Hard Probes in A-A collisions: jet-quenching

Andrea Beraudo

Physics Department - Theory Unit - CERN

Quark-Gluon Plasma and heavy-ion collisions: past, present and future,
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Outline

- The QCD lagrangian
- QCD in elementary collisions: soft-gluon radiation
- QCD in A-A collisions: medium-induced gluon radiation and jet-quenching

The QCD Lagrangian: construction

Let us start from the *free quark Lagrangian* (diagonal in flavor!)

$$\mathcal{L}_q^{\rm free} = \overline{q}_f(x)[i\partial\!\!\!/ - m_f]q_f(x).$$

The *quark field* is actually a *vector in color space* ($N_c = 3$):

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The free quark Lagrangian is invariant under *global* $SU(3)$ (i.e. $V^\dagger V = 1$ and $\det(V)=1$) color transformations, namely:

$$q(x) \longrightarrow V\,q(x) \quad \text{and} \quad \overline{q}(x) \longrightarrow \overline{q}(x)\,V^\dagger,$$

with

$$V = \exp\left[i\theta^a t^a\right] \quad \text{and} \quad [t^a, t^b] = i\,f^{abc}t^c \quad (a = 1, \ldots N_c^2 - 1).$$

f^{abc}: real, antisymmetric *structure constants* of the $su(3)$ algebra.

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f^{abc}: real, antisymmetric *structure constants* of the $su(3)$ algebra.
We want to build a lagrangian invariant under *local color transformations*:

$$q(x) \longrightarrow V(x)\,q(x) \quad \overline{q}(x) \longrightarrow \overline{q}(x)\,V^\dagger(x),$$

where now $V(x) = \exp\left[i\theta^a(x)t^a\right]$.

Due to the derivative term, $\mathcal{L}_q^{\rm free}$ is not invariant under local $SU(N_c)$ transformations:

$$\mathcal{L}_q^{\rm free} \longrightarrow \mathcal{L'}_q^{\rm free} = \mathcal{L}_q^{\rm free} + \overline{q}(x) V^\dagger(x) \left[i \partial\!\!\!/ V(x) \right] q(x) \qquad (1)$$

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The solution is to couple the quarks to the gauge field $A_\mu \equiv A_\mu^a t^a$ through the *covariant derivative*

$$\partial_\mu \longrightarrow \mathcal{D}_\mu(x) \equiv \partial_\mu - igA_\mu(x),$$

getting:

$$\mathcal{L}_q = \overline{q}(x)[i \mathcal{D}\!\!\!/(x) - m]q(x) = \mathcal{L}_q^{\text{free}} + g\overline{q}(x) A\!\!\!/(x) q(x).$$

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The transformation of A_μ under local $SU(N_c)$ must be such to compensate the extra term in Eq. (1):

$$A_\mu \longrightarrow A'_\mu = VA_\mu V^\dagger - \frac{i}{g}(\partial_\mu V)V^\dagger.$$

Exercise: verify that $\mathcal{L}_q$ is now invariant under local $SU(N_c)$ transformations. In particular:

$$\mathcal{D}_\mu q \longrightarrow V\mathcal{D}_\mu q \quad \Longrightarrow \quad \mathcal{D}_\mu \longrightarrow V\mathcal{D}_\mu V^\dagger \qquad (2)$$

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We must now construct the lagrangian for the gauge-field A_μ

Remember the ($U(1)$ invariant) QED lagrangian of the e.m. field

$$\mathcal{L}_{\rm gauge}^{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad \text{with} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

The field-strength $F_{\mu\nu}$ can be expressed through the covariant derivative

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$$F_{\mu\nu} = \frac{i}{g} [\mathcal{D}_\mu, \mathcal{D}_\nu] \quad \longrightarrow \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig [A_\mu, A_\nu] .$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c \quad \text{(verify!)}$$

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$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu \quad \text{(verify!)}$$

From the transformation of the covariant derivative in Eq. (2) one has

$$F_{\mu\nu} \longrightarrow VF_{\mu\nu}V^\dagger, \quad \text{not invariant!}$$

so that the proper $SU(N_c)$-invariant generation of the QED lagrangian is

$$\mathcal{L}_{\rm gauge}^{QCD} = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a}$$

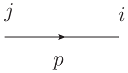
where we have used $\mathrm{Tr}(t^a t^b) = (1/2)\delta^{ab}$.

The QCD Lagrangian and Feynman rules

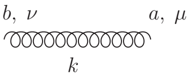
The final form of the QCD Lagrangian is then

$$\mathcal{L}^{QCD} = \sum_f \overline{q}_f [i\not{D} - m_f] q_f - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a},$$

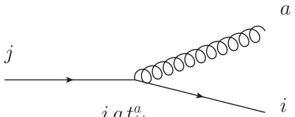
leading to the following Feynman rules (ex: derive them!)



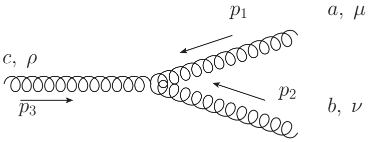
$$\delta^{ij} \frac{i(p_\mu \gamma^\mu + m)}{p^2 - m^2 + i\epsilon}$$



$$\delta^{ab} \frac{i(-g^{\mu\nu} + ...)}{k^2 + i\epsilon}$$



$$i\, g\, t^a_{ij}$$



$$g\, f^{abc}[g^{\mu\nu}(p_1 - p_2)^\rho + g^{\nu\rho}(p_2 - p_3)^\mu + g^{\rho\mu}(p_3 - p_1)^\nu]$$

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- One can verify (try!) that this choice satisfies the $su(3)$ algebra

$$[T^a, T^b]_{ce} = if^{abd}(T^d)_{ce}$$

  Suggestion: exploit the relation among the structure constants

$$f^{abd}f^{dce} + f^{bcd}f^{dae} + f^{cad}f^{dbe} = 0,$$

  coming from the (trivial) Jacobi identity

$$[[t^a, t^b], t^c] + [[t^b, t^c], t^a] + [[t^c, t^a], t^b] = 0$$

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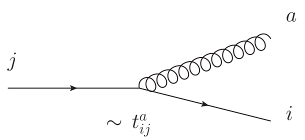
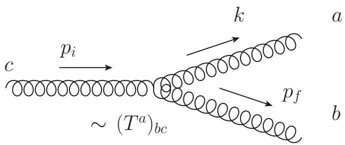
- This allows us to reinterpret the $g \to gg$ Feynman diagram





Color-flow in QCD processes

Graphical shortcuts (exact in the *large-N_c limit*) allows one to follow the color-flow in QCD processes and to evaluate color factors:

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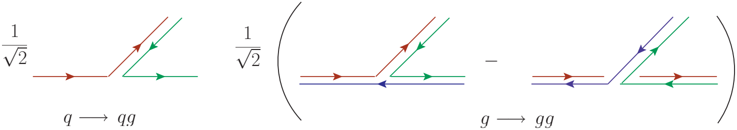

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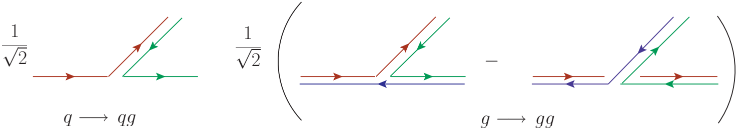
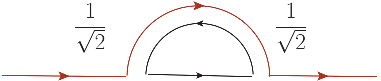


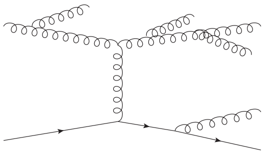
- Rad. prob. involves the factor $T_R^a T_R^a = C_R$ ($C_F = (N_c^2 - 1)/2N_c$ and $C_A = N_c$): $d\sigma_g^{\mathrm{rad}} \approx 2 d\sigma_q^{\mathrm{rad}}$ (gluon can radiate from 2 colored lines!)



$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}N_c = \frac{N_c}{2} \approx C_F$$

QCD in elementary collisions

In elementary collisions (e^+e^-, pp, $p\overline{p}$...) QCD allows one



- to calculate the hard-process ($qg \to qg$, $gg \to q\overline{q}g$...) in which high-p_T partons are produced;
- to resum the (mostly soft and collinear) gluons radiated by the accelerated color charges.

We will focus on the last item, which – in a second stage – we will generalize to deal with the additional radiation induced by the presence of a medium

Notation

It will convenient, depending on the cases, to employ different coordinate systems:

- Minkowski coordinates (more transparent physical meaning)

$$a=(a^0,\vec{a}),\quad b=(b^0,\vec{b}),\quad \text{with}\quad a\cdot b=a^0b^0-\vec{a}\cdot\vec{b}$$

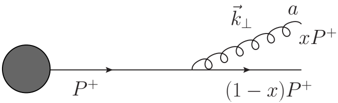
- Light-cone coordinates (calculations ∼10 times easier)

$$a=[a^+,a^-,\vec{a}_\perp],\quad b=[b^+,b^-,\vec{b}_\perp],\quad \text{with}\quad a\cdot b=a^+b^-+a^-b^+-\vec{a}_\perp\cdot\vec{b}_\perp$$

where $a^\pm\equiv[a^0\pm a^z]/\sqrt{2}$ (verify the consistency!).

