Axions and lattice QCD

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Axion

A hypotethical elementary particle introduced to solve a

puzzle with the parity transformation

in particle physics and a leading candidate for the

dark matter particle .

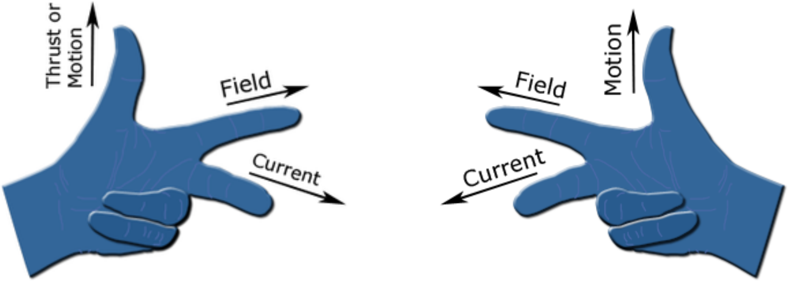
Parity transformation (P)

Flip the sign of all spatial coordinates.

$$x \to -x, \quad y \to -y, \quad z \to -z$$

Parity transforms an object to its mirror image

$$L \to R, \quad R \to L$$



Is P a symmetry?

Are the laws of physics the same in the mirror world?

Or can I tell the difference between the original and the mirror image?

Is P a symmetry?

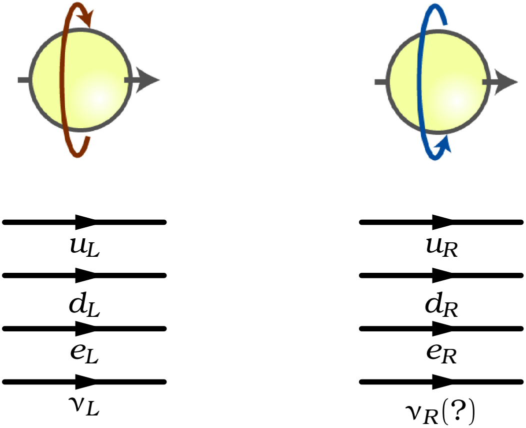
Particle physics 101

Particles: up quark, down quark, electron and neutrino.

Interactions: strong, electromagnetic, weak and Higgs.

Parity and particles

Particles have spin and are massless $\rightarrow$



P exchanges left-handed particles with right-handed

Electromagnetic interaction

$$\nabla E = 4\pi\rho, \quad \nabla B = 0, \quad \nabla \times E + \partial_t B = 0, \quad \nabla \times B - \partial_t E = 4\pi j$$

$$P : E \to -E, B \to B, \rho \to \rho, j \to -j, \nabla \to -\nabla$$

Classical electrodynamics

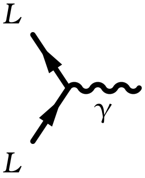
↓

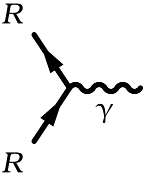
P symmetric

↑

Quantum electrodynamics

$$L_{QED} = -\frac{1}{4}F_{\mu\nu}^2 + \psi_L^\dagger \sigma_\mu D_\mu \psi_L + \psi_R^\dagger \overline{\sigma}_\mu D_\mu \psi_R$$



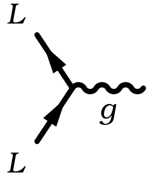


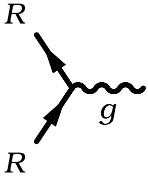
Strong interaction

Generalized QED: 3x3-matrices and 3-vectors instead of numbers

$$F_{\mu\nu} \to \begin{pmatrix} G^{00}_{\mu\nu} & G^{01}_{\mu\nu} & G^{02}_{\mu\nu} \\ G^{10}_{\mu\nu} & G^{11}_{\mu\nu} & G^{12}_{\mu\nu} \\ G^{20}_{\mu\nu} & G^{21}_{\mu\nu} & G^{22}_{\mu\nu} \end{pmatrix} \qquad \psi \to \begin{pmatrix} \psi^0 \\ \psi^1 \\ \psi^2 \end{pmatrix}$$

