Hadron Resonance Gas model

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Study of QCD thermodynamics

Theoretical investigations of QCD thermodynamics make use of different methods and tools

From first principles:

- Lattice QCD
- Perturbation theory (large T and/or μ)

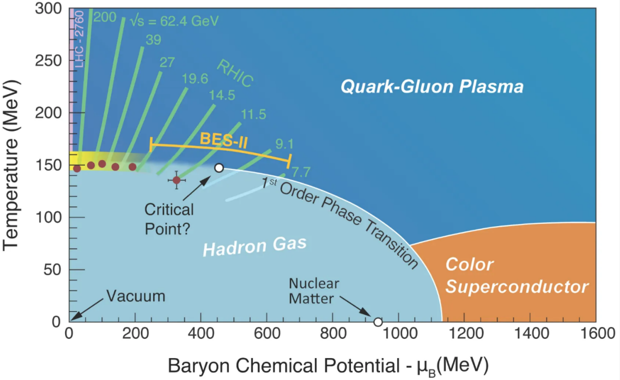
Models:

- Nambu-Jona-Lasinio (NJL) -type models (**Nambu and Jona-Lasinio, Phys. Rev. 122 (1961) 345, Phys. Rev. 124 (1961) 246**)
- **Hadron Resonance Gas (HRG) -type models** (**Hagedorn, Nuovo Cim. Suppl. 3 (1965), 147**)
- Functional methods (functional renormalization group - FRG, Dyson-Schwinger equations, etc...)
- ...

The phase diagram of QCD

Different phases of QCD matter (in equilibrium) are depicted in (temperature vs baryo-chemical potential) phase diagram

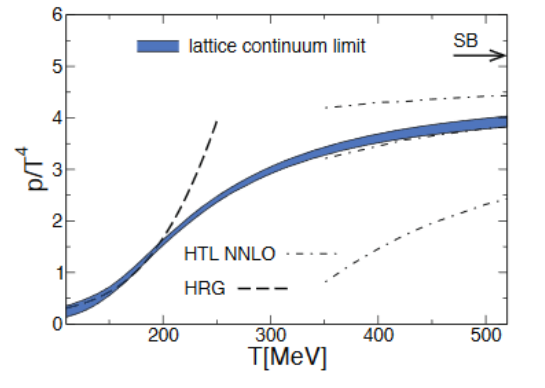
- **Hadron gas** at low-T and/or low-μ_B
- **Quark Gluon Plasma (QGP)** at large T and (possibly) at large μ_B
- **More exotic phases** proposed at low-T and high-μ_B (color superconductivity, etc...)



Equation of state at $\mu_B = 0$

A combination of methods gives us good understanding of the EoS at $\mu_B = 0$ at all temperatures

- Perturbative QCD at high temperature
  $\rightarrow$ "pure quark-gluon phase"
- **HRG model at low temperature**
  $\rightarrow$ "pure hadron phase"
- Lattice QCD bridges between regimes and captures the transition



Borsányi *et al.*, PLB 370 (2014) 99-104

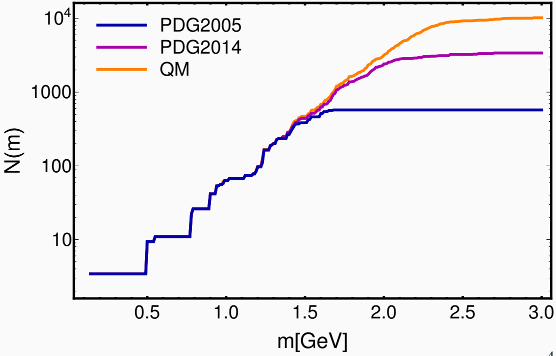
From Hagedorn to hadron resonance gas

- In its modern interpretation, the model does not have a continuous spectrum $\rho(m)$, like the one postulated by Hagedorn. Instead, the discrete set of known (or predicted) hadrons is considered, and summed over
- Nonetheless, hadronic states indeed seem to populate an exponentially increasing spectrum, when spin and isospin multiplicities are taken into account

Cumulative number of states

$$N(m) = \sum_i g_i \Theta(m - m_i)$$

S. Godfrey *et al.*, PRD 32, 189 (1985); S. Capstick *et al.*, PRD 34, 2809 (1986); Particle Data Group PTEP 2020, 083C01 (2020) and earlier versions



Hadron Resonance Gas model

The basic idea is the same as Hagedorn's: approximate a gas of interacting hadrons in their ground state through a *non-interacting* gas of hadrons and all their resonant states

Being non-interacting, it is formally extremely simple:

$$\ln \mathcal{Z}(T, V, \vec{\mu}) = \sum_{i \in \text{hadrons}} \ln \mathcal{Z}_i(T, V, \vec{\mu}) \ ,$$

where $\vec{\mu} = (\mu_B, \mu_Q, \mu_S)$.
The one-particle partition function reads

$$\ln \mathcal{Z}_i(T, V) = \frac{V \eta_i g_i}{(2\pi)^3} \int d^3\mathrm{p} \ln \left[1 + \eta_i z_i e^{-\epsilon_i/T}\right] \ ,$$

where:

