Heavy-ion collisions: theory review

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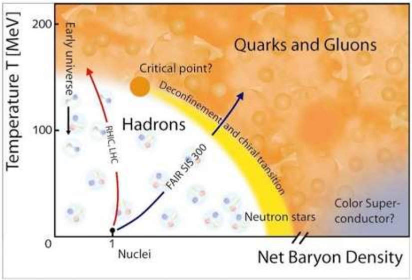
Outline

- The motivation: exploring the QCD phase diagram
- Virtual experiment: lattice-QCD simulations
- Real experiments: heavy-ion collisions
    - *Soft* observables;
    - *Hard* probes

Heavy-ion collisions: exploring the QCD phase-diagram



- Critical line (cross-over + C.E.P. + 1^{st}-order) from lQCD and effective lagrangians (NJL, linear sigma model..)

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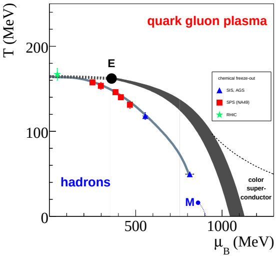

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- Experimental points from fit of final hadron multiplicities

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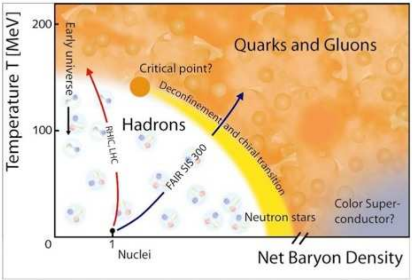

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Region explored at LHC: *high-T/low-density* (early universe, $n_B/n_\gamma \sim 10^{-9}$)

- From QGP (color deconfinement, chiral symmetry restored)
- to hadronic phase (confined, chiral symmetry breaking)

NB $\langle \overline{q}q \rangle \neq 0$ responsible for most of the baryonic mass of the universe: *only* $\sim$*35 MeV of the proton mass from* $m_{u/d} \neq 0$

Virtual experiments: lattice-QCD simulations

- The best (unique?) tool to study QCD in the non-perturbative regime
- Limited to the study of equilibrium quantities

QCD on the lattice

The QCD partition function

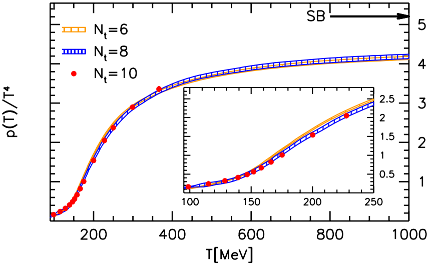
$$\mathcal{Z} = \int [dU] \exp\left[-\beta S_g(U)\right] \prod_q \det\left[M(U, m_q)\right]$$

is evaluated on the lattice through a MC sampling of the field configurations, where

- $\beta = 6/g^2$
- S_g is the gauge action, weighting the different field configurations;
- $U \in SU(3)$ is the link variable connecting two lattice sites;
- M is the Dirac operator

QCD on the lattice: results

From the partition function on gets all the thermodynamical quantities[1]:



- Pressure: $P = (T/V) \ln \mathcal{Z}$;

[1] Wuppertal group, JHEP 1011 (2010) 077

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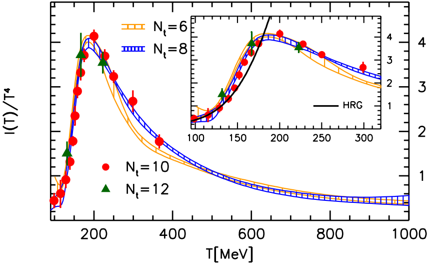

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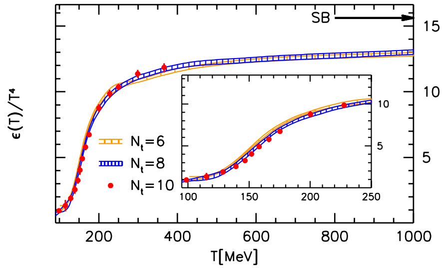

