

# Testing the Standard Model with the lepton $g-2$

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# Outline

- 1.  $\mu$ : The muon g-2: recent theory progress
- 2.  $e$ : Testing the Standard Model with the electron g-2
- 3.  $\tau$ : The tau g-2: opportunities or fantasies?

# 0. Lepton magnetic moments: the basics

## The beginning: $g = 2$

- Uhlenbeck and Goudsmit in 1925 proposed:

$$\vec{\mu} = g \frac{e}{2mc} \vec{s}$$
$$g = \underline{2} \quad (\text{not } 1!)$$

- Dirac 1928:

$$(i\partial_\mu - eA_\mu) \gamma^\mu \psi = m\psi$$

- A Pauli term in Dirac's eq would give a deviation...

$$a \frac{e}{2m} \sigma_{\mu\nu} F^{\mu\nu} \psi \quad \rightarrow \quad g = 2(1 + a)$$

...but there was no need for it!  $g=2$  stood for ~20 yrs.

# Theory of the g-2: Quantum Field Theory

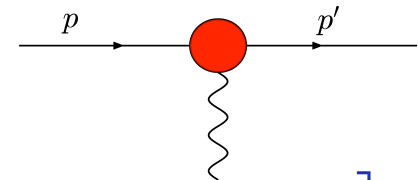
- Kusch and Foley 1948:

$$\mu_e^{\text{exp}} = \frac{e\hbar}{2mc} (1.00119 \pm 0.00005)$$

- Schwinger 1948 (triumph of QED!):

$$\mu_e^{\text{th}} = \frac{e\hbar}{2mc} \left(1 + \frac{\alpha}{2\pi}\right) = \frac{e\hbar}{2mc} \times 1.00116$$

- Keep studying the lepton- $\gamma$  vertex:



$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p') \left[ \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m} F_2(q^2) + \dots \right] u(p)$$

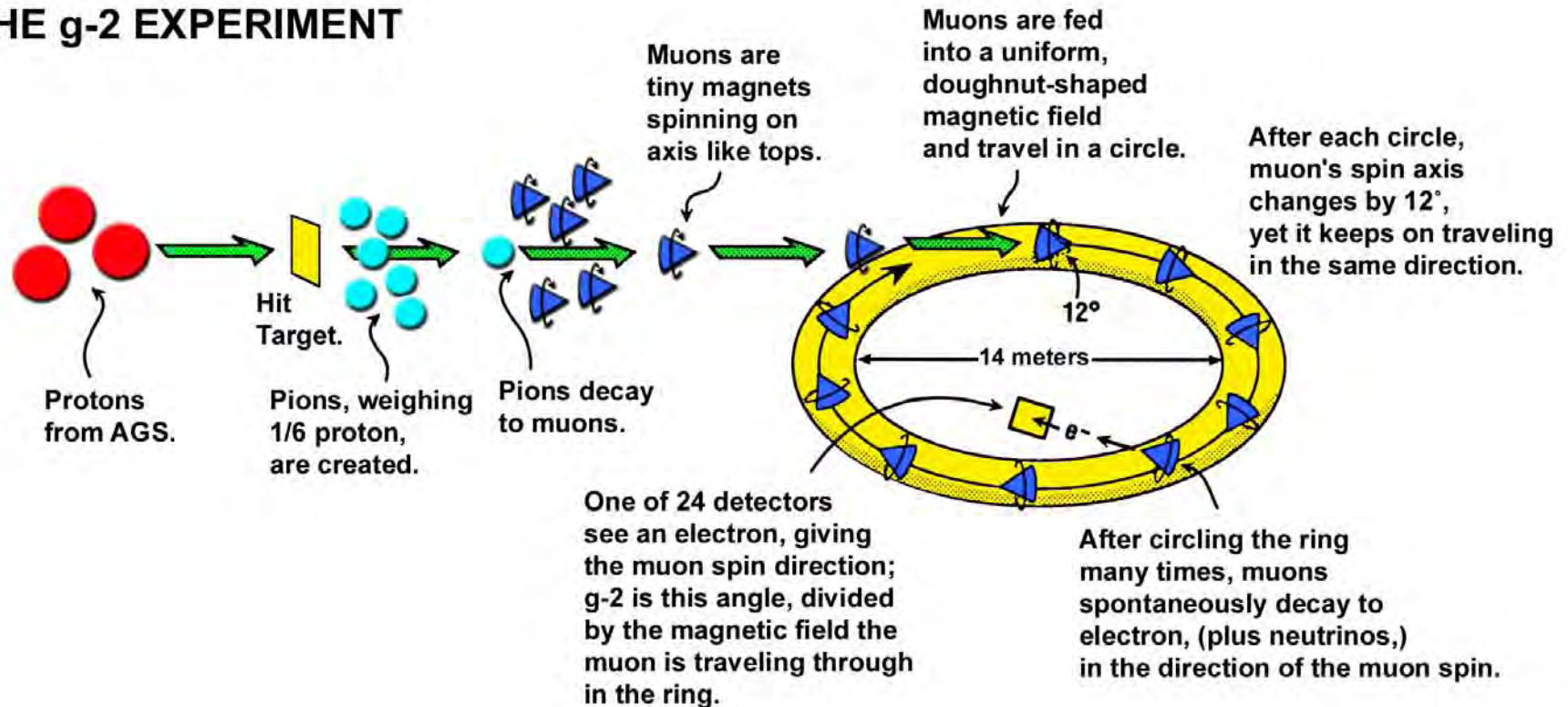
$$F_1(0) = 1 \quad F_2(0) = a_l$$

A pure “quantum correction” effect!