- g_i is spin degeneracy
- $\eta_i = (-1)^{B_i+1} = 1(-1)$ for baryons (mesons)
- relativistic particles $\epsilon_i = \sqrt{p^2 + m_i^2}$
- $z_i = \exp[\mu_i/T] = \exp[(\mu_B B_i + \mu_Q Q_i + \mu_S S_i)/T]$ is the fugacity

Hadron Resonance Gas model

The pressure follows trivially

$$P(T,\vec{\mu}) = -T\frac{\partial \ln \mathcal{Z}}{\partial V} = -\frac{T}{V}\ln \mathcal{Z} = \sum_i T\frac{\eta_i g_i}{2\pi^2}\int_0^\infty dp\, p^2 \ln\left[1+\eta_i e^{-\frac{\epsilon_i-\mu_i}{T}}\right] \;,$$

the number density

$$n(T,\vec{\mu}) = \sum_i \frac{g_i}{2\pi^2}\int_0^\infty dp\, p^2 \frac{1}{\eta_i + e^{\frac{\epsilon_i-\mu_i}{T}}} \;,$$

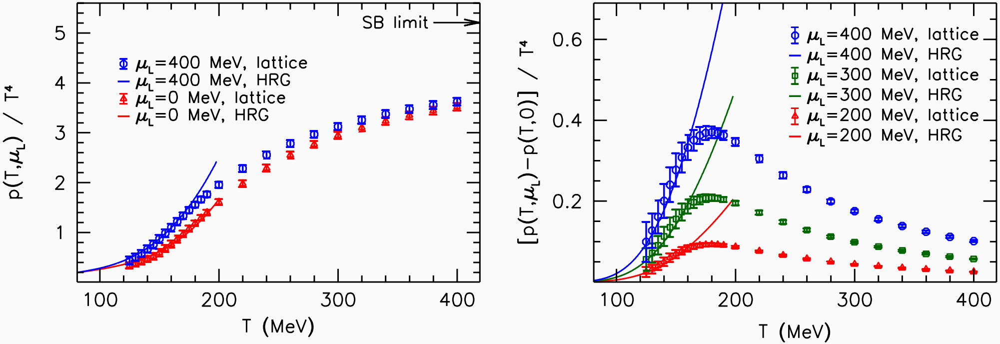
energy density

$$\epsilon(T,\vec{\mu}) = \sum_i \frac{g_i}{2\pi^2}\int_0^\infty dp\, p^2 \frac{\epsilon_i}{\eta_i + e^{\frac{\epsilon_i-\mu_i}{T}}} \;,$$

and so on..

HRG model vs lattice QCD

Agreement between HRG model and lattice QCD is excellent up to around the transition temperature for virtually every observable



NOTE: the temperature where deviations start to occur decreases with increasing μ_B, as expected from the shape of the transition line

Borsányi *et al.*, JHEP 08 (2012) 053

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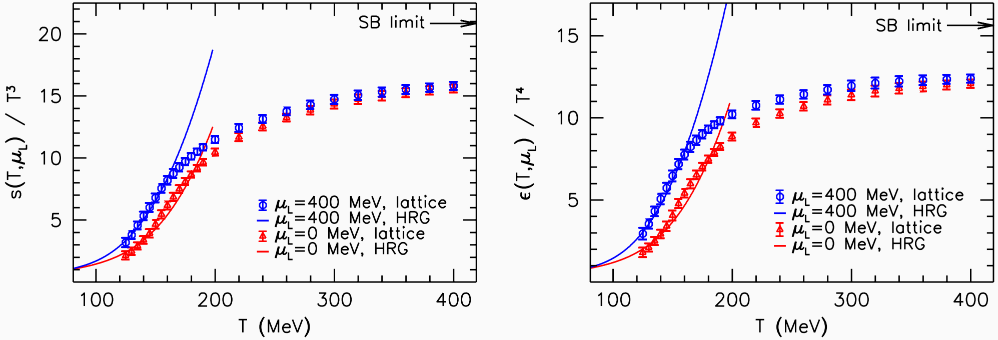

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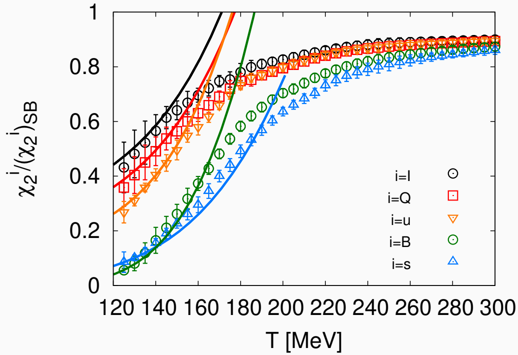


Borsanyi *et al.* JHEP 1201 (2012) 138

Why the HRG model?