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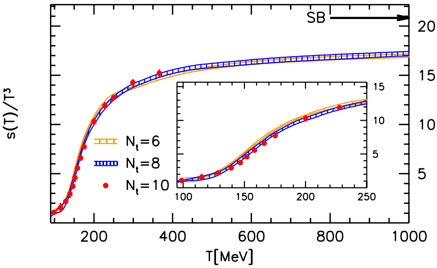

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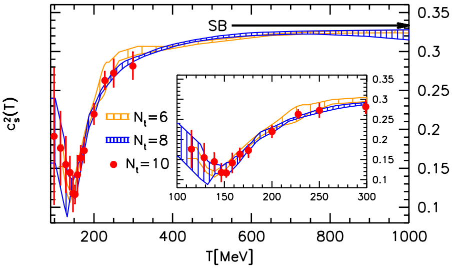

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- Entropy density: $s = (\epsilon + P)/T$;
- Speed of sound: $c_s^2 = dP/d\epsilon$

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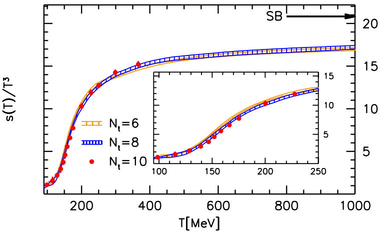
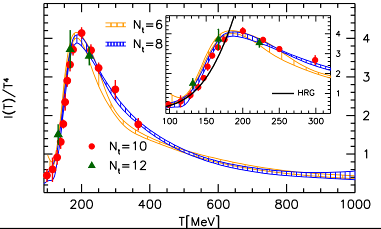
lattice-QCD results: some comments

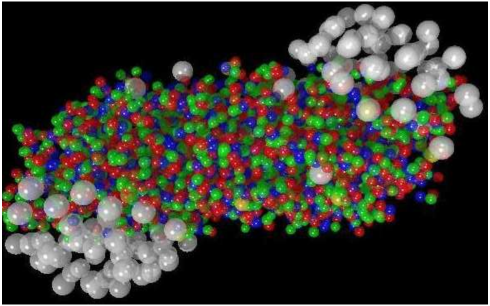




- One observes a $\sim$20% deviation from the SB limit even at large T: how to interpret it?
- $T^{\mu}_{\nu} \equiv \mathrm{diag}(\epsilon, -P, -P, -P)$: the trace anomaly $I \equiv \epsilon - 3P$ gives a measure of the breaking of conformal invariance (a challenge for approaches based on AdS/CFT correspondence?)

Real experiments: heavy-ion collisions

Heavy-ion collisions: a typical event



- Valence quarks of participant nucleons act as sources of strong color fields giving rise to *particle production*
- Spectator nucleons don't participate to the collision;

Almost all the energy and baryon number carried away by the remnants

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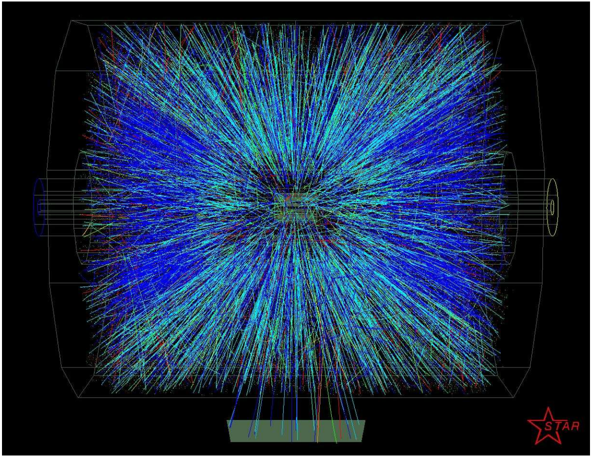
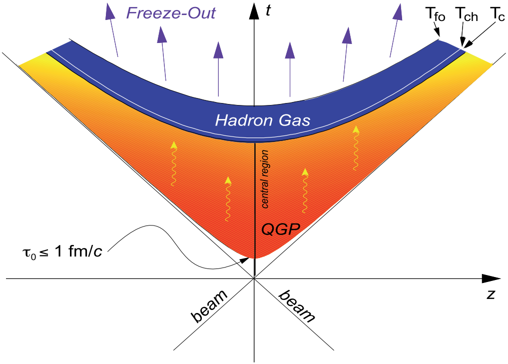


Heavy-ion collisions: a cartoon of space-time evolution



- Soft probes (low-p_T hadrons): collective behavior of the *medium*;
- Hard probes (high-p_T particles, heavy quarks, quarkonia): produced in *hard pQCD processes* in the initial stage, allow to perform a tomography of the medium

Soft probes and hydrodynamics

Some references...

