Non-exponential decay law

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Seminar

11/12/2015

Outline

1. Decay law: general properties, Zeno effect, experimental evidence
2. Lee Hamiltonian: a QFT-like quantum mechanical approach
3. Decays in Quantum Field Theory
4. Decay of a moving particle

Part 1: General discussion and exp. evidence

Exponential decay law

- N_0 : Number of unstable particles at the time t = 0.

$$N(t) = N_0 e^{-\Gamma t} \quad , \quad \tau = 1/\Gamma \text{ mean lifetime}$$

Confirmend in countless cases!

- For a single unstable particle:

$$p(t) = e^{-\Gamma t}$$

is the survival probability for a single unstable particle created at t=0.
(Intrinsic probabilty, see Schrödinger´s cat).

For small times: $p(t) = 1 - \Gamma t + ...$

Basic definitions

Let $|S\rangle$ be an unstable state prepared at $t = 0$.

Survival probabilty ammplitude at $t > 0$:

$$a(t) = \langle S | e^{-iHt} | S \rangle \qquad \hbar = 1$$

Survival probability: $p(t) = |a(t)|^2$

Survival probability also called nondecay probability: $p(t) = p_{nd}(t)$.

Rep. Prog. Phys., Vol. 41, 1978. Printed in Great Britain

Decay theory of unstable quantum systems

L FONDA, G C GHIRARDI and A RIMINI

Deviations from the exp. law at short times

Taylor expansion of the amplitude:

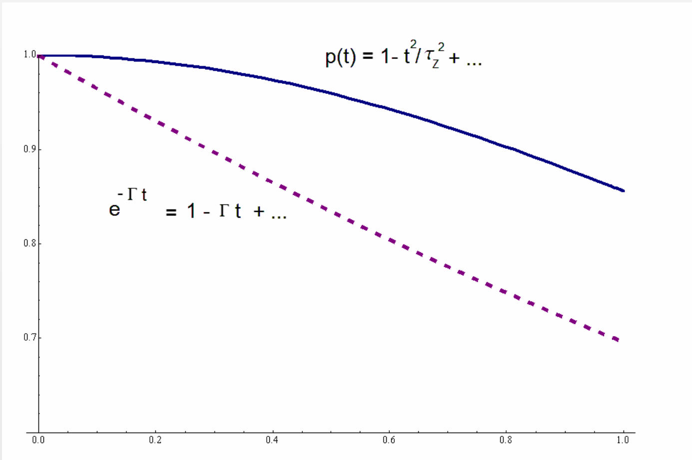
$$a(t) = \langle S | e^{-iHt} | S \rangle = 1 - it\langle S|H|S\rangle - \frac{t^2}{2}\langle S|H^2|S\rangle + \ldots$$

$$a^*(t) = \langle S | e^{-iHt} | S \rangle = 1 + it\langle S|H|S\rangle - \frac{t^2}{2}\langle S|H^2|S\rangle + \ldots$$

It follows:

$$\mathrm{p(t)} = |\mathrm{a(t)}|^2 = \mathrm{a^*(t)a(t)} = 1 - \mathrm{t}^2\left(\langle S|H^2|S\rangle - \langle S|H|S\rangle^2\right) + \ldots = 1 - \frac{\mathrm{t}^2}{\tau_Z^2} + \ldots$$

where $\tau_Z = \dfrac{1}{\sqrt{\langle S|H^2|S\rangle - \langle S|H|S\rangle^2}}$.



p(t) decreases quadratically (not linearly);

no exp. decay for short times.

τ_Z is the `Zeno time´.

Time evoluition and energy distribution (1)

The unstable state $|S\rangle$ is not an eigenstate of the Hamiltonian H.

Let $d_S(E)$ be the energy distribution of the unstable state $|S\rangle$.

Normalization holds: $\int_{-\infty}^{+\infty} d_S(E) dE = 1$

$$a(t) = \int_{-\infty}^{+\infty} d_S(E) e^{-iEt} dE$$

In stable limit : $d_S(E) = \delta(E - M_0) \rightarrow a(t) = e^{-iM_0 t} \rightarrow p(t) = 1$

Time evoluition and energy distribution (2)

Breit-Wigner distribution:

$$d_S(E) = \frac{\Gamma}{2\pi} \frac{1}{(E - M_0)^2 + \Gamma^2/4} \to a(t) = e^{-iM_0 t - \Gamma t/2} \to p(t) = e^{-\Gamma t}.$$

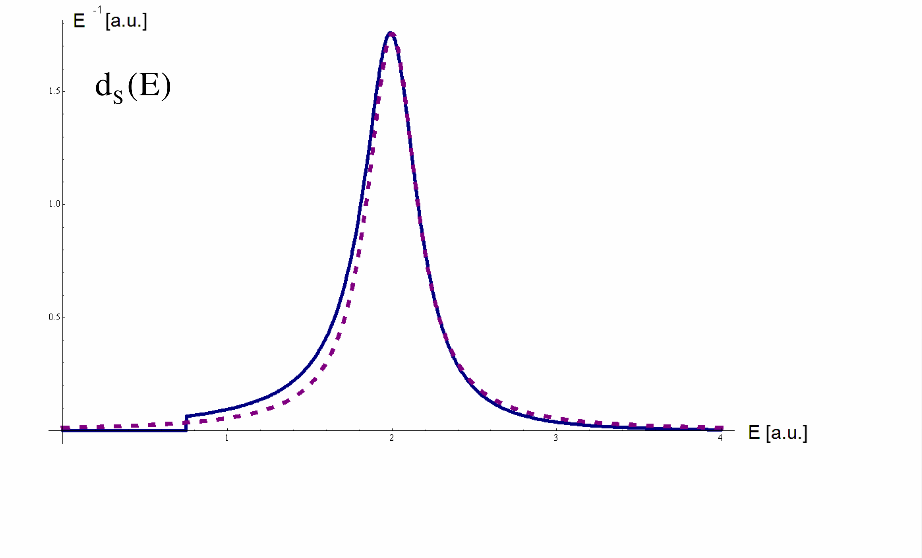
The Breit-Wigner energy distribution cannot be exact.

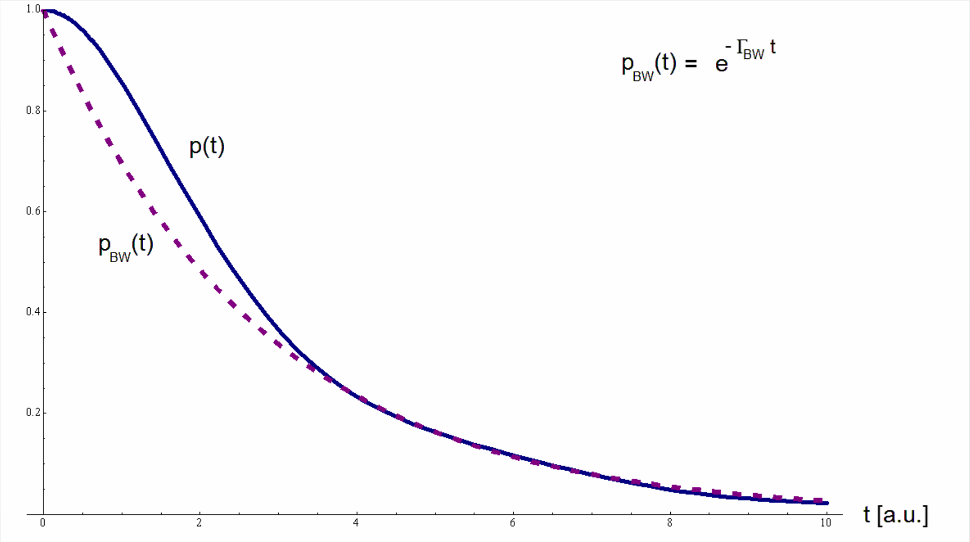
Two physical conditions for a realistic $d_S(E)$ are:

1) Minimal energy: $d_S(E) = 0 \text{ for } E < E_{\min}$

2) Mean energy finite: $\langle \mathrm{E} \rangle = \int_{-\infty}^{+\infty} d_S(E) E dE = \int_{E_{\min}}^{+\infty} d_S(E) E dE < \infty$

A very simple numerical example





$M_0 = 2;\ E_{min} = 0.75;\ \ \Gamma = 0.4; \Lambda = 3$

$$d_S(E) = N_0 \frac{\Gamma}{2\pi} \frac{e^{-(E^2 - E_0^2)/\Lambda^2} \theta(E - E_{min})}{(E - M_0)^2 + \Gamma^2/4}$$

$$d_{BW}(E) = \frac{\Gamma_{BW}}{2\pi} \frac{1}{(E - M_0)^2 + \Gamma_{BW}{}^2/4}$$

Γ_{BW}, such that $d_{BW}(M_0) = d_S(M_0)$

$$a(t) = \int_{-\infty}^{+\infty} d_S(E) e^{-iEt} dE\ ; \quad p(t) = |a(t)|^2$$

$$p_{BW}(t) = e^{-\Gamma_{BW} t}$$

The quantum Zeno effect

We perform N inst. measurements:

the first one at time $t = t_0$, the second at time $t = 2t_0$, ..., the N-th at time $T = Nt_0$.

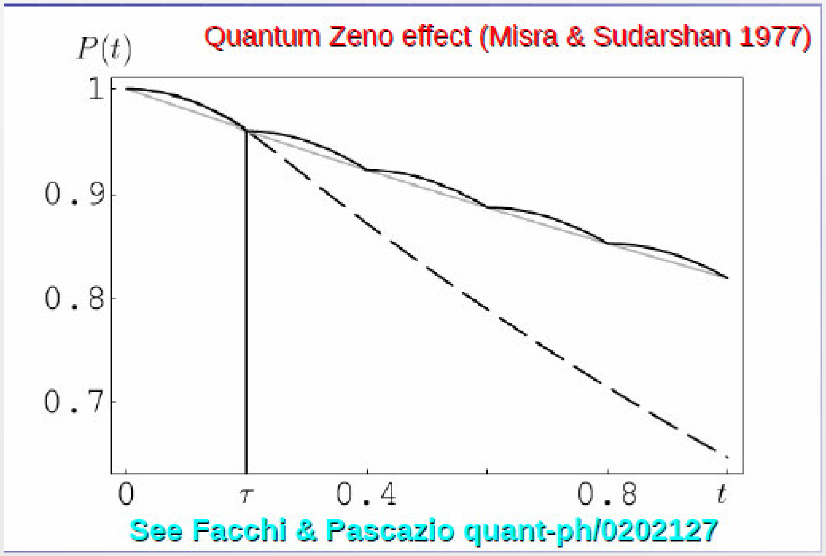
$$p_{\text{after N measurements}} = p(t_0)^N \approx \left(1 - \frac{t_0^2}{\tau_Z^2}\right)^N = \left(1 - \frac{T^2}{N^2 \tau_Z^2}\right)^N$$

under the assumption that t_0 is small enough.

If $N >> 1$ (at fixed T): $p_{\text{after N measurements}} \approx e^{-\frac{T^2}{N\tau_Z^2}} \approx 1.$

For large but finite N :

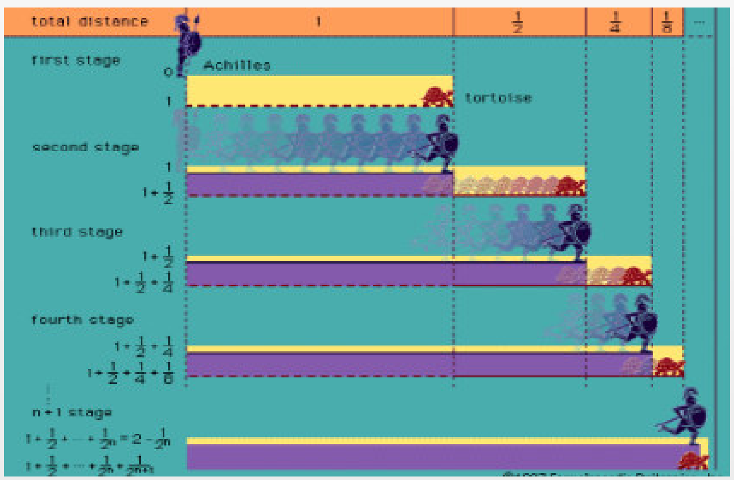
$\rightarrow$ slowing down of the decay.

