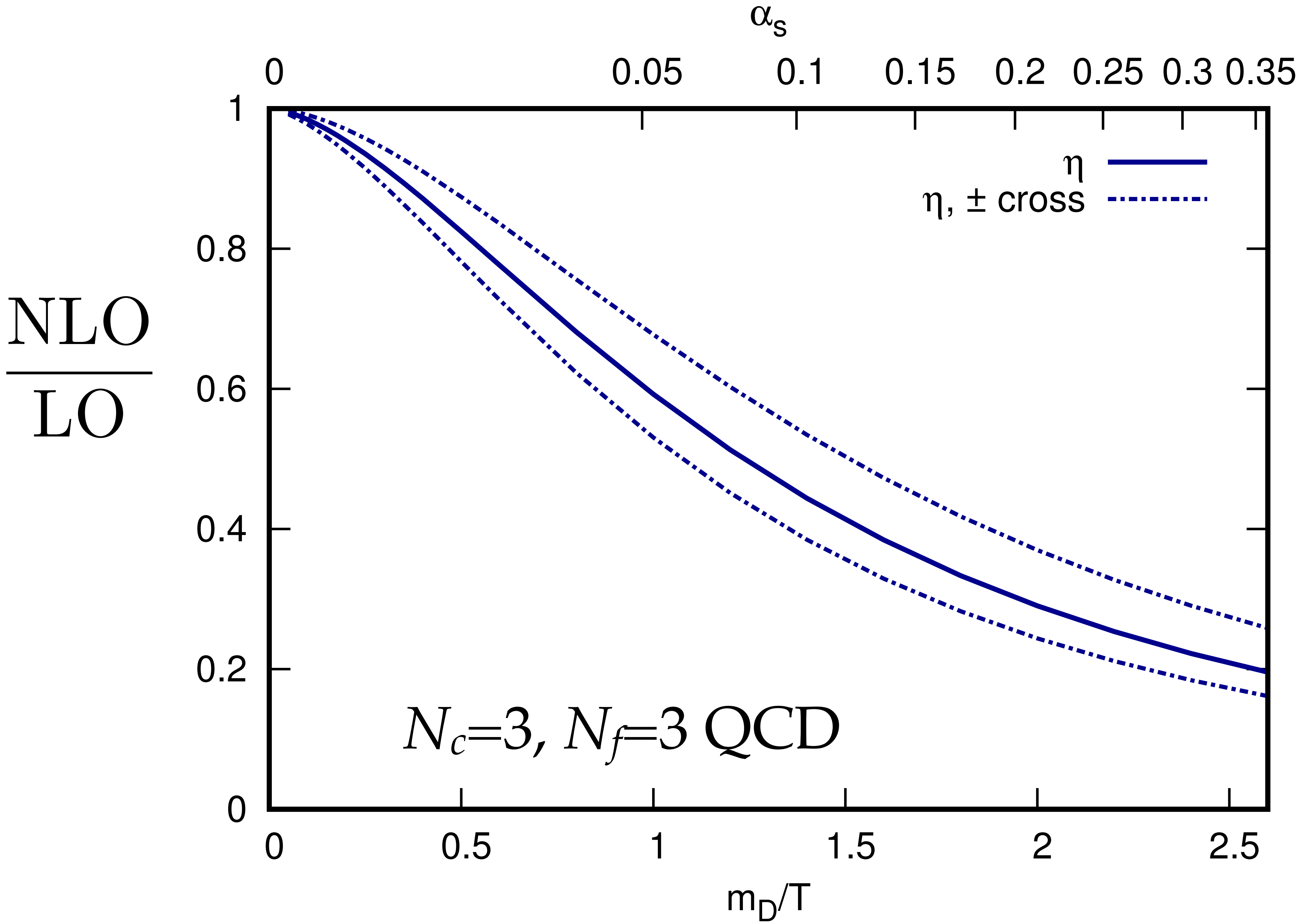
- Convergence realized at $m_D \sim 0.5T$