- J.Y. Ollitrault, "*Phenomenology of the little bang*", J.Phys.Conf.Ser. 312 (2011) 012002;
- J.Y. Ollitrault, "*Relativistic hydrodynamics for heavy-ion collisions*", Eur.J.Phys. 29 (2008) 275-302
- U.W. Heinz, "Hydrodynamic description of ultrarelativistic heavy ion collisions", in *Hwa, R.C. (ed.) et al.: Quark gluon plasma* 634-714

Hydrodynamics and heavy-ion collisions

The success of hydrodynamics in describing particle spectra in heavy-ion collisions measured *at RHIC came as a surprise*!

- The general setup and its implications
- Predictions
  - Radial flow
  - Elliptic flow
- What can we learn?
  - Initial conditions
  - Event-by-event fluctuations and consequences
  - QCD EOS

Hydrodynamics: the general setup

- Hydrodynamics is applicable in a situation in which $\lambda_{\rm mfp} \ll L$
- In this limit the behavior of the system is entirely governed by the *conservation laws*

$$\underbrace{\partial_\mu T^{\mu\nu} = 0}_{\rm four-momentum}\,, \qquad \underbrace{\partial_\mu j_B^\mu = 0}_{\rm baryon\ number}\,,$$

where

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu} \quad \text{and} \quad j_B^\mu = n_B u^\mu$$

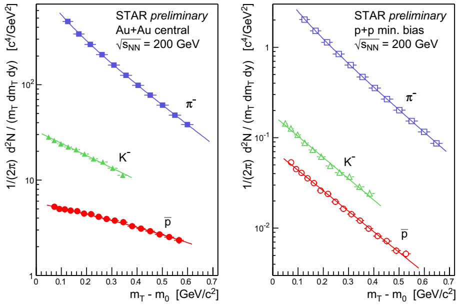
- Information on the medium is *entirely encoded* into the EOS

$$P = P(\epsilon)$$

- The transition from fluid to particles occurs at the *freeze-out hypersuface* $\Sigma^{\rm fo}$ (e.g. at $T = T_{\rm fo}$)

$$E(dN/d\vec{p}) = \int_{\Sigma^{\rm fo}} p^\mu d\Sigma_\mu \, \exp[-(p \cdot u)/T]$$

Hydro predictions: radial flow (I)



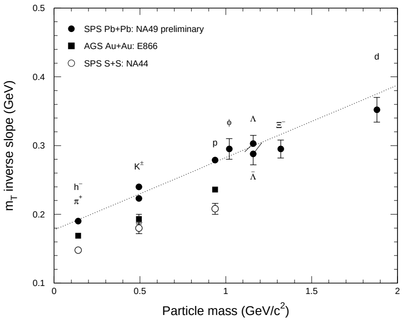
$$\frac{dN}{m_T dm_T} \sim e^{-m_T/\mathrm{T_{slope}}} = e^{-\sqrt{p_T^2+m^2}/\mathrm{T_{slope}}}$$

- $T_{\rm slope}(\sim 167\,\mathrm{MeV})$ *universal* in pp collisions;
- $T_{\rm slope}$ *growing with m* in AA collisions: spectrum gets harder!