# 1. The muon $g-2$ : recent theory progress

# The old experiment E821

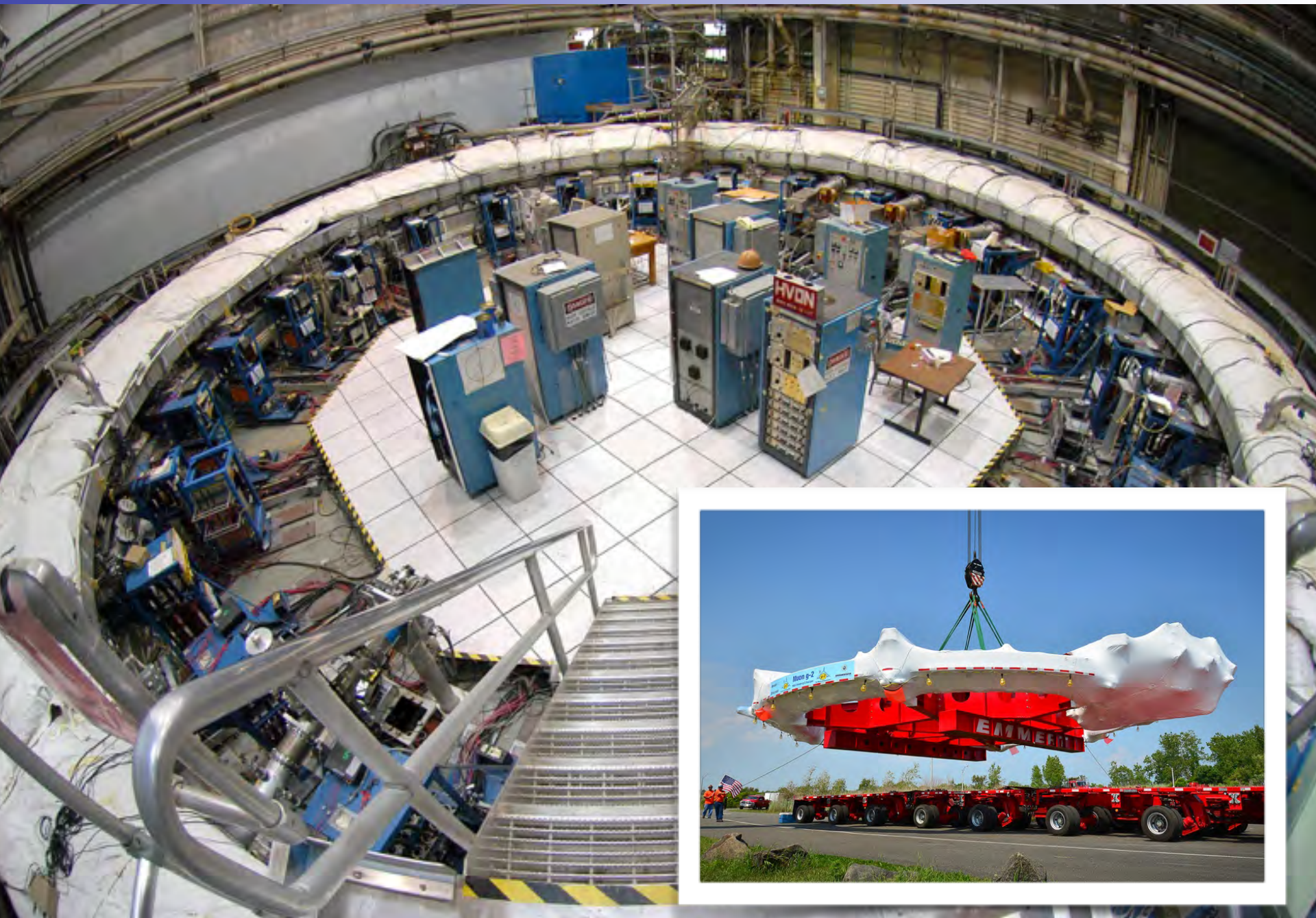
## LIFE OF A MUON: THE g-2 EXPERIMENT



E821 @ BNL



# The old experiment E821 (2)



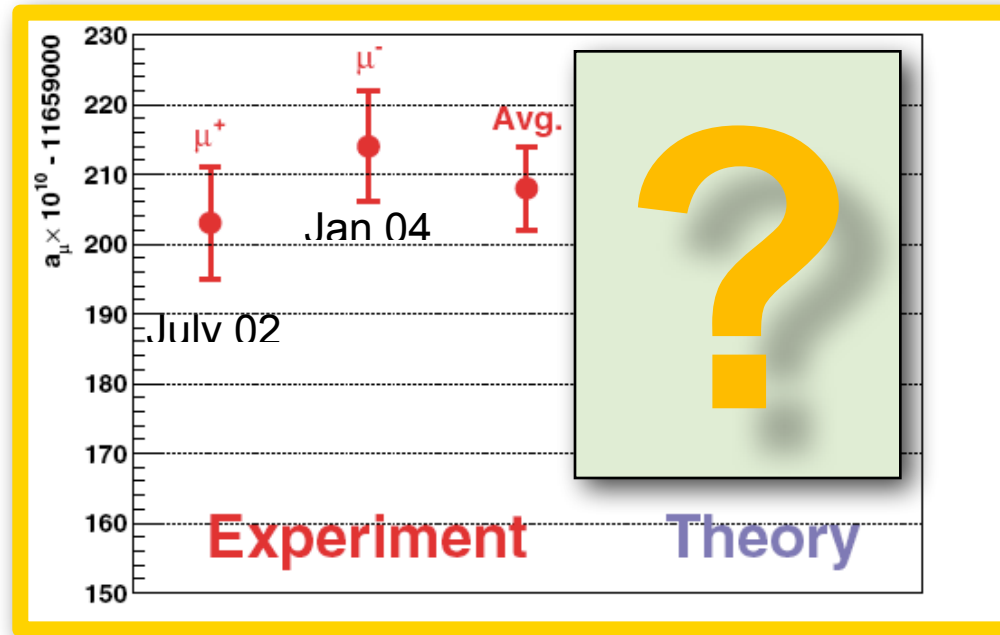




**The Big Move:  
BNL to Fermilab  
Summer 2013**



# The muon g-2: the experimental result



- Today:  $a_\mu^{\text{EXP}} = (116592089 \pm 54_{\text{stat}} \pm 33_{\text{sys}}) \times 10^{-11}$  [0.5ppm].
- Future: new muon g-2 experiments proposed at:
  - Fermilab E989, aiming at  $\pm 16 \times 10^{-11}$ , ie 0.14ppm
  - J-PARC aiming at 0.1 ppm
- Are theorists ready for this (amazing) precision? No(t yet)

# The muon g-2: the QED contribution

$$a_{\mu}^{\text{QED}} = (1/2)(\alpha/\pi) \quad \text{Schwinger 1948}$$

$$+ 0.765857426 (16) (\alpha/\pi)^2$$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$$+ 24.05050988 (28) (\alpha/\pi)^3$$

Remiddi, Laporta, Barbieri ... ; Czarnecki, Skrzypek; MP '04;  
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$$+ 130.8796 (63) (\alpha/\pi)^4$$

Kinoshita & Lindquist '81, ... , Kinoshita & Nio '04, '05;  
Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012,  
Steinhauser et al. 2013 (analytic, in progress).

$$+ 753.29 (1.04) (\alpha/\pi)^5 \quad \text{COMPLETED!}$$

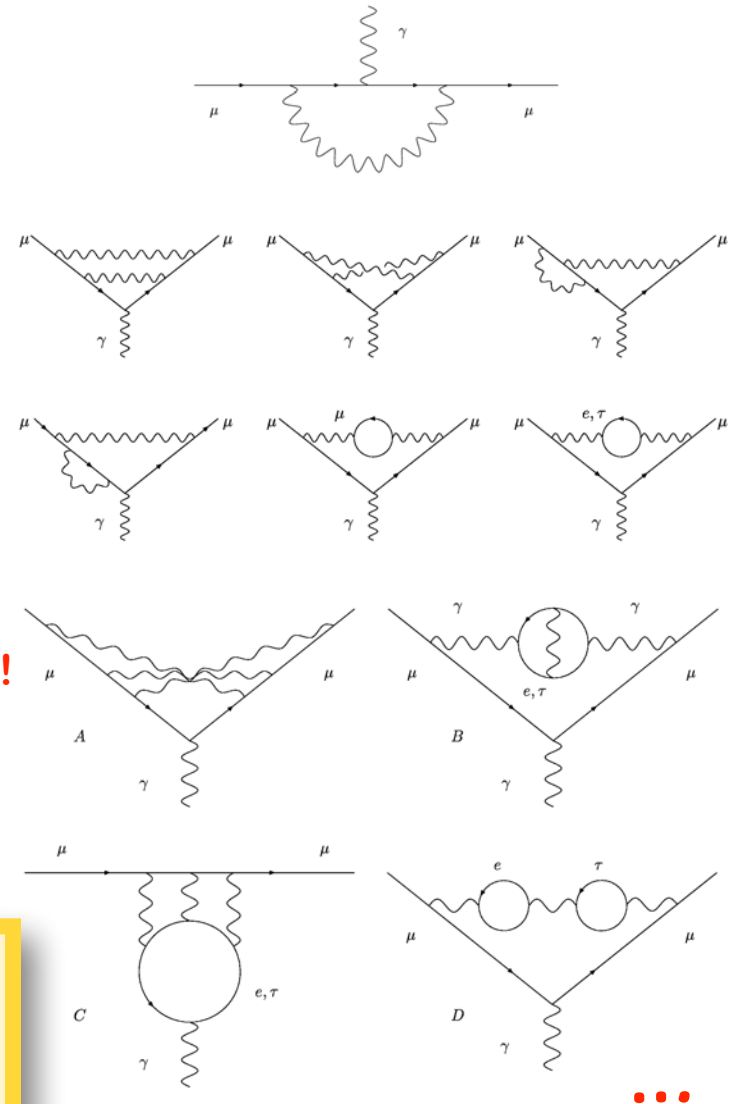
Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,  
Karshenboim, ..., Kataev, Kinoshita & Nio '06, Kinoshita et al. 2012

Adding up, we get:

$$a_{\mu}^{\text{QED}} = 116584718.951 (22)(77) \times 10^{-11}$$

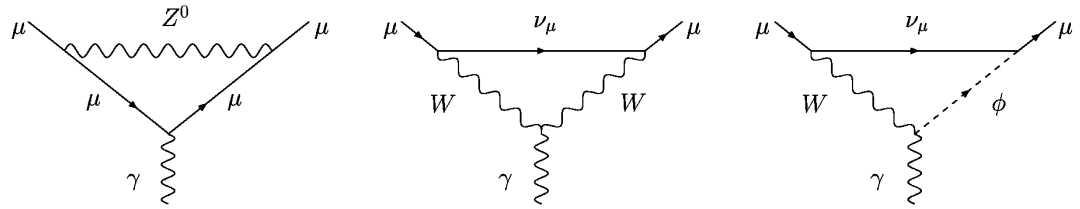
from coeffs, mainly from 4-loop unc ↙ ↘ from  $\delta\alpha(\text{Rb})$

with  $\alpha = 1/137.035999049(90)$  [0.66 ppb]



# The muon g-2: the electroweak contribution

## ● One-loop term:



$$a_{\mu}^{\text{EW}}(1\text{-loop}) = \frac{5G_{\mu}m_{\mu}^2}{24\sqrt{2}\pi^2} \left[ 1 + \frac{1}{5} (1 - 4\sin^2\theta_W)^2 + O\left(\frac{m_{\mu}^2}{M_{Z,W,H}^2}\right) \right] \approx 195 \times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda; Studenikin et al. '80s

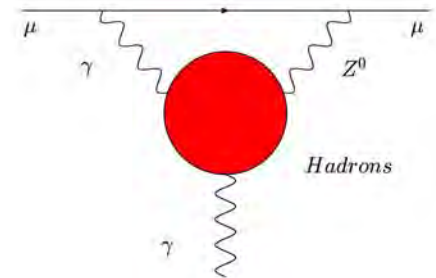
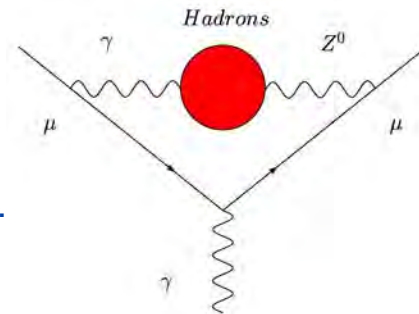
## ● One-loop plus higher-order terms:

$$a_{\mu}^{\text{EW}} = 153.6 (1) \times 10^{-11}$$

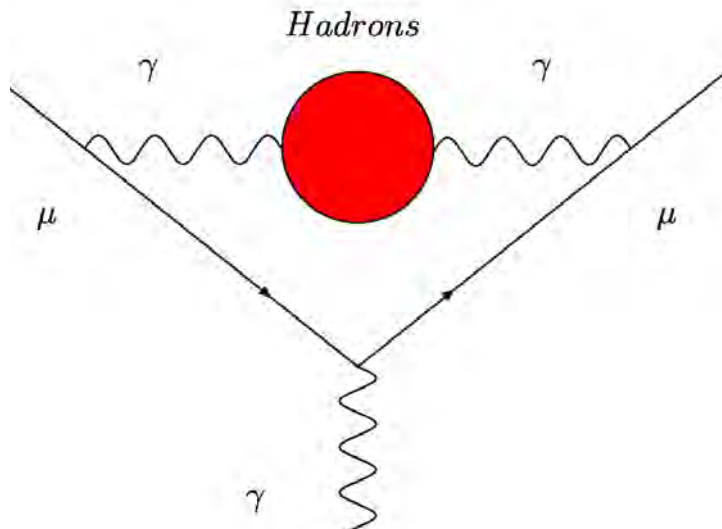
with  $M_{\text{Higgs}} = 125.6 (1.5) \text{ GeV}$

Hadronic loop uncertainties and 3-loop nonleading logs.