η/s convergence



- The ~entirety of the downward shift comes from NLO O(g) corrections to $\hat{q}$

Ratio



α_s
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
1
0.8
0.6
0.4
0.2
0
NLO / LO
η
η, ± cross
N_c=3, N_f=3 QCD
0
0.5
1
1.5
2
2.5
m_D/T

Conclusions

- Effective kinetic theory of hard quasi-particles and a soft background
- Can be employed to describe jets, transport coefficients and thermalization
- The interactions with the soft background are encoded in Wilson-line operators, which
  - can be evaluated more easily through the analytic properties of light-like amplitude
  - some of them can now be computed on the lattice

Conclusions

- NLO corrections are large, η down by a factor of ~5 in the phenomenological region
- Convergence below m_D~$0.5T$
- Quark number diffusion coefficient D and second-order hydro τ_Π will be available in the papers
- Corrections dominated by NLO $\hat{q}$. Could it be that observables directly sensitive to transverse momentum broadening show bad convergence and those who are not show good convergence? Why? #statisticswithsmallnumbers

Backup

Euclideanization of light-cone soft physics

- For $t/x_z = 0$: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n, p) e^{i\mathbf{p}\cdot\mathbf{x}}$$

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

- For $t/x_z=0$: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\int_p G_E(\omega_n,p)e^{i\mathbf{p}\cdot\mathbf{x}}$$

- Consider the more general case $|t/x^z|<1$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp - p^0 x^0)}\left(\frac{1}{2}+n_{\rm B}(p^0)\right)\left(G_R(P)-G_A(P)\right)$$

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

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- Consider the more general case $|t/x^z| < 1$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp - p^0 x^0)} \left(\frac{1}{2} + n_{\mathrm{B}}(p^0)\right)(G_R(P) - G_A(P))$$

- Change variables to $\tilde{p}^z = p^z - p^0(t/x^z)$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 d\tilde{p}^z d^2p_\perp e^{i(\tilde{p}^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp)} \left(\frac{1}{2} + n_{\mathrm{B}}(p^0)\right)(G_R(p^0, \mathbf{p}_\perp, \tilde{p}^z + (t/x^z)p^0) - G_A)$$

Caron-Huot **PRD79** (2009)

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- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

Caron-Huot **PRD79** (2009)

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- Change variables to $\tilde{p}^z = p^z - p^0(t/x^z)$

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- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

$$G_{rr}(t,\mathbf{x}) = T\sum_n \int dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp)} G_E(\omega_n, p_\perp, p^z + i\omega_n t/x^z)$$

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

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- Consider the more general case $|t/x^z| < 1$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp - p^0 x^0)} \left(\frac{1}{2} + n_{\rm B}(p^0)\right)(G_R(P) - G_A(P))$$

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$$G_{rr}(t,\mathbf{x}) = T\sum_n \int dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp)} G_E(\omega_n, p_\perp, p^z + i\omega_n t/x^z)$$

- Soft physics dominated by $n=0$ (and t-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

- For t/x_z =0: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\int_p G_E(\omega_n, p) e^{i\mathbf{p}\cdot\mathbf{x}}$$

- Consider the more general case $|t/x^z| < 1$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp - p^0 x^0)} \left(\frac{1}{2} + n_{\rm B}(p^0)\right)(G_R(P) - G_A(P))$$

- Change variables to $\tilde{p}^z = p^z - p^0(t/x^z)$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 d\tilde{p}^z d^2p_\perp e^{i(\tilde{p}^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp)} \left(\frac{1}{2} + n_{\rm B}(p^0)\right)(G_R(p^0, \mathbf{p}_\perp, \tilde{p}^z + (t/x^z)p^0) - G_A)$$