Hydro predictions: radial flow (II)

Physical interpretation:

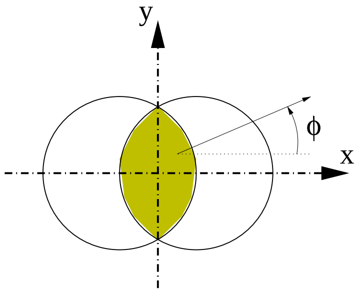
Thermal emission on top of a collective flow



$$\frac{1}{2}m\langle \mathbf{v}_\perp^2\rangle = \frac{1}{2}m\left\langle (\mathbf{v}_{\perp th} + \mathbf{v}_{\perp flow})^2\right\rangle$$
$$= \frac{1}{2}m\langle \mathbf{v}_{\perp th}^2\rangle + \frac{1}{2}m\mathbf{v}_{\perp flow}^2$$

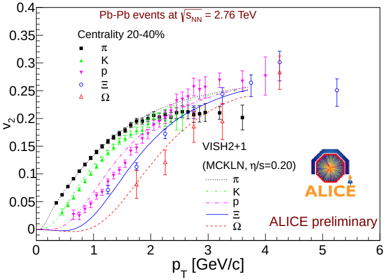
$$\Longrightarrow \quad T_{\rm slope} = T_{\rm fo} + \frac{1}{2}m\mathbf{v}_{\perp flow}^2$$

Hydro predictions: elliptic flow



- In *non-central collisions* particle emission is not azimuthally-symmetric!

Hydro predictions: elliptic flow



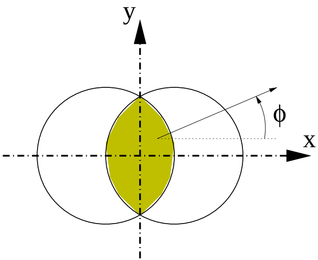
- In *non-central collisions* particle emission is not azimuthally-symmetric!
- The effect can be quantified through the *Fourier coefficient* v_2

$$\frac{dN}{d\phi} = \frac{N_0}{2\pi}\left(1 + 2v_2 \cos[2(\phi - \psi_{RP})] + \dots\right)$$

$$v_2 \equiv \langle \cos[2(\phi - \psi_{RP})] \rangle$$

- $v_2(p_T) \sim 0.2$ gives a modulation 1.4 vs 0.6 for in-plane vs out-of-plane particle emission!

Elliptic flow: physical interpretation



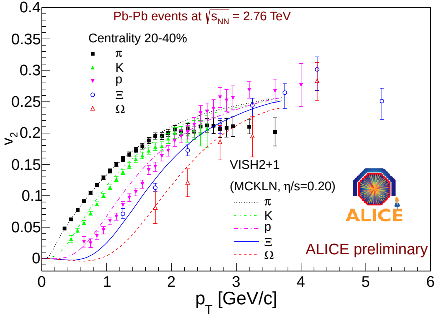
- Matter behaves like a fluid whose *expansion* is *driven by pressure gradients*

$$\frac{\partial}{\partial t}\left[(\epsilon + P)v^i\right] = -\frac{\partial P}{\partial x^i};$$

- Spatial anisotropy is converted into momentum anisotropy;
- At freeze-out particles are mostly emitted along the reaction-plane.

Elliptic flow: mass ordering

The mass ordering of v_2 is a direct consequence of the hydro expansion



- Particles emitted according to a thermal distribution $\sim\exp[-p\cdot u(x)/T_{\rm fo}]$ in the local rest-frame of the fluid-cell;
- Parametrizing the fluid velocity as

$$u^\mu \equiv \gamma_\perp(\cosh Y, \mathbf{u}_\perp, \sinh Y),$$

one gets $(v_z \equiv \tanh Y)$

$$p\cdot u = \gamma_\perp[m_\perp \cosh(y-Y) - \mathbf{p}_\perp \cdot \mathbf{u}_\perp]$$

- Dependence on m_T at the basis of *mass ordering* at fixed p_T

Initial conditions: "Bjorken" estimate

- It is useful to describe the evolution in term of the variables

$$\tau \equiv \sqrt{t^2 - z^2} \quad \text{and} \quad \eta_s \equiv \frac{1}{2} \ln \frac{t+z}{t-z}$$

- Assuming a boost-invariant purely longitudinal expansion $(v_z = z/t)$ entropy conservation implies:

$$s\,\tau = s_0\,\tau_0 \quad \longrightarrow \quad s_0 = (s\,\tau)/\tau_0$$

- Entropy density is defined in the local fluid rest-frame:

$$s \equiv \left. \frac{dS}{d\mathbf{x}_\perp dz} \right|_{z=0} = \frac{1}{\tau} \frac{dS}{d\mathbf{x}_\perp d\eta_s}$$