So, the HRG model provides an accurate description of the thermodynamics in the confined region. **Why is it important?**

I. **Theory-experiment comparison:** HRG offers a number of advantages over lattice QCD results
   - No sign problem: can be used at finite chemical potential
   - Lattice QCD gets increasingly demanding at lower T (it is good if we can avoid simulating low temperatures)
   - Individual particle contributions can be isolated
   - Additional effects typical of experimental setup can be included:
     - Decays of resonances
     - Finite acceptance in detectors
     - etc.

   $\Rightarrow$ <u>Better for comparison to experimental results</u>

II. **Theory-theory comparison:** HRG can be used to understand results from lattice QCD at low T in terms of hadronic degrees of freedom, and to reverse-engineer from lattice results

Fluctuations of conserved charges

Looking at observables which are particularly sensitive to NLO effects (interactions, exotic states), one can test the HRG model description in detail

Fluctuations of conserved charges

They are defined as:

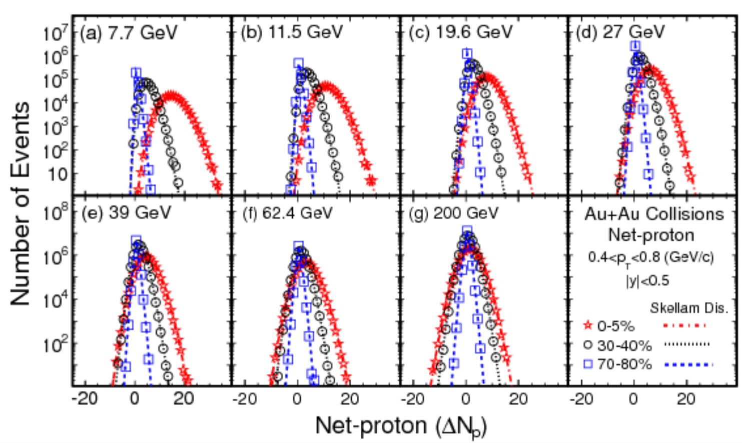
$$\chi_{ijk}^{BQS}(T,\mu_B,\mu_Q,\mu_S) = \frac{\partial^{i+j+k} P\left(T,\mu_B,\mu_Q,\mu_S\right)/T^4}{\partial\left(\mu_B/T\right)^i \partial\left(\mu_Q/T\right)^j \partial\left(\mu_S/T\right)^k}$$

and are related to the moments of net-particle distributions:

mean:	$M = \chi_1$	variance:	$\sigma^2 = \chi_2$
skewness:	$S = \chi_3/\left(\chi_2\right)^{3/2}$	kurtosis:	$\kappa = \chi_4/\left(\chi_2\right)^2$

Heavy-ion collisions: event-by-event fluctuations

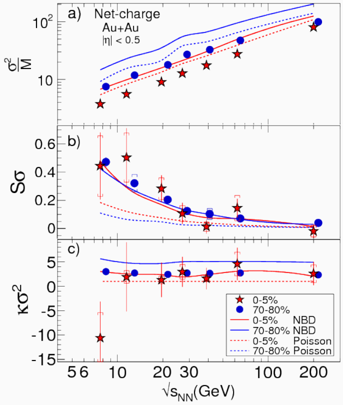
Due to limited acceptance and efficiency of detectors, conserved charges are conserved only on average. Possible to **measure the moments of the distribution**

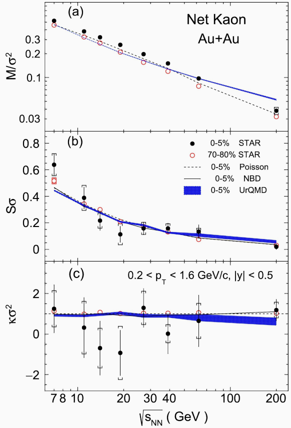


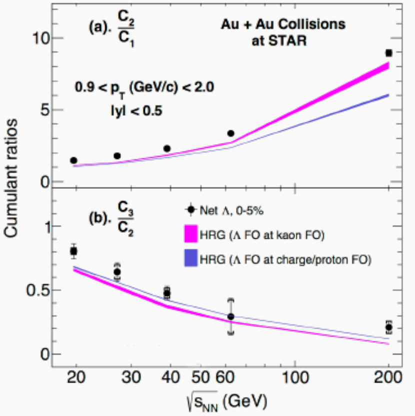
STAR Collaboration: PRL 112 (2014) 032302

Fluctuations of conserved charges - Experiment

Event-by-event net-particle distributions allow to measure different cumulants (and ratios thereof):





