Glauber model for heavy-ion collisions

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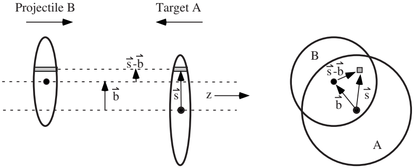
PhD course on Advanced Nuclear Physics

Collision Geometry: the Glauber Model

- For a nice overview: M.L. Miller *et al.*, nucl-ex/0701025;
- For some references to *pp* physics: T. Sjöstrand and M. van ZijL, PRD 36, 2019 (1987)

Glauber Model: outline



a) Side View

b) Beam-line View

- Nuclei are extended/composite objects: they can cross at different impact parameter b and with a different number of elementary binary collisions N_{coll};
- the Glauber Model (optical or MC) is used to describe the geometry of the collision

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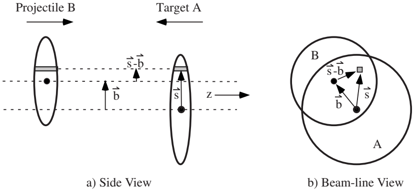

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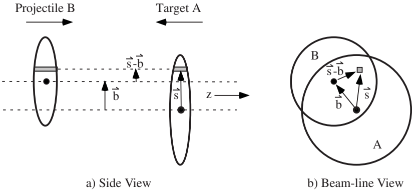
Modeling collision geometry important to interpret the data

- Thicker/denser medium going *from peripheral to central collisions* (higher particle multiplicity, larger jet quenching...);
- *Initial eccentricity and fluctuations* leave their fingerprints in final hadronic observables

Analogies with modeling of UE and MPI in *pp* collisions

Glauber Model: the optical limit



- Nuclear "thickness function" [Area^{-1}]:

$$\hat{T}_A(\boldsymbol{s}) \equiv \int dz_A \, \rho_A(\boldsymbol{s}, z_A)$$

- Nuclear "overlap function" [Area^{-1}]:

$$\hat{T}_{AB}(\boldsymbol{b}) \equiv \int d\boldsymbol{s} \, \hat{T}_A(\boldsymbol{s}) \hat{T}_B(\boldsymbol{s} - \boldsymbol{b})$$

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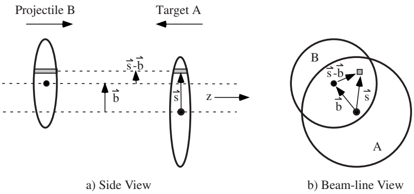

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- Probability of elementary inelastic collision: $p_{\rm coll}^{NN}(b) = \sigma_{\rm in}^{NN}\, \widehat{T}_{AB}(b)$
- Collisions *at a given impact parameter* b is described by a *binomial distribution*:

$$P(n, b) = \binom{AB}{n} [p_{\rm coll}^{NN}(b)]^n [1 - p_{\rm coll}^{NN}(b)]^{AB-n}$$

Glauber Model: results in the optical limit

- Number of binary collisions (per $A-B$ crossing, $\sum_{n=0}^{AB} P(n,b)=1$):

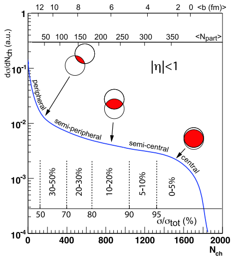
$$N_{\rm coll}(b) = \sum_{n=1}^{AB} n\, P(n,b) = AB\, \widehat{T}_{AB}(b)\, \sigma_{\rm in}^{NN}$$

- Number of participants:

$$N_{\rm part}(b) = A \int d\boldsymbol{s}\, \widehat{T}_A(\boldsymbol{s}) \left\{1 - [1 - \widehat{T}_B(\boldsymbol{s}-\boldsymbol{b})\sigma_{\rm in}^{NN}]^B\right\} + B \int d\boldsymbol{s}\, \widehat{T}_B(\boldsymbol{s}-\boldsymbol{b}) \left\{1 - [1 - \widehat{T}_A(\boldsymbol{s})\sigma_{\rm in}^{NN}]^A\right\}$$

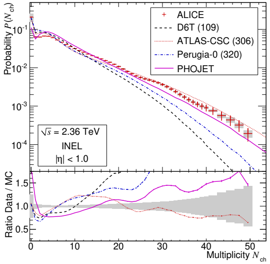
- Total inelastic cross section $\sigma_{\rm in}^{AB} = \int_0^\infty 2\pi\, bdb\, p_{\rm in}^{AB}(b)$ obtained integrating the *probability of having at least one inelastic interaction*

$$p_{\rm in}^{AB}(b) = \sum_{n=1}^{AB} P(n,b) = 1 - [1 - \widehat{T}_{AB}(b)\sigma_{\rm in}^{NN}]^{AB}$$



- *Centrality classes* defined from measured $dN_{\mathrm{evt}}/dN_{\mathrm{ch}}$, dividing total inelastic cross-section in percentiles;