Soft gluon radiation off hard partons

A hard parton with $p_i \equiv \left[p^+, Q^2/2p^+, \mathbf{0}\right]$ loses its virtuality Q through gluon-radiation. In *light-cone coordinates*, with $p^\pm \equiv [E \pm p_z]/\sqrt{2}$:



$$k \equiv \left[xp^+, \frac{\mathbf{k}^2}{2xp^+}, \mathbf{k}\right] \qquad \epsilon_g = \left[0, \frac{\boldsymbol{\epsilon}_g \cdot \mathbf{k}}{xp^+}, \boldsymbol{\epsilon}_g\right]$$

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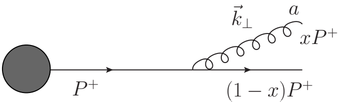

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Let us evaluate the radiation amplitude (notice that $\epsilon_g \cdot k = 0$)

$$\mathcal{M}^{\rm rad} = \overline{u}(p_f)(igt^a)\not{\epsilon}_g \frac{i(\not{p}_f + \not{k})}{(p_f + k)^2} \mathcal{M}^{\rm hard} \underset{\rm soft}{\approx} \overline{u}(p_f)(igt^a)\not{\epsilon}_g \frac{i\,\not{p}_f}{2p_f \cdot k} \mathcal{M}^{\rm hard}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad \longrightarrow \quad \not{\epsilon}_g \not{p}_f = 2p_f \cdot \epsilon_g - \not{p}_f \not{\epsilon}_g = 2p_f \cdot \epsilon_g \quad ({\rm since}\ \overline{u}(p_f)\not{p}_f = 0)$$

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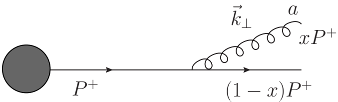

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The amplitude for soft ($x \ll 1$) gluon radiation reads then

$$\mathcal{M}^{\rm rad} \underset{x \ll 1}{\sim} g\left(\frac{p_f \cdot \epsilon_g}{p_f \cdot k}\right) t^a \mathcal{M}^{\rm hard} \qquad (3)$$

- Notice that the soft-gluon radiation amplitude

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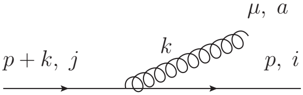
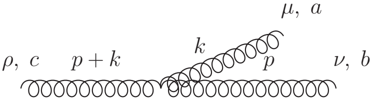
- One can derive *effective radiation vertexes* treating the quarks as complex scalar fields, getting rid of the Dirac algebra:

$$\mathcal{L}_{SQCD} = (\mathcal{D}_\mu \Phi)^* (\mathcal{D}^\mu \Phi) - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a}.$$

From $\epsilon_g \cdot k = 0$ (radiated gluons are transverse!) one gets (verify!)



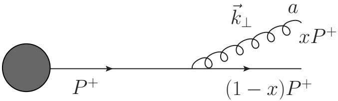
$$ig\, t^a_{ij}\, (2p+k)^\mu = ig\, t^a_{ij}\, 2p^\mu$$



$$g f^{abc}(-2p-k)^\mu g^{\nu\rho} = -g f^{abc} 2p^\mu g^{\nu\rho}$$

All soft-gluon radiation amplitudes (both in-vacuum and in-medium) can be derived within this approximation!

One gets (verify!)



$$\left(\frac{p_f \cdot \epsilon_g}{p_f \cdot k}\right) = 2\,(1-x)\,\frac{\boldsymbol{\epsilon}_g \cdot \mathbf{k}}{\mathbf{k}^2}$$

Squaring and *summing over the polarizations of the gluon* ($\sum_{\rm pol} \epsilon_g^i \epsilon_g^j = \delta^{ij}$) one gets the soft radiation cross-section:

$$d\sigma^{\rm rad}_{\rm vac} \underset{x\to 0}{\sim} d\sigma^{\rm hard}\,\frac{\alpha_s}{\pi^2}\,C_F\,\frac{dk^+}{k^+}\,\frac{d\mathbf{k}}{\mathbf{k}^2}$$

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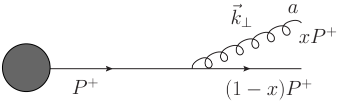

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$$d\sigma^{\rm rad}_{\rm vac} \underset{x\to 0}{\sim} d\sigma^{\rm hard}\,\frac{\alpha_s}{\pi^2}\,C_F\,\frac{dk^+}{k^+}\,\frac{d\mathbf{k}}{\mathbf{k}^2}$$

- Radiation spectrum (our benchmark): IR and collinear divergent!
- $k_\perp$ *vs* virtuality: $\mathbf{k}^2 = x\,(1-x)\,Q^2$;
- Time-scale (*formation time*) for gluon radiation:

$$\Delta t_{\rm rad} \sim Q^{-1}(E/Q) \sim 2\omega/\mathbf{k}^2 \quad (x \approx \omega/E)$$

Formation times will become important in the presence of a medium, whose thickness L will provide a scale to compare with!

Soft-gluon emission: color coherence

We have seen how the radiation of soft (i.e. *long wavelength*) gluon is not sensitive to short-distance details (e.g. *the spin* of the radiator), but only to the the color-charge of the emitter: *this will have deep consequences on the angular distribution of the radiation*.

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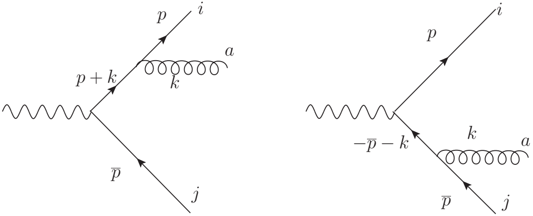
Let us consider the decay of a color-singlet ($\gamma^\star, Z, W, H$) into a $q\overline{q}$ pair: the suddenly accelerated color-charges can radiate gluons



Employing the effective soft-gluon vertexes one gets:

$$\mathcal{M}^{\rm rad} \approx g t^a_{ij} \left(\frac{p \cdot \epsilon_g}{p \cdot k} - \frac{\overline{p} \cdot \epsilon_g}{\overline{p} \cdot k} \right) \mathcal{M}^{\rm Born}.$$

In order to evaluate the radiation cross-section one must square the amplitude and integrate over the gluon phase-space. From the sum over the gluon polarizations (in Feynman gauge)

$$\sum_{\rm pol} \epsilon_\mu \epsilon_\nu^\star = -g_{\mu\nu}$$

one gets, for $k = (\omega, \vec{k})$,

$$d\sigma^{\rm rad} = d\sigma^{\rm Born}\, g^2 C_F \frac{d\vec{k}}{(2\pi)^3} \frac{1}{2\omega} \frac{2(p \cdot \overline{p})}{(p \cdot k)(\overline{p} \cdot k)}$$
$$= d\sigma^{\rm Born} \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\phi}{2\pi} \underbrace{\frac{1 - \cos\theta_{ij}}{(1 - \cos\theta_{ik})(1 - \cos\theta_{jk})}}_{W_{[ij]}} d\cos\theta$$

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$$\begin{aligned} d\sigma^{\mathrm{rad}} &= d\sigma^{\mathrm{Born}}\, g^2 C_F \frac{d\vec{k}}{(2\pi)^3} \frac{1}{2\omega} \frac{2(p\cdot\overline{p})}{(p\cdot k)(\overline{p}\cdot k)} \\ &= d\sigma^{\mathrm{Born}} \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\phi}{2\pi} \underbrace{\frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})}}_{W_{[ij]}} d\cos\theta \end{aligned}$$

One would like to obtain a *probabilistic interpretation*, possibly to insert into an Monte-Carlo setup. Non trivial request, since (in Feynman gauge) $d\sigma^{\mathrm{rad}}$ comes entirely from the interference term! However...

$$W_{[ij]} = \frac{1}{2}\left[\frac{\cos\theta_{ik}-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} + \frac{1}{1-\cos\theta_{ik}}\right] + \frac{1}{2}[i\leftrightarrow j] \equiv W_{[i]} + W_{[j]}.$$

This will help to achieve our goal!

$$W_{[i]} = \frac{1}{2}\left[\frac{\cos\theta_{ik} - \cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} + \frac{1}{1-\cos\theta_{ik}}\right]$$

allows one to give a probabilistic interpretation. In fact:

- $$W_{[i]} \underset{\theta_{ik}\to 0}{\sim} \frac{1}{1-\cos\theta_{ik}} \quad \text{and} \quad W_{[i]} \underset{\theta_{jk}\to 0}{\sim} \text{finite}$$

  and analogously for $W_{[j]}$.

- After azimuthal average:

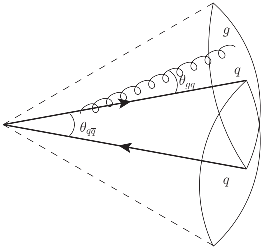
  $$\int_0^{2\pi}\frac{d\phi}{2\pi}W_{[i]} = \frac{\Theta(\theta_{ij}-\theta_{ik})}{1-\cos\theta_{ik}} \quad \text{and} \quad \int_0^{2\pi}\frac{d\phi}{2\pi}W_{[j]} = \frac{\Theta(\theta_{ij}-\theta_{jk})}{1-\cos\theta_{jk}}$$

  The quark can radiate a gluon within the cone of opening angle θ_{ij} obtained rotating the antiquark and vice versa.