- Entropy is related to the *final multiplicity of charged particles* $(S \sim 3.6\,N$ for pions), so that:

$$s_0 = \frac{1}{\tau_0} \frac{3.6}{\pi R_A^2} \frac{dN_{\rm ch}}{d\eta} \frac{3}{2}$$

"Bjorken" estimate: results

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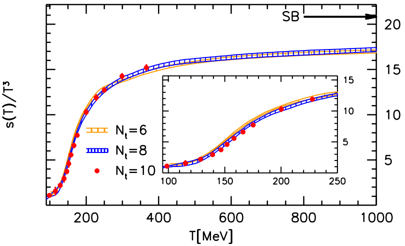

- From $dN_{\rm ch}/d\eta \approx 1600$ measured by ALICE at LHC and $R_{\rm Pb} \approx 6$ fm one gets:

$$s_0 \approx (80\,{\rm fm}^{-2})/\tau_0$$

- τ_0 is found to be quite small:

$$0.1 < \tau_0 < 1\ {\rm fm} \longrightarrow 80 < s_0 < 800\ {\rm fm}^{-3}$$

- This should be compared with l-QCD

$$s(T = 200\,{\rm MeV}) \approx 10\,{\rm fm}^{-3}$$

Initial conditions: Glauber model

- Within the Glauber model, given the nuclear *thickness function*

$$T_A(\mathbf{x}) \equiv \int_{-\infty}^{+\infty} dz\, \rho_A(\mathbf{x}, z)$$

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- participants: $s(\tau_0, \mathbf{x}; \mathbf{b}) = K(\tau_0)\,[n_{\rm part}^A(\mathbf{x}; \mathbf{b}) + n_{\rm part}^B(\mathbf{x}; \mathbf{b})]$, with

$$n_{\rm part}^A(\mathbf{x}; \mathbf{b}) = T_A(\mathbf{x} + \mathbf{b}/2)\left[1 - \left(1 - \sigma_{pp}^{\rm in} T_B(\mathbf{x} - \mathbf{b}/2)/B\right)^B\right]$$

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- binary collisions: $s(\tau_0, \mathbf{x}; \mathbf{b}) = K'(\tau_0)\, n_{\rm bin}(\mathbf{x}; \mathbf{b})$, where

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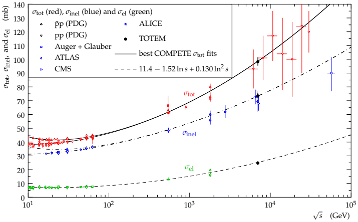
- An essential input is the *inelastic pp cross section* $\sigma_{pp}^{\rm in}(\sqrt{s})$

Glauber model and heavy-ion collisions



- $\sigma_{pp}^{\text{in}} \approx 40 - 60$ mb at RHIC-LHC energies;

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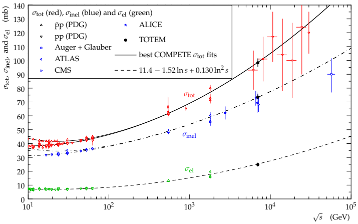
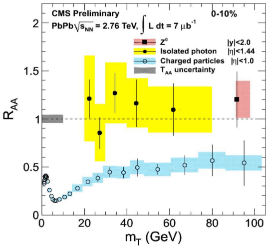




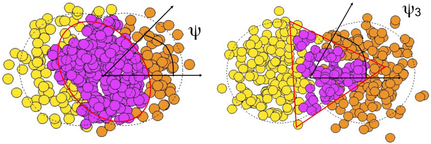
- $\sigma_{pp}^{\rm in} \approx 40 - 60$ mb at RHIC-LHC energies;
- The Glauber model seems to work pretty well: *nuclear modification factor*

$$R_{AA}(p_T) \equiv \frac{(dN/dp_T)_{AA}}{\langle N_{\rm coll}\rangle (dN/dp_T)_{pp}}$$

close to 1 for color-neutral probes!