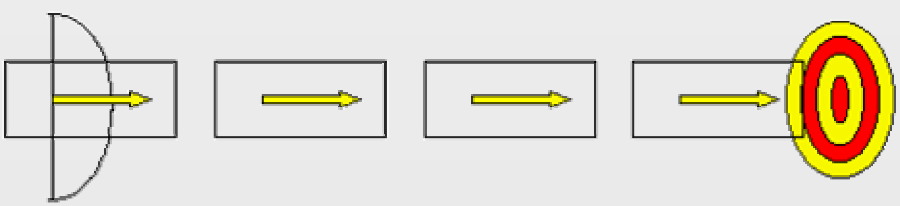


Zeno of Elea

489/431 a.c., Elea

Paradoxes:





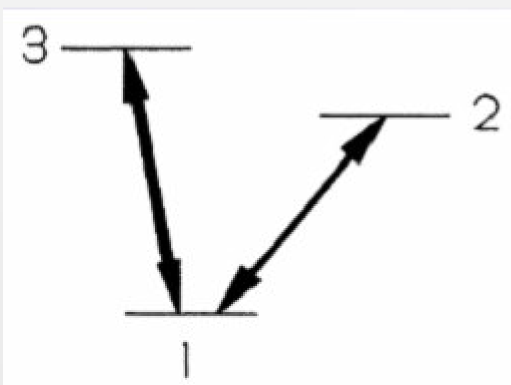
Experimental confirmation of the quantum Zeno effect - Itano et al (1)

PHYSICAL REVIEW A VOLUME 41, NUMBER 5 1 MARCH 1990

Quantum Zeno effect

Wayne M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland
Time and Frequency Division, National Institute of Standards and Technology, Boulder, Colorado 80303
(Received 12 October 1989)

The quantum Zeno effect is the inhibition of transitions between quantum states by frequent measurements of the state. The inhibition arises because the measurement causes a collapse (reduction) of the wave function. If the time between measurements is short enough, the wave function usually collapses back to the initial state. We have observed this effect in an rf transition between two $^9Be^+$ ground-state hyperfine levels. The ions were confined in a Penning trap and laser cooled. Short pulses of light, applied at the same time as the rf field, made the measurements. If an ion was in one state, it scattered a few photons; if it was in the other, it scattered no photons. In the latter case the wave-function collapse was due to a null measurement. Good agreement was found with calculations.



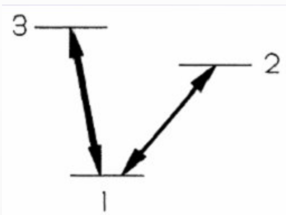
(Undisturbed) survival probability

At $t = 0$, the electron is in $|1\rangle$.

$$p(t) = \cos^2\left(\frac{\Omega t}{2}\right) = 1 - \frac{\Omega^2 t^2}{4} + \ldots$$

$$p(T) = 0 \text{ für } T = \pi/\Omega$$

Experimental confirmation of the quantum Zeno effect - Itano et al (2)



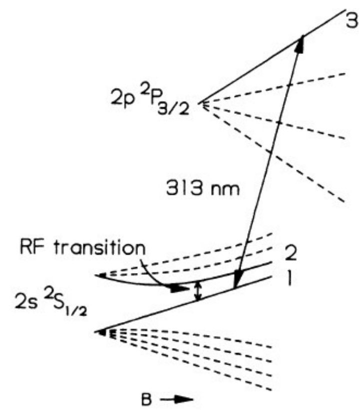


FIG. 2. Diagram of the energy levels of $^9Be^+$ in a magnetic field B. The states labeled 1, 2, and 3 correspond to those in Fig. 1 .

5000 Ions in a Penning trap

Short laser pulses 1-3 work as measurements.

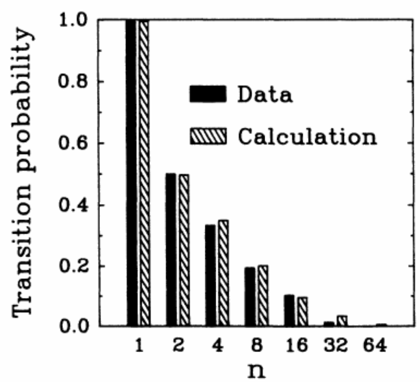


FIG. 3. Graph of the experimental and calculated $1 \to 2$ transition probabilities as a function of the number of measurement pulses n. The decrease of the transition probabilities with increasing n demonstrates the quantum Zeno effect.

$$p(t) = \cos^2(\Omega t/2) = 1 - \frac{\Omega^2 t^2}{4} + \ldots ; \quad p(T) = 0 \text{ für } T = \pi/\Omega$$

(Transition probability (without measuring) at time T) : $1 - p(T) = 1$.

Witn n measurements in between the transition probabilty decreases!
The electron stays in state 1.

Other experiments about Zeno

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ELSEVIER
1 June 2000
OPTICS COMMUNICATIONS
Optics Communications 180 (2000) 115–120
www.elsevier.com/locate/optcom

The quantum Zeno effect – evolution of an atom impeded by measurement

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OPTICS COMMUNICATIONS
Optics Communications 211 (2002) 235–241
www.elsevier.com/locate/optcom

A relaxationless demonstration of the Quantum Zeno paradox on an individual atom

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Continuous and Pulsed Quantum Zeno Effect

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PRL **101,** 180402 (2008) PHYSICAL REVIEW LETTERS week ending 31 OCTOBER 2008

Freezing Coherent Field Growth in a Cavity by the Quantum Zeno Effect

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Experimental confirmation of non-exponential decays (1)

NATURE | VOL 387 | 5 JUNE 1997

Experimental evidence for non-exponential decay in quantum tunnelling

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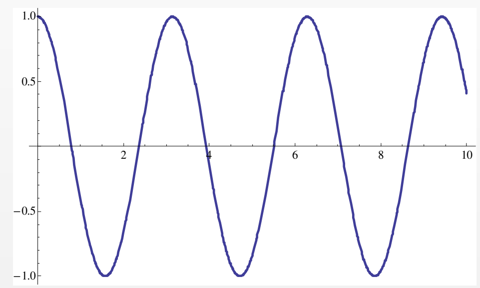
An exponential decay law is the universal hallmark of unstable systems and is observed in all fields of science. This law is not, however, fully consistent with quantum mechanics and deviations from exponential decay have been predicted for short as well as long times[1–8]. Such deviations have not hitherto been observed experimentally. Here we present experimental evidence for short-time deviation from exponential decay in a quantum tunnelling experiment. Our system consists of ultra-cold sodium atoms that are trapped in an accelerating periodic optical potential created by a standing wave of light. Atoms can escape the wells by quantum tunnelling, and the number that remain can be measured as a function of interaction time for a fixed value of the well depth and acceleration. We observe that for short times the survival probability is initially constant before developing the characteristics of exponential decay. The conceptual simplicity of the experiment enables a detailed comparison with theoretical predictions.

Cold Na atoms in a optical potential

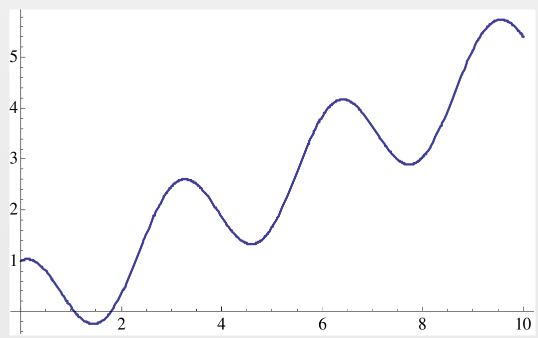
$V(x,0)$



$$V(x,t) = V_0 \cos\left(2k_L x - k_L a t^2\right)$$

x [a.u.]

$U(x')$



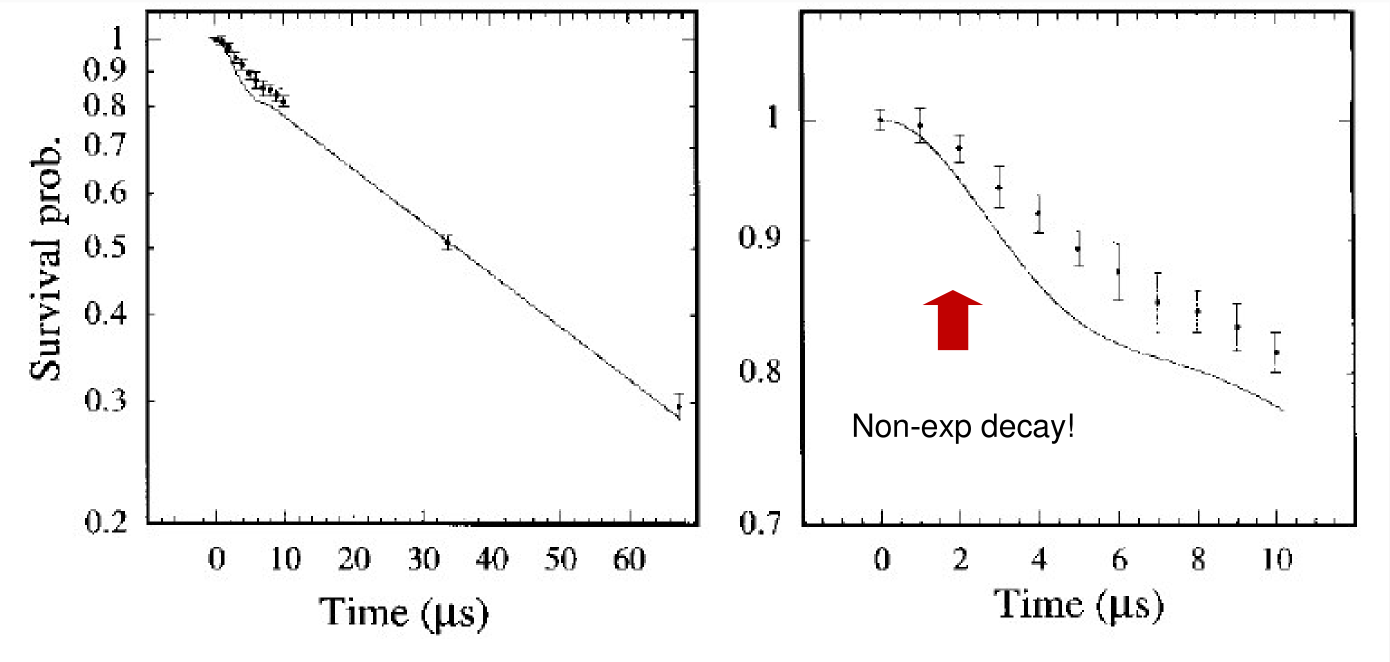
$$x' = x - \frac{1}{2} a t^2$$

$$U(x') = V_0 \cos\left(2k_L x'\right) + Max'$$

x' [a.u.]

Experimental confirmation of non-exponential decays (2)

Measured survival probabilty p(t)



Experimental confirmation of non-exponential decays and Zeno /Anti-Zeno effects

Uniwersytet Jana Kochanowskiego w Kielcach

VOLUME 87, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JULY 2001

Observation of the Quantum Zeno and Anti-Zeno Effects in an Unstable System

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(Received 30 March 2001; published 10 July 2001)

We report the first observation of the quantum Zeno and anti-Zeno effects in an unstable system. Cold sodium atoms are trapped in a far-detuned standing wave of light that is accelerated for a controlled duration. For a large acceleration the atoms can escape the trapping potential via tunneling. Initially the number of trapped atoms shows strong nonexponential decay features, evolving into the characteristic exponential decay behavior. We repeatedly measure the number of atoms remaining trapped during the initial period of nonexponential decay. Depending on the frequency of measurements we observe a decay that is suppressed or enhanced as compared to the unperturbed system.