One gets:

$$d\sigma^{\rm rad} = d\sigma^{\rm Born}\frac{\alpha_s C_F}{\pi}\frac{d\omega}{\omega}\left[\Theta(\theta_{ij}-\theta_{ik})\frac{d\cos\theta_{ik}}{1-\cos\theta_{ik}} + \Theta(\theta_{ij}-\theta_{jk})\frac{d\cos\theta_{jk}}{1-\cos\theta_{jk}}\right]$$

Angular ordering: physical interpretation



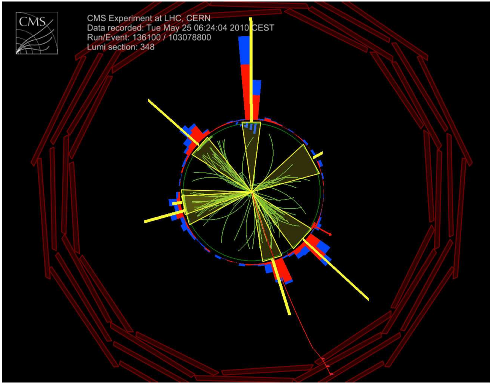
Radiation pattern of a $q\bar{q}$ antenna *in the vacuum*

- Formation-time required for gluon radiation: $t_f = 2\omega/k_\perp^2 \sim 1/\omega\theta_{gq}^2$;
- Transverse wave-length of the gluon $\lambda_\perp \sim 1/k_\perp \sim 1/\omega\theta_{gq}$ ...
- ... must be sufficient to *resolve* the transverse separation $d_\perp = t_f \theta_{q\bar{q}}$ reached meanwhile by the pair:

$$1/\omega\theta_{gq} \sim \lambda_\perp < d_\perp \sim \theta_{q\bar{q}}/\omega\theta_{gq}^2$$

- Gluon forced to be radiated within the cone $\theta_{gq} < \theta_{q\bar{q}}$

Angular ordering in parton branching: jet production



Angular ordering of QCD radiation in the vacuum *at the basis of the development of collimated jets*

Angular ordering: Hump-backed Plateau

- In order to resolve the color charges of the antenna

$$\lambda_\perp < d_\perp = t_f \, \theta_{q\bar{q}} \quad \longrightarrow \quad 1/k_\perp < (2\omega/k_\perp^2)\, \theta_{q\bar{q}}$$

- The request $k_\perp > \Lambda_{\rm QCD}$ leads to the constraint $\omega > \Lambda_{\rm QCD}/\theta_{q\bar{q}}$

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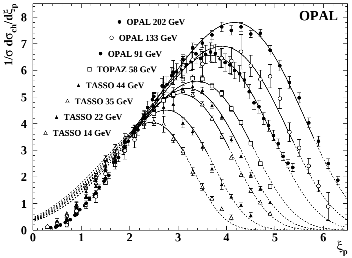
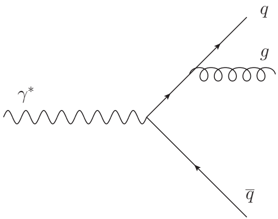


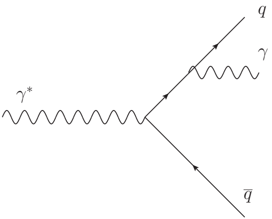
$$\xi \equiv -\ln\left(p^h/E^{\rm jet}\right)$$

(OPAL collab. – EPJC 27 (2003), 467)

Angular ordering responsible for the *suppression of soft-hadron production* in jet-fragmentation in the vacuum

Color-coherence in QCD: the string effect in e^+e^-





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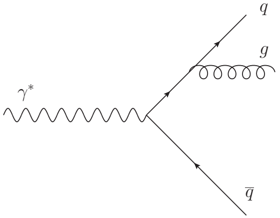
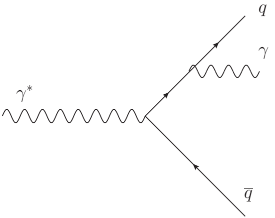
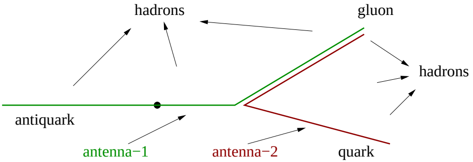
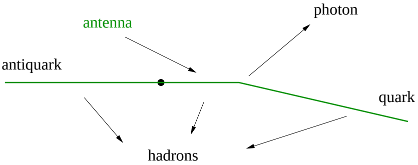




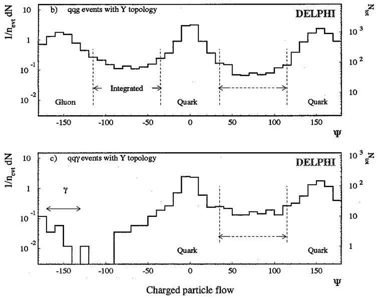


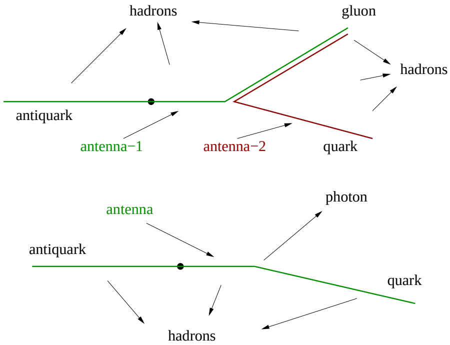


$e^+e^- \to q\bar{q}g$ vs $e^+e^- \to q\bar{q}\gamma$

Exactly the same kinematics, but *different color flow*

Color-coherence in QCD: the string effect in e^+e^-





Depletion vs enhancement of particle production within the $q - \overline{q}$ angle

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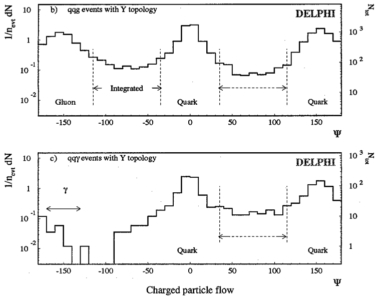

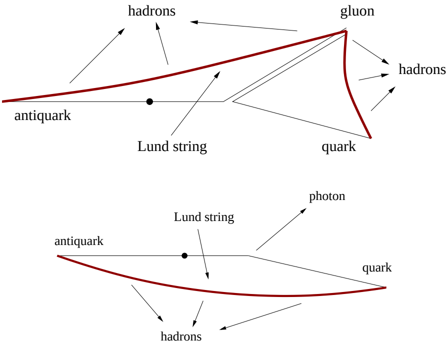

Depletion vs enhancement of particle production within the $q - \overline{q}$ angle

NB Alternative (complementary, still based on *color-flow*!) interpretation in terms of different string-breaking pattern when going from partonic to hadronic d.o.f. in the two cases

A first lesson

- We have illustrated some aspects of soft-gluon radiation (in particular angular-ordering and color-flow) essential to describe *basic qualitative predictions of QCD* in elementary collisions:
  - Development of collimated jets (the experimentally accessible observable closest to quarks and gluons);
  - *Intra-jet* coherence (soft-hadron suppression inside the jet-cone: Hump-backed Plateau);
  - *Inter-jet* coherence (angular pattern of soft particles outside the jets: string effect)

  Without explaining the above effects could QCD have been promoted to be THE theory of strong interactions?

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  Without explaining the above effects could QCD have been promoted to be THE theory of strong interactions?
- Hence the interest in studying how the above picture gets modified due to the interaction (i.e. *color-exchange*) with a medium

Ubi maior minor cessat: some references...

- R.K. Ellis, W.J. Stirling and B.R. Webber, *QCD and Collider Physics*, Cambridge University Press;
- G. Dissertori, I.G. Knowles and M. Schmelling, *Quantum Chromodynamics: High Energy Experiments and Theory*, Oxford University Press;
- Michelangelo Mangano, *QCD Lectures*, 1998 European School of High Energy Physics, St Andrews, Scotland;
- Yuri Dokshitzer, *Perturbative QCD for beginners*, Cargese NATO school 2001.

QCD radiation in A-A collisions

We have seen how suddenly accelerated color-charges can radiate soft gluons. In A-A collisions the presence of a medium where high-energy partons can scatter (changing *momentum* and *color*) can enhance the probability of gluon radiation.