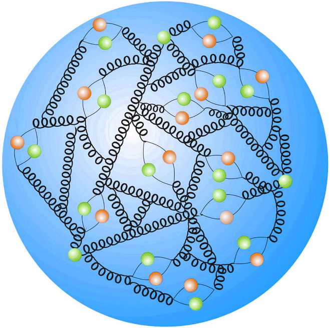
$$L_{QCD} = -\frac{1}{4}\mathrm{Tr}(G^2_{\mu\nu}) + (\psi^\dagger_L, \sigma_\mu D_\mu \psi_L) + (\psi^\dagger_R, \overline{\sigma}_\mu D_\mu \psi_R)$$





P symmetric

Neutron electric dipole moment



P is symmetry $[H, P] = 0$

$\downarrow$

neutron is P-eigenstate $P|n\rangle \propto |n\rangle$

$\downarrow$

$\langle n|\vec{d}|n\rangle = 0$

$\uparrow$

EDM is P-odd $P \cdot \vec{d} \cdot P = -\vec{d}$

Expt: $\langle n|\vec{d}|n\rangle = -0.2(1.9) \times 10^{-26}$ ecm [Pendlebury '15]

Higgs interaction

Higgs mechanism

Higgs interaction

figs/bush.png

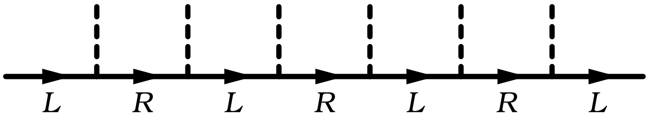
Higgs interaction

figs/bush.png

"Left hand knows what the right hand is doing."

Higgs interaction

"Left hand knows what the right hand is doing."



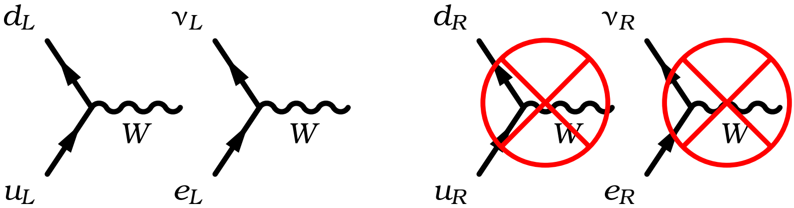
$$L_{Higgs} = m\psi_L^\dagger \psi_R + m\psi_R^\dagger \psi_L$$

Generate mass by combining massless L,R particles

P symmetric

Weak interaction

Maximally violates parity, only interacts with L-particles.



P violating

Left:= the handedness of particles to which W couples

The P puzzle

electromagnetic	✓
strong	✓
Higgs	✓
weak	✗

P is violated by the weak interaction $\rightarrow$ it is not symmetry of Nature.

Why P is not violated by the others?

Try: P-invariance is consequence of remaining symmetries (Lorentz invariance, internal).

The P puzzle in QED

$$L_{QED} = -\frac{1}{4}F_{\mu\nu}^2 + \psi_L^\dagger \sigma_\mu D_\mu \psi_L + \psi_R^\dagger \overline{\sigma}_\mu D_\mu \psi_R$$

What is the most general Lagrangian with Lorentz invariance and gauge symmetry?

$$L = L_{QED} + \theta \cdot F\widetilde{F} \quad \text{with} \quad F\widetilde{F} \equiv F_{\mu\nu} F_{\rho\sigma} \epsilon_{\mu\nu\rho\sigma}$$

Violates parity

$$F\widetilde{F} \to -F\widetilde{F}$$

Total derivative

$$F\widetilde{F} = \partial_\mu K_\mu \quad \text{with} \quad K_\mu = \epsilon_{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma}$$

then by Gauss-theorem

$$\int d^4x \, F\widetilde{F} = \oint dn_\mu K_\mu = 0$$

P-invariance follows from Lorentz+gauge

The P puzzle in QCD

Most general SU(3) symmetric Lagrangian

$$L = L_{QCD} + \theta \cdot G\widetilde{G} \quad \text{with} \quad G\widetilde{G} = \frac{1}{8\pi^2}\mathrm{Tr}\left(G_{\mu\nu}G_{\rho\sigma}\epsilon_{\mu\nu\rho\sigma}\right)$$