Same exp. setup,
but with measurements in between

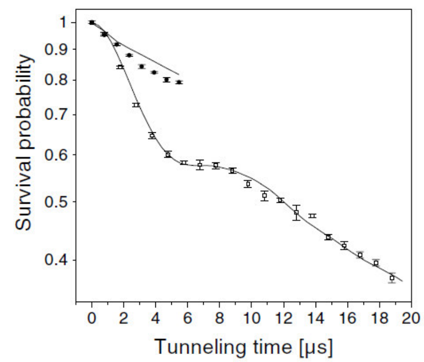


FIG. 3. Probability of survival in the accelerated potential as a function of duration of the tunneling acceleration. The hollow squares show the noninterrupted sequence, and the solid circles show the sequence with interruptions of 50 μs duration every 1 μs. The error bars denote the error of the mean. The data have been normalized to unity at $t_{\mathrm{tunnel}} = 0$ in order to compare with the simulations. The solid lines are quantum mechanical simulations of the experimental sequence with no adjustable parameters. For these data the parameters were $a_{\mathrm{tunnel}} = 15\,000$ m/s^2, $a_{\mathrm{interr}} = 2000$ m/s^2, $t_{\mathrm{interr}} = 50$ μs, and $V_0/h = 91$ kHz, where h is Planck's constant.

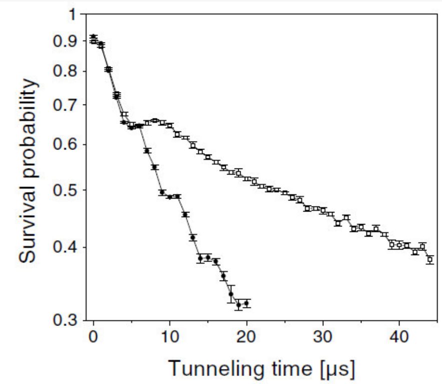


FIG. 4. Survival probability as a function of duration of the tunneling acceleration. The hollow squares show the noninterrupted sequence, and the solid circles show the sequence with interruptions of 40 μs duration every 5 μs. The error bars denote the error of the mean. The experimental data points have been connected by solid lines for clarity. For these data the parameters were: $a_{\mathrm{tunnel}} = 15\,000$ m/s^2, $a_{\mathrm{interr}} = 2800$ m/s^2, $t_{\mathrm{interr}} = 40$ μs, and $V_0/h = 116$ kHz.

Zeno effekt

Anti-Zeno effect

Part 2: Lee Hamiltonian

Lee Hamiltonian

$$H = H_0 + H_1$$

$$H_0 = M_0 |S\rangle\langle S| + \int_{-\infty}^{+\infty} dk\, \omega(k) |k\rangle\langle k|$$

$$H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk (g \cdot f(k))(|S\rangle\langle k| + |k\rangle\langle S|)$$

|S> is the initial unstable state, coupled to an infinity of final states |k>. (Poincare-time is infinite. Irreversible decay). General approach, similar Hamiltonians used in many areas of Physics.

Example/1: spontaneous emission. |S> represents an atom in the excited state, |k> is the ground-state plus photon.

Example/2: pion decay. |S> represents a neutral pion, |k> represents two photons (flying back-to-back)

Propagator and spectral function

$$H = H_0 + H_1 \; ; \; H_0 = M_0 |S\rangle\langle S| + \int_{-\infty}^{+\infty} dk\,\omega(k) |k\rangle\langle k| \; ; \; H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk (g \cdot f(k)) (|S\rangle\langle k| + |k\rangle\langle S|)$$

$$G_S(E) = \left\langle S \left| (E - H + i\varepsilon)^{-1} \right| S \right\rangle = (E - M_0 + \Pi(E) + i\varepsilon)^{-1}$$

$$\Pi(E) = -\int_{-\infty}^{+\infty} \frac{dk}{2\pi} \frac{g^2 f(k)^2}{E - \omega(k) + i\varepsilon}$$

$$d_S(E) = \frac{1}{\pi} \operatorname{Im} G_S(E) \; ;$$

$$a(t) = \left\langle S \left| e^{-iHt} \right| S \right\rangle = \int_{-\infty}^{+\infty} dE\, d_S(E) e^{-iEt}$$

It follows:

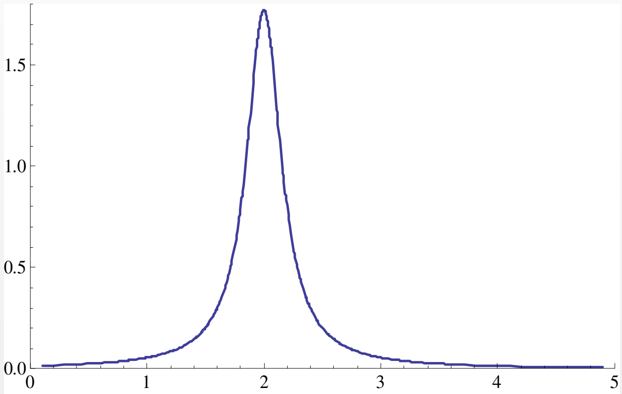
$$\int_{-\infty}^{+\infty} dE\, d_S(E) = 1$$

Fermi golden rule: $\Gamma = \operatorname{Im}[\Pi(M)]/2$.

Exponential limit

$$H = H_0 + H_1 \; ; \; H_0 = M_0 |S\rangle\langle S| + \int_{-\infty}^{+\infty} dk\,\omega(k) |k\rangle\langle k| \; ; \; H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk (g \cdot f(k)) (|S\rangle\langle k| + |k\rangle\langle S|)$$

$$\omega(k) = k \; ; \quad f(k) = 1 \quad \Rightarrow \quad \Pi(E) = ig^2/2 \; ; \; \Gamma = g^2$$



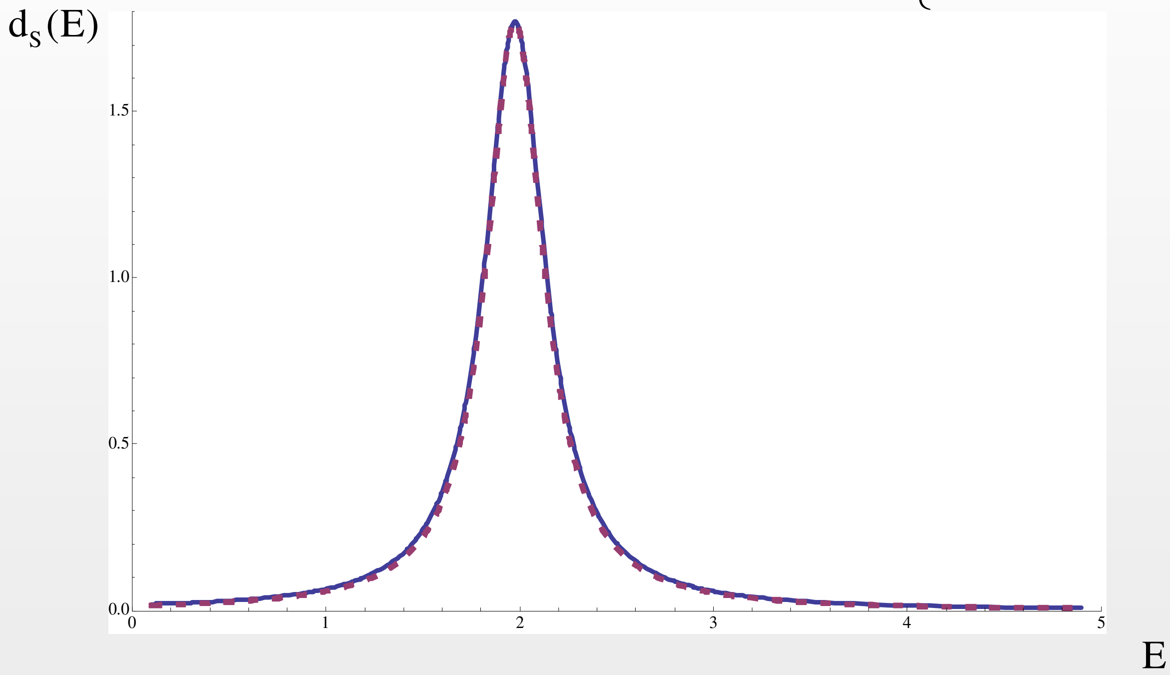
$$d_S(E) = \frac{\Gamma}{2\pi} \frac{1}{(E - M_0)^2 + \Gamma^2/4}$$

$$\Rightarrow a(t) = e^{-i(M_0 - i\Gamma/2)t} \Rightarrow p(t) = e^{-\Gamma t}$$

Non-exponential case (1)

$$H_1 = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dk(g \cdot f(k))(|S\rangle\langle k| + |k\rangle\langle S|)$$

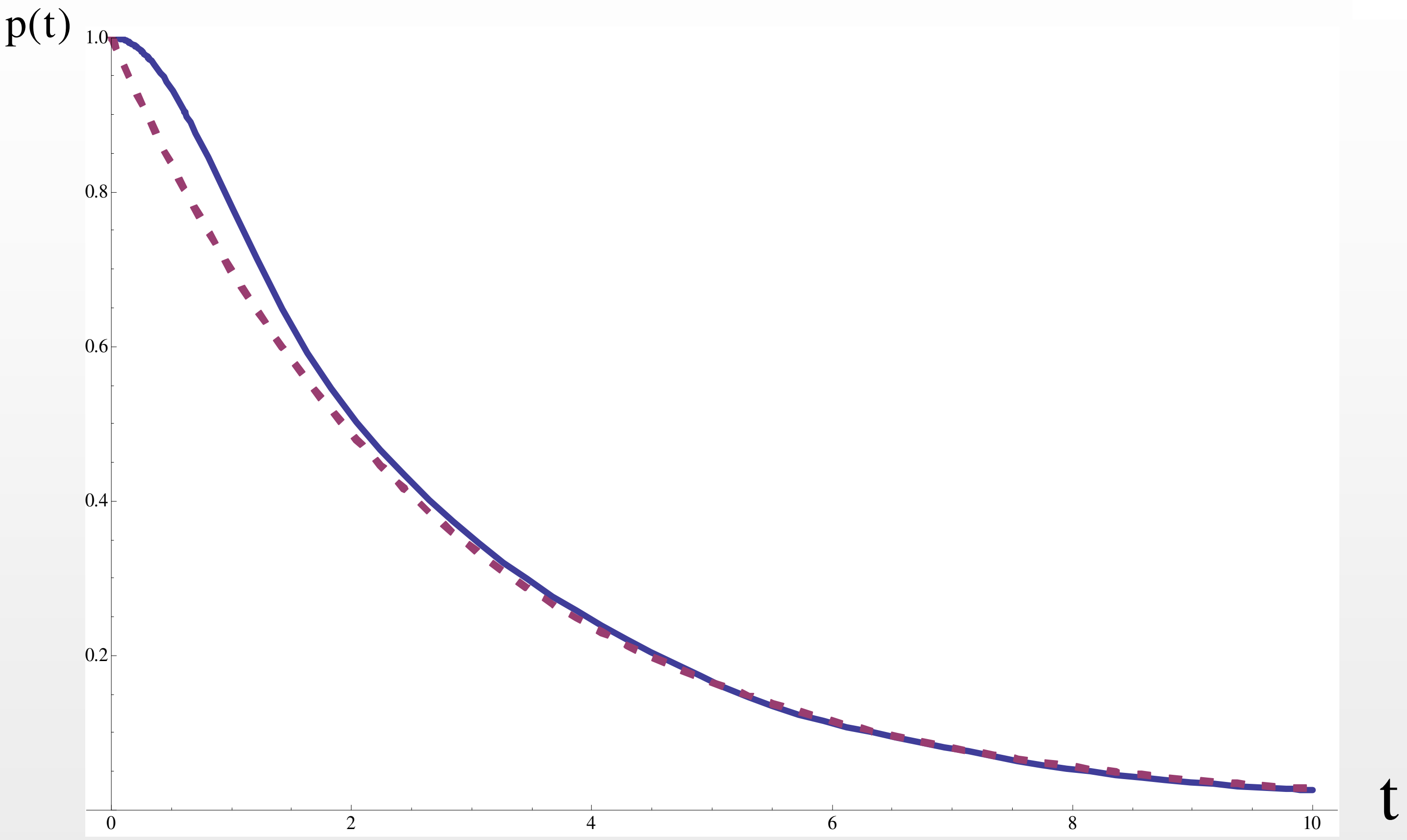
$$f(k) = \begin{cases} 0 \text{ for } k < E_{min} \\ 1 \text{ for } E_{min} \leq k \leq E_{max} \\ 0 \text{ for } k > E_{max} \end{cases}$$



$$M_0 = 2;\ E_{min} = 0;\ E_{max} = 5;\ g^2 = 0.36 \text{ (all in a.u. of energy)}$$