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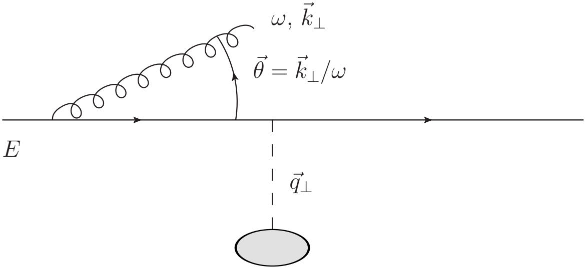
The elementary brick to consider will be the radiation due to a single elastic scattering in the medium



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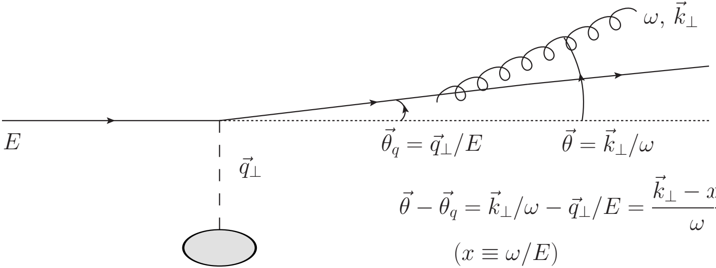

$$\vec{\theta} - \vec{\theta}_q = \vec{k}_\perp/\omega - \vec{q}_\perp/E = \frac{\vec{k}_\perp - x\vec{q}_\perp}{\omega}$$

$$(x \equiv \omega/E)$$

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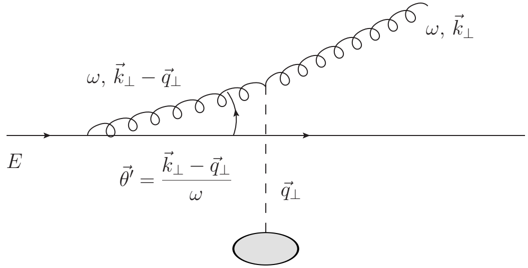

The modelling of the medium (I)

The modelling of the medium in radiative energy-loss studies is usually quite elementary. It is just given by a color-field $A^\mu(x)$ arising from a collection of scattering centers, mimicking the elastic collisions suffered by the high-energy parton with the color-charges present in the medium. In the *axial gauge* $A^+ = 0$ one has:

$$A^-(x) \equiv \sum_{n=1}^{N} \int \frac{d\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{x}_n)} \mathcal{A}(\mathbf{q})\, \delta(x^+ - x_n^+)\; T_{(n)}^{a_n} \otimes T_{(R)}^{a_n}$$

- $T_{(n)}^{a_n}$ describes the color rotation of the $n^{\rm th}$ scattering center in the representation n;
- $T_{(R)}^{a_n}$ describes the color rotation of high-energy projectile, in the representation R;
- $\mathcal{A}(\mathbf{q})$ is a generic interaction potential responsible for the transverse-momentum transfer $\mathbf{q}$. Its specific form in not important, what matters is that *the medium is able to provide a momentum kick and to exchange color with the projectile.*

The modelling of the medium (II)

- It will be convenient to express the color-field in Fourier space:

$$A^-(x) \equiv \sum_{n=1}^{N} (2\pi)\delta(q^+) e^{iq^- x^+} e^{-i\mathbf{q}\cdot\mathbf{x}_n} \mathcal{A}(\mathbf{q})\ T^{a_n}_{(n)} \otimes T^{a_n}_{(R)}$$

$\mathcal{A}(\mathbf{q})$ is often taken as Debye-screened potential $\mathcal{A}(\mathbf{q}) = \frac{g^2}{\mathbf{q}^2 + \mu_D^2}$:in this case μ_D^2 ($\sim \alpha_s T^2$ in weak-coupling) will represent the typical $\mathbf{q}^2$-transfer from the medium.

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- In squaring the amplitudes one will have to evaluate the traces

$$\mathrm{Tr}\left(T^{a_1}_{(n)} T^{a_2}_{(n')}\right) = \delta_{nn'}\delta^{a_1 a_2} C(n) \quad (C(\mathrm{fund}) = 1/2 \;\text{ and }\; C(\mathrm{adj}) = N_c)$$

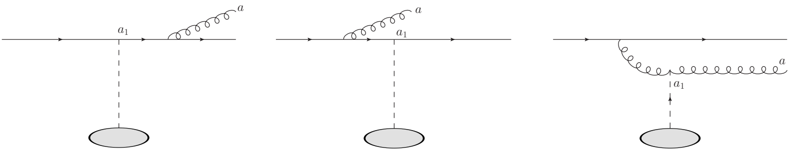
and (averaging over the d_R and d_n colors of proj. R and targ. n)

$$\frac{1}{d_R d_n}\mathrm{Tr}\,(T^{a_1}_R T^{a_2}_R)\,(T^{a_1}_n T^{a_2}_n) = \frac{C_R C(n)}{d_n} \longrightarrow \frac{d\sigma^{\mathrm{el}}(R,n)}{d\mathbf{q}} = \frac{C_R C(n)}{d_n}\frac{\mathcal{A}(\mathbf{q})}{(2\pi)^2}$$

Medium-induced gluon radiation: projectile from $-\infty$

We consider the radiation off a on-shell high-E parton $p_i = [p^+, 0, \mathbf{0}]$, *induced by a single elastic scattering* ($N=1$ opacity expansion)

$$p_f = \left[(1-x)p^+, \frac{(\mathbf{q}-\mathbf{k})^2}{2(1-x)p^+}, \mathbf{q}-\mathbf{k}\right], \quad k = \left[xp^+, \frac{\mathbf{k}^2}{2xp^+}, \mathbf{k}\right], \quad \epsilon_g = \left[0, \frac{\boldsymbol{\epsilon}_g \cdot \mathbf{k}}{xp^+}, \boldsymbol{\epsilon}_g\right]$$



(a) (b) (c)

$$i\mathcal{M}_{(a)} = -ig\,(t^a t^{a_1}) \sum_n \left(\frac{p_f \cdot \epsilon_g}{p_f \cdot k}\right) (2p^+)\mathcal{A}(\mathbf{q})\, e^{iq\cdot x_n}\; T^{a_1}_{(n)}$$

$$= -ig\,(t^a t^{a_1}) \sum_n 2(1-x) \underbrace{\frac{\boldsymbol{\epsilon}_g \cdot (\mathbf{k}-x\mathbf{q})}{(\mathbf{k}-x\mathbf{q})^2}}_{\sim\, \vec{\theta}-\vec{\theta}_q} (2p^+)\mathcal{A}(\mathbf{q})\, e^{iq\cdot x_n}\; T^{a_1}_{(n)}$$

The three different amplitudes reads (verify!)

$$
\begin{aligned}
i\mathcal{M}_{(a)} &= -ig\,(t^a t^{a_1}) \sum_n 2(1-x)\,\frac{\boldsymbol{\epsilon}_g\cdot(\mathbf{k}-x\mathbf{q})}{(\mathbf{k}-x\mathbf{q})^2}\,(2p^+)\mathcal{A}(\mathbf{q})\,e^{iq\cdot x_n}\;T^{a_1}_{(n)} \\
i\mathcal{M}_{(b)} &= ig\,(t^{a_1} t^a) \sum_n 2(1-x)\,\frac{\boldsymbol{\epsilon}_g\cdot\mathbf{k}}{\mathbf{k}^2}\,(2p^+)\mathcal{A}(\mathbf{q})\,e^{iq\cdot x_n}\;T^{a_1}_{(n)} \\
i\mathcal{M}_{(c)} &= ig\,[t^a, t^{a_1}] \sum_n 2(1-x)\,\frac{\boldsymbol{\epsilon}_g\cdot(\mathbf{k}-\mathbf{q})}{(\mathbf{k}-\mathbf{q})^2}\,(2p^+)\mathcal{A}(\mathbf{q})\,e^{iq\cdot x_n}\;T^{a_1}_{(n)}.
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\end{aligned}
$$

Neglecting $\mathcal{O}(x)$ corrections in (a) one gets the compact expression:

$$
i\mathcal{M}^{\rm rad} = -2ig\,[t^a, t^{a_1}] \sum_n \left[\frac{\epsilon_g\cdot\mathbf{k}}{\mathbf{k}^2} - \frac{\epsilon_g\cdot(\mathbf{k}-\mathbf{q})}{(\mathbf{k}-\mathbf{q})^2}\right](2p^+)\mathcal{A}(\mathbf{q})\,e^{iq\cdot x_n}\;T^{a_1}_{(n)}
$$

leading to the *Gunion-Bertsch* spectrum:

$$
k^+\frac{dN_g}{d\mathbf{k}dk^+} \equiv \frac{1}{\sigma^{\rm el}}k^+\frac{d\sigma^{\rm rad}}{d\mathbf{k}dk^+} = C_A\frac{\alpha_s}{\pi^2}\left\langle[\mathbf{K}_0-\mathbf{K}_1]^2\right\rangle = C_A\frac{\alpha_s}{\pi^2}\left\langle\frac{\mathbf{q}^2}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}\right\rangle
$$

$$
\text{where}\quad \mathbf{K}_0 \equiv \frac{\mathbf{k}}{\mathbf{k}^2},\quad \mathbf{K}_1 \equiv \frac{\mathbf{k}-\mathbf{q}}{(\mathbf{k}-\mathbf{q})^2}\quad \text{and}\quad \langle\dots\rangle \equiv \int d\mathbf{q}\frac{1}{\sigma^{\rm el}}\frac{d\sigma^{\rm el}}{d\mathbf{q}}
$$

Medium-induced radiation: the QED case

In the case of QED-radiation one would have just 2 amplitudes to sum:

$$\mathcal{M}_{(a)} \sim -g \sum_n 2\,\frac{\epsilon_\gamma \cdot (\mathbf{k} - x\mathbf{q})}{(\mathbf{k} - x\mathbf{q})^2} \mathcal{A}(\mathbf{q})\, e^{iq\cdot x_n}, \quad \mathcal{M}_{(b)} \sim g \sum_n 2\,\frac{\epsilon_\gamma \cdot \mathbf{k}}{\mathbf{k}^2} \mathcal{A}(\mathbf{q})\, e^{iq\cdot x_n}$$

getting the *Bethe-Heitler* spectrum

$$k^+ \frac{dN_\gamma}{d\mathbf{k} dk^+} = \frac{\alpha_{\mathrm{QED}}}{\pi^2} \left\langle \frac{x^2 \mathbf{q}^2}{\mathbf{k}^2 (\mathbf{k} - x\mathbf{q})^2} \right\rangle .$$