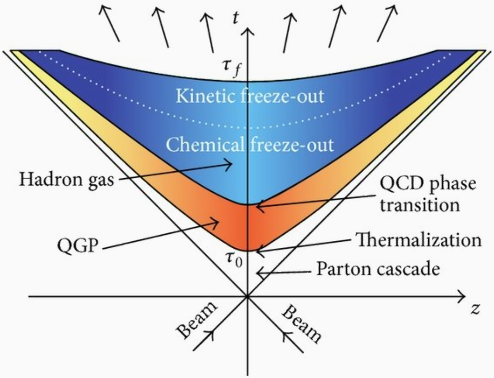


STAR: Phys. Rev.Lett. 113 92301 (2014); Phys. Lett. B 785 551 (2018); arXiv: 2001.06419 [nucl-ex]

Freeze-out in heavy-ion collisions

The stages of a heavy-ion collision

- **Thermalization**: after a short time τ_0 the system thermalizes to a QGP (if the energy density is sufficient)
- **Hadronization**: when the system reaches T_C, hadrons are formed
- **Chemical freeze-out**: all inelastic collision cease and chemical composition is fixed (**yields, fluctuations**)
- **Kinetic freeze-out**: elastic collisions cease and spectra are fixed → free streaming to the detectors



Hui Wang's PhD thesis [Wang:2012jua]

Freeze-out parameters extraction

Thanks to measurements of net-particle moments (mean, variance, etc.), given two numbers, one can get T, μ_B

Several effects are taken into account

- **Resonance decays**, considering (strongly) stable hadrons:

$$N_i = N_i^0 + \sum_R P_{R\to i} N_R$$

where $P_{R\to i} = \mathrm{BR}_{R\to i}\, n_i^R$ is the average number of particles i produced by a particle R

- Include **acceptance cuts** on the kinematics of measured particles:

$$p_T^{\mathrm{m}} \le p_T \le p_T^{\mathrm{M}} \qquad\qquad |y| < y^* \quad (\text{or } |\eta| < \eta^*)$$

(it is not clear how to include correct radipity/p_T distributions)

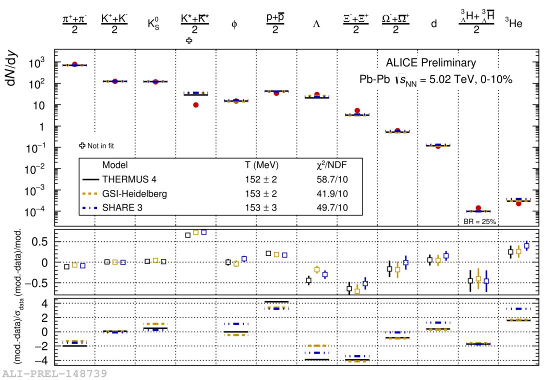
- Impose **strangeness neutrality** to constrain chemical potentials:

$$\langle n_S \rangle = 0 \qquad\qquad \langle n_Q \rangle = 0.4 \langle n_B \rangle$$

Note: strictly speaking, there is an overall factor which corresponds to the system volume V. It is often (not always) removed by considering ratios of moments.

Thermal fits to particle yields: a success story

The simplest case: number of particles (1^{st} moment). With one temperature yields are reproduced fairly well over 7 orders of magnitude



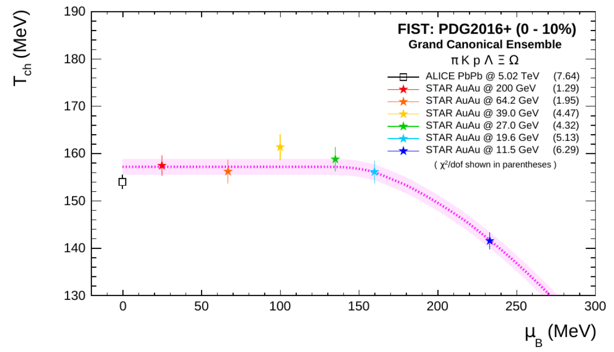
ALICE Collaboration, NPA971 (2018); NPA982 (2019)

At LHC collision energies, the chemical potential is often not fitted, but rather fixed to $\mu_B = 1\,\text{MeV}$ or 0

Thermal fits to yields across energies

Systematic analysis of yields over energy scan (freeze-out parameters are now T, μ_B):

- ALICE, Pb-Pb at $\sqrt{s} = 5.02\,\text{TeV}$ (ALICE Collaboration, NPA 982 (2019))
- STAR, Au-Au at $\sqrt{s} = 200 - 11.5\,\text{GeV}$ (STAR Collaboration, PRC 96 (2017) 044904; 1906.03732)

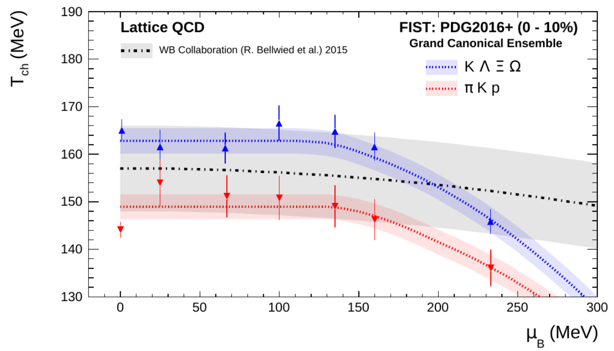


Flor *et al.*, PLB 814 (2021) 136098

Freeze-out at $\mu_B = 0$ occurs at $T_{\text{FO}} = 158.0 \pm 3.8\,\text{MeV}$

Thermal fits: a double freeze-out scenario

It's been hypothesized that strange and non-strange particles freeze-out at different stages (thus temperatures) during the system's evolution: **flavour hierarchy**