F. Giacosa, PRA 88 (2013) 5, 052131 [arXiv:1305.4467 [quant-ph]].

Non-exponential case (2)

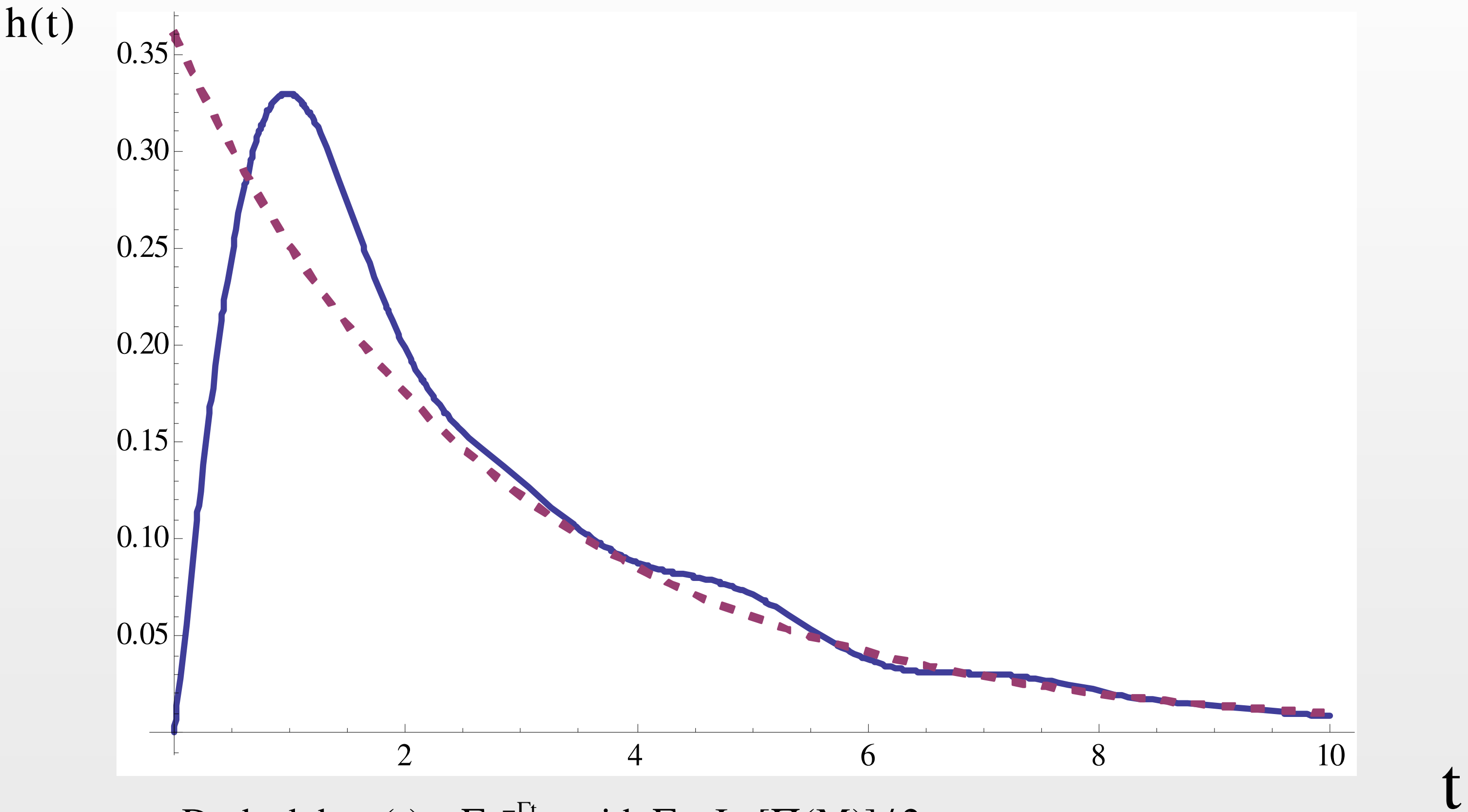


Dashed: $p_{BW}(t) = e^{-\Gamma t}$ with $\Gamma = \mathrm{Im}[\Pi(M)]/2$

Non-exponential case (3)

$$h(t) = -\frac{dp(t)}{dt}$$

Namley: $h(t)dt = p(t) - p(t+dt)$ is the probability that the particles decays between t and t+dt

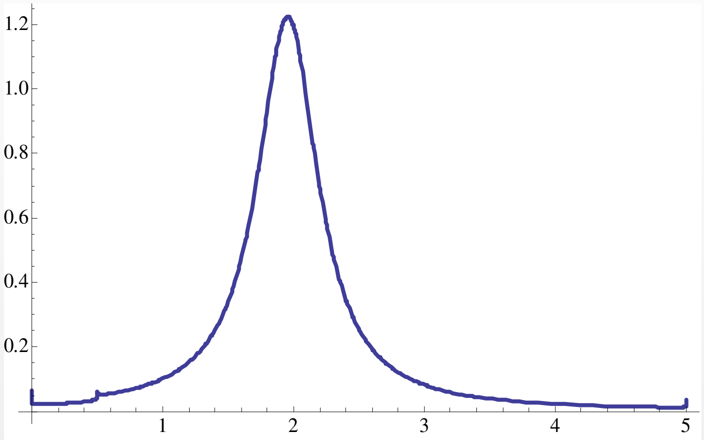


Dashed: $h_{BW}(t) = \Gamma e^{-\Gamma t}$ with $\Gamma = \mathrm{Im}[\Pi(M)]/2$

Two-channel case (1)

$$H_1 = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dk (g_1 \cdot f_1(k))(|S\rangle\langle k,1| + |k,1\rangle\langle S|) + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dk (g_2 \cdot f_2(k))(|S\rangle\langle k,2| + |k,2\rangle\langle S|)$$

$$f_i(k) = \begin{cases} 0 \text{ for } k < E_{i,\min} \\ 1 \text{ for } E_{i,\min} \le k \le E_{i,\max} \\ 0 \text{ for } k > E_{i,\max} \end{cases}$$



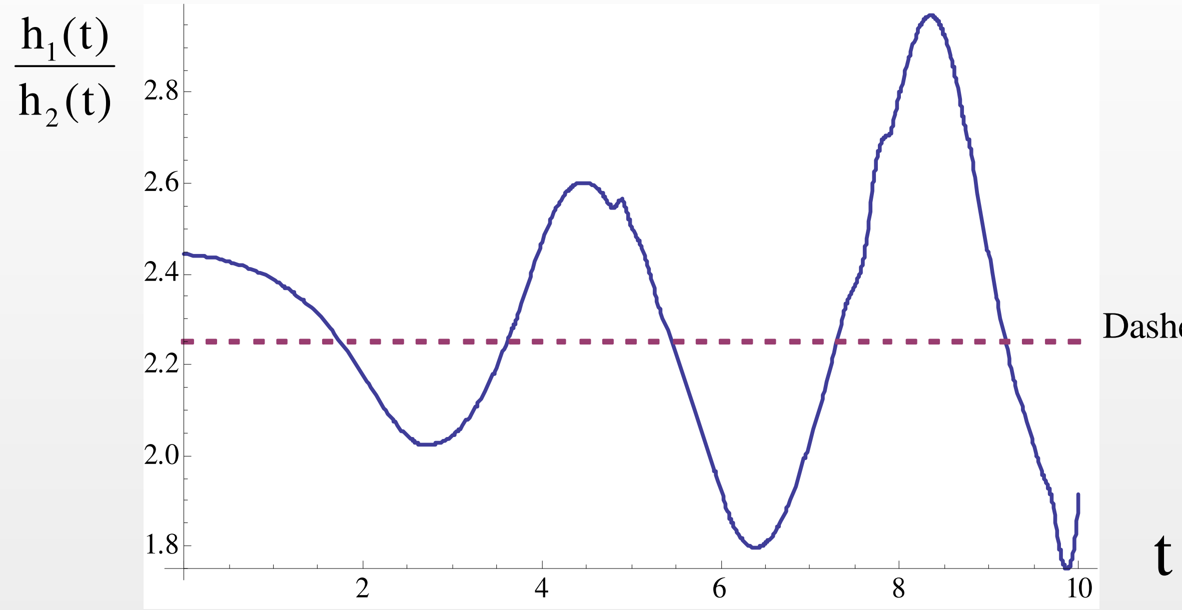
$M_0 = 2;\ E_{1,\min} = 0;\ E_{2,\min} = 0\ ;\ E_{1,\max} = E_{2,\max} = 5;$

$g_1^2 = 0.36\ ;\ g_2^2 = 0.16$ (all in a.u. of energy)

Two-channel case (2)

$h_1(t)dt =$ probabilty that the state $|S\rangle$ decays in the first channel between (t,t+dt)

$h_2(t)dt =$ probabilty that the state $|S\rangle$ decays in the second channel between (t,t+dt)



Dashed: $\frac{h_{1,BW}(t)}{h_{2,BW}(t)} = \frac{\Gamma_1}{\Gamma_2} = \text{const}$

Measurable effect???

Details in:

F. G., Non-exponential decay in quantum field theory and in quantum mechanics: the case of two (or more) decay channels, Found. Phys. 42 (2012) 1262 [arXiv:1110.5923].

Part 3: Quantum field theory

Francesco Giacosa

Quantum field theory: textbook treatment

$$d\Gamma = \frac{(2\pi)^4}{2M} |\mathcal{M}|^2 \, \delta(p - k_1 - k_2) \frac{d^3k_1}{(2\pi)^3 2E_1} \frac{d^3k_2}{(2\pi)^3 2E_2}$$

see e.g. Peskin-Schroeder ord PDG

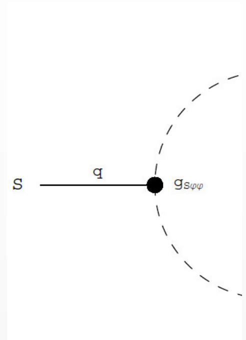
Care is needed:

- An unstable state is not an asymptotic state
- The formula is valid only for $\Gamma \ll M$
- Within this treatment the decay is purely exponential
- One needs to go beyond to study non-exp. decays

Quantum field theory: spectral function

$$L_{int} = gS\varphi^2$$

[g] =[Energy]; QFT super-renorm.

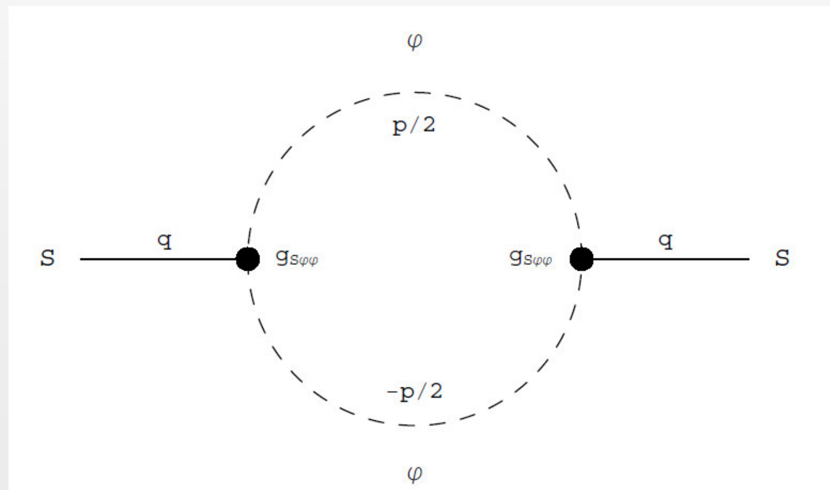


Propagator:

$$\Delta_S(p^2) = \frac{1}{p^2 - M_0^2 + \Pi(p^2) + i\varepsilon}$$

Spectral function (or energy distribution):