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- Notice that the photon radiation is suppressed in the $x \to 0$ limit, in which $\mathbf{k}-x\mathbf{q}\approx\mathbf{k}$. This corresponds to $\vec{\theta}-\vec{\theta}_q \approx \vec{\theta}$, neglecting the recoil angle of the quark (it cannot radiate photons if it doesn't change direction!);

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getting the *Bethe-Heitler* spectrum

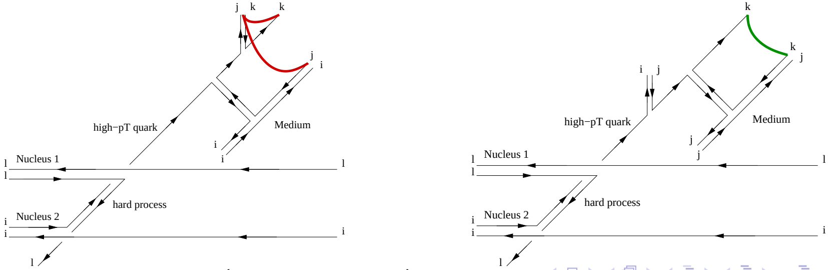
$$k^+ \frac{dN_\gamma}{d\mathbf{k} dk^+} = \frac{\alpha_{\rm QED}}{\pi^2} \left\langle \frac{x^2 \mathbf{q}^2}{\mathbf{k}^2 (\mathbf{k} - x\mathbf{q})^2} \right\rangle .$$

- Notice that the photon radiation is suppressed in the $x \to 0$ limit, in which $\mathbf{k} - x\mathbf{q} \approx \mathbf{k}$. This corresponds to $\vec{\theta} - \vec{\theta}_q \approx \vec{\theta}$, neglecting the recoil angle of the quark (it cannot radiate photons if it doesn't change direction!);
- However in QCD, even neglecting the recoil (i.e. the quark goes on propagating straight-line), the quark rotates in color and hence can radiate gluons, yielding a non-vanishing spectrum even in the strict $x \to 0$ limit.

Medium-induced radiation: color flow

The 3-gluon amplitude $\mathcal{M}_{(c)}$ has the color structure $[t^a, t^{a_1}]$, which can be decomposed as $t^a t^{a_1} - t^{a_1} t^a$, corresponding to the two color flows

The relevant color channels to consider are then just two:



We will investigate (see next lecture) the implications at hadronization!

The radiation amplitude can be decomposed in the two color channels

$$\mathcal{M}^{\rm rad} = \mathcal{M}^{aa_1} + \mathcal{M}^{a_1 a}$$

In squaring the amplitude interference terms between the two color channels are suppressed by $\mathcal{O}(1/N_c^2)$, since (verify!)

$$\mathrm{Tr}(t^a t^{a_1} t^{a_1} t^a) = C_F^2 N_c \quad \text{and} \quad \mathrm{Tr}(t^a t^{a_1} t^a t^{a_1}) = -(1/2N_c) C_F N_c .$$

The radiation spectrum in the two color channels reads then:

$$k^+ \frac{dN_g}{d\mathbf{k} dk^+}\bigg|_{aa_1} = \frac{N_c}{2} \frac{\alpha_s}{\pi^2} \left\langle \left[\overline{\mathbf{K}}_0 - \mathbf{K}_1\right]^2 \right\rangle , \quad k^+ \frac{dN_g}{d\mathbf{k} dk^+}\bigg|_{a_1 a} = \frac{N_c}{2} \frac{\alpha_s}{\pi^2} \left\langle \left[\mathbf{K}_0 - \mathbf{K}_1\right]^2 \right\rangle$$

where $\overline{\mathbf{K}}_0 \equiv \dfrac{\mathbf{k} - x\mathbf{q}}{(\mathbf{k} - x\mathbf{q})^2}$. Notice that, in the soft $x \to 0$ limit, the two channel contributes equally to the spectrum.

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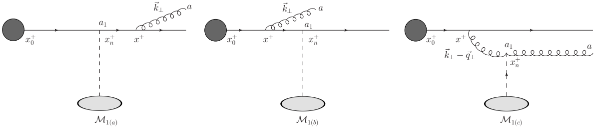
In the soft limit the sum returns the inclusive Gunion Bertsch spectrum

$$k^+ \frac{dN_g}{d\mathbf{k}dk^+}\bigg|_{aa_1} + k^+ \frac{dN_g}{d\mathbf{k}dk^+}\bigg|_{a_1 a} \underset{x\to 0}{\sim} C_A \frac{\alpha_s}{\pi^2}\left\langle \frac{\mathbf{q}^2}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2} \right\rangle$$

Radiation off a parton produced in the medium



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$$d\sigma^{\rm rad} = d\sigma^{\rm vac} + d\sigma^{\rm ind}$$

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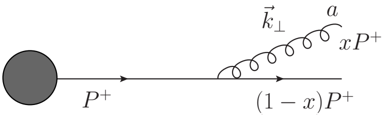

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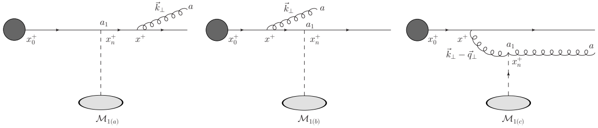

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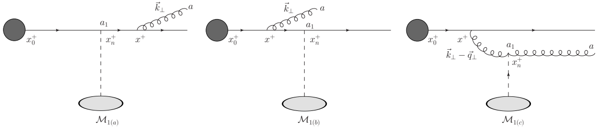

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- The *medium length* L introduces a scale to compare with the *gluon formation-time* $t_{\rm form}$ $\longrightarrow$ non-trivial *interference effects*!
  In the vacuum (no other scale!) $t_{\rm form}^{\rm vac} \equiv 2\omega/\mathbf{k}^2$ played no role.

Calculating the spectrum: opacity expansion

Gluon-spectrum $d\sigma^{\mathrm{rad}}$ written as an *expansion in powers of* $(L/\lambda^{\mathrm{el}})$

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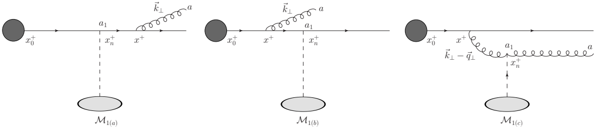

$\langle|\mathcal{M}_1|^2\rangle$: contribution to the radiation spectrum involving *color-exchange* with the medium

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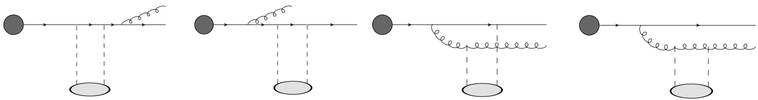

$2{\rm Re}\langle\mathcal{M}_2^{\rm virt}\rangle\mathcal{M}_0^*$: reducing the contribution to the spectrum by vacuum radiation, involving *no color-exchange* with the medium

The medium-induced spectrum: physical interpretation

$$\omega\frac{d\sigma^{\mathrm{ind}}}{d\omega d\mathbf{k}} = d\sigma^{\mathrm{hard}} C_R \frac{\alpha_s}{\pi^2}\left(\frac{L}{\lambda_g^{\mathrm{el}}}\right)\left\langle\left[(\mathbf{K}_0-\mathbf{K}_1)^2+\mathbf{K}_1^2-\mathbf{K}_0^2\right]\left(1-\frac{\sin(\omega_1 L)}{\omega_1 L}\right)\right\rangle$$

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  The full radiation spectrum can be organized as

$$d\sigma^{\rm rad} = d\sigma^{\rm GB} + d\sigma^{\rm vac}_{\rm gain} + d\sigma^{\rm vac}_{\rm loss}$$

where

$$d\sigma^{\rm GB} = d\sigma^{\rm hard} C_R \frac{\alpha_s}{\pi^2}\left(L/\lambda_g^{\rm el}\right)\left\langle(\mathbf{K}_0-\mathbf{K}_1)^2\right\rangle(d\omega d\mathbf{k}/\omega)$$

$$d\sigma^{\rm vac}_{\rm gain} = d\sigma^{\rm hard} C_R \frac{\alpha_s}{\pi^2}\left(L/\lambda_g^{\rm el}\right)\langle\mathbf{K}_1^2\rangle(d\omega d\mathbf{k}/\omega)$$

$$d\sigma^{\rm vac}_{\rm loss} = \left(1-L/\lambda_g^{\rm el}\right) d\sigma^{\rm hard} C_R \frac{\alpha_s}{\pi^2}\mathbf{K}_0^2(d\omega d\mathbf{k}/\omega)$$

(for a detailed derivation see e.g. JHEP 1207 (2012) 144)

In-medium gluon formation time

Behavior of the induced spectrum depending on the *gluon formation-time*

$$t_{\rm form} \equiv \omega_1^{-1} = 2\omega/(\mathbf{k} - \mathbf{q})^2$$

differing from the vacuum result $t_{\rm form}^{\rm vac} \equiv 2\omega/\mathbf{k}^2$, due to the transverse $\mathbf{q}$-kick received from the medium. Why such an expression?