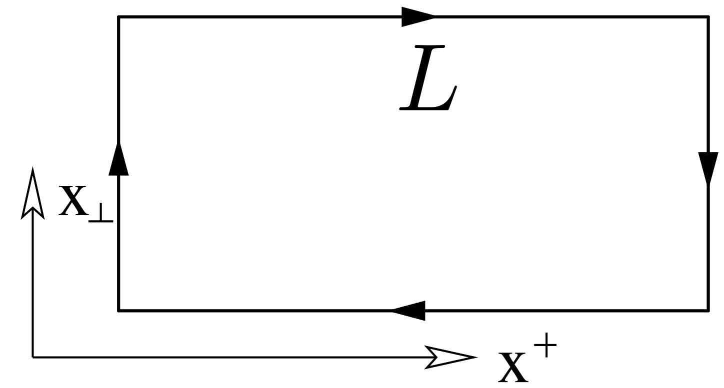
- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

$$G_{rr}(t,\mathbf{x})_{\rm soft} = T\int d^3p\, e^{i\mathbf{p}\cdot\mathbf{x}}\, G_E(\omega_n = 0, \mathbf{p})$$

- Soft physics dominated by $n=0$ (and t-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

LPM resummation



$\propto e^{\mathcal{C}(x_\perp)L}$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- All points at spacelike or lightlike separation, only preexisting correlations
- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008)
    - Can be "easily" computed in perturbation theory
    - Possible lattice measurements Laine **EPJC72** (2012) Laine Rothkopf **JHEP1307** (2013) Panero Rummukainen Schäfer **1307.5850**

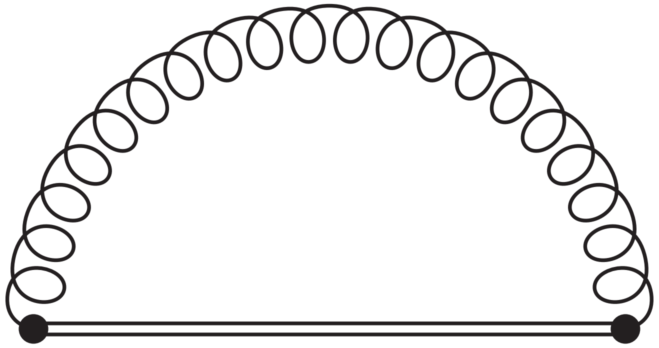
Longitudinal momentum diffusion

- Field-theoretical lightcone definition (justifiable with SCET)

$$\hat{q}_L \equiv \frac{g^2}{d_R}\int_{-\infty}^{+\infty} dx^+ \mathrm{Tr}\left\langle U(-\infty,x^+)F^{+-}(x^+)U(x^+,0)F^{+-}(0)U(0,-\infty)\right\rangle$$