$$d_S(m) = \frac{2m}{\pi} \mathrm{Im}[\Delta_S(p^2 = m^2)]$$

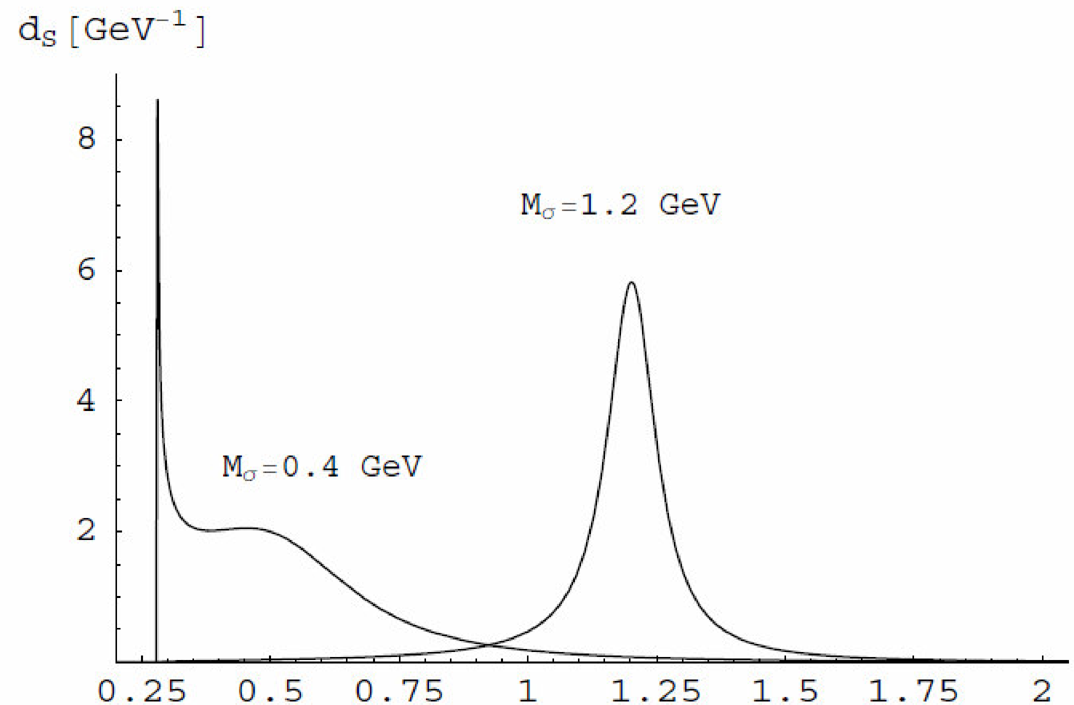


Normalization follows authomatically:

$$\int_0^\infty dm d_S(m) = 1$$

F.G. and G. Pagliara, *On the spectral functions of scalar mesons*, Phys. Rev. C 76 (2007) 065204 [arXiv:0707.3594].

Quantum field theory: two examples of spectral functions

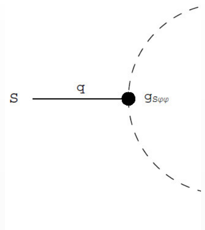


Two examples of scalar resonances:
f_0(1370) is approx. a relativistic BW resonance
f_0(500) is very far from it!!!! (Relevant for chiral theories, nuclear matter….)

Further study of f0(500): position of the pole
F.G. and T. Wolkanowski,
Mod. Phys. Lett. A 27 (2012) 1250229
[arXiv:1209.2332].

Quantum field theory:decay width

$$L_{int} = gS\varphi^2$$



$$\Gamma_{tl}(m) = \frac{\sqrt{\frac{m^2}{4} - \mu^2}}{4\pi m^2} g^2 \, ;$$

$\Gamma_{tl}(M)$ is the tree-level decay width

$$\Gamma = \int_0^\infty \Gamma_{tl}(m) d_S(m) dm$$

It is an effective inclusion of loop effects!

Applications to hadrons (eLSM):

D. Parganlija, F. G. and D. H. Rischke,
Vacuum Properties of Mesons in a Linear Sigma Model with Vector Mesons and Global Chiral Invariance,,
Phys. Rev. D 82 (2010) 054024 [arXiv:1003.4934 [hep-ph]].

F. Divotgey, L. Olbrich and F. G.,
Phenomenology of axial-vector and pseudovector mesons and their mixing in the kaonic sector,
to appear in EPJA, arXiv:1306.1193 [hep-ph].

…

Quantum field theory: the decay law

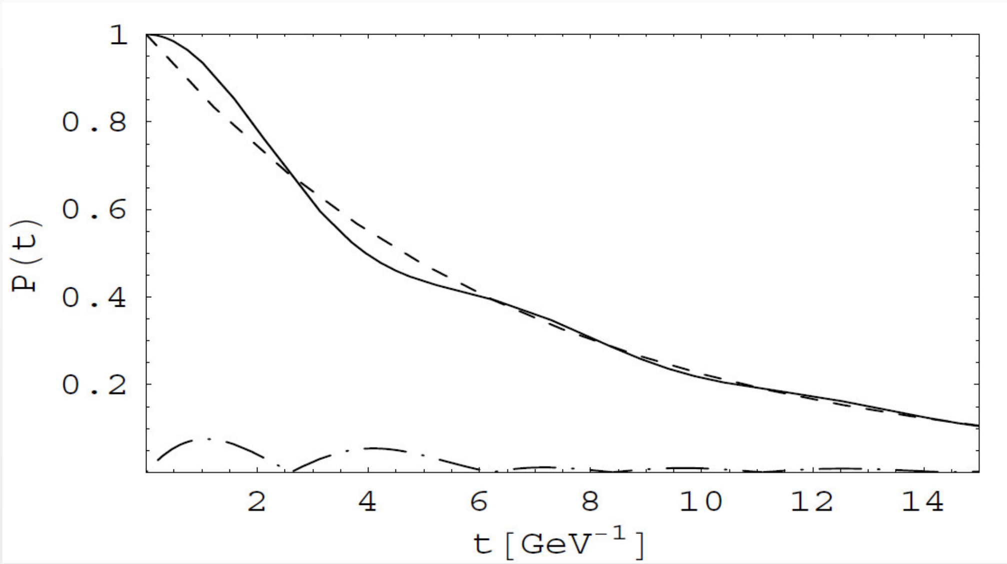
Example: p(t) for the ρ meson

Survival probability amplitude:

$$a(t) = \int_0^\infty dm\, d_S(m) e^{-imt}$$

Just as in QM: non-trivial result!

No dep. on cutoff for a superrenormalizable field theory



Details in: F. G. and G. Pagliara,
Deviation from the exponential decay law in relativistic quantum field theory: the example of strongly decaying particles,
Mod. Phys. Lett. A **26** (2011) 2247 [arXiv:1005.4817 [hep-ph]].

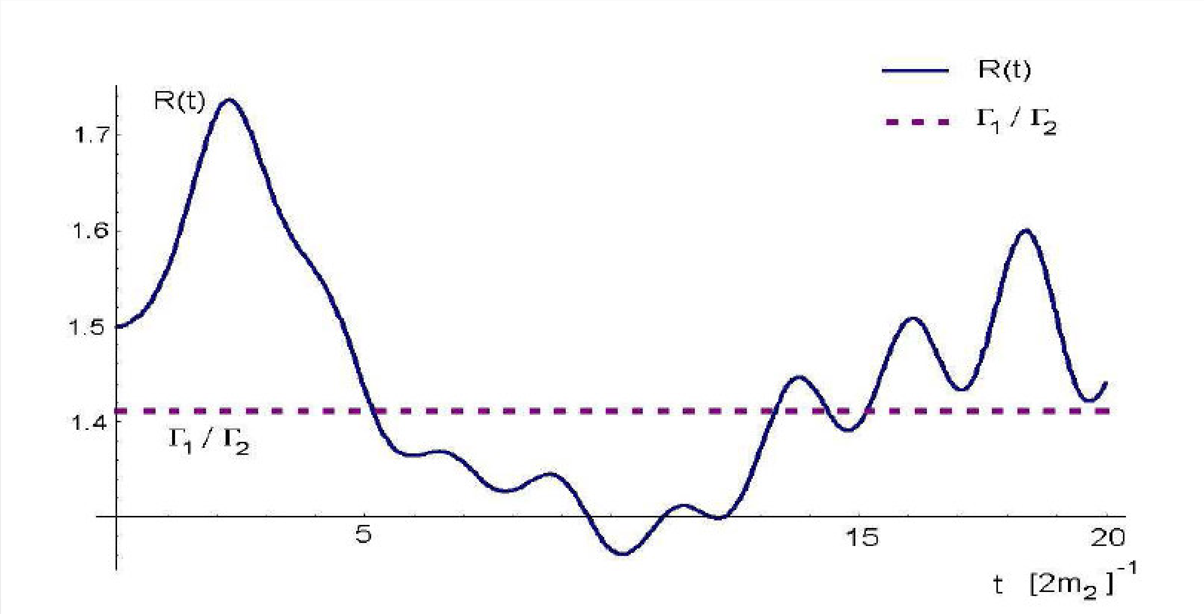
Quantum field theory: two-channel case

$$L_{int} = g_1 S \varphi_1^2 + g_2 S \varphi_2^2$$

$h_1(t)dt =$ probabilty that the state $|S\rangle$ decays in the first channel between (t,t+dt)

$h_2(t)dt =$ probabilty that the state $|S\rangle$ decays in the second channel between (t,t+dt)

$$\frac{h_1(t)}{h_2(t)}$$



Details in: F. Giacosa,
Non-exponential decay in quantum field theory and in quantum mechanics: the case of two (or more) decay channels,'
Found. Phys. **42** (2012) 1262 [arXiv:1110.5923 [nucl-th]].

Quantum field theory: is there a maximal energy scale? (1)

Infinities, renormalization, high energy scale,…

From A. Zee, Quantum field Theory in a nutshell: *"I emphasize that Λ should be thought of as physical, parametrizing our threshold of ignorance, and not as a mathematical construct. Indeed, physically sensible quantum field theories should all come with an implicit Λ. If anyone tries to sell you a field theory claiming that it holds up to arbitrarily high energies, you should check to see if he sold used cars for a living*

$L_{\text{int}} = gH\overline{\psi}\psi$ This is a renorm. theory.

Calculation of the energy distribution $d_H(m)$

Quantum field theory: is there a “maximal energy scale? (2)

$$\int_0^{\Lambda} d_H(m)dm = 1 \qquad\qquad d_H(m) \propto 1/(m \cdot \ln^2 m) \quad \text{for large m}$$

no matter how large is Λ...

but if one tries to do $\Lambda \to \infty$ one encounters problems:

normalization, etc.

Finite outcome: even for a renorm. QFT the existence of a maximal energy scale (i.e., a minimal length) is needed.

F. G. and G.Pagliara, *Spectral function of a scalar boson coupled to fermions*, Phys. Rev. D 88 (2013) 025010 [arXiv:1210.4192].

Part 4: Decay of a moving particle

Unstable particle with momentum p

We work in the exp. limit

M = rest mass; Γ= decay width in the rest frame.

An unstable particle moves with definite momentum p.

Which is its decay width? The stanard expression is:

$$\tilde{\Gamma}_p = \frac{\Gamma}{\gamma} \equiv \frac{\Gamma M}{\sqrt{p^2 + M^2}}$$

Important but sublte point:
in QM and QFT a state with definite momentum has not definite velocity.

Unstable particle with momentum p: unexpected result

$$|S,p\rangle = U_p \, |S,0\rangle$$

$$|S,p\rangle = \int_0^\infty dm a_S(m) \, |m,p\rangle$$

The non-decay probability: $P_{nd}(t) = e^{-\Gamma_p t}$

$$\Gamma_p = \sqrt{2}\sqrt{\left[\left(M^2 - \frac{\Gamma^2}{4} + p^2\right)^2 + M^2\Gamma^2\right]^{1/2} - \left(M^2 - \frac{\Gamma^2}{4} + p^2\right)}$$

F. G. arXiv:1512.00232 [hep-ph]

$$\Gamma_p \neq \tilde{\Gamma}_p = \Gamma M / \sqrt{p^2 + M^2}.$$

But this is not a breaking of relativity!
It is a different setup.