Initial conditions: event-by-event fluctuations

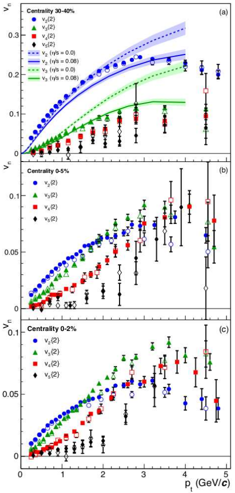
- Flow coefficients are defined as $v_n \equiv \langle\langle \cos[n(\phi - \Psi_n)]\rangle\rangle$.
- For hydro simulations with smooth initial conditions
  - $\Psi_n \equiv \Psi_{\rm RP}$ known exactly;
  - all odd-harmonics vanish.
- Real life is more complicated...



Odd harmonics appear, angles Ψ_n are not directly measured.

- Glauber-MC initial conditions mandatory to study these effects

Event-by-event fluctuations: experimental consequences



Fluctuating initial conditions giving rise to[a]:

- Non-vanishin v_2 in central collisions;
- Odd harmonics (v_3 and v_5)

[a] ALICE, Phys.Rev.Lett. 107 (2011) 032301

Initial conditions: Color Glass Condensate

Basic idea:

s_0 related to the *rapidity density of produced gluons*

Spectrum of produced gluons evaluated within k_T-factorization:

$$s_0 \sim \frac{dN_g}{d\mathbf{r}_\perp dy} \sim \int \frac{d\mathbf{p}_\perp}{\mathbf{p}_\perp^2} \int d\mathbf{k}_\perp \, \alpha_s \, \phi_A(x_1, \mathbf{k}_\perp^2) \, \phi_B(x_2, (\mathbf{p}_\perp - \mathbf{k}_\perp)^2)$$

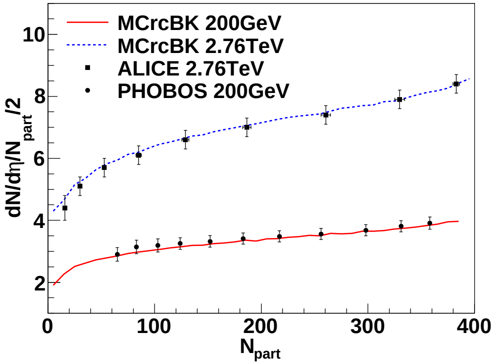
where $\phi(x, \mathbf{k}_\perp^2)$ is an *unintegrated gluon distribution*

- It can be expressed through the *dipole scattering amplitude* $\mathcal{N}(x, \mathbf{r}_\perp)$
- The small-x evolution of the latter is described by the BK-equation

$$\partial \mathcal{N} \sim \underbrace{\mathcal{N}}_{\text{BFKL}} - \underbrace{\mathcal{N}^2}_{\text{saturation}}$$

A unique setup able to describe data from DIS up to A-A collisions?

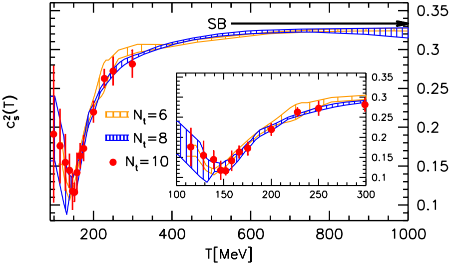
CGC and particle production



Particle density and its evolution with centrality nicely accomodated[2]

[2] J.L. Albacete, A. Dumitru and Y. Nara, J.Phys.Conf.Ser. 316 (2011) 012011.

Hydro evolution: the role of the Equation of State



In ideal hydro the dependence on the EOS enters through *speed of sound*:

$$\frac{\partial v^i}{\partial t} = -\frac{1}{\epsilon + P}\frac{\partial P}{\partial x^i} = -c_s^2 \frac{\partial \ln s}{\partial x^i};$$

For the transverse expansion one gets:

$$v_x = \frac{c_s^2 x}{\sigma_x^2} t, \quad v_y = \frac{c_s^2 y}{\sigma_y^2} t$$

The larger the speed of sound, the larger the *radial flow*!