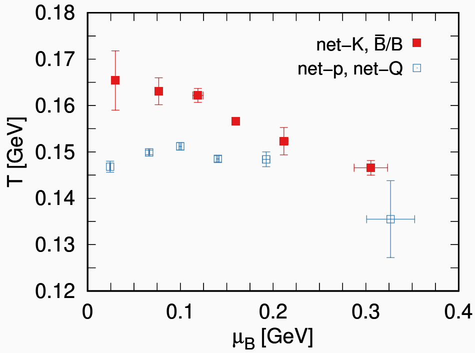
Flor *et al.*, PLB 814 (2021) 136098

Strange and non-strange states are fitted separately, and a much better fit quality is obtained $\Rightarrow$ **in 2FO scenario:** $T_{\text{FO}}^{\text{light}} = 150.0 \pm 2.5\,\text{MeV}$, $T_{\text{FO}}^{\text{strange}} = 163.0 \pm 4.0\,\text{MeV}$

Freeze-out parameters from fluctuations

Thanks to measurements of higher order flutuations (variance, etc.), these quantities too could be used to extract freeze-out parameters

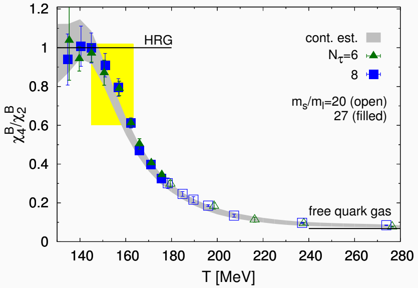
- Blue points: net-proton and net-charge (p,π,K) with M/σ^2
- Red points: net-kaon M/σ^2 and strange antibaryon-baryon ratios
- A hierarchy in the freeze-out temperatures seems to appear from fits to fluctuations too

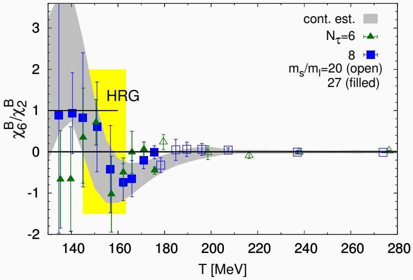


Alba *et al.* Phys.Lett.B 738 (2014) 305-310; Bluhm *et al.*, EPJ C79 (2019) no.2, 155

Theory-theory comparison

- The simplicity of the HRG model is also in the fact that it does not have free parameters. One "free parameter" is the hadronic spectrum utilized to sum over. Although "bulk" quantities like pressure, entropy etc. depend on it fairly mildly, more specific quantities can really test its content





The agreement is pushed to lower temperatures. Interactions, which play a larger role in higher moments, are likely the reason of the disagreement

Bazavov *et al.*, PRD 95 (2017) 054504

Theory-theory comparison

Different improvements have been proposed to restore agreement with lattice QCD at larger temperatures for more specific observables:

- **Different hadron spectra:** new hadron states are discovered routinely, their inclusion does improve agreement for some observables
  Bazavov *et al.*, PRL 113 (2014) 072001; Alba *et al.*, PRD 96 (2017) 034517; Alba *et al.*, PRC 101 (2020) 054905
- **Excluded volume and van der Waals interactions:** repulsive (excluded volume) and attractive interactions have been introduced. Agreement is improved **Vovchenko *et al.*, PRL 118 (2017) 182301; Vovchenko, IJMPE 29 (2020) 05, 2040002**
- **S-matrix formalism:** inclusion of information from scattering experiments on the partial waves expansion of hadron-hadron scatterings. Problem is not much info is available from such experiments
  Dashen, Ma, Bernstein, Phys. Rev. 187 (1969) 345; Pok Man Lo, EPJC 77 (2017) 8, 533