Unstable particle with momentum p: previous works

L. A. Khalfin, Theory of unstable particles and relativity, PDMI Preprint/1997

M. I. Shirokov, JIMR E2 10614 (1977), Int. J. Theor. Phys. 43 (2004) 1541.

E. V. Stefanovich, Int. Jour. Theor. Phys, 35 12 (1996)

K. Urbanowski, Phys. Lett. B 737 (20014) 346.

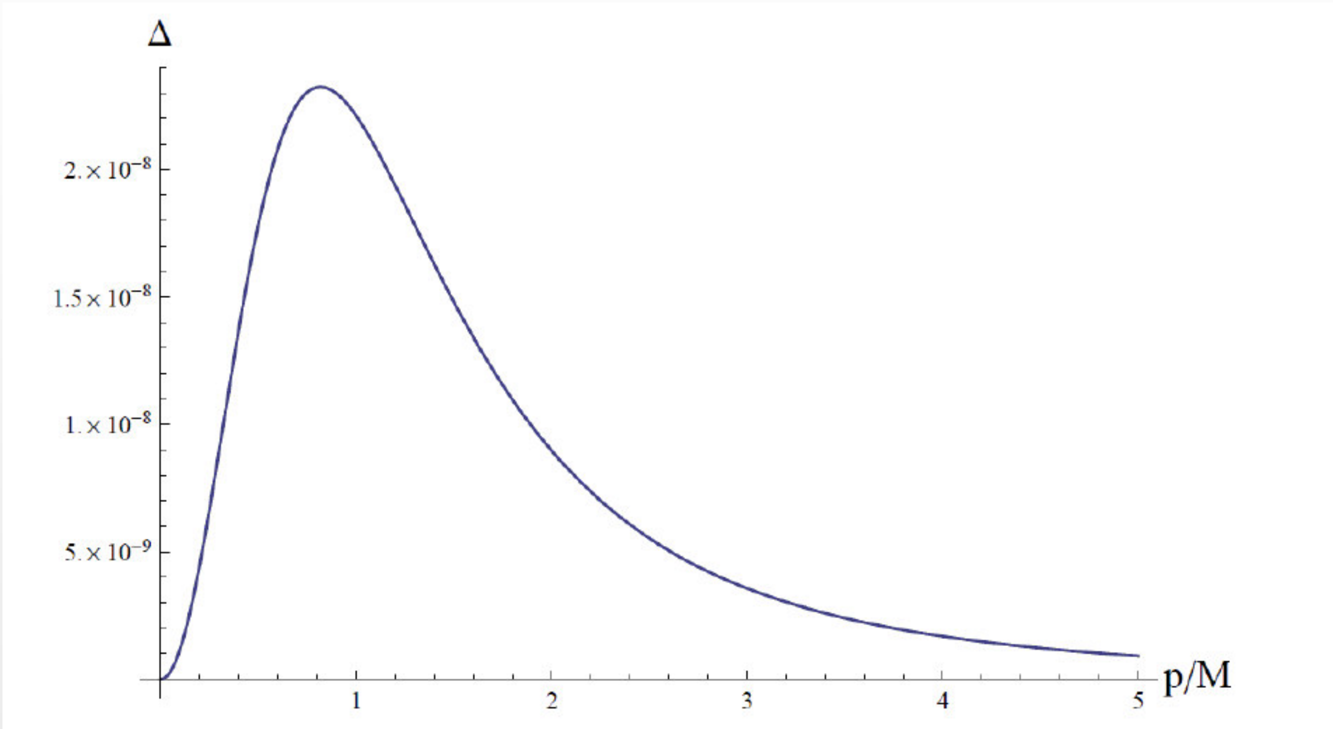
See also the negative result

S. A. Alavi and C. Giunti, Europhys. Lett. 109 (2015) 6, 6001

My recent paper: F. G. arXiv:1512.00232 [hep-ph]

Unstable particle with momentum p:deviation

$$\Delta = \frac{\Gamma_p - \tilde{\Gamma}_p}{M}$$



$$\frac{p_{\max}}{M} = \sqrt{\frac{2}{3}} \simeq 0.816$$

$$\Delta_{\max} = \frac{\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}}}{M} \simeq \frac{3}{100}\sqrt{\frac{3}{5}}\left(\frac{\Gamma}{M}\right)^3$$

Unstable particle with momentum p: some examples of deviations

Muon
M = 105.65 MeV
Γ = $2.99 \cdot 10^{-16}$ MeV

$$\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}} \simeq 5.598 \cdot 10^{-53} \text{ MeV}$$

Neutral pion
M = 134.98 MeV
Γ = $7.72 \cdot 10^{-6}$ MeV

$$\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}} \simeq 5.81 \cdot 10^{-22} \text{ MeV}$$

Rho meson
M = 775.26 MeV
Γ = 147.8 MeV

$$\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}} \simeq 0.125 \text{ MeV}$$

Very small deviations!

Wave packet

$$|\Psi\rangle = \int_{-\infty}^{+\infty} dp B(p)\,|S,p\rangle$$

the quantity $\langle \Psi \,|e^{-iHt}|\, \Psi \rangle$ is *not* what we are looking for.

$$P_{nd}(t) = \int_{-\infty}^{+\infty} dp\, \left|\langle S,p\,|e^{-iHt}|\,\Psi\rangle\right|^2$$

$$P_{nd}(t) = \int_{-\infty}^{+\infty} dp\, |B(p)|^2\, e^{-\Gamma_p t}$$

Inclusion of spatial wave function is simple.

Boost: state with definite velocity

$$U_v \left| S, 0 \right\rangle \equiv \left| S, v \right\rangle$$

$$\left| S, v \right\rangle = \int_0^\infty dm a_S(m) \sqrt{m} \gamma^{3/2} \left| m, m\gamma v \right\rangle$$

$$P_{nd}(t) = 0$$

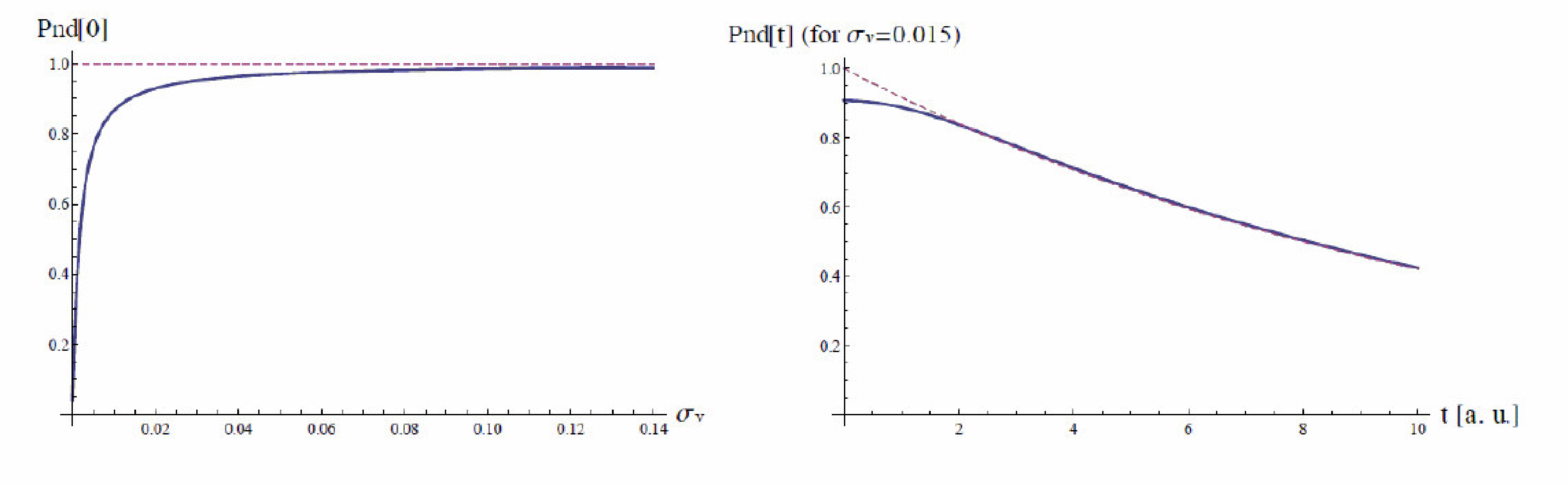
A boosted muon consists of an electron and two neutrinos!

Boost: wave packet in velocity (is qualitatively different!)

$$|\Phi\rangle = \int_{-1}^{+1} dv C(v) |S, v\rangle$$

$$C(v) = N e^{-(v-v_0)^2/(4\sigma_v^2)}$$

$$P_{nd}(t) = \int_{-\infty}^{+\infty} dp \left| \langle S, p | e^{-iHt} | \Phi \rangle \right|^2$$



Summary and outlook

- The decay is never exponential! This is a fact.

- QM: Lee Hamiltonian, deviations easily explained;
  final state energy spectrum broadens at short t
  two-channel case: the ratio!

- QFT: qualitatively just as in QM!
  Deviations from exp. in particle physics.
  Two-channel decay also here interesting.
  Minimal length scale.

Summary and outlook

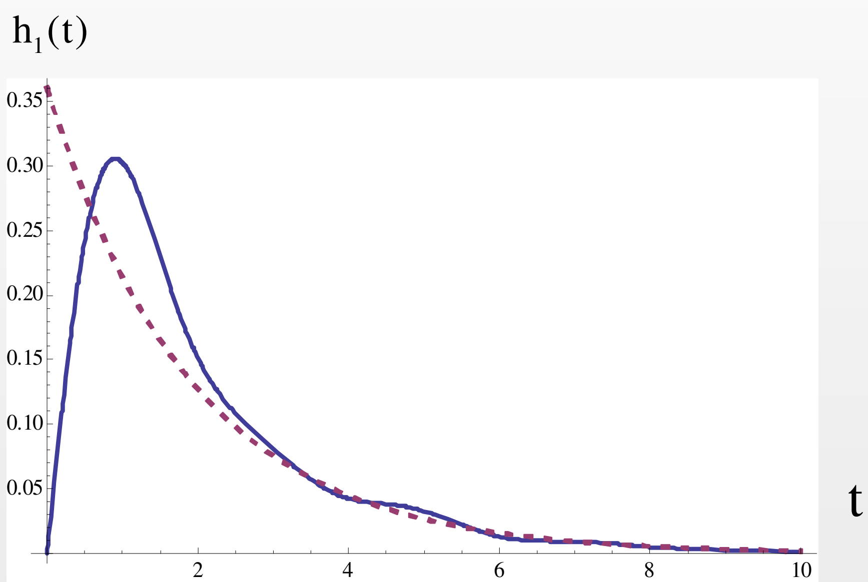
- Decay of a moving particle: interesting link between relativitu and QM and QFT.
- For a particle with definite momentum p (for the measuring observer) there is a different formula.
- A boost is a very subtle operation in QM and QFT.

Thank You!