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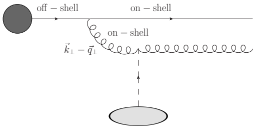
Consider for instance the $\langle \mathbf{K}_1^2 \rangle$ contribution, with the hard parton produced off-shell $p_i \equiv [p_+, Q^2/2p_+, \mathbf{0}]$ and radiating an on-shell gluon, which then scatters in the medium



$$k_g \equiv \left[x p_+, \frac{(\mathbf{k}-\mathbf{q})^2}{2xp_+}, \mathbf{k}-\mathbf{q} \right]$$

$$p_f = \left[(1-x)p_+, \frac{(\mathbf{k}-\mathbf{q})^2}{2(1-x)p_+}, \mathbf{q}-\mathbf{k} \right]$$

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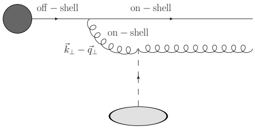

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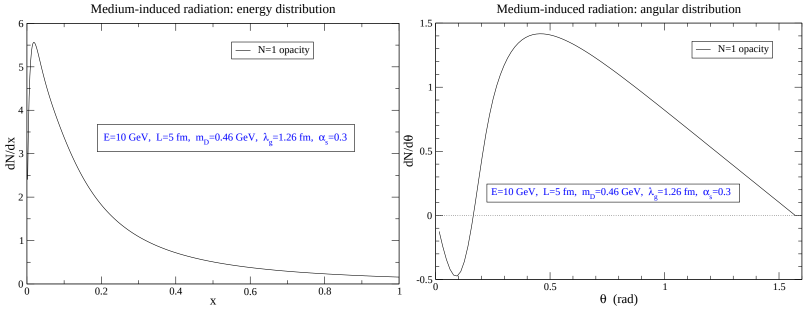
The radiation will occur in a time set by the uncertainty principle:

$$t_{\rm form} \sim Q^{-1}(E/Q) \sim 2\omega/(\mathbf{k}-\mathbf{q})^2$$

$\longrightarrow$ if $t_{\rm form} \gtrsim L$ the process is suppressed!

Medium-induced radiation spectrum: numerical results



At variance with vacuum-radiation, medium induced spectrum

- Infrared safe (vanishing as $\omega \to 0$);
- Collinear safe (vanishing as $\theta \to 0$).

Depletion of gluon spectrum at small angles due to their rescattering in the medium!

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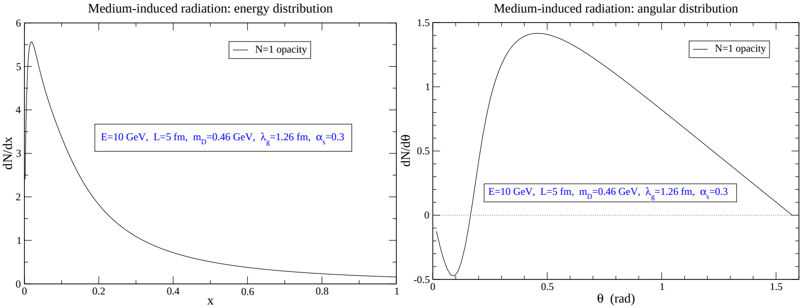

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In general $\langle N \rangle > 1$, so that addressing *multiple gluon emission* becomes mandatory

Average energy-loss: analytic estimate

Integrating the lost energy ω over the inclusive gluon spectrum one gets, for an extremely energetic parton,

$$\langle \Delta E \rangle = \int d\omega \int d\mathbf{k}\, \omega \frac{dN_g^{\text{ind}}}{d\omega d\mathbf{k}} \underset{L \ll \sqrt{E/\hat{q}}}{\sim} \frac{C_R \alpha_s}{4} \left(\frac{\mu_D^2}{\lambda_g^{\text{el}}} \right) L^2$$

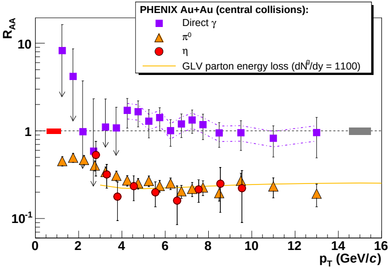
- L^2 dependence on the medium-length (as long as the medium is sufficiently thin);
- In the same limit $\langle \Delta E \rangle$ *independent on the parton energy*;
- μ_D: Debye screening mass of color interaction $\sim$ *typical momentum exchanged in a collision*;
- $\mu_D^2/\lambda_g^{\text{el}}$ often replaced by the *transport coefficient* $\hat{q}$, so that

$$\langle \Delta E \rangle \sim \alpha_s \hat{q} L^2$$

$\hat{q}$: average $q_\perp^2$ acquired per unit length

Inclusive hadron spectra: the nuclear modification factor

Historically, the *first "jet-quenching" observable*



$$R_{AA} \equiv \frac{\left(dN^h/dp_T\right)^{AA}}{\langle N_{\rm coll}\rangle \left(dN^h/dp_T\right)^{pp}}$$

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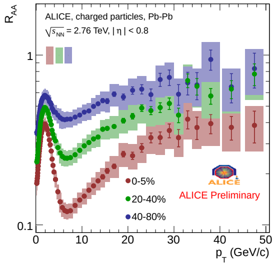

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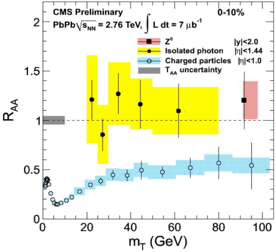

$$R_{AA} \equiv \frac{\left(dN^h/dp_T\right)^{AA}}{\langle N_{\rm coll}\rangle \left(dN^h/dp_T\right)^{pp}}$$

Hard-photon $R_{AA} \approx 1$

- supports the Glauber picture (binary-collision scaling);
- entails that quenching of inclusive hadron spectra is a *final state effect* due to in-medium energy loss.

Some CAVEAT:

- At variance wrt e^+e^- collisions, in hadronic collisions one starts with a parton p_T-distribution ($\sim 1/p_T^\alpha$) so that inclusive hadron spectrum simply reflects *higher moments of FF*

$$\frac{dN^h}{dp_T} \sim \frac{1}{p_T^\alpha} \sum_f \int_0^1 dz\, z^{\alpha-1} D_{f\to h}(z)$$

carrying limited information on FF (but very sensitive to hard tail!).

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$$\begin{aligned}
\frac{dN^h}{dp_T} &= \sum_f \int_0^1 dz \int dp'_T\, D_{f\to h}(z)\, \delta(p_T - z p'_T)\, \frac{dN^q}{dp'_T} \\
&= \sum_f \int_0^1 dz \int dp'_T\, D_{f\to h}(z)\, \frac{1}{z} \delta(p'_T - p_T/z)\, \frac{1}{(p'_T)^\alpha} \\
&= \frac{1}{p_T^\alpha} \sum_f \int_0^1 dz\, z^{\alpha-1} D_{f\to h}(z)
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$$D^{\rm med}_{f\to h}(z) = \int_0^1 d\epsilon\, P(\epsilon) \int_0^1 dz'\, \delta[z-(1-\epsilon)z']\, D^{\rm vac}_{f\to h}(z')$$

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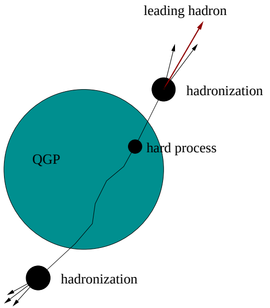
- Final *spectrum sensitive to small energy losses* $\epsilon \ll 1$

$$\frac{dN^h}{dp_T} = \frac{1}{p_T^\alpha} \sum_f \int_0^1 dz\, z^{\alpha-1} \int_0^{1-z} \frac{d\epsilon}{1-\epsilon} P(\epsilon)\, D_{f\to h}^{\text{vac}}\left(\frac{z}{1-\epsilon}\right)$$

Surface bias:



Quenched spectrum does not reflect $\langle L_{\mathrm{QGP}} \rangle$ crossed by partons distributed in the transverse plane according to $n_{\mathrm{coll}}(\mathbf{x})$ scaling, but *due to its steeply falling shape* is biased by the enhanced contribution of the ones *produced close to the surface and losing a small amount of energy*!

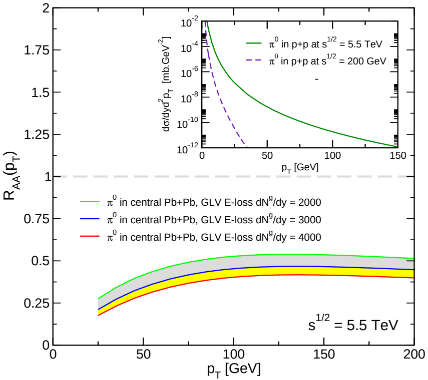
From theory to experiment...

We have seen that

- $\langle N \rangle > 1$ makes mandatory to deal with multiple gluon radiation;
- $\langle \Delta E \rangle$ is *not sufficient to characterize the quenching* of the spectra, but one needs the full $P(\Delta E)$, in particular for $\Delta E \ll E$.