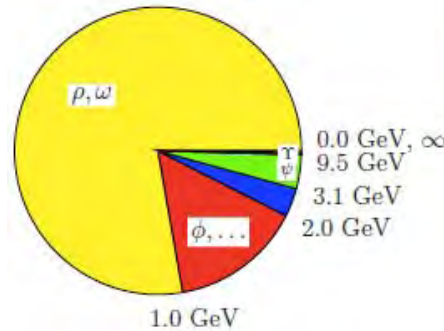
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.



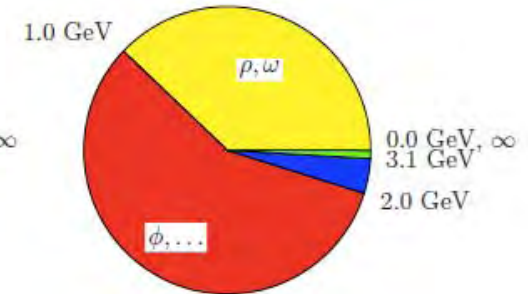
# The muon g-2: the hadronic LO contribution (HLO)



Central values



Errors<sup>2</sup>



F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)}$$

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^\infty ds K(s) \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty \frac{ds}{s} K(s) R(s)$$

$$a_\mu^{\text{HLO}} = 6903 (53)_{\text{tot}} \times 10^{-11}$$

F. Jegerlehner, A. Nyffeler, Phys. Rept. 477 (2009) 1

$$= 6923 (42)_{\text{tot}} \times 10^{-11}$$

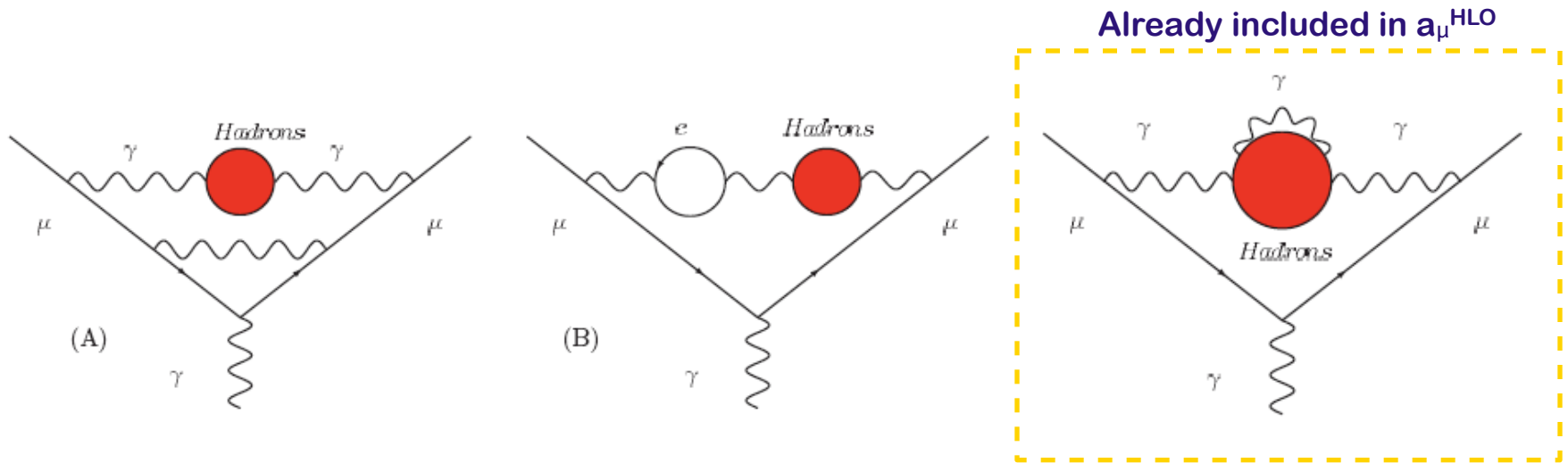
Davier et al, EPJ C71 (2011) 1515 (incl. BaBar & KLOE10 2π)

$$= 6949 (37)_{\text{exp}} (21)_{\text{rad}} \times 10^{-11}$$

Hagiwara et al, JPG 38 (2011) 085003



- HNLO: Vacuum Polarization**



$O(\alpha^3)$  contributions of diagrams containing hadronic vacuum polarization insertions:

$$a_\mu^{\text{HNLO}}(\text{vp}) = -98 (1) \times 10^{-11}$$

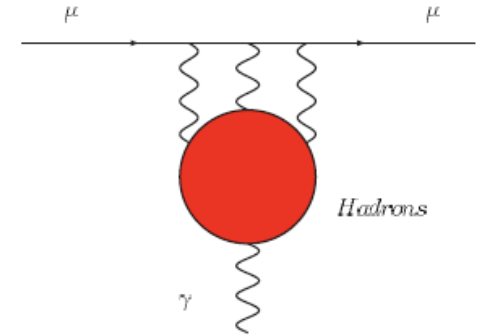
Krause '96, Alemany et al. '98, Hagiwara et al. 2011



## ● HNLO: Light-by-light contribution

🔊 Unlike the HLO term, the hadronic l-b-l term relies at present on theoretical approaches.

🔊 This term had a troubled life! Latest values:



$a_{\mu}^{\text{HNLO}}( b ) = +80 (40) \times 10^{-11}$	Knecht & Nyffeler '02
$a_{\mu}^{\text{HNLO}}( b ) = +136 (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$a_{\mu}^{\text{HNLO}}( b ) = +105 (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein '09
$a_{\mu}^{\text{HHO}}( b ) = +116 (39) \times 10^{-11}$	Jegerlehner & Nyffeler '09

Results based also on Hayakawa, Kinoshita '98 & '02; Bijmens, Pallante, Prades '96 & '02

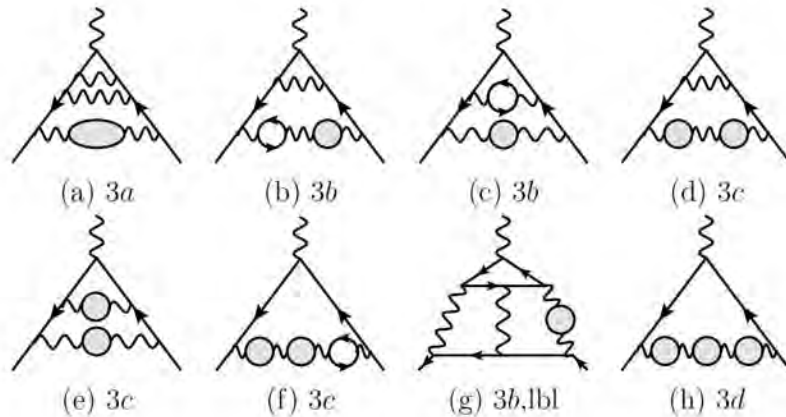
🔊 **“Bound”**  $a_{\mu}^{\text{HNLO}}(|b|) < \sim 160 \times 10^{-11}$  Erler, Sanchez '06, Pivovarov '02; also Boughezal, Melnikov '11

🔊 **Pion exch. in holographic QCD agrees.** D.K.Hong, D.Kim '09; Capiello, Catà, D'Ambrosio '11

🔊 **Lattice? Very hard but promising** Tom Blum @ Quark Confinement 2014

🔊 **Dispersive approach recently proposed** Colangelo, Hoferichter, Procura, Stoffer 1402.7081

## ● HNNLO: Vacuum Polarization



$O(\alpha^4)$  contributions of diagrams containing hadronic vacuum polarization insertions:

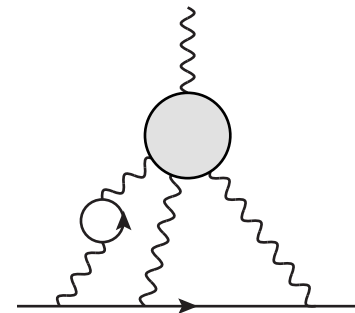
$$a_{\mu}^{\text{HNNLO}}(\text{vp}) = 12.4 (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

## ● HNNLO: Light-by-light

$$a_{\mu}^{\text{HNNLO}}(\text{lbl}) = 3 (2) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014



# The muon g-2: SM vs. Experiment

Adding up all contributions, we get the following SM predictions and comparisons with the measured value:

$$a_{\mu}^{\text{EXP}} = 116592091 (63) \times 10^{-11}$$

E821 – Final Report: PRD73 (2006) 072 with latest value of  $\lambda=\mu_{\mu}/\mu_{\text{p}}$  from CODATA'10

$a_{\mu}^{\text{SM}} \times 10^{11}$	$\Delta a_{\mu} = a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}}$	$\sigma$
116 591 809 (66)	282 (91) $\times 10^{-11}$	3.1 [1]
116 591 829 (57)	262 (85) $\times 10^{-11}$	3.1 [2]
116 591 855 (58)	236 (86) $\times 10^{-11}$	2.8 [3]

with the “conservative”  $a_{\mu}^{\text{HNLO}}(|b|) = 116 (39) \times 10^{-11}$  and the LO hadronic from:

- [1] Jegerlehner & Nyffeler, Phys. Rept. 477 (2009) 1
- [2] Davier et al, EPJ C71 (2011) 1515 (includes BaBar & KLOE10  $2\pi$ )
- [3] Hagiwara et al, JPG38 (2011) 085003 (includes BaBar & KLOE10  $2\pi$ )

**Note that the th. error is now about the same as the exp. one**

# The muon g-2: connection with the SM Higgs mass

- $\Delta a_\mu$  can be explained by errors in QED, EW, HNLO, g-2 EXP, HLO, or, we hope, by **New Physics!**
- Can  $\Delta a_\mu$  be due to **hypothetical mistakes** in the hadronic  $\sigma(s)$ ?
- An upward shift of  $\sigma(s)$  also induces an increase of  $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$ .
- **Consider:**

$$\begin{aligned} a_\mu^{\text{HLO}} &\rightarrow a = \int_{4m_\pi^2}^{s_u} ds f(s) \sigma(s), & f(s) &= \frac{K(s)}{4\pi^3}, \quad s_u < M_Z^2, \\ \Delta\alpha_{\text{had}}^{(5)} &\rightarrow b = \int_{4m_\pi^2}^{s_u} ds g(s) \sigma(s), & g(s) &= \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)}, \end{aligned}$$

and the increase

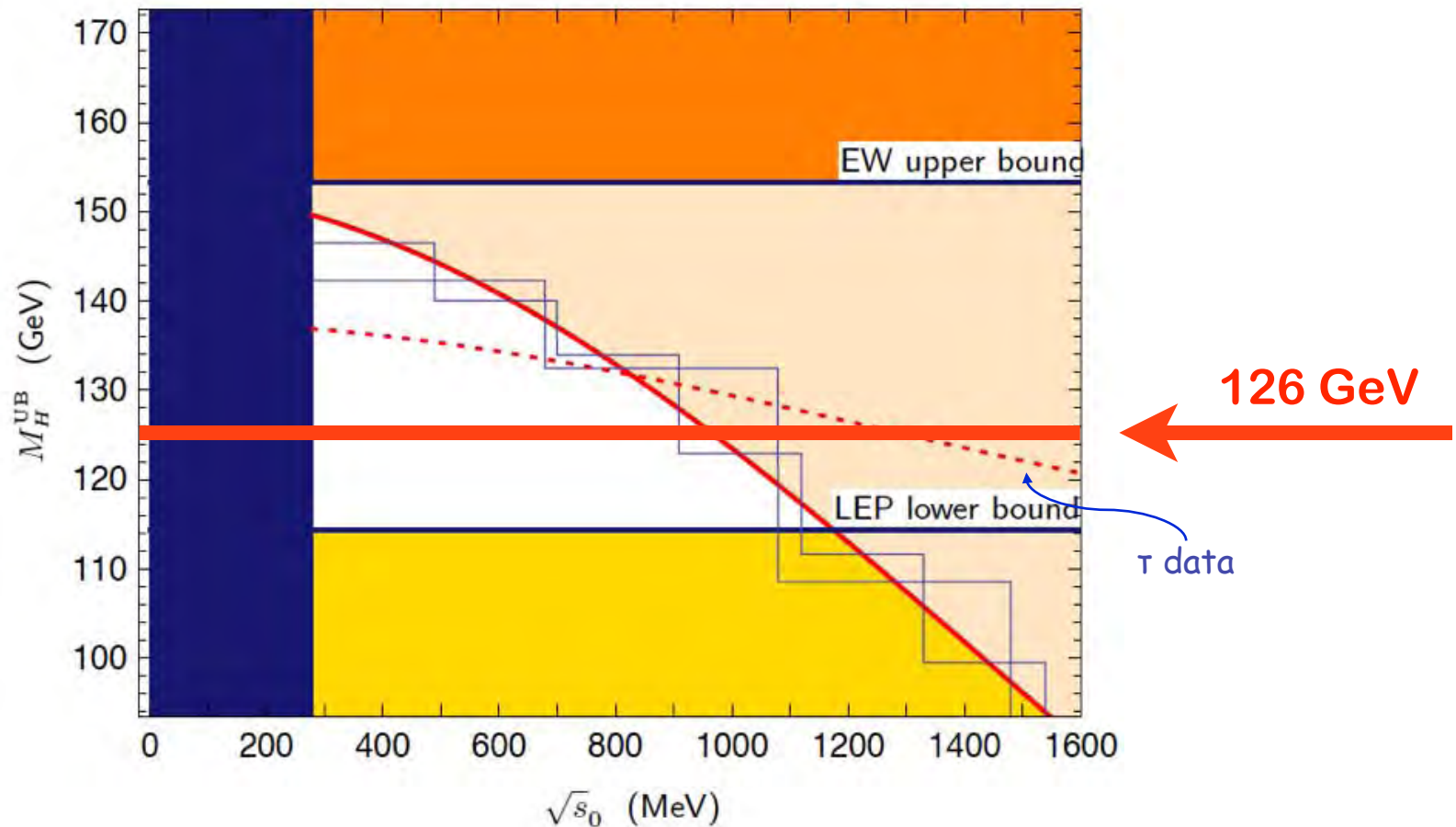
$$\Delta\sigma(s) = \epsilon\sigma(s)$$

( $\epsilon > 0$ ), in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2] \quad \longrightarrow$$

## The muon g-2: connection with the SM Higgs mass (2)

- How much does the  $M_H$  upper bound from the EW fit change when we shift  $\sigma(s)$  by  $\Delta\sigma(s)$  [and thus  $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$ ] to accommodate  $\Delta a_\mu$  ?



W.J. Marciano, A. Sirlin, MP, 2008 & 2010

## The muon $g-2$ : connection with the SM Higgs mass (3)

- Given the quoted exp. uncertainty of  $\sigma(s)$ , the possibility to explain the muon  $g-2$  with these very large shifts  $\Delta\sigma(s)$  appears to be very unlikely.
- Also, given a **126 GeV SM Higgs**, these hypothetical shifts  $\Delta\sigma(s)$  could only occur at very low energy (below  $\sim 1$  GeV).
- Vice versa, assuming we now have a SM Higgs with  $M_{\text{Higgs}} = 126$  GeV, if we bridge the  $M_{\text{Higgs}}$  discrepancy in the EW fit decreasing the low-energy hadronic cross section, **the muon  $g-2$  discrepancy increases.**



## 2. Testing the SM with the electron $g-2$

G.F. Giudice, P. Paradisi & MP, arXiv:1208.6583 (JHEP 2012)

M. Fael & MP, arXiv:1402.1575 (PRD 2014)

# The QED prediction of the electron g-2

$$a_e^{\text{QED}} = + (1/2)(\alpha/\pi) - 0.328\,478\,444\,002\,55(33)(\alpha/\pi)^2$$

Schwinger 1948 Sommerfeld; Petermann; Suura&Wichmann '57; Elend '66; CODATA Mar '12

$$A_1^{(4)} = -0.328\,478\,965\,579\,193\,78\dots$$

$O(10^{-18})$  in  $a_e$

$$A_2^{(4)}(m_e/m_\mu) = 5.197\,386\,68(26) \times 10^{-7}$$

$$A_2^{(4)}(m_e/m_\tau) = 1.837\,98(33) \times 10^{-9}$$

$$+ 1.181\,234\,016\,816(11)(\alpha/\pi)^3$$

$O(10^{-19})$  in  $a_e$

Kinoshita; Barbieri; Laporta, Remiddi, ... , Li, Samuel; MP '06; Giudice, Paradisi, MP 2012

$$A_1^{(6)} = 1.181\,241\,456\,587\dots$$

$$A_2^{(6)}(m_e/m_\mu) = -7.373\,941\,62(27) \times 10^{-6}$$

$$A_2^{(6)}(m_e/m_\tau) = -6.5830(11) \times 10^{-8}$$

$$A_3^{(6)}(m_e/m_\mu, m_e/m_\tau) = 1.909\,82(34) \times 10^{-13}$$

$$- 1.9097(20)(\alpha/\pi)^4$$

$0.6 \cdot 10^{-13}$  in  $a_e$

Kinoshita & Lindquist '81, ... , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio 2012; Kurz, Liu, Marquard & Steinhauser 2014: analytic mass dependent part.

$$+ 9.16(58)(\alpha/\pi)^5 \quad \text{Complete Result! (12672 mass indep. diagrams!)}$$

Aoyama, Hayakawa, Kinoshita, Nio, PRL 109 (2012) 111807; work in progress to reduce the error.

$0.4 \cdot 10^{-13}$  in  $a_e$

NB:  $(\alpha/\pi)^6 \sim O(10^{-16})$

# The SM prediction of the electron g-2

The SM prediction is:

$$a_e^{\text{SM}}(\alpha) = a_e^{\text{QED}}(\alpha) + a_e^{\text{EW}} + a_e^{\text{HAD}}$$

The EW (1&2 loop) term is: Czarnecki, Krause, Marciano '96 [value from CODATA10]

$$a_e^{\text{EW}} = 0.2973(52) \times 10^{-13}$$

The Hadronic contribution (LO+NLO+NNLO) is:

Nomura & Teubner '12, Jegerlehner & Nyffeler '09; Krause'97; Kurz, Liu, Marquard & Steinhauser 2014

$$a_e^{\text{HAD}} = 17.10(17) \times 10^{-13}$$

$$a_e^{\text{HLO}} = +18.66(11) \times 10^{-13}$$

$$a_e^{\text{HNLO}} = [-2.234(14)_{\text{vac}} + 0.39(13)_{\text{lbl}}] \times 10^{-13}$$

$$a_e^{\text{HNNLO}} = +0.28(1) \times 10^{-13}$$

Which value of  $\alpha$  should we use to compute  $a_e^{\text{SM}}$ ?