Example: hadron spectrum

hadron	m_i (GeV)	d_i	B_i	S_i	I_i	hadron	m_i (GeV)	d_i	B_i	S_i	I_i
π	0.140	3	0	0	1	N (1535)	1.530	4	1	0	1/2
K	0.496	2	0	1	1/2	π_1 (1600)	1.596	9	0	0	1
$\overline{K}$	0.496	2	0	-1	1/2	Δ (1600)	1.600	16	1	0	3/2
η	0.543	1	0	0	0	Λ (1600)	1.600	2	1	-1	0
ρ	0.776	9	0	0	1	Δ (1620)	1.630	8	1	0	3/2
ω	0.782	3	0	0	0	η_2 (1645)	1.617	5	0	0	0
K^*	0.892	6	0	1	1/2	N (1650)	1.655	4	1	0	1/2
$\overline{K}^*$	0.892	6	0	-1	1/2	ω (1650)	1.670	3	0	0	0
N	0.939	4	1	0	1/2	Σ (1660)	1.660	6	1	-1	1
η'	0.958	1	0	0	0	Λ (1670)	1.670	2	1	-1	0
f_0	0.980	1	0	0	0	Σ (1670)	1.670	2	1	-1	1
a_0	0.980	3	0	0	1	ω_3 (1670)	1.667	7	0	0	0
ϕ	1.020	3	0	0	0	π_2 (1670)	1.672	15	0	0	1
Λ	1.116	2	1	-1	0	Ω^-	1.672	4	1	-3	0
h_1	1.170	3	0	0	1	N (1675)	1.675	12	1	0	1/2
Σ	1.189	6	1	-1	1	ϕ (1680)	1.680	3	0	0	0
a_1	1.230	9	0	0	1	K^* (1680)	1.717	6	0	1	1/2
b_1	1.230	9	0	0	1	$\overline{K}^*$ (1680)	1.717	6	0	-1	1/2
Δ	1.232	16	1	0	3/2	N (1680)	1.685	12	1	0	1/2
f_2	1.270	5	0	0	0	ρ_3 (1690)	1.688	21	0	0	1
K_1	1.273	6	0	1	1/2	Λ (1690)	1.690	4	1	-1	0
$\overline{K}_1$	1.273	6	0	-1	1/2	Ξ (1690)	1.690	8	1	-2	1/2
f_1	1.285	3	0	0	1	ρ (1700)	1.720	9	0	0	1
η (1295)	1.295	1	0	0	0	N (1700)	1.700	8	1	0	1/2
π (1300)	1.300	3	0	0	1	$\Delta(1700)$	1.700	16	1	0	3/2

Hadron spectrum: additional states?

Strangeness-related observables show conflicting results when further states are added to the spectrum: ⇒ Systematic analysis of hadron spectrum

Different lists

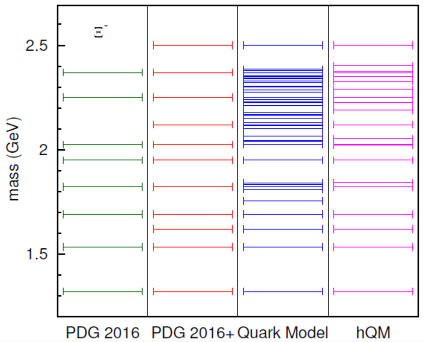
- **Particle Data Group** (PDG) lists particle according to experimental evidence
- **Quark models** predict many additional states (especially in the strange sector)
- Lattice QCD can help determine which states exist
- In figure:

  PDG 2016 (**, *** and **** states): 608 states

  PDG 2016+ (*, **, *** and **** states): 738 states

  Quark Model: 1517 states

  Hypercentral QM (hQM): 985 states



Alba *et al.* PRD 96 (2017) 034517

Hadron spectrum: partial pressures

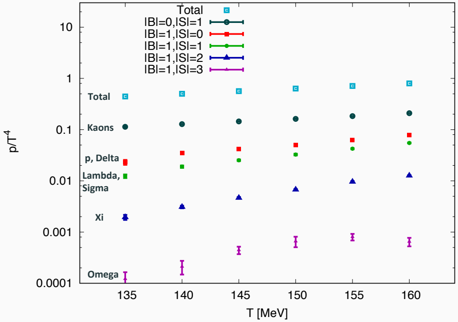
Separate contribution to the pressure from different quantum numbers (in Boltzmann approximation):

$$\frac{P}{T^4} = \sum_i (-1)^{B_i+1} \frac{d_i}{2\pi^2 T^3} \int dk\, k^2 \ln\Big(1 + (-1)^{B_i+1} \exp\left[-\left(\epsilon_i - \mu_i\right)/T\right]\Big) \simeq$$

$$\simeq \sum_i \frac{d_i}{2\pi^2 T^3} \int dk\, k^2 e^{-(\epsilon_i - \mu_i)/T} \simeq \sum_i e^{\mu_i/T} \frac{d_i}{2\pi^2 T^3} \int dk\, k^2 e^{-\epsilon_i/T}$$

The pressure then becomes:

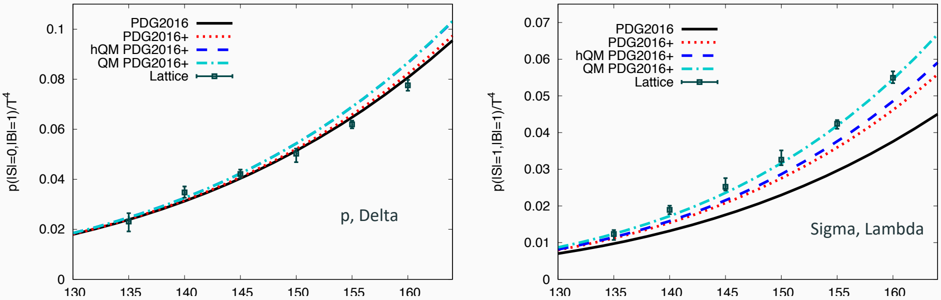
$$P(T, \frac{\mu_B}{T}, \frac{\mu_S}{T}) = P_{00}^{BS} + P_{10}^{BS} \cosh(\frac{\mu_B}{T}) +$$
$$+ P_{01}^{BS} \cosh(\frac{\mu_S}{T}) +$$
$$+ P_{11}^{BS} \cosh(\frac{\mu_B}{T} - \frac{\mu_S}{T}) +$$
$$+ P_{12}^{BS} \cosh(\frac{\mu_B}{T} - 2\frac{\mu_S}{T}) +$$
$$+ P_{13}^{BS} \cosh(\frac{\mu_B}{T} - 3\frac{\mu_S}{T})$$



Alba *et al.* PRD 96 (2017) 034517

Hadron spectrum: partial pressures

Comparison between lattice QCD and HRG for the different lists:



- No significant difference between lists for $|S| = 0$ baryons
- All lists except QM list contain too few $|S| = 1$ states

Alba *et al.* PRD 96 (2017) 034517

Hadron spectrum: partial pressures

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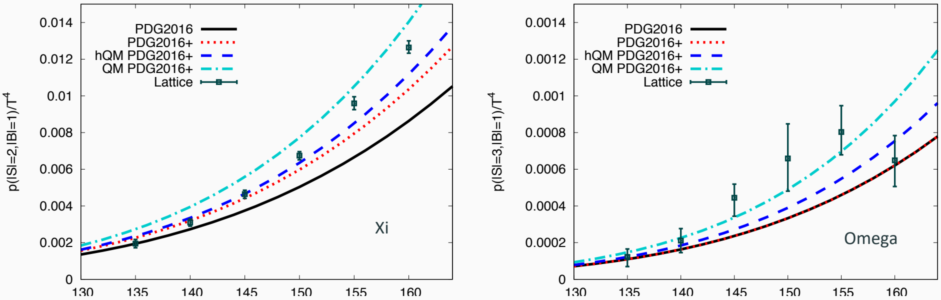


- The QM list contains too many $|S| = 2$ states, but works well for $|S| = 3$ baryons
- Again the PDG2016 list is not enough in both cases

Alba *et al.* PRD 96 (2017) 034517

Extension to the HRG model: excluded volume

In order to take into account interactions for a refined model, excluded volume interactions (know from measurements)

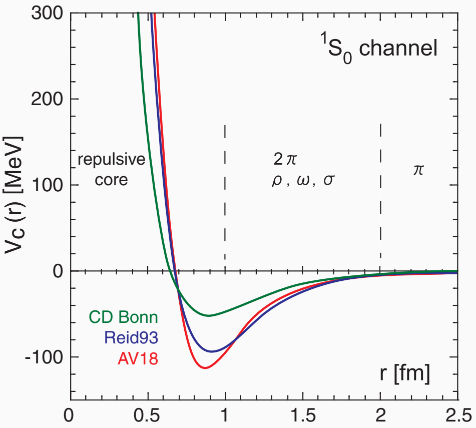
The pressure is corrected via a modification in the effective chemical potential

$$p(T, \mu_B) = \sum_i p^{\text{id}}(T, \mu_i - v_i p)$$

which yields:

$$n_i(T, \mu_B) = \frac{n_i^{\text{id}}(T, \mu_i - v_i p)}{1 + \sum_j v_j n_j^{\text{id}}(T, \mu_j - v_j p)}$$

where different assumption can be made on the excluded volumes v_i



Vovchenko, IJMPE 29 (2020) 2040002

Extension to the HRG model: EV + van der Waals

Further extensions have been considered, like the addition of attractive interactions

The pressure takes on the form:

$$p(T,\mu_B) = \sum_i p^{\mathrm{id}}(T,\mu_i^*) - \sum_{ij} a_{ij} n_i n_j$$

where all sums run over all species, and:

- the modified chemical potential satisfies:

$$\mu_i^* + \sum_j \tilde{b}_{ij} p^{\mathrm{id}}(T,\mu_j^*) - \sum_j (a_{ij} + a_{ji}) n_j = \mu_i$$