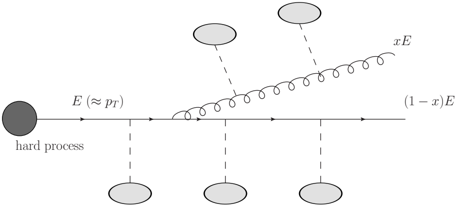
In case of *uncorrelated gluon radiation* (strong assumption! it's not the case for vacuum-radiation)

$$P(\Delta E) = \sum_{n=0}^{\infty} \frac{e^{-\langle N_g \rangle}}{n!} \prod_{i=1}^{n} \left[\int d\omega_i \frac{dN_g}{d\omega_i} \right] \times \delta\left(\Delta E - \sum_{i=1}^{n} \omega_i \right),$$



(see I. Vitev, PLB 639 (2006), 38-45)

Some heuristic estimates



In general the projectile system (high-E parton + rad. gluon) can interact several times with the medium. One can then estimate the *gluon formation-length* as

$$l_{\rm f} \sim \frac{\omega}{(\mathbf{k}-\mathbf{q})^2} \quad \longrightarrow \quad l_{\rm f} \sim \frac{\omega}{(\mathbf{k}-\sum_n \mathbf{q}_n)^2} \approx \frac{\omega}{N_{\rm scatt}\langle \mathbf{q}_n^2\rangle} = \frac{\omega}{l_{\rm f}\langle \mathbf{q}_n^2\rangle/\lambda_{\rm mfp}}.$$

Hence, one can identify $l_{\rm f} \equiv \sqrt{\omega/\hat{q}}$: soft gluon are formed earlier!

From $1 = \hbar c = 0.1973\,\mathrm{GeV}\cdot\mathrm{fm} \longrightarrow 1\,\mathrm{GeV}\cdot\mathrm{fm} \approx 5$...

- Gluon radiation is suppressed if $l_{\rm form}(\omega) > L$, which occurs above the *critical frequency* ω_c. Medium induces radiation of gluons with

$$l_{\rm form}(\omega) = \sqrt{\omega/\hat{q}} < L \quad \longrightarrow \quad \omega < \omega_c \equiv \hat{q} L^2$$

For $\hat{q} \approx 1$ GeV^2/fm and $L \approx 5$ fm one gets $\omega_c \approx 125$ GeV.

- One can estimate the *typical angle at which gluons are radiated*:

$$\langle \mathbf{k}^2 \rangle \approx \hat{q}\, l_{\rm form}(\omega) = \sqrt{\hat{q}\omega} \longrightarrow \langle \theta^2 \rangle = \frac{\langle \mathbf{k}^2 \rangle}{\omega^2} = \sqrt{\frac{\hat{q}}{\omega^3}} \longrightarrow \overline{\theta} = \left(\frac{\hat{q}}{\omega^3} \right)^{1/4}$$

For a typical $\hat{q} \approx 1$ GeV^2/fm one has (verify!):

$$\omega = 2\,\mathrm{GeV} \longrightarrow \overline{\theta} \approx 0.4 \qquad \omega = 5\,\mathrm{GeV} \longrightarrow \overline{\theta} \approx 0.2$$

Soft gluons radiated at larger angles!

- Below the *Bethe-Heitler* frequency $\omega_{\rm BH}$ one has $l_{\rm form}(\omega) < \lambda_{\rm mfp}$ and coherence effects are no longer important:

$$l_{\rm form}(\omega_{\rm BH}) = \sqrt{\omega_{\rm BH}/\hat{q}} = \lambda_{\rm mfp} \quad \longrightarrow \quad \omega_{\rm BH} \equiv \hat{q} \lambda_{\rm mfp}^2$$

Energy-loss: heuristic derivation

Let us estimate the spectrum of radiated gluons *in the coherent regime* $\omega_{\rm BH} < \omega < \omega_c$. One has to express the medium thickness L *in units of the gluon formation length* $l_{\rm form} = \sqrt{\omega/\hat{q}}$, getting the *effective numbers of radiators*:

$$\omega \frac{dN_g}{d\omega} \sim \alpha_s C_R \frac{L}{l_{\rm form}(\omega)} = \alpha_s C_R \sqrt{\frac{\omega_c}{\omega}}$$

Hence, for the *average energy-loss* one gets:

$$\langle \Delta E \rangle \sim \alpha_s C_R \sqrt{\omega_c} \int_{\omega_{\rm BH}}^{\omega_c} \frac{d\omega}{\sqrt{\omega}} \underset{\omega_{\rm BH} \ll \omega_c}{\sim} \alpha_s C_R \, \omega_c = \alpha_s \, C_R \, \hat{q} L^2$$

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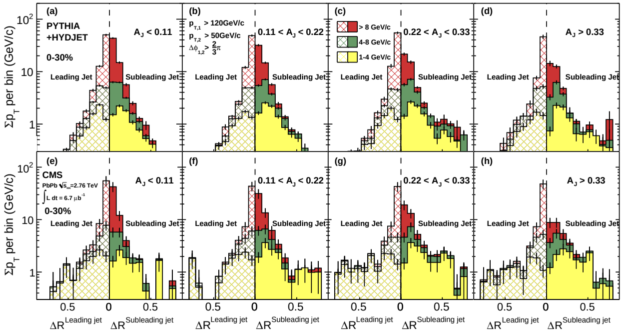
One can show (try!) that the contribution from the *incoherent regime* $\omega < \omega_c$ in which

$$\omega \frac{dN_g}{d\omega} \sim \alpha_s C_R \frac{L}{\lambda_{\rm mfp}}$$

is subleading by a factor $\lambda_{\rm mfp}/L$.

Dijet measurements (with tracking information)

Tracks in a ring of radius $\Delta R \equiv \sqrt{\Delta\phi^2 + \Delta\eta^2}$ and width 0.08 *around the subleading-jet axis*:



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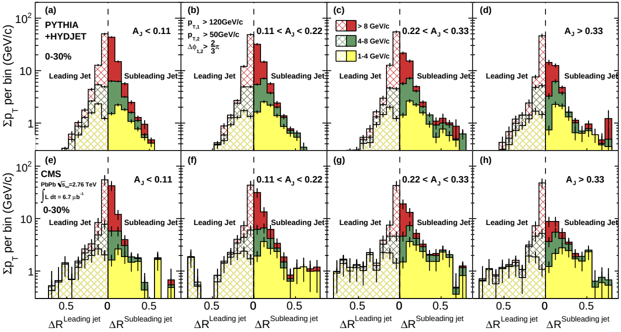

Increasing A_J a sizable fraction of energy around subleading jet carried by *soft* ($p_T < 4$ GeV) *tracks* with a *broad angular distribution*

- So far we have considered a purely partonic description, assuming a direct connection with the final hadronic observables. In particular, based on time-scale considerations

$$\Delta t_{\rm rest}^{\rm hadr} \sim 1/Q \quad \longrightarrow \quad \Delta t_{\rm lab}^{\rm hadr} \sim (E/Q)(1/Q) \underset{E\to\infty}{>>} \tau_{\rm QGP},$$

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high-energy partons are expected to fragment *outside the medium*. Hence one could think of neglecting medium effects at the hadronization stage;

- However high-energy partons exchange color with the medium and *this can modify the color flow in the shower*, no matter when this occurred, affecting the final hadron spectra and the jet-fragmentation pattern!

...Hence the interest in studying medium-modification of color-flow for high-p_T probes[1] focusing on

- leading-hadron spectra...
- ...but considering also more differential observables (e.g. jet-fragmentation pattern)

Essential ideas presented here in a $N = 1$ opacity calculation

[1] A.B, J.G.Milhano and U.A. Wiedemann, Phys. Rev. C85 (2012) 031901 and JHEP 1207 (2012) 144

From partons to hadrons

The *final stage of* any *parton shower* has to be interfaced with some hadronization routine. Keeping track of color-flow one identifies *color-singlet* objects whose decay will give rise to hadrons

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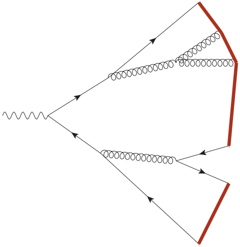

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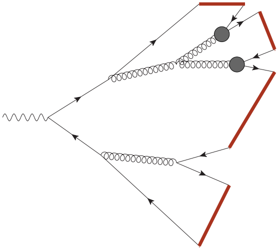
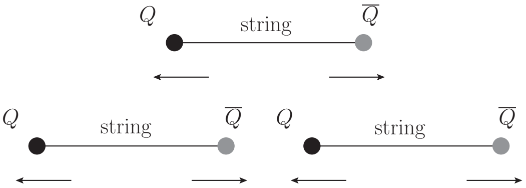


- In PYTHIA hadrons come from the fragmentation of *$q\bar{q}$ strings*, with gluons representing kinks along the string (Lund model);
- In HERWIG the shower is evolved up to a softer scale, all gluons are *forced to split* in $q\bar{q}$ pair (large-N_c!) and *singlet clusters* (usually of *low invariant mass*!) are thus identified.

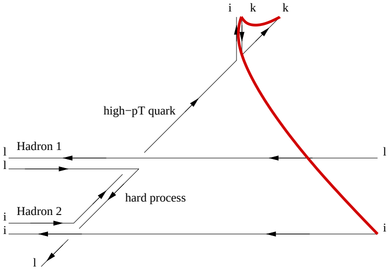
PYTHIA vs HERWIG

- The PYTHIA hadronization routine is *based on the Lund string model*, in which a string is stretched between a $Q\overline{Q}$ pair until the energy $E = \sigma R$ makes more favorable to excite a new $Q\overline{Q}$ pair from the vacuum



- The HERWIG hadronization routine is based on the *decay of color-singlet low-mass cluster*, e.g. $\mathcal{C} \to \pi^+\pi^-$, $\mathcal{C} \to K^+K^-$...Being most of the clusters light ($M \sim 1$ GeV) one has usually just a *2-body decay*.