# The electron g-2 gives the best determination of alpha

- The 2008 measurement of the electron g-2 is:

$$a_e^{\text{EXP}} = 11596521807.3 (2.8) \times 10^{-13} \quad \text{Hanneke et al, PRL100 (2008) 120801}$$

vs. old (factor of 15 improvement,  $1.8\sigma$  difference):

$$a_e^{\text{EXP}} = 11596521883 (42) \times 10^{-13} \quad \text{Van Dyck et al, PRL59 (1987) 26}$$

- Equate  $a_e^{\text{SM}}(\alpha) = a_e^{\text{EXP}}$  → best determination of alpha:

$$\alpha^{-1} = 137.035\,999\,177 (34) \quad [0.25 \text{ ppb}]$$

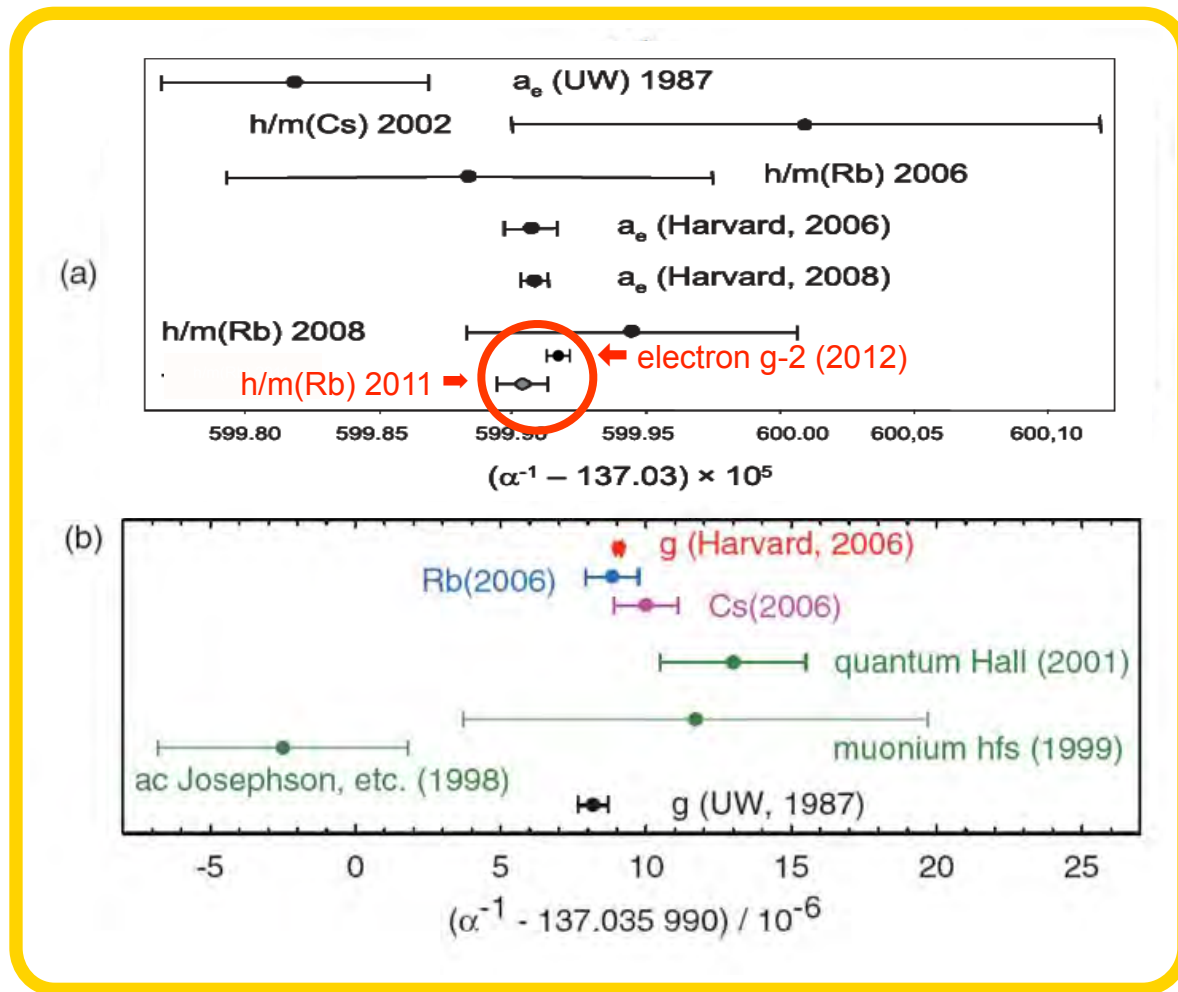
- Compare it with other determinations (independent of  $a_e$ ):

$$\alpha^{-1} = 137.036\,000\,0 (11) \quad [7.7 \text{ ppb}] \quad \text{PRA73 (2006) 032504 (Cs)}$$

$$\alpha^{-1} = 137.035\,999\,049 (90) \quad [0.66 \text{ ppb}] \quad \text{PRL106 (2011) 080801 (Rb)}$$

**Excellent agreement → beautiful test of QED at 4-loop level!**

# Old and new determinations of alpha



Gabrielse, Hanneke, Kinoshita, Nio & Odom, PRL99 (2007) 039902  
 Hanneke, Fogwell & Gabrielse, PRL100 (2008) 120801  
 Bouchendiria et al, PRL106 (2011) 080801

# The electron g-2: SM vs Experiment

- Using  $\alpha = 1/137.035\,999\,049\,(90)$  [ $^{87}\text{Rb}$ , 2011], the SM prediction for the electron g-2 is

$$a_e^{\text{SM}} = 115\,965\,218\,18.1\,(0.6)\,(0.4)\,(0.2)\,(7.6) \times 10^{-13}$$

$\delta C_4^{\text{qed}}$   $\delta C_5^{\text{qed}}$   $\delta a_e^{\text{had}}$  from  $\delta\alpha$

- The EXP-SM difference is (note the negative sign):

$$\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = -10.8\,(8.1) \times 10^{-13}$$

The SM is in very good agreement with experiment ( $1.3\sigma$ ).

NB: The 4-loop contrib. to  $a_e^{\text{QED}}$  is  $-556 \times 10^{-13} \sim 70 \delta\Delta a_e!$

(the 5-loop one is  $6.2 \times 10^{-13}$ )



## The electron g-2 sensitivity and NP tests

- The present sensitivity is  $\delta\Delta a_e = 8.1 \times 10^{-13}$ , ie ( $10^{-13}$  units):

$$(0.6)_{\text{QED4}}, \quad (0.4)_{\text{QED5}}, \quad (0.2)_{\text{HAD}}, \quad (7.6)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\text{EXP}}}$$

$$(0.7)_{\text{TH}} \quad \leftarrow \text{may drop to 0.2 or 0.3}$$

- The  $(g-2)_e$  exp. error may soon drop below  $10^{-13}$  and work is in progress for a significant reduction of that induced by  $\delta\alpha$ .

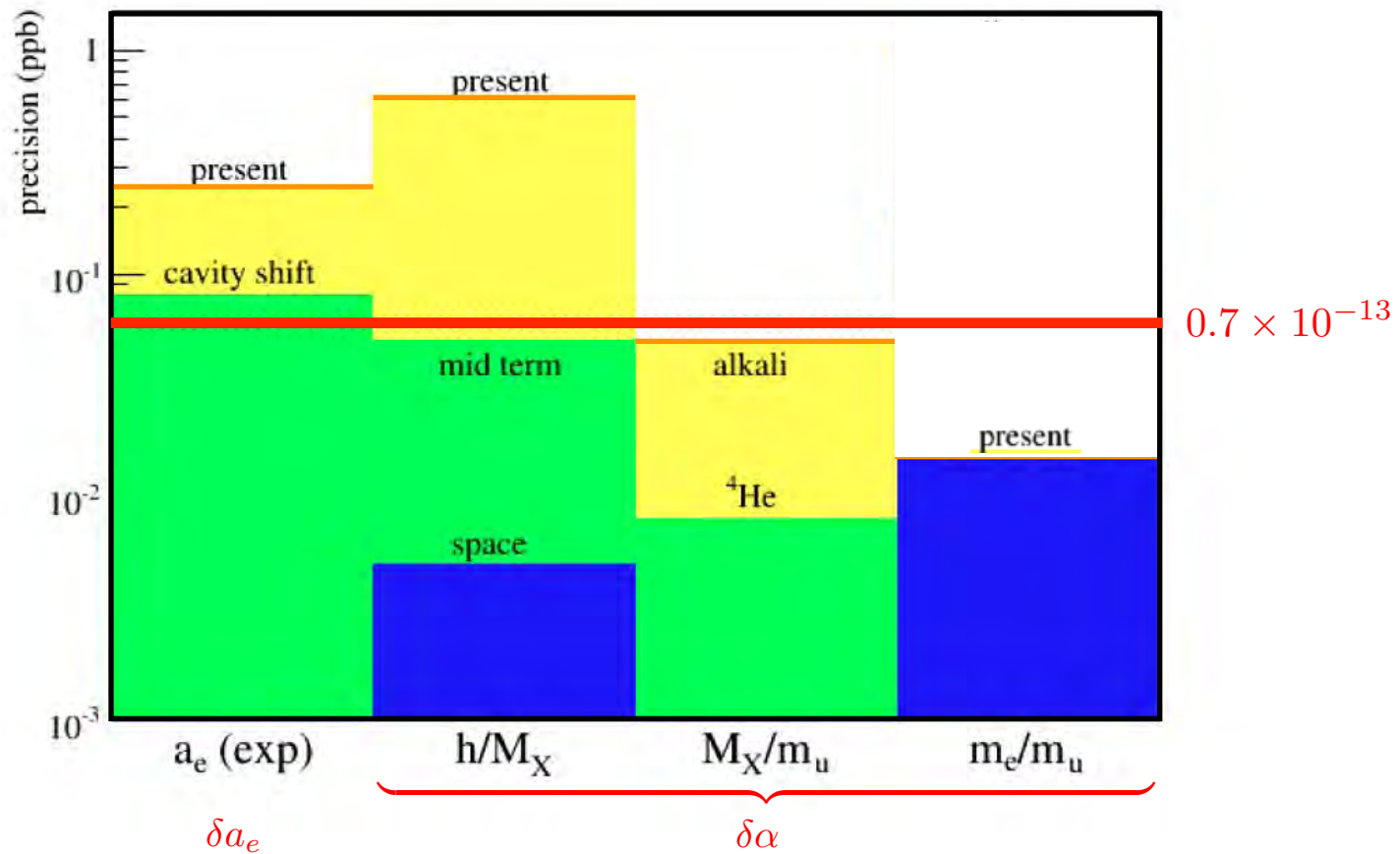
→ sensitivity of  $10^{-13}$  may be reached with ongoing exp. work