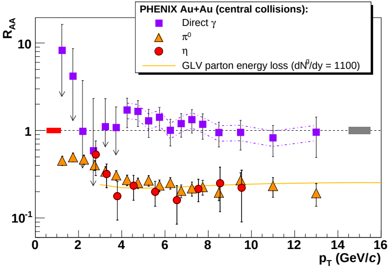
Hard probes

- A few experimental results
  - Jet-quenching
  - Heavy-flavor
- The physical interpretation (with some novel ideas)

Jet-quenching

(in a broad sense: jet-reconstruction in AA possible only recently)

Inclusive hadron spectra: the nuclear modification factor



$$R_{AA} \equiv \frac{\left(dN^h/dp_T\right)^{AA}}{\langle N_{\rm coll}\rangle \left(dN^h/dp_T\right)^{pp}}$$

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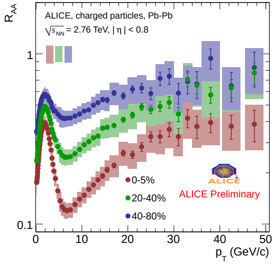

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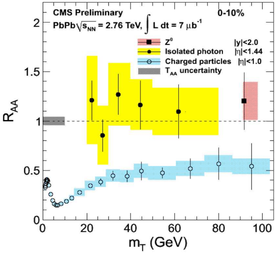
Inclusive hadron spectra: the nuclear modification factor



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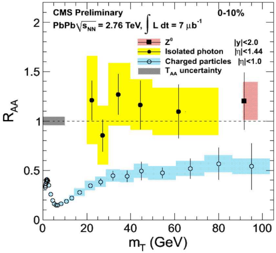

$$R_{AA} \equiv \frac{\left(dN^h/dp_T\right)^{AA}}{\langle N_{\rm coll}\rangle \left(dN^h/dp_T\right)^{pp}}$$

Hard-photon $R_{AA} \approx 1$

- supports the Glauber picture (binary-collision scaling);
- entails that quenching of inclusive hadron spectra is a *final state effect* due to in-medium energy loss.

Some CAVEAT:

- At variance wrt e^+e^- collisions, in hadronic collisions one starts with a parton p_T-distribution ($\sim 1/p_T^\alpha$) so that inclusive hadron spectrum simply reflects *higher moments of FF*

$$\frac{dN^h}{dp_T} \sim \frac{1}{p_T^\alpha} \sum_f \int_0^1 dz\, z^{\alpha-1} D^{f\to h}(z)$$

carrying limited information on FF (but very sensitive to hard tail!)

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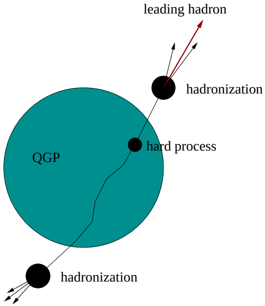
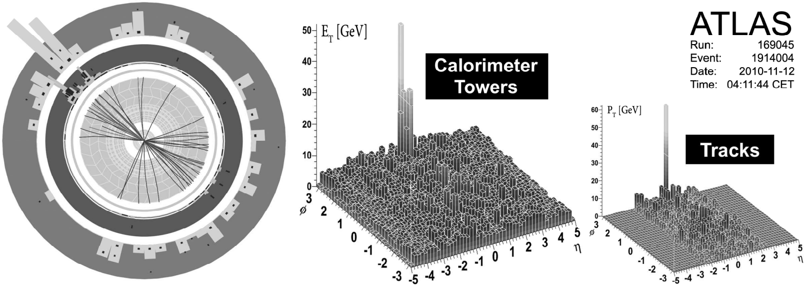


Quenched spectrum does not reflect $\langle L_{\rm QGP}\rangle$ crossed by partons distributed in the transverse plane according to $n_{\rm coll}(\mathbf{x})$ scaling, but *due to its steeply falling shape* is biased by the enhanced contribution of the ones *produced close to the surface and losing a small amount of energy*!

Di-jet imbalance at LHC: looking at the event display

An important fraction of events display a *huge mismatch* in E_T between the leading jet and its away-side partner



Possible to observe event-by-event, without any analysis!