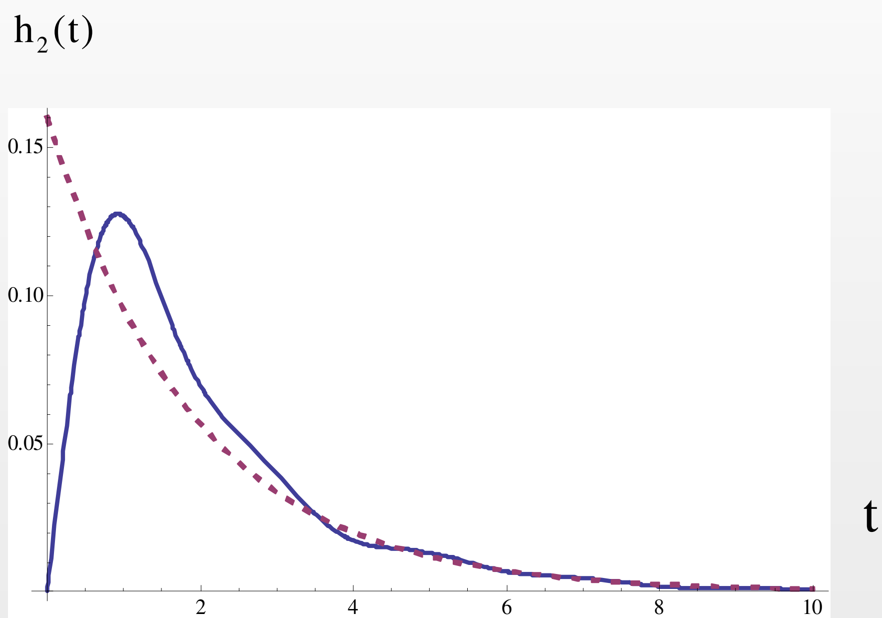
Two-channel case (2)

$h_1(t)dt =$ probabilty that the state $|S\rangle$ decays in the first channel between (t,t+dt)

$h_2(t)dt =$ probabilty that the state $|S\rangle$ decays in the second channel between (t,t+dt)



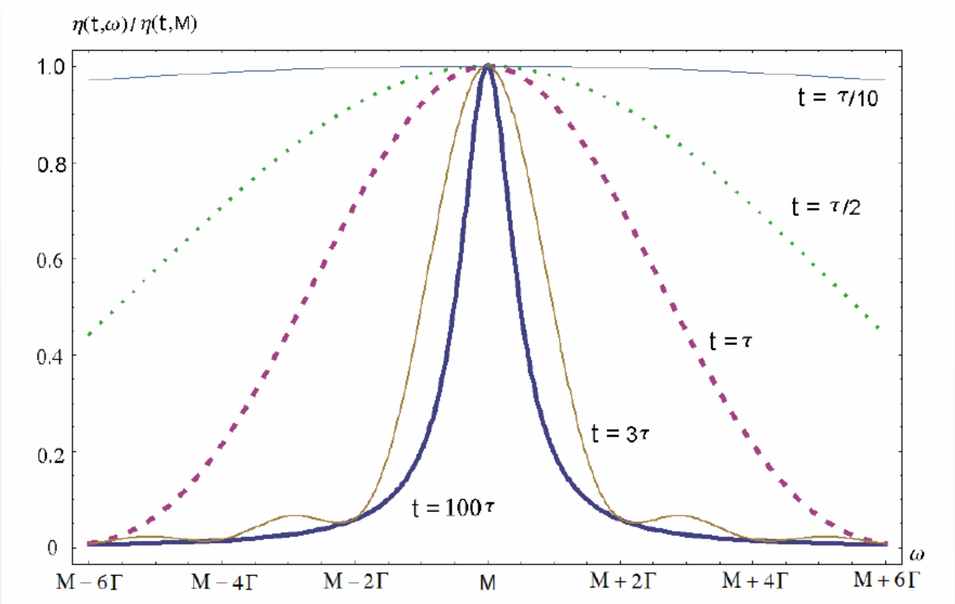
Dashed: $h_{1,BW}(t) = \Gamma_1 e^{-\Gamma t}$ with $\Gamma_1 = \mathrm{Im}[\Pi_1(M)]/2$



Dashed: $h_{2,BW}(t) = \Gamma_2 e^{-\Gamma t}$ with $\Gamma_2 = \mathrm{Im}[\Pi_2(M)]/2$

Final state energy spectrum (3)

$$\eta(t,\omega) = \frac{\Gamma}{2\pi}\left|\frac{e^{-i\omega t} - e^{-i(M_0 - i\Gamma/2)t}}{E - M_0 + i\Gamma/2}\right|^2$$



Details in: F. G., arXiv:1305.4467 [quant-ph].

Mathematical details for non-exp decay

1) There is an energy threshold:

$d_S(E) = 0$ for $E < E_{min} \Rightarrow p(t)$ is for large times not exp.

2) $d_S(E)$ converges faster than $1/E^2$ for large E (form factors):

$$\langle E \rangle = \int_{-\infty}^{+\infty} v(E) E dE = \int_{E_{min}}^{+\infty} v(E) E dE \quad \text{ist endlich} \;\Rightarrow p(t) \text{ ist für kleine Zeiten keine exp. Funktion} \;\; (p'(t=0)=0)$$

If we also assume that $\langle E^2 \rangle$ is finite:

$$p(t) = 1 - t^2 \left(\langle E^2 \rangle - \langle E \rangle^2 \right) + \ldots$$

Details in:

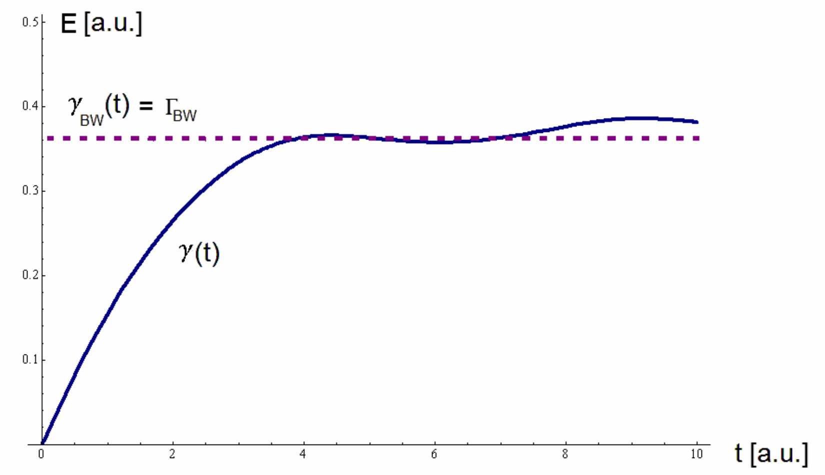
Rep. Prog. Phys., Vol. 41, 1978. Printed in Great Britain

Decay theory of unstable quantum systems

L FONDA, G C GHIRARDI and A RIMINI

General description of the Zeno and anti-Zeno effects

$$p(t) = e^{-\gamma(t)t} \Rightarrow \gamma(t) = -\frac{1}{t}\ln p(t)$$



Survival probabilty after a single measurement

$$p(T) = e^{-\gamma(T)T}$$

Survival probability after N measurments:

$$p(\tau)^N = e^{-\gamma(\tau)\tau N} = e^{-\gamma(\tau)T} > e^{-\gamma(T)T} \quad \text{wenn} \quad \gamma(\tau) < \gamma(T)$$ Zeno effect

Und für $\tau \to 0, \gamma(\tau \to 0) \to 0, \; p(\tau)^N \to 1$

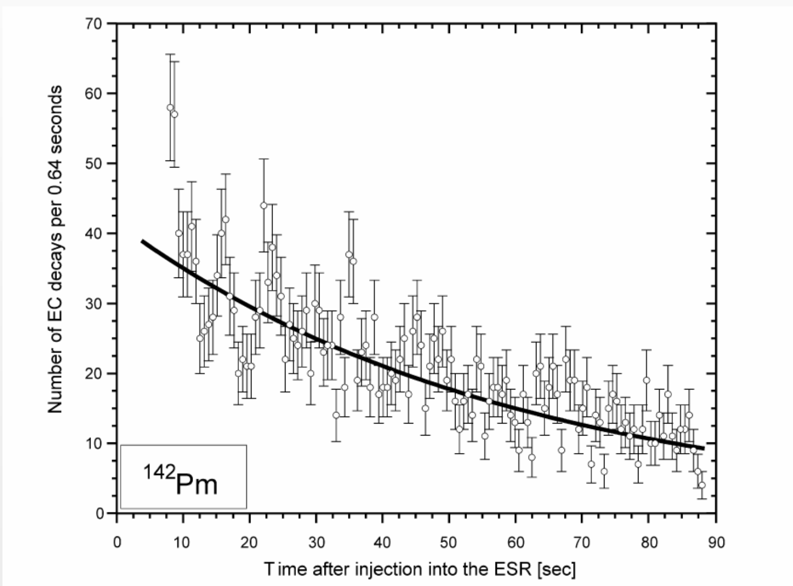
How it is also possible that: $\gamma(\tau) > \gamma(T) : \; p(\tau)^N = e^{-\gamma(\tau)\tau N} = e^{-\gamma(\tau)T} < e^{-\gamma(T)T}$ Anti-Zeno-Effekt

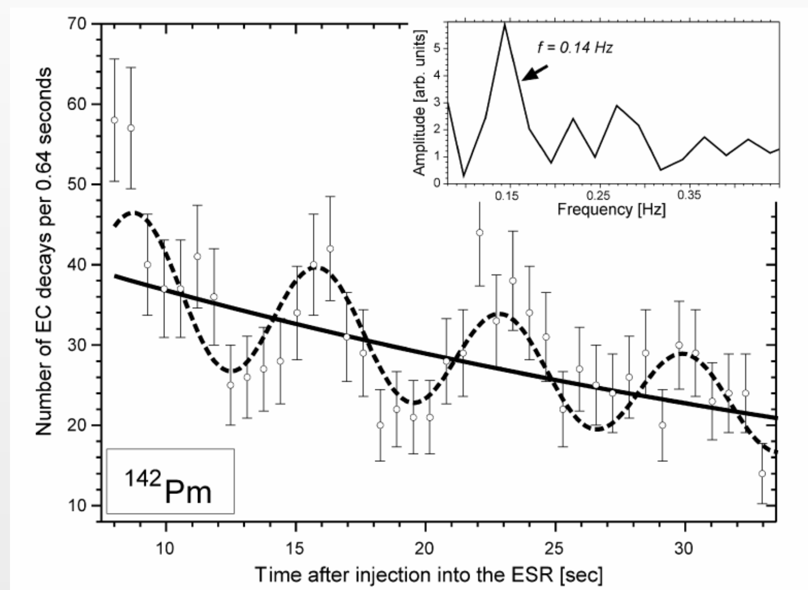
GSI-Anomalie (details)

$$\underbrace{^{142}_{61}\text{Pm} + e^-}_{\text{ein Zustand}} \to \nu_e + {}^{142}_{60}\text{Nd}$$

Gemessen wurde:

$$\frac{dN_{\text{Zerfälle}}}{\text{dt}} \propto -\frac{dp(t)}{dt}$$





$$\frac{dN_{\text{EC}}(t)}{dt} = N(0) \cdot e^{-\lambda t} \cdot \widetilde{\lambda_{\text{EC}}}(t),$$

$$\widetilde{\lambda_{\text{EC}}}(t) = \lambda_{\text{EC}} \cdot [1 + a \cdot \cos(\omega t + \phi)]$$

Fit parameters of ^{142}Pm data

Eq.	$N_0\lambda_{\text{EC}}$ [s^{-1}]	λ [s^{-1}]	a	ω [s^{-1}]	ϕ	χ^2/DoF
(1)	41.5(17)	0.0170(9)	–	–	–	173/124
(1)	46.8(40)*	0.0240(42)*	–	–	–	63.77/38*
(2)	46.0(39)*	0.0224(42)*	0.23(4)*	0.885(31)*	−1.6(5)*	31.82/35*

Two-channel case: formal eqs

$$H = H_0 + H_1$$

$$H_0 = M_0 |S\rangle\langle S| + \int_{-\infty}^{+\infty} dk\omega_1(k)|k,1\rangle\langle k,1| + \int_{-\infty}^{+\infty} dk\omega_2(k)|k,2\rangle\langle k,2|$$

$$H_1 = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dk(g_1 \cdot f_1(k))(|S\rangle\langle k,1| + |k,1\rangle\langle S|) + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dk(g_2 \cdot f_2(k))(|S\rangle\langle k,2| + |k,2\rangle\langle S|)$$

$$G_S(E) = \langle S|(E - H + i\varepsilon)^{-1}|S\rangle = (E - M_0 + \Pi(E) + i\varepsilon)^{-1}$$

$$d_S(E) = \tfrac{1}{\pi}\operatorname{Im} G_S(E)$$

$$a(t) = \langle S|e^{-iHt}|S\rangle = \int_{-\infty}^{+\infty} dE d_S(E) e^{-iEt}$$

Quantum corrections:

$$\Pi(E) = \Pi_1(E) + \Pi_2(E)$$

$$d_S(E) = d_S^1(E) + d_S^2(E)$$

$$d_S^1(E) = \frac{1}{\pi}\frac{\operatorname{Im}\Pi_1(E)}{(E - M + \operatorname{Re}\Pi(E))^2 + (\operatorname{Im}\Pi(E))^2};$$

$$a(t) = a_1(t) + a_2(t)$$

Two-channel case: decay probabilities

$h_1(t)dt =$ probabilty that the state $|S\rangle$ decays in the first channel between (t,t+dt)

$h_2(t)dt =$ probabilty that the state $|S\rangle$ decays in the second channel between (t,t+dt)

$$h_{1,BW}(t) = \Gamma_1 e^{-\Gamma t} \quad \text{with } \Gamma_1 = \text{Im}[\Pi_1(M)]/2 \qquad h_{2,BW}(t) = \Gamma_2 e^{-\Gamma t} \quad \text{with } \Gamma_2 = \text{Im}[\Pi_2(M)]/2$$

$$a_i(t) = \int_{-\infty}^{+\infty} dE d_S^i(E) e^{-iEt}$$

$$A_1(t) = |a_1(t)|^2 ;\ A_2(t) = |a_2(t)|^2 \ ;\ A_{mix}(t) = \text{Re}[a_1(t) a_2^*(t)]$$

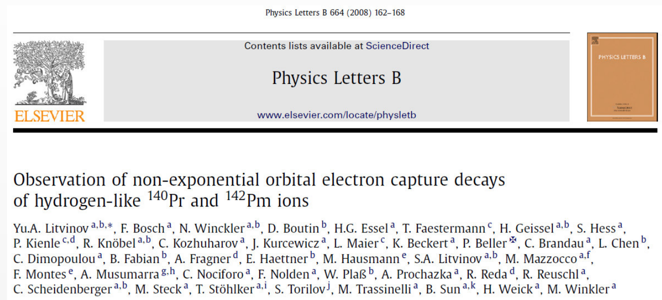
Conjecture of the solutions:

$$h_1(t) = -\frac{d}{dt}\left(A_1(t) + A_{mix}(t)\right)$$

$$h_2(t) = -\frac{d}{dt}\left(A_2(t) + A_{mix}(t)\right)$$

GSI-Anomaly (1)

Measurement of weak decays of ions.