- In a broad class of BSM theories, contributions to  $a_l$  scale as

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left( \frac{m_{\ell_i}}{m_{\ell_j}} \right)^2 \quad \text{This Naive Scaling leads to:}$$

$$\Delta a_e = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.7 \times 10^{-13}; \quad \Delta a_\tau = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.8 \times 10^{-6}$$

# The electron g-2 sensitivity and NP tests (2)



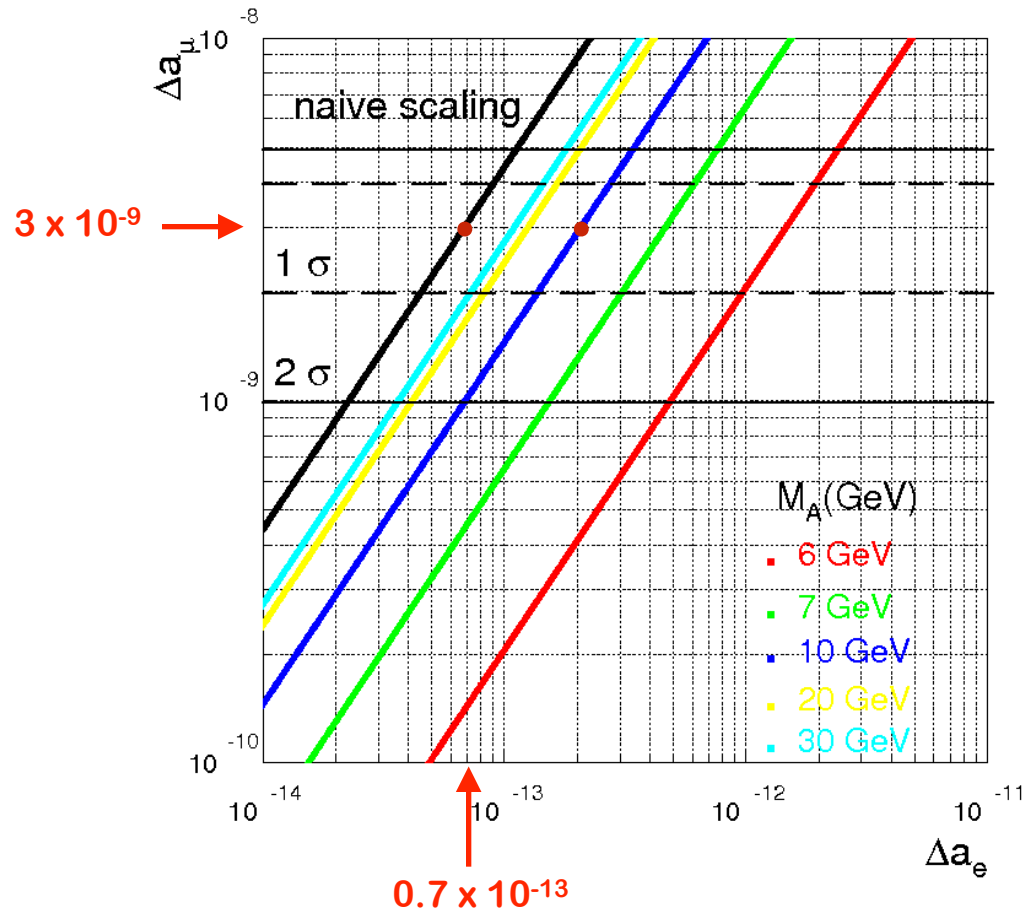
Summary of the exp. contributions to the relative uncertainty of  $\Delta a_e$  (in ppb)

F. Terranova & G.M. Tino, PRA89 (2014) 052118

## The electron $g-2$ sensitivity and NP tests (3)

- The experimental sensitivity in  $\Delta a_e$  is not very far from what is needed to **test if the discrepancy in  $(g-2)_\mu$  also manifests itself in  $(g-2)_e$**  under the naive scaling hypothesis.
- NP scenarios exist which **violate Naive Scaling**. They can lead to larger effects in  $\Delta a_e$  and contributions to EDMs, LFV or lepton universality breaking observables.
- Example: In the MSSM with non-degenerate but aligned sleptons (vanishing flavor mixing angles),  $\Delta a_e$  can reach  $10^{-12}$  (at the limit of the present exp sensitivity). For these values one typically has breaking effects of lepton universality at the few per mil level (within future exp reach).

# The electron g-2 sensitivity and NP tests (4)



- Example: light pseudoscalars. Interplay between 1-loop and 2-loop contributions. NS systematically violated,  $\Delta a_e$  always lies above its naive expectation.

# The electron g-2 sensitivity and NP tests (5)

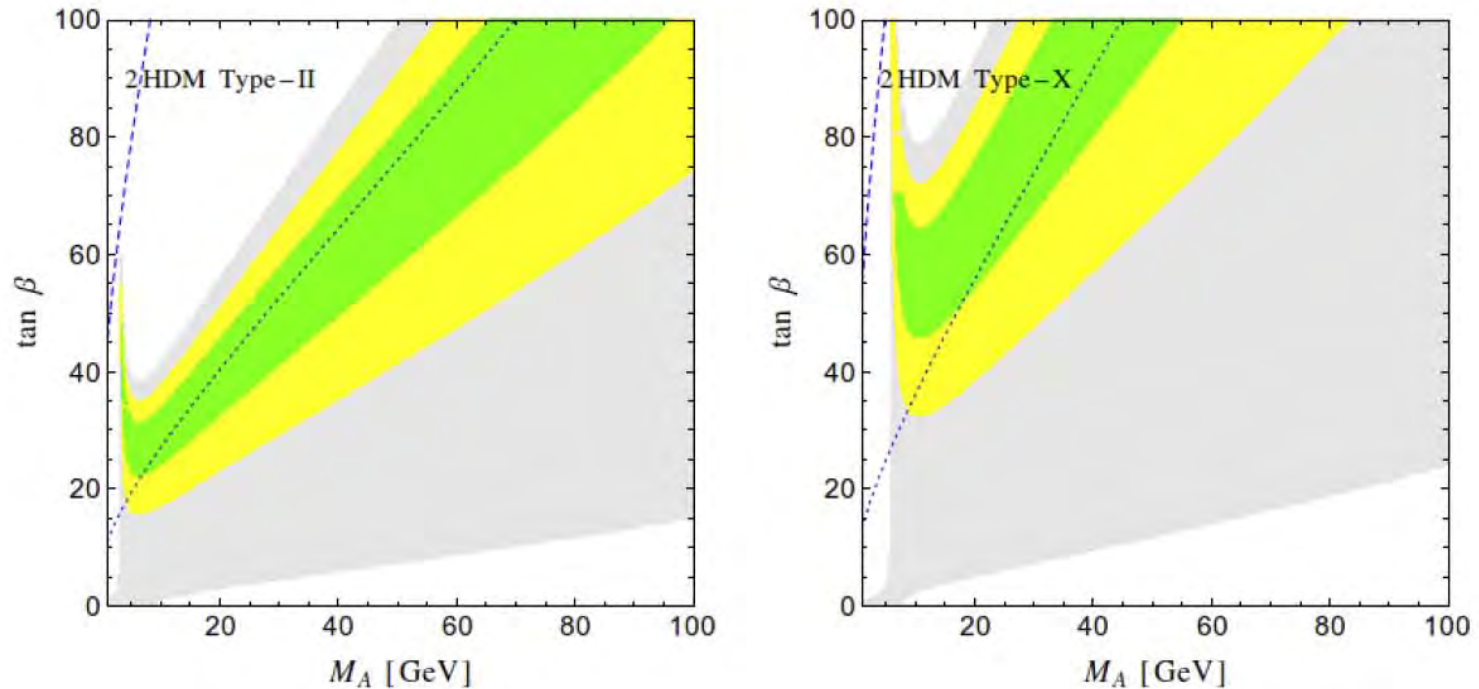


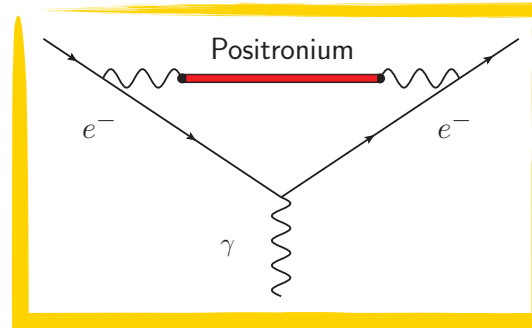
FIG. 4. The  $1\sigma$ ,  $2\sigma$  and  $3\sigma$  regions allowed by  $\Delta a_\mu$  in the  $M_A$ - $\tan\beta$  plane taking the limit of  $\beta - \alpha = \pi/2$  and  $M_{h(H)} = 125$  (200) GeV in type II (left panel) and type X (right panel) 2HDMs. The regions below the dashed (dotted) lines are allowed at  $3\sigma$  ( $1.4\sigma$ ) by  $\Delta a_e$ .