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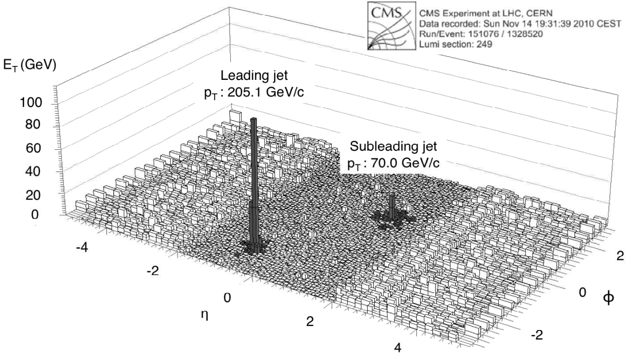

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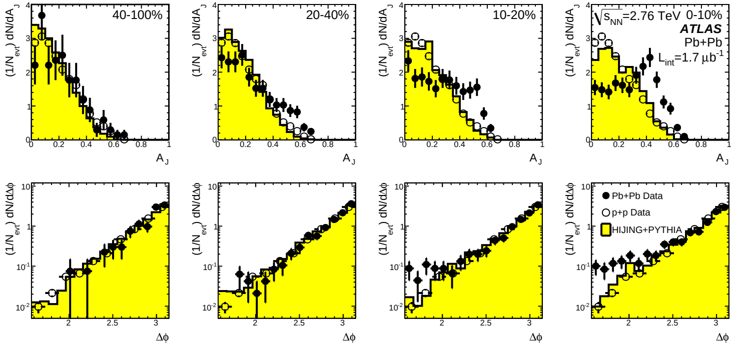
Dijet correlations: results



- Dijet asymmetry $A_j \equiv \frac{E_{T_1} - E_{T_2}}{E_{T_1} + E_{T_2}}$ enhanced wrt to p+p and increasing with centrality;
- $\Delta\phi$ distribution unchanged wrt p+p (jet pairs $\sim$ back-to-back)

Dijet correlations: adding tracking information

Tracks in a ring of radius $\Delta R \equiv \sqrt{\Delta\phi^2 + \Delta\eta^2}$ and width 0.08 *around the subleading-jet axis*:

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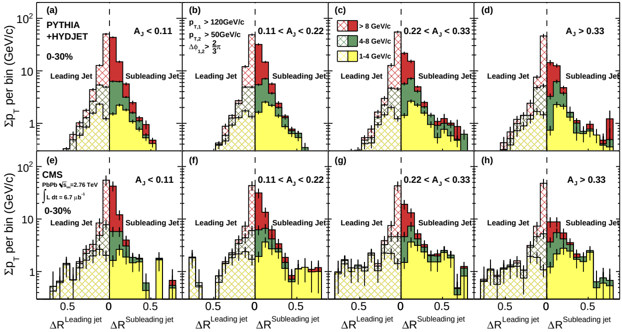

Increasing A_J a sizable fraction of energy around subleading jet carried by *soft* ($p_T < 4$ GeV) *tracks* with a *broad angular distribution*

Dijet measurements: Fragmentation Fuctions

$$\xi \equiv -\ln z \equiv -\ln\left(p_T^{\mathrm{track}}/p_T^{\mathrm{jet}}\right), \qquad p_T^{\mathrm{track}} > 4\mathrm{GeV}$$

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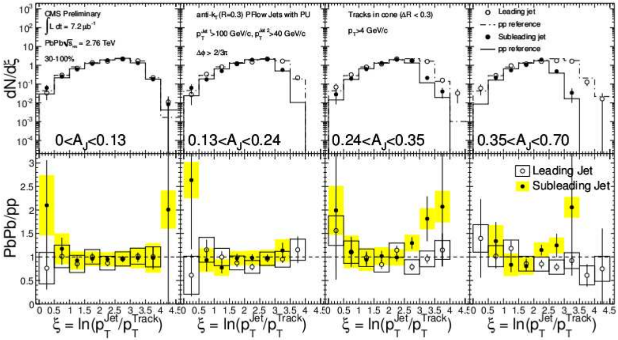

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