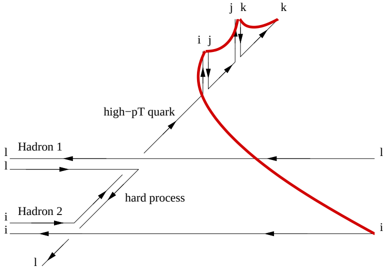
Vacuum radiation: color flow (in large-N_c)



Final hadrons from the fragmentation of the Lund string (in red)

- First endpoint attached to the final quark fragment;
- Radiated gluon – *color connected* with the other daughter of the branching – belongs to the same string forming a kink on it;
- Second endpoint of the string here attached to the beam-remnant (very low p_T, very far in rapidity)

Vacuum radiation: color flow (in large-N_c)



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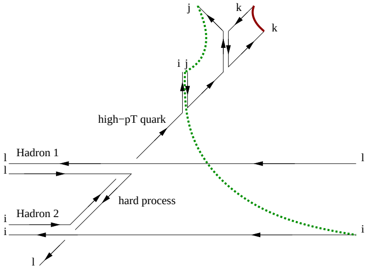
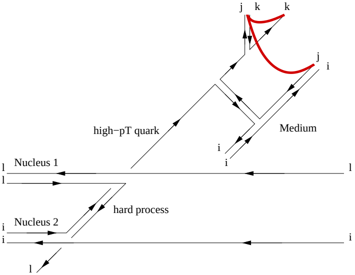


- Most of the radiated gluons in a shower remain color-connected with the projectile fragment;
- Only $g \to q\overline{q}$ splitting can break the color connection, BUT

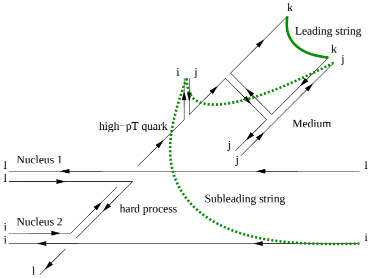
$$P_{qg} \sim \left[z^2 + (1-z)^2\right] \quad vs \quad P_{gg} \sim \left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right]$$

less likely: no soft (i.e. $z \to 1$) enhancement!

AA collisions: in-medium parton shower

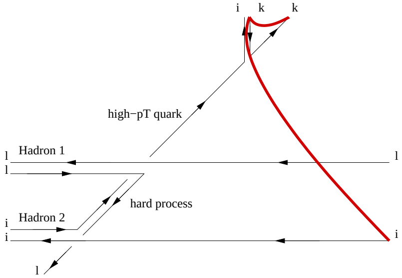


“Final State Radiation”
(gluon $\in$ leading string)
Gluon contributes to leading hadron



“Initial State Radiation”
(gluon decohered: lost!)
Gluon contributes to *enhanced soft multiplicity* from subleading string

From partons to hadrons...



In the following slides we will hadronize partonic configurations with

- the same kinematics
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  - $q_{\mathrm{proj}} g \overline{q}_{\mathrm{beam}}$;

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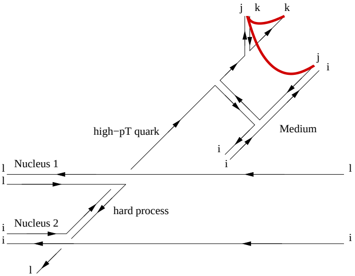

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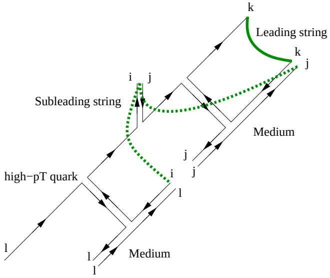

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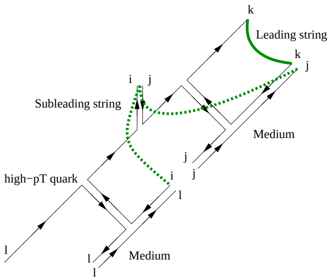

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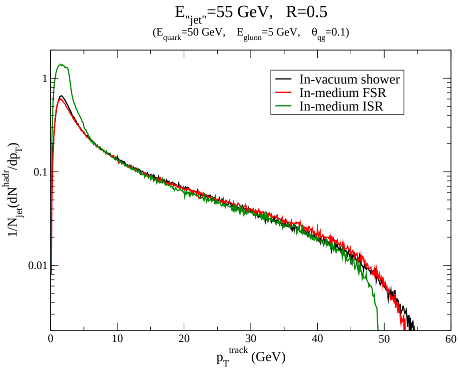
Hadronization performed with Lund-string model of PYTHIA 6.4

"Jet"-Fragmentation



- FSR overlapping with vacuum-shower;
- ISR characterized by:
  - Depletion of hard tail of FF (gluon decohered!);
  - Enhanced soft multiplicity from the subleading string

"Jet"-FF: higher moments and hadron spectra

At variance wrt e^+e^- collisions, in hadronic collisions one starts with a parton p_T-distribution ($\sim 1/p_T^\alpha$) so that inclusive hadron spectrum simply reflects *higher moments of FF*

$$\frac{dN^h}{dp_T} \sim \frac{1}{p_T^\alpha} \sum_f \int_0^1 dz\, z^{\alpha-1} D^{f\to h}(z)$$

carrying limited information on FF (but very sensitive to hard tail!)

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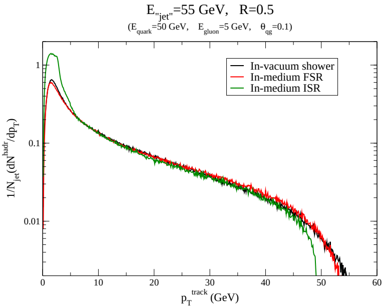

- Quenching of hard tail of FF affects higher moments: e.g.
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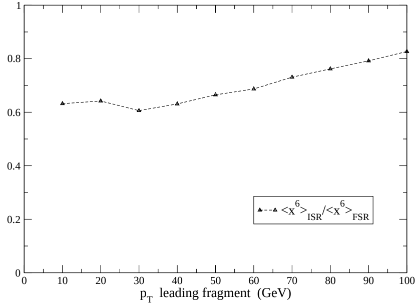

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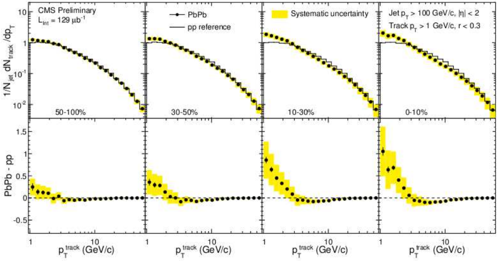
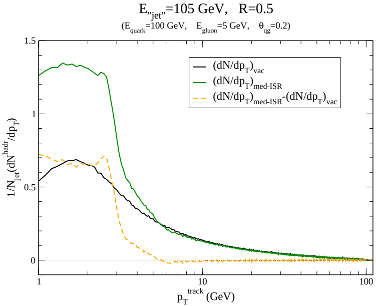
- Ratio of the two channels suggestive of the effect on the hadron spectrum

"Jet"-FF: *AA* vs *pp*



CMS Jet-FF ($p_T^{\rm track} > 1$ GeV)



Same parton kinematics, but *different color-connections*: enhanced soft-hadron multiplicity from the decay of subleading strings (decohered gluons give rise to new strings!)

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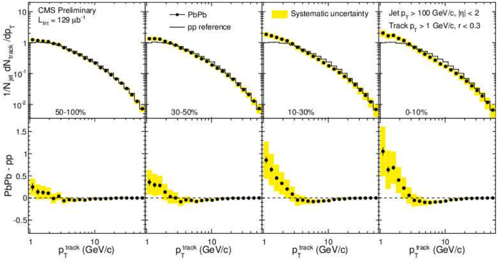

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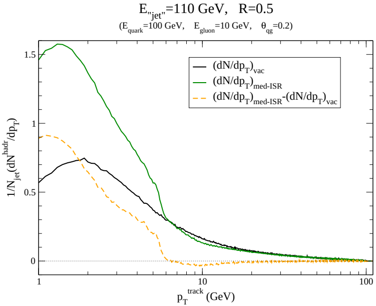

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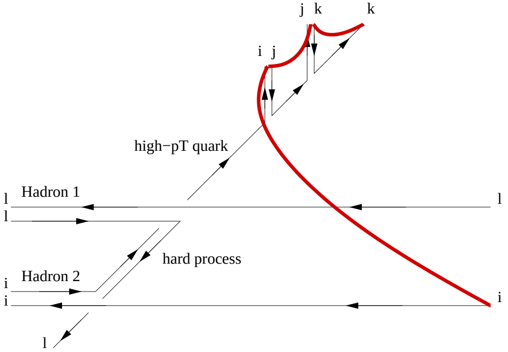
Relevance for info on medium properties

- Hadronization schemes developed to reproduce data from elementary collisions: a situation in which most of the radiated gluons are still color-connected with leading high-p_T fragment;



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- Color-decoherence of radiated gluon might contribute to reproduce the observed high-p_T suppression with milder values of the medium transport coefficients (e.g. $\hat{q}$).

Some references...

- Y. Mehtar-Tani, J.G. Milhano and K. Tywoniuk, *Jet physics in heavy-ion collisions*, Int.J.Mod.Phys. A28 (2013) 1340013;
- S. Peigne and A.V. Smilga, *Energy losses in a hot plasma revisited*, arXiv:0810.5702;
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- Jorge Casalderrey-Solana and Carlos A. Salgado, *Introductory lectures on jet quenching in heavy ion collisions*, Acta Phys.Polon. B38 (2007) 3731-3794.