Physics Letters B 664 (2008) 162–168
Contents lists available at ScienceDirect
Physics Letters B
www.elsevier.com/locate/physletb

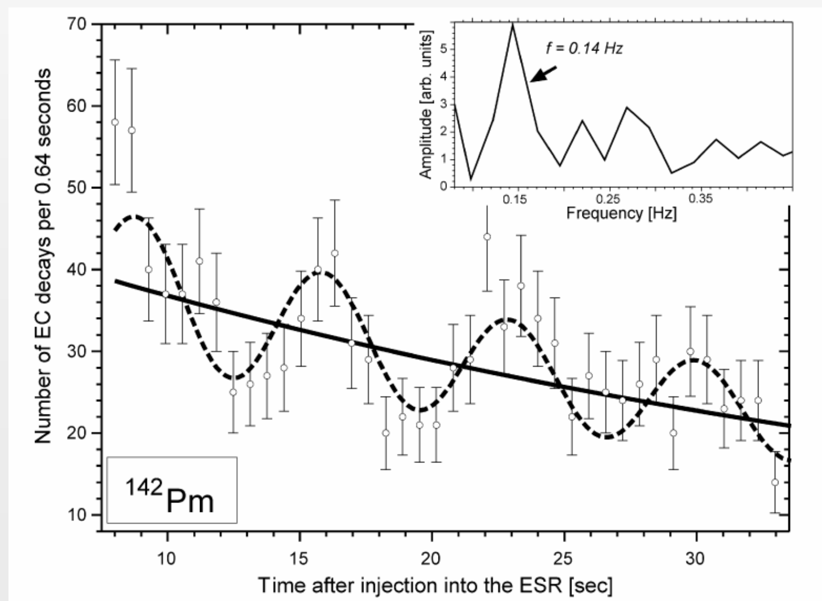
Observation of non-exponential orbital electron capture decays of hydrogen-like ^{140}Pr and ^{142}Pm ions

Yu.A. Litvinov[a,b,*], F. Bosch[a], N. Winckler[a,b], D. Boutin[b], H.G. Essel[a], T. Faestermann[c], H. Geissel[a,b], S. Hess[a], P. Kienle[c,d], R. Knöbel[a,b], C. Kozhuharov[a], J. Kurcewicz[a], L. Maier[c], K. Beckert[a], P. Beller[✠], C. Brandau[a], L. Chen[b], C. Dimopoulou[a], B. Fabian[b], A. Fragner[d], E. Haettner[b], M. Hausmann[e], S.A. Litvinov[a,b], M. Mazzocco[a,f], F. Montes[e], A. Musumarra[g,h], C. Nociforo[a], F. Nolden[a], W. Plaß[b], A. Prochazka[a], R. Reda[d], R. Reuschl[a], C. Scheidenberger[a,b], M. Steck[a], T. Stöhlker[a,i], S. Torilov[j], M. Trassinelli[a], B. Sun[a,k], H. Weick[a], M. Winkler[a]

$$\underbrace{{}^{142}_{61}\mathrm{Pm} + e^-}_{\text{One state}} \rightarrow \upsilon_e + {}^{142}_{60}\mathrm{Nd}$$

Measurement was:

$$\frac{dN_{decays}}{dt} \propto -\frac{dp(t)}{dt}$$



Oscillations very recently confirmed!

arXiv:1309.7294 [nucl-ex].

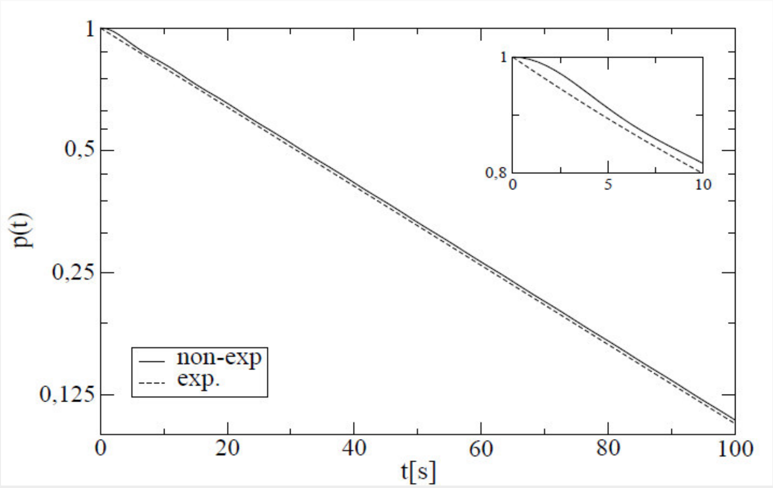
Francesco Giacosa

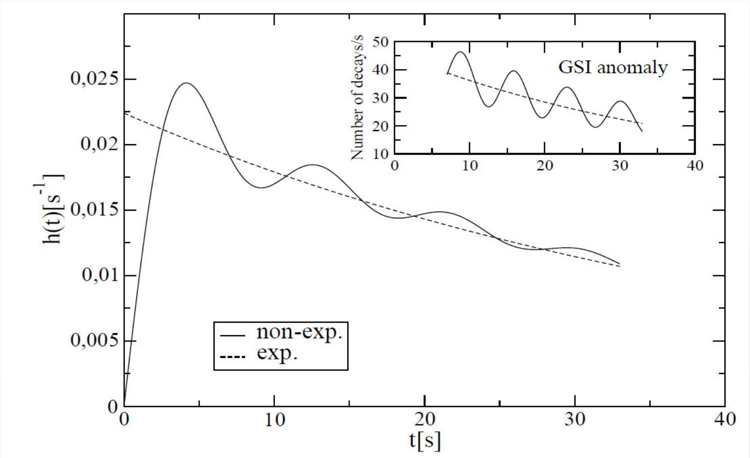
GSI-Anomaly (2)

• Up to now: no explanation of these oscillations!

Neutrino oscillations, (coherent/incoherent sum…), quantum beats,… V. P. Krainov, J. of Exp. and Theor. Phys., Vol.**115**, 68-75

• Simple idea: non-exp. decay due to deviations from the Breit-Wigner limit: Cutoff





$$h(t) = -\frac{dp}{dt}$$

$$\Lambda = 32\Gamma$$

$$d_S(E) = N\frac{\theta(\Lambda^2 - (E-M)^2)}{(E-M)^2 + \Gamma^2/4}$$

Details in: F. G. and G.Pagliara,
Oscillations in the decay law: A possible quantum mechanical explanation of the anomaly in the experiment at the GSI facility,
Quant. Matt **2** (2013) 54 [arXiv:1110.1669 [nucl-th]].

Exponential limit and final state spectrum (1)

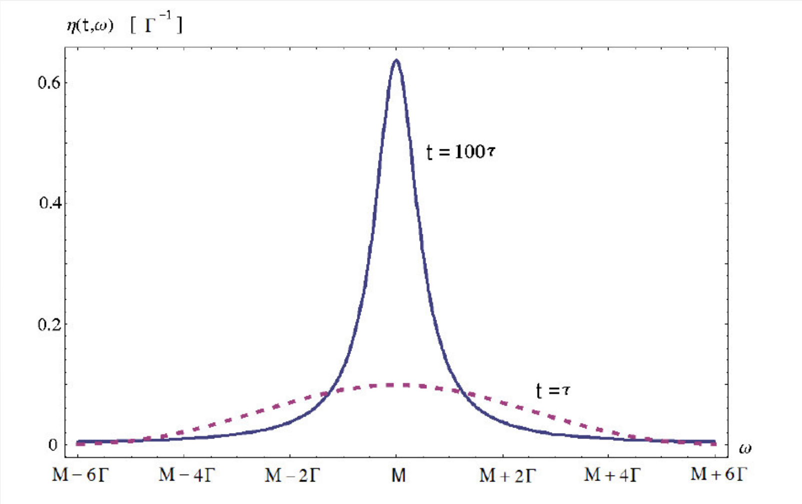
$\left|\left\langle k\right|e^{-iHt}\left|S\right\rangle\right|^2$ is the prob. that $\left|S\right\rangle$ transforms into $\left|k\right\rangle$

Translating into energy:

$$\eta(t,\omega)=\frac{\Gamma}{2\pi}\left|\frac{e^{-i\omega t}-e^{-i(M_0-i\Gamma/2)t}}{E-M_0+i\Gamma/2}\right|^2 \quad ;$$

In spont. emission:

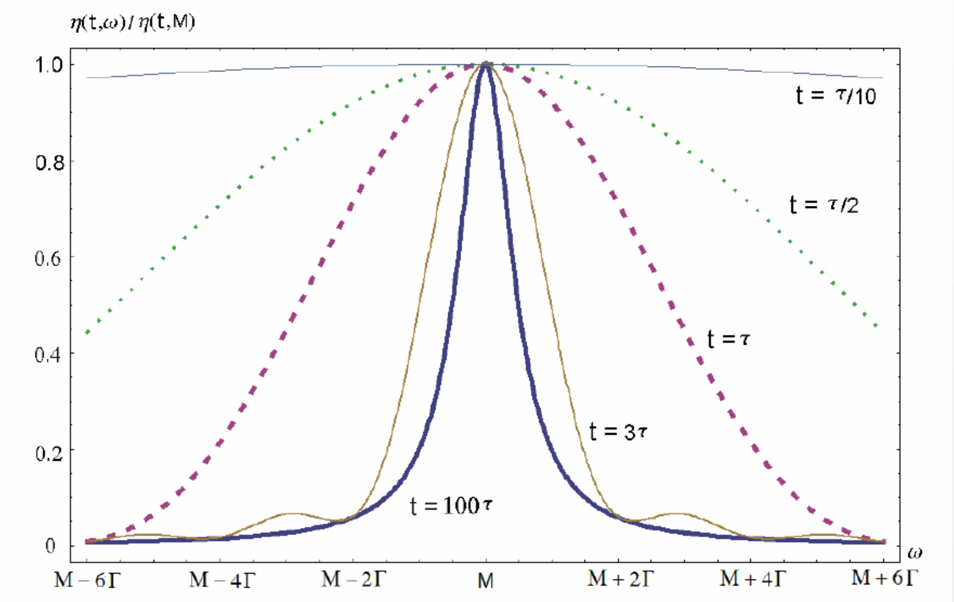
$\eta(t,\omega)d\omega$ is the prob. that the outgoing photon has an energy between ω and $\omega+d\omega$



Details in: F. G.,
Energy uncertainty of the final state of a decay process
arXiv:1305.4467 [quant-ph].

Exponential limit and final state spectrum (2)

$$\eta(t,\omega) = \frac{\Gamma}{2\pi} \left| \frac{e^{-i\omega t} - e^{-i(M_0 - i\Gamma/2)t}}{E - M_0 + i\Gamma/2} \right|^2$$



Details in: F. G., arXiv:1305.4467 [quant-ph].