$F^{+-}=E^z$, longitudinal Lorentz force correlator

- At leading order

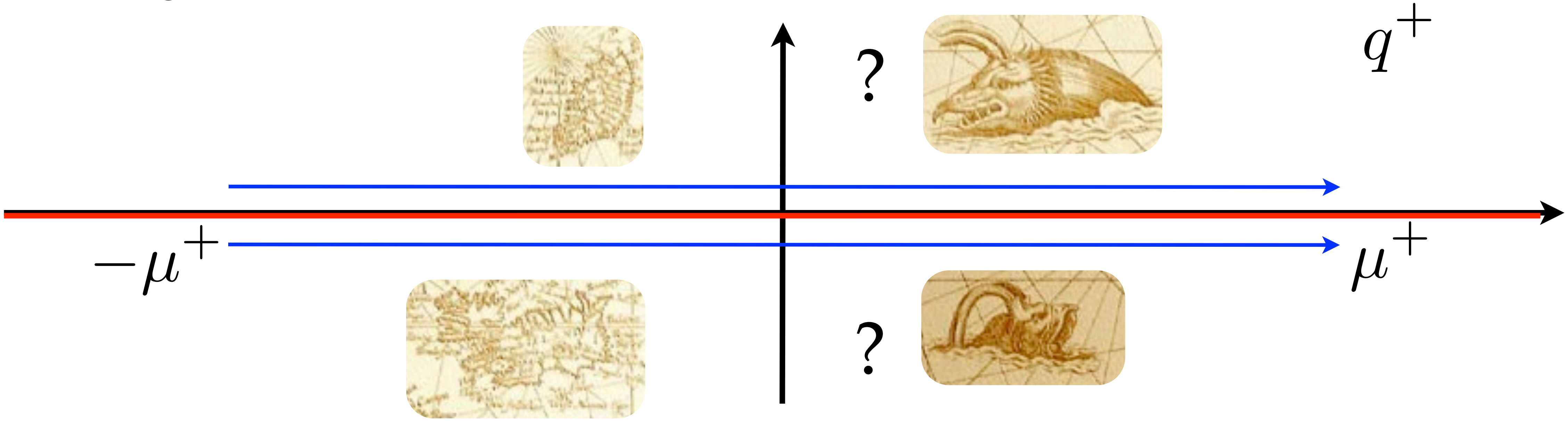


$$\hat{q}_L \propto \int \frac{dq^+ d^2q_\perp}{(2\pi)^3}(q^+)^2 G^>_{++}(q^+,q_\perp,0)$$

$$= \int \frac{dq^+ d^2q_\perp}{(2\pi)^3} T q^+ (G^R_{++}(q^+,q_\perp,0) - G^A)$$

Longitudinal momentum diffusion

$$\hat{q}_L\Big|_{\rm LO} = g^2 C_R \int \frac{dq^+ d^2q_\perp}{(2\pi)^3} T q^+ \left(G_R^{--}(q^+, q_\perp) - G_A^{--}(q^+, q_\perp)\right)$$



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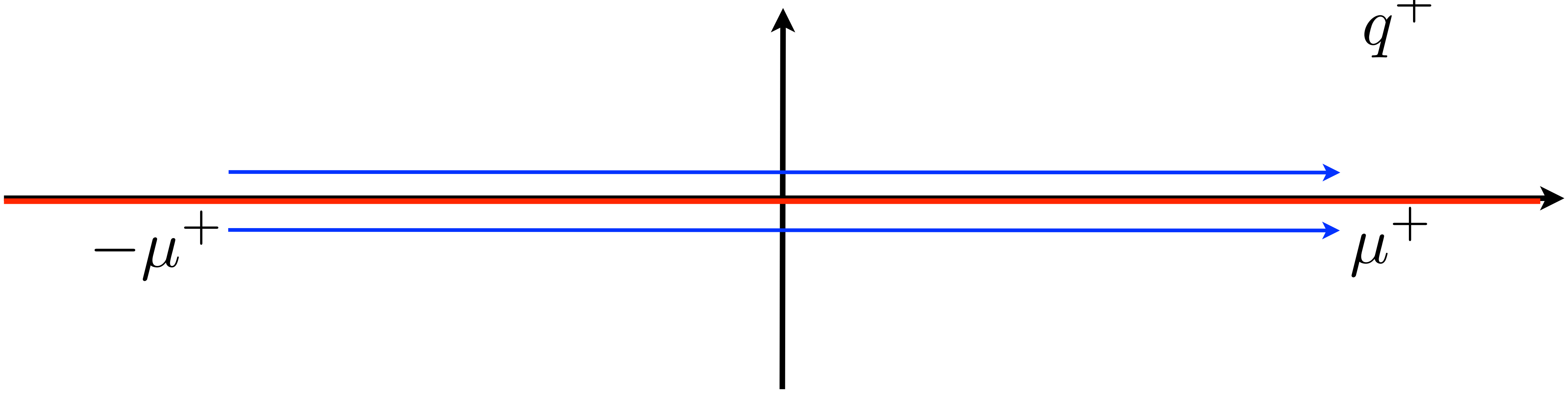