Violates parity

$$G\widetilde{G} \to -G\widetilde{G}$$

Total derivative

$$G\widetilde{G} = \partial_\mu K_\mu \quad \text{with} \quad K_\mu = \mathrm{Tr}\epsilon_{\mu\nu\rho\sigma}\left(A_\nu G_{\rho\sigma} + \frac{2}{3}A_\nu A_\rho A_\sigma\right)$$

then by Gauss-theorem

$$\int d^4x\, G\widetilde{G} = \oint dn_\mu K_\mu \neq 0$$

K_μ can be non-zero at ∞

P could be violated by $\theta \cdot G\widetilde{G}$. Why not?

nEDM experiments $\to \theta \lesssim 0.000000001$

P-violation in Higgs

$$L_{Higgs} = m\left(\psi_L^\dagger\psi_R + \psi_R^\dagger\psi_L\right) \quad \text{with real } m$$

Most general Lagrangian has complex mass:

$$L = m\psi_L^\dagger\psi_R + m^*\psi_R^\dagger\psi_L$$

It violates parity, but can be transformed away by an axial transformation:

$$\psi_L \to \psi_L, \quad \psi_R \to e^{-i\arg m}\psi_R$$

but P-violation does not go away:

$$L \to L + \arg m \cdot F\widetilde{F} + \arg m \cdot G\widetilde{G}$$

Higgs P violation can be transformed to strong $G\widetilde{G}$

The P puzzle

electromagnetic	✓
strong	✓
Higgs	✓
weak	✗

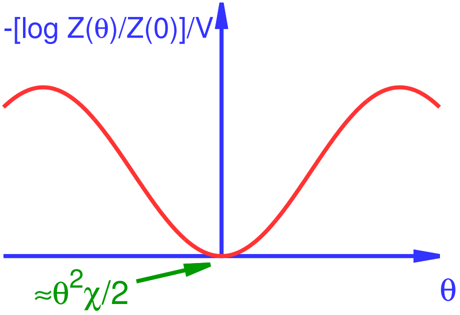
$\longleftarrow$ by Lorentz+gauge invariance

why $G\widetilde{G}$ not appears in Nature?

θ dependence of QCD

Calculate the Feynman path integral!

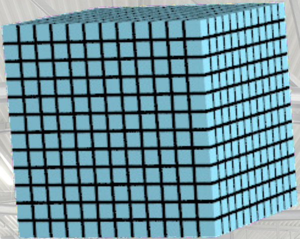
$$Z(\theta) = \int [dG][d\psi^\dagger][d\psi] \exp\left(i \int d^4x \; (L + \theta G\widetilde{G}) \right)$$



Has a minimum at $\theta = 0$!

Lattice QCD computation

Discretize!



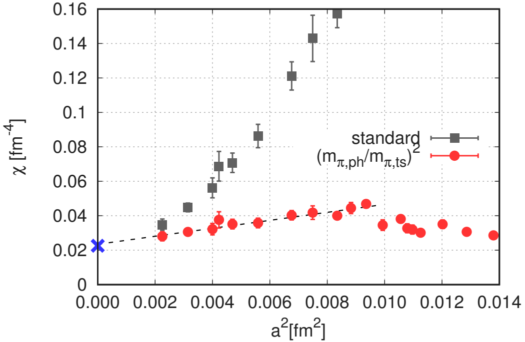
lattice spacing $a \lesssim \frac{1}{10}$ proton size

Calculate!

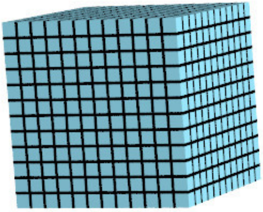
10^9 dimensional integrals using supercomputers

θ dependence from lattice QCD

$$-\frac{1}{V}\log Z(\theta)/Z(0) = \frac{1}{2}\theta^2\chi + \ldots$$



a=0.134 fm
2.000 PCyears



100.000 years on a PC, 1 year on a supercomputer.