- $\tilde{b}_{ij}$ and a_{ij} are the generalized excluded volume and attractive interaction parameters, which in general might depend upon the pair of particles

note: in general, a real vdW gas predicts a critical point at:

$$T_c = \frac{8a}{27b} \qquad n_c = \frac{1}{3b} \qquad p_c = \frac{a}{27b^2}$$

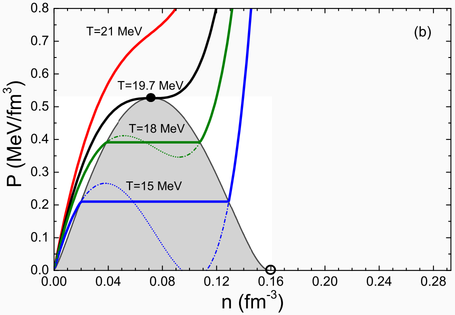
Vovchenko *et al.*, PRC 96 (2017) 045202

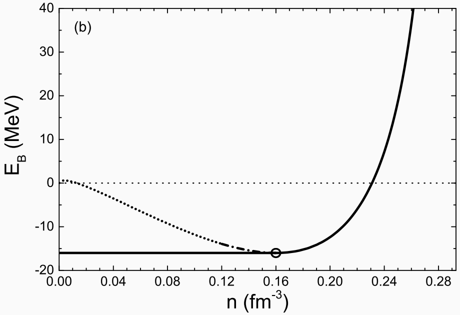
Extension to the HRG model: EV + van der Waals

In this model, it is sufficient to include interactions only among nucleons, with:

$$b_{NN} = 3.42\,\mathrm{fm}^3 \qquad a_{NN} = 329\,\mathrm{MeV\,fm}^3$$

to reproduce the features of nuclear matter, namely the binding energy $E/A \simeq 16\,\mathrm{MeV}$ and saturation density $n_0 = 0.16\,\mathrm{fm}^{-3}$.

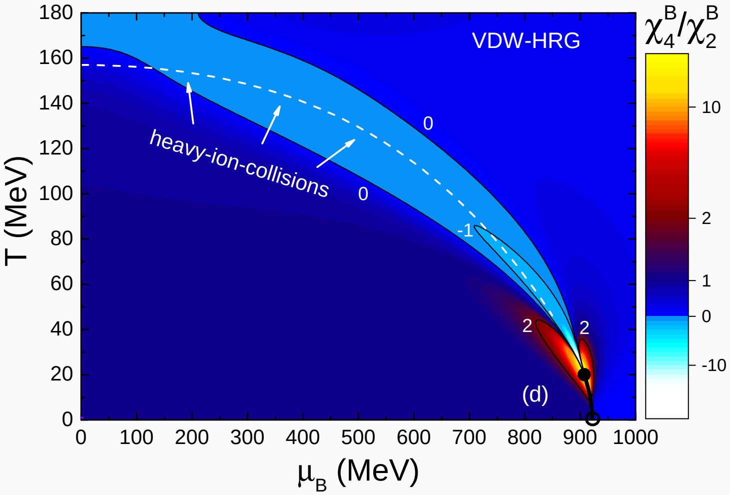




Vovchenko *et al.*, PRC 96 (2017) 045202

Extension to the HRG model: EV + van der Waals

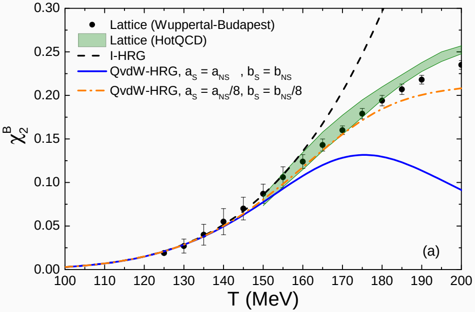
This predicts a critical point at $T \simeq 19.7\,\text{MeV}$ and $\mu_B \simeq 908\,\text{MeV}$ (with $n_c \simeq 0.05\,\text{fm}^{-3}$)

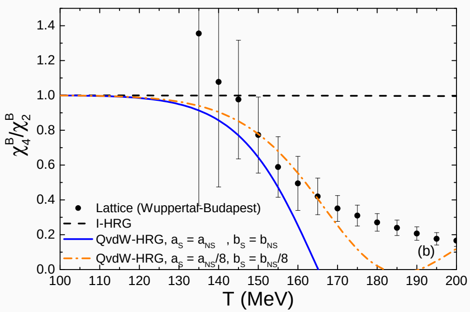


Vochenko *et al.*, PRL 118 (2017) 182301

Theory-theory comparison with vdW HRG

Including interactions unsurprisingly inproves the agreement of HRG with lattice QCD results around the transition





Vovchenko *et al.*, PRC 96 (2017) 045202

Summary

- Hadron resonance gas model proves excellent at describing low-T thermodynamics results from lattice QCD
- At $T \leq 120\,\mathrm{MeV}$ lattice results are expensive (thus rare) $\rightarrow$ HRG effectively trusted to be the correct description
- HRG can give insight in what degrees of freedom contribute to certain observables
- Comparison with experiment has proven fruitful for a long time. Fits to yields and fluctuations give us knowledge on freeze-out locations at different collision energies
- Many experimental effects can be at least partially incorporated
- Improvements to the model have been studied, yielding info on e.g. interactions and even the phase structure of QCD (at low T)