● **Example: 2HDMs** Broggio, Chun, MP, Patel, Vempati, arXiv:1409.3199

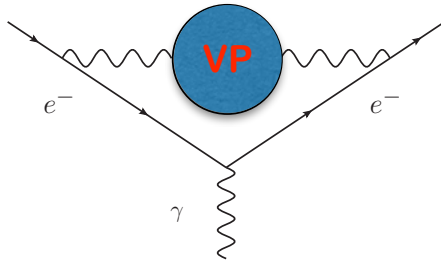


- The leading contribution of positronium to  $a_e$  comes from:

Mishima 1311.7109; Fael & MP 1402.1575; Melnikov et al. 1402.5690; Eides 1402.5860; Hayakawa 1403.0416



- The  $e^+e^-$  bound states appear as poles in the vac. pol.  $\Pi(q^2)$  right below the branch-point  $q^2 = (2m)^2$ . Their contribution is:



$$\Rightarrow a_e(\text{vp}) = \frac{\alpha}{\pi^2} \int_0^\infty \frac{ds}{s} \text{Im}\Pi(s + i\epsilon) K(s)$$

$$a_e^{\text{P}} = \frac{\alpha^5}{4\pi} \zeta(3) \left( 8 \ln 2 - \frac{11}{2} \right) = 0.9 \times 10^{-13} = 1.3 \left( \frac{\alpha}{\pi} \right)^5$$

Mishima 1311.7109

- Of the same magnitude of the exp. unc. of  $a_e$  & the naively rescaled muon  $\Delta a_\mu$ . Of the same order of  $\alpha$  as the 5-loop term!



- Melnikov, Vainshtein & Voloshin (MVV) 1402.5690 determined a nonpert. contrib. of the  $e^+e^-$  continuum right above threshold that cancels one-half of  $a_e^P$ :

$$a_e(\text{vp})^{\text{cont,np}} = -\frac{|\alpha|^5}{8\pi} \zeta(3) \left( 8 \ln 2 - \frac{11}{2} \right)$$

- In fact the **total positronium poles + continuum** nonperturbative contribution to  $a_e$  arising from the threshold region at LO in  $\alpha$  is:

$$a_e^{\text{thr}}(\text{vp}) = -\frac{\alpha}{\pi} K(4m^2) \text{Re} A(1)$$

with

$$A(\beta) = -\frac{\alpha^2}{2} \left[ \gamma + \psi \left( 1 - \frac{i\alpha}{2\beta} \right) \right] = \frac{\alpha^2}{2} \sum_{k=1}^{\infty} \zeta(k+1) \left( \frac{i\alpha}{2\beta} \right)^k$$

so that

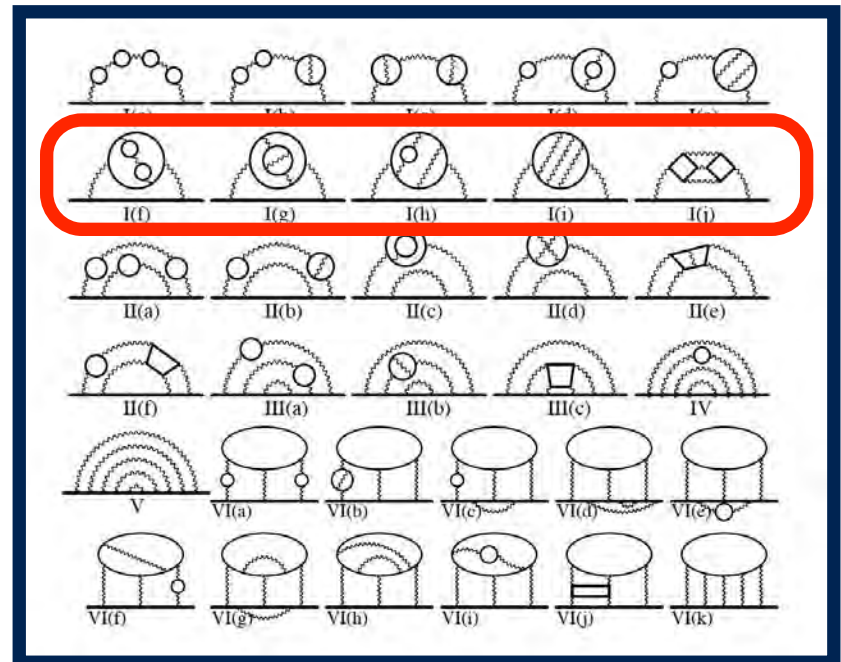
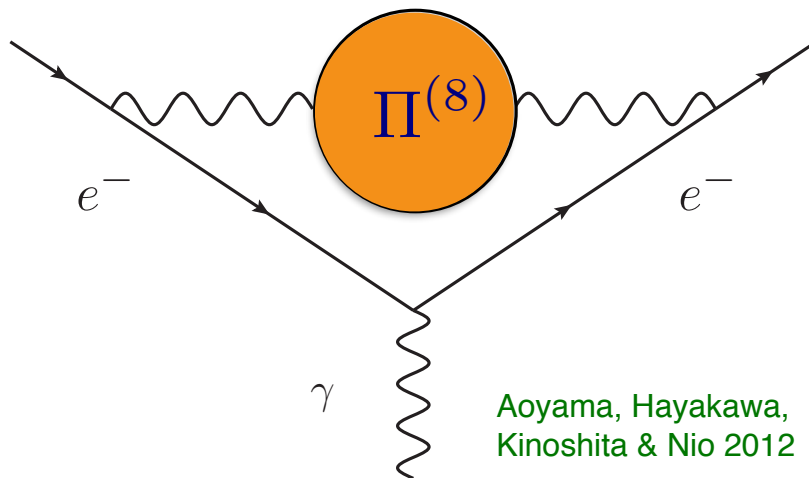
$$a_e^{\text{thr}}(\text{vp}) = \frac{\alpha^5}{8\pi} \zeta(3) K(4m^2) = \frac{a_e^P}{2}$$



# Positronium contribution to the electron $g-2$ (III)



- So, should we add this total threshold contribution  $a_e^P/2$  to the perturbative QED 5-loop result of Kinoshita and collaborators?
- Using the Coulomb Green's function, MVV 1402.5690 argued that it is already contained in the contribution of  $O(\alpha^5)$ .
- Hayakawa 1403.0416 claimed that positronium contributes to  $a_e$  only through a specific class of diagrams of  $O(\alpha^7)$ .
- To address this question: study the 5-loop QED contribution to  $a_e$  arising from the insertion of the 4-loop VP in the photon line. This has been computed via:



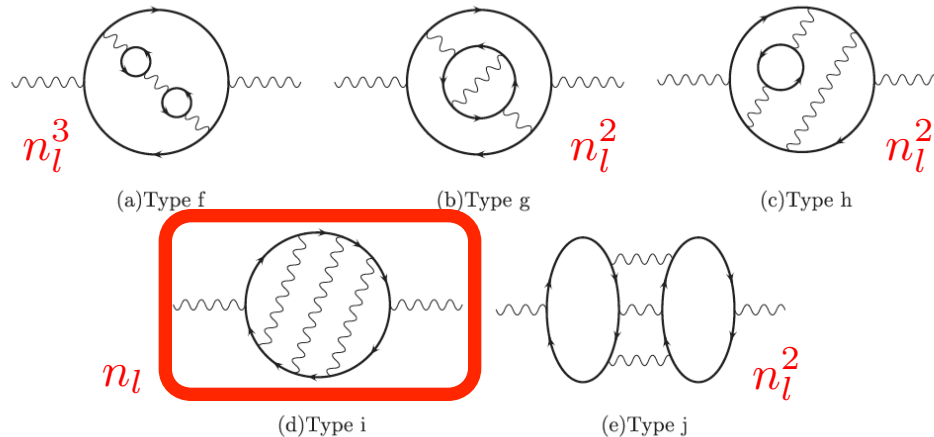


- Using explicit expressions for  $\Pi^{(8)}(q^2)$  (Baikov, Maier, Marquard '13) in

$$a_e^{(10)}(\text{vp}) = -\frac{\alpha}{\pi} \int_0^1 dx (1-x) \Pi^{(8)}\left(-\frac{m^2 x^2}{1-x}\right)$$

we obtain:

$$a_e^{(10)}(\text{vp}) = n_e \frac{\alpha^5}{8\pi} \zeta(3) K(4m^2) + \dots = \frac{a_e^{\text{P}}}{2} + \dots$$



- $a_e^{\text{P}}/2$  is already included in the 5-loop contrib. of class I(i).
- There is no additional contrib of QED bound states beyond PT!