θ dependence from lattice QCD

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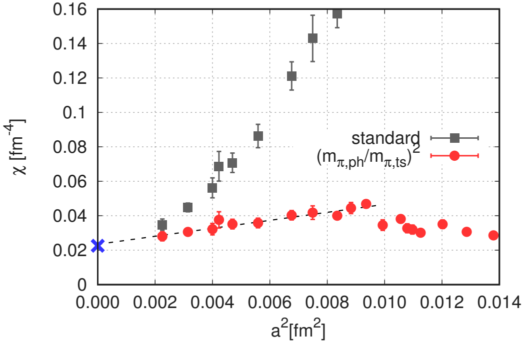
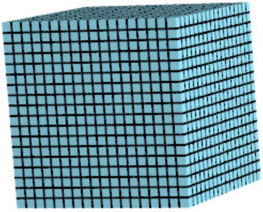


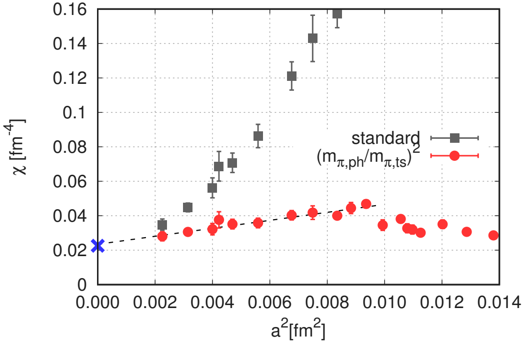
a=0.095 fm
12.000 PCyears



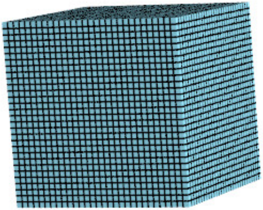
100.000 years on a PC, 1 year on a supercomputer.

θ dependence from lattice QCD

$$-\frac{1}{V}\log Z(\theta)/Z(0) = \frac{1}{2}\theta^2\chi + \dots$$



a=0.064 fm
86.000 PCyears



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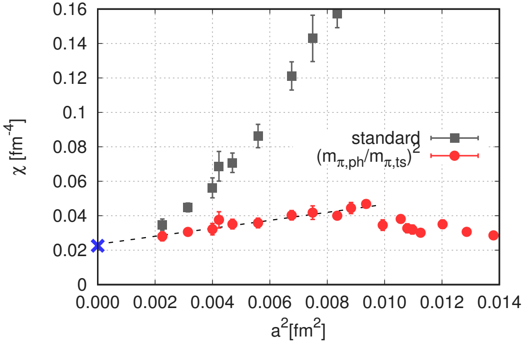
θ dependence from lattice QCD

$$-\frac{1}{V}\log Z(\theta)/Z(0) = \frac{1}{2}\theta^2\chi + \ldots$$



Continuum limit

$$\chi^{1/4} = 76(2)(1)\mathrm{MeV}$$

100.000 years on a PC, 1 year on a supercomputer.

A solution by Peccei-Quinn

Make a dynamical field from the parameter!

figs/thetapot/plot.gif

$$L + \theta \cdot G\widetilde{G} + \frac{1}{2} f^2 \cdot (\partial_\mu \theta)^2 + V(\theta, \partial_\mu \theta)$$

with $V(\theta, \partial_\mu \theta)$ such, that minimum stays at $\theta = 0$.

Dynamical field $\rightarrow$ new particle

The axion

"Cleaning up the problem with the axial transformation"

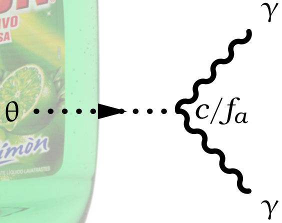
[Weinberg,Wilczek]

$$L_a = \theta \cdot G\widetilde{G} + \tfrac{1}{2} f_a^2 \cdot (\partial_\mu \theta)^2 + V(\theta, \partial_\mu \theta)$$

Mass ↔ Scale $m_a^2 = \chi / f_a^2$ with $\chi = 76(2)(1)\text{MeV}$

Interactions are model dependent

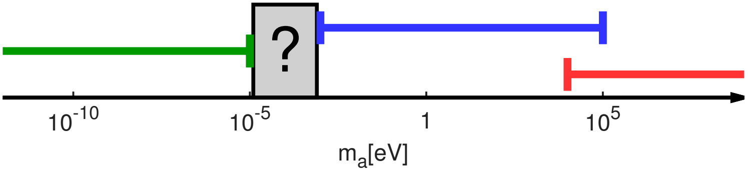
$$V(\theta, \partial\theta) = c \cdot \theta \cdot F\widetilde{F} + V_1(\partial\theta)$$



Smaller mass more elusive.

The axion window

Searching for axions is hard, since mass is unknown.



Exclusions on m_a from

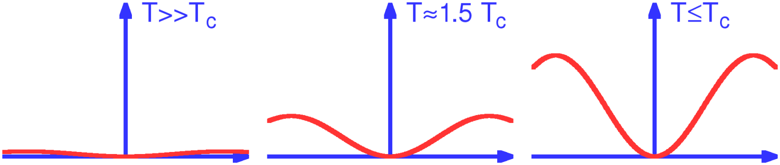
Early laboratory searches

Astrophysics (supernovae, red giants)

Axion is dark matter

Axion production in the early Universe

Potential becomes flat at QCD transition ($T_c \approx 150$MeV)



Calculate the number of axions produced!

Rolling down the potential ($\to \chi(T)$) + damped by expansion ($\to \epsilon(T), p(T)$ equation of state).

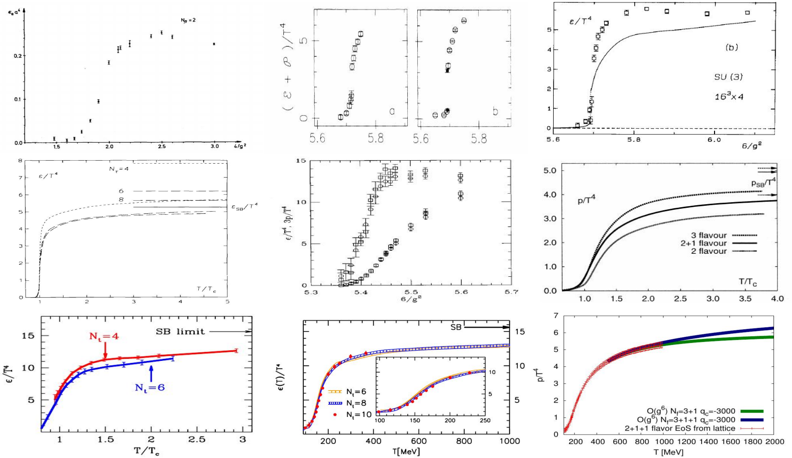
Need lattice QCD!

Calculation of the axion mass based on high-temperature lattice quantum chromodynamics

S. Borsanyi[1], Z. Fodor[1,2,3], J. Guenther[1], K.-H. Kampert[1], S. D. Katz[3,4], T. Kawanai[2], T. G. Kovacs[5], S. W. Mages[2], A. Pasztor[1], F. Pittler[3,4], J. Redondo[6,7], A. Ringwald[8] & K. K. Szabo[1,2]

doi.org/10.1038/nature20115

Equation of state from lattice QCD

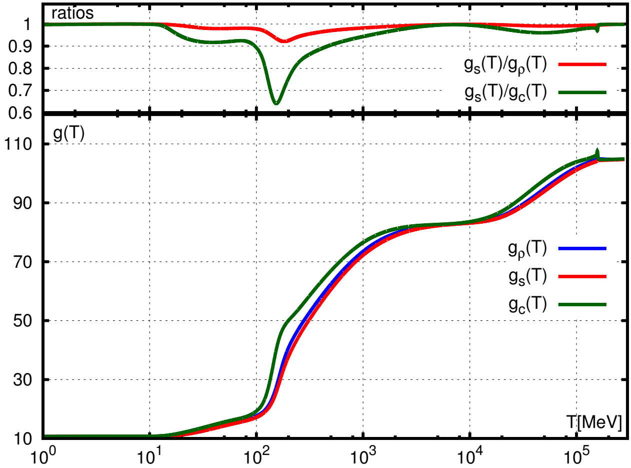


1981: pure SU(2), $N_t = 2$

2016: SU(3) + u,d,s,c,b; cont. extrap. from $N_t = 6 \ldots 16$

First time without “left for future work”!

Equation of state

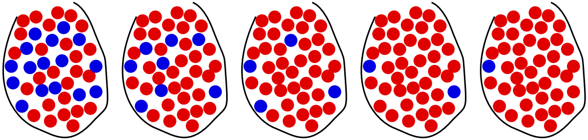


Full result= lattice QCD + weak [Laine,Meyer,Schroeder] +
photon + neutrinos + leptons

Determination of axion potential

Challenge

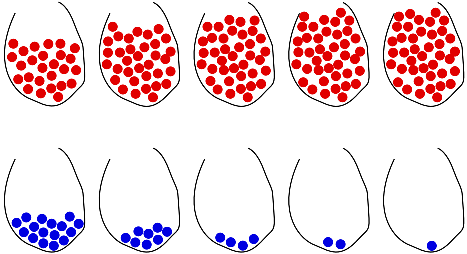
Determine the blue/red ratio by random pick!



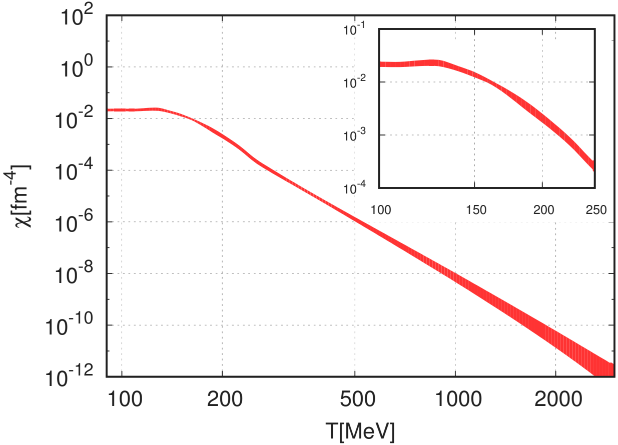
$\longrightarrow$ getting very difficult with T $\longrightarrow$

Solution

Separate colors and determine the rate of change with T!



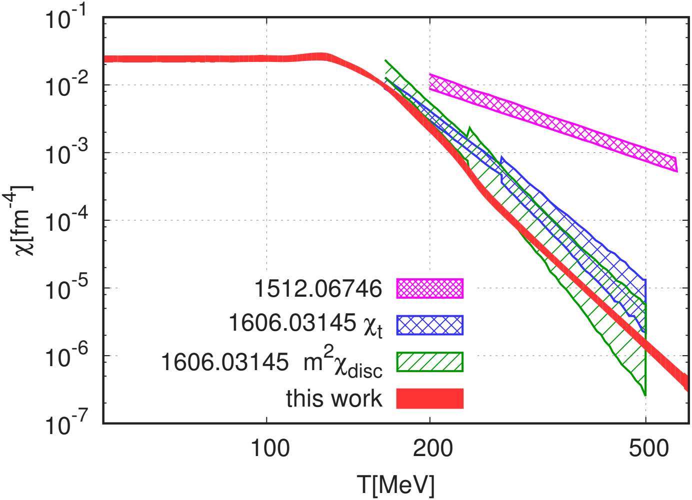
Axion potential $\chi(T)$



Two challenges to solve:

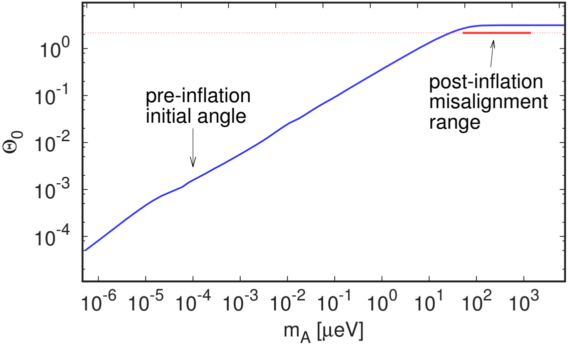
1. signal is small 2. lattice artefacts are large

Comparison to others



Results

All dark matter is axion: $\Omega_{DM} \equiv \Omega_a(m_a, \theta_0) \rightarrow m_a(\theta_0)$



post-inflation: average all possible θ_0 values $\rightarrow$

$$\langle m_a(\theta_0) \rangle = 28(1)\mu\text{eV}$$

(If not all DM is axion, then this is a lower bound.)

pre-inflation: single θ_0 in Universe, m_a can be anything