Glauber Model: centrality classes



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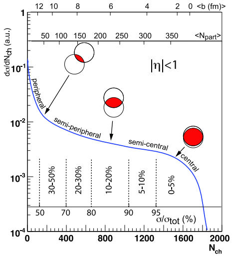

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- Which is the range of impact parameters (*to use in a theory calculation*!) corresponding to a given centrality class?
- A simple geometrical picture arises from the Glauber Model, e.g.

$$\frac{\int_0^{b_{0.1}} bdb\{1-[1-\widehat{T}_{AB}(b)\sigma_{\rm in}^{NN}]^{AB}\}}{\int_0^{\infty} bdb\{1-[1-\widehat{T}_{AB}(b)\sigma_{\rm in}^{NN}]^{AB}\}} = 0.1$$

defines the 0-10% centrality class

Glauber model for hard processes

Hard pQCD processes ($c\overline{c}$ production, high-p_T scattering...) scale with N_{coll}, hence the interest of estimating $\langle N_{\mathrm{coll}} \rangle$ in a given centrality class

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- Binary collisions *per inelastic event at given b*:

$$N_{\rm coll}^{\rm in.evt}(b) = N_{\rm coll}(b)/p_{\rm in}^{AB}(b)$$

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- Average over all inelastic events at different b:

$$\langle N_{\mathrm{coll}} \rangle_{b_1-b_2} \equiv \frac{\int_{b_1}^{b_2} bdb\, N_{\mathrm{coll}}^{\mathrm{in.evt}}(b)\, p_{\mathrm{in}}^{AB}(b)}{\int_{b_1}^{b_2} bdb\, p_{\mathrm{in}}^{AB}(b)} = \frac{\int_{b_1}^{b_2} bdb\, N_{\mathrm{coll}}(b)}{\int_{b_1}^{b_2} bdb\, p_{\mathrm{in}}^{AB}(b)}$$

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One can then compare hard observables in *AA* collisions with a proper *rescaled pp* benchmark

Modeling of MPI in pp: some similarities

In QCD $\sigma_{\rm hard}(p_T^{\rm min}) > \sigma_{\rm tot}^{\rm pp}$ for small $p_T^{\rm min}$;
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- Interactions at given b assumed to follow a Poisson distribution

$$P_n(b) = \frac{[\overline{n}(b)]^n}{n!}\,\exp[-\overline{n}(b)], \quad \text{with} \quad \overline{n}(b) = k\,\underbrace{\mathcal{O}(b)}_{\text{overlap}}$$

NB: Poisson vs Binomial distribution in AB collisions

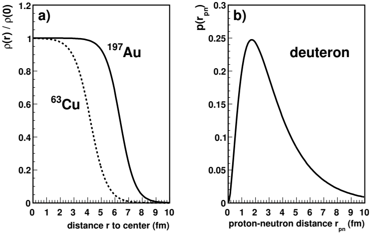
- Number of interactions *per inelastic event at given b*:

$$\langle n(b)\rangle = \frac{\overline{n}(b)}{p_{\rm in}(b)} = \frac{k\mathcal{O}(b)}{1-\exp[-k\mathcal{O}(b)]}$$

- Average number of interactions *per inelastic event*:

$$\langle n\rangle = \frac{\int bdb\,\langle n(b)\rangle\,p_{\rm in}(b)}{\int bdb\,p_{\rm in}(b)} = \frac{\int bdb\,\overline{n}(b)}{\int bdb\,p_{\rm in}(b)} = \frac{\sigma_{\rm hard}}{\sigma_{\rm ND}}$$

Glauber Model: Monte Carlo implementation



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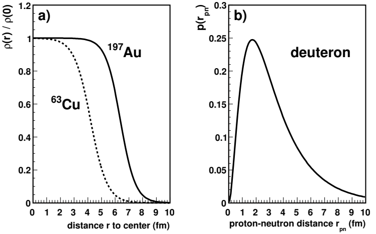

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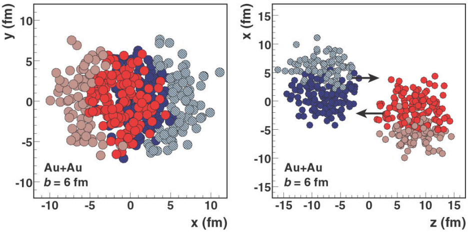

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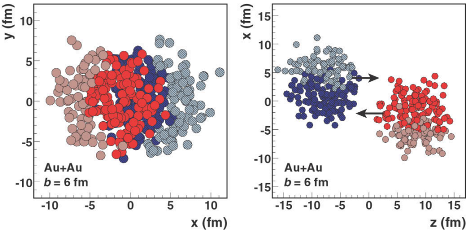

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- Overall agreement except for most peripheral collisions;
- MC-Glauber provides *more granular* initial conditions