M.A. Braun 1968; Barbieri, Christillin, Remiddi 1973

### 3. The tau $g-2$ : opportunities or fantasies?

Work in progress in collaboration with  
S. Eidelman, D. Epifanov, M. Fael, L. Mercolli

arXiv:1301.5302

arXiv:1310.1081

# The SM prediction of the tau g-2

The Standard Model prediction of the tau g-2 is:

$$\begin{aligned} a_{\tau}^{\text{SM}} &= 117324 \quad (2) && \times 10^{-8} && \text{QED} \\ &+ 47.4 \quad (0.5) && \times 10^{-8} && \text{EW} \\ &+ 337.5 \quad (3.7) && \times 10^{-8} && \text{HLO} \\ &+ 7.6 \quad (0.2) && \times 10^{-8} && \text{HHO (vac)} \\ &+ 5 \quad (3) && \times 10^{-8} && \text{HHO (lbl)} \end{aligned}$$

$$a_{\tau}^{\text{SM}} = 117721 \quad (5) \times 10^{-8}$$

Eidelman & MP  
2007

$(m_{\tau}/m_{\mu})^2 \sim 280$ : great opportunity to look for New Physics, and a “clean” NP test too...

	Muon	Tau
$a_{\text{EW}}/a_{\text{H}}$	1/45	1/7
$a_{\text{EW}}/\delta a_{\text{H}}$	3	10

... if only we could measure it!!

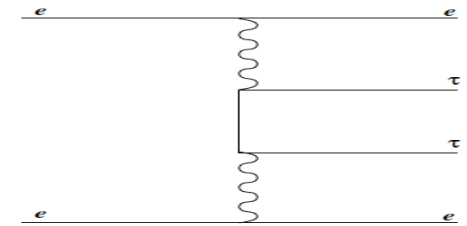
## The tau g-2: experimental bounds

- The very short lifetime of the tau makes it very difficult to determine  $a_\tau$  measuring its spin precession in a magnetic field.

- DELPHI's result, from  $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$  total cross-section measurements at LEP 2 (the PDG value):

$$a_\tau = -0.018 (17)$$

PDG 2014



- With an effective Lagrangian approach, using data on tau lepton production at LEP1, SLC, and LEP2:

$$-0.004 < a_\tau^{\text{NP}} < 0.006 \quad (95\% \text{ CL})$$

Escribano & Massó 1997

$$-0.007 < a_\tau^{\text{NP}} < 0.005 \quad (95\% \text{ CL})$$

González-Sprinberg et al 2000

- Bernabéu et al, propose the measurement of  $F_2(q^2=M_\gamma^2)$  from  $e^+e^- \rightarrow \tau^+\tau^-$  production at B factories. NPB 790 (2008) 160

# The tau g-2 via its radiative leptonic decays: a proposal

- **Tau radiative leptonic decays at LO:**

$$\frac{d^3\Gamma}{dx dy d\cos\theta} = \frac{\alpha M_\tau^5 G_F^2 y \sqrt{x^2 - 4r^2}}{2\pi(4\pi)^6} G_0(x, y, \cos\theta, r)$$

Kinoshita & Sirlin PRL2(1959)177; Kuno & Okada, RMP73(2001)151

$$\left. \frac{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau \gamma)}{\Gamma_{\text{total}}} \right|_{E_\gamma > 10\text{MeV}} = 1.836\% \quad \text{vs} \quad 1.75(18)\%$$

$$\left. \frac{\Gamma(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau \gamma)}{\Gamma_{\text{total}}} \right|_{E_\gamma > 10\text{MeV}} = 0.367\% \quad \text{vs} \quad 0.361(38)\%$$

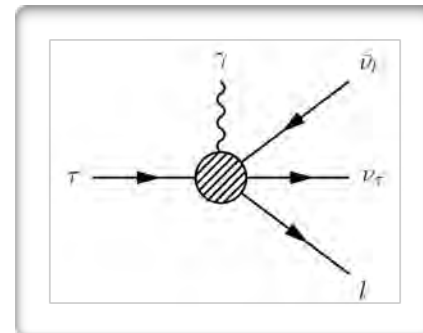
CLEO 2000

- **Add the contribution of the effective coupling and the QED corrections:**

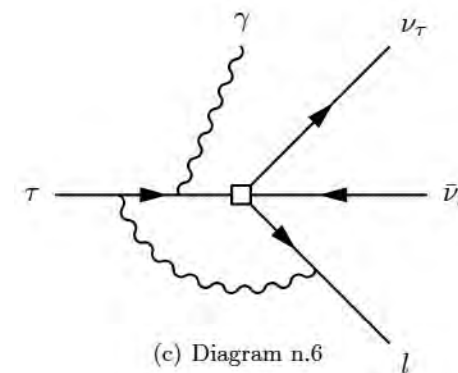
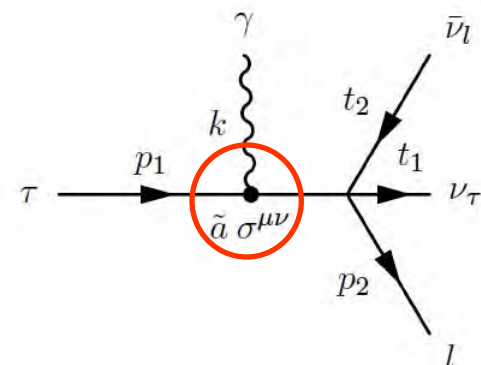
$$G_0 \rightarrow G_0 + \tilde{a}_\tau G_a + \frac{\alpha}{\pi} G_{\text{RC}}$$

- **Measure  $d^3\Gamma$  precisely and get  $\tilde{a}_\tau$ !**

[see also Laursen, Samuel, Sen, PRD29 (1984) 2652]



$$x = \frac{2E_l}{M_\tau}, \quad y = \frac{2E_\gamma}{M_\tau}, \quad r = \frac{m_l}{M_\tau}$$

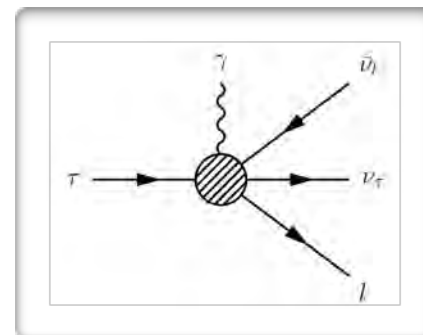


# The tau g-2 via its radiative leptonic decays: a proposal

- **Tau radiative leptonic decays at LO:**

$$\frac{d^3\Gamma}{dx dy d\cos\theta} = \frac{\alpha M_\tau^5 G_F^2 y \sqrt{x^2 - 4r^2}}{2\pi(4\pi)^6} G_0(x, y, \cos\theta, r)$$

Kinoshita & Sirlin PRL2(1959)177; Kuno & Okada, RMP73(2001)151



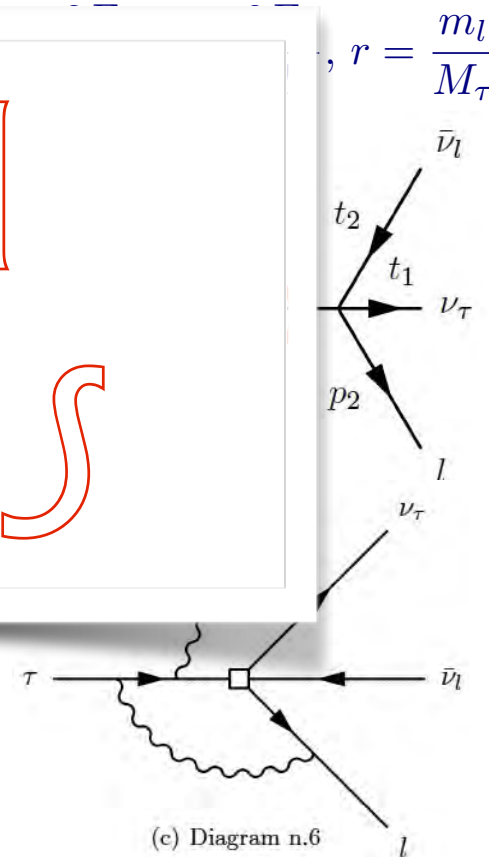
$\Gamma(\tau^- \rightarrow \nu_\tau^- l \bar{l} \nu_l)$

$\Gamma(\tau^- \rightarrow \nu_\tau^- l \bar{l} \nu_l \gamma)$

- **Add coupling**

WORK IN PROGRESS

- **Measure  $d^3\Gamma$  precisely and get  $\tilde{a}_\tau$ !**  
[see also Laursen, Samuel, Sen, PRD29 (1984) 2652]





# Conclusions

- The lepton  $g-2$  provide beautiful examples of interplay between theory and experiment.
- The muon discrepancy is  $\Delta a_\mu \sim 3 \div 3.5 \sigma$ . Is it NP? New upcoming  $g-2$  experiment: QED & EW terms ready for the challenge; how about the hadronic one? Future of hadronic LBL??
- Could  $\Delta a_\mu$  be due to **mistakes in the hadronic  $\sigma(s)$** ? Very unlikely. Also, given a 126 GeV SM Higgs, these hypothetical shifts  $\Delta\sigma(s)$  could only occur at very low energies (below  $\sim 1\text{GeV}$ ).
- The sensitivity of the electron  $g-2$  has improved. It may soon be possible to **test if  $\Delta a_\mu$  manifests itself also in the electron  $g-2$** ! A robust and ambitious experimental program is under way to improve  $\alpha$  &  $a_e$ . **The positronium contribution shouldn't be added!**
- The tau  $g-2$  is essentially unknown: we propose to measure it at Belle II via its radiative leptonic decays.

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**The End**