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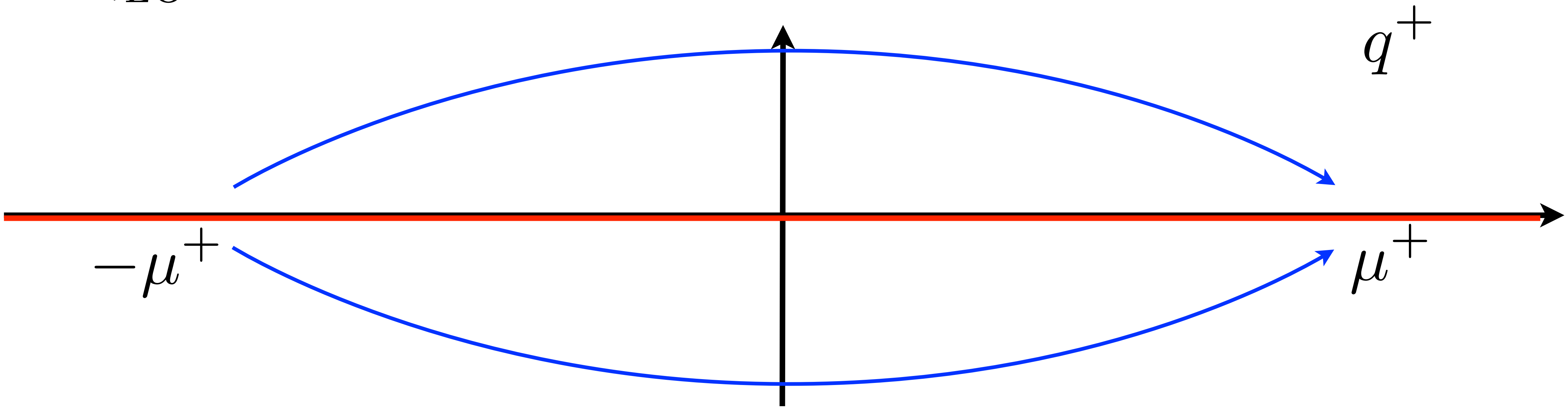

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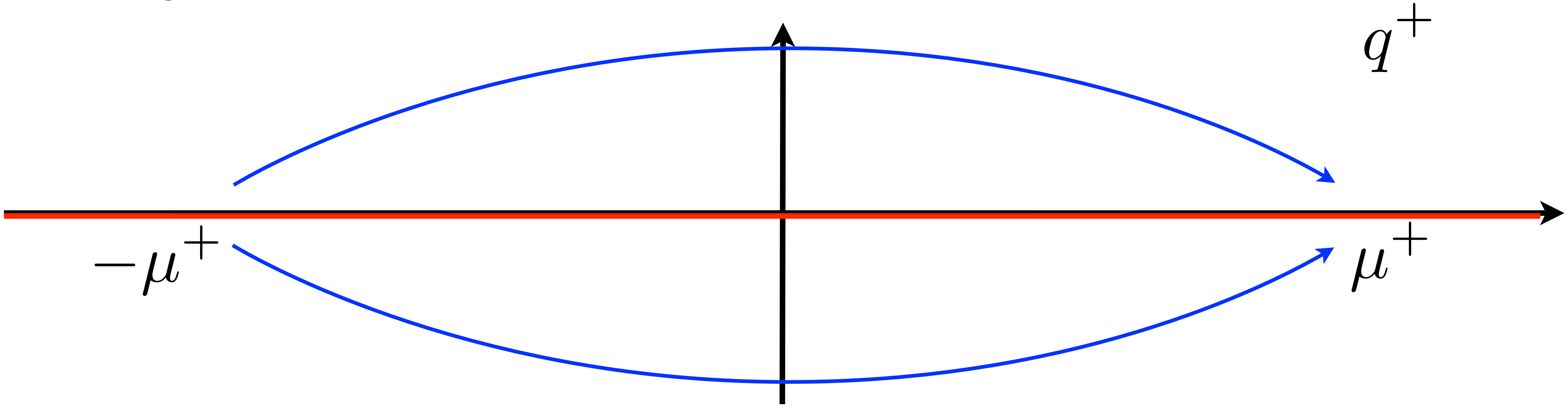

- Use analyticity to deform the contour away from the real axis and keep $1/q^+$ behaviour

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R T \int \frac{d^2q_\perp}{(2\pi)^2} \frac{M_\infty^2}{q_\perp^2 + M_\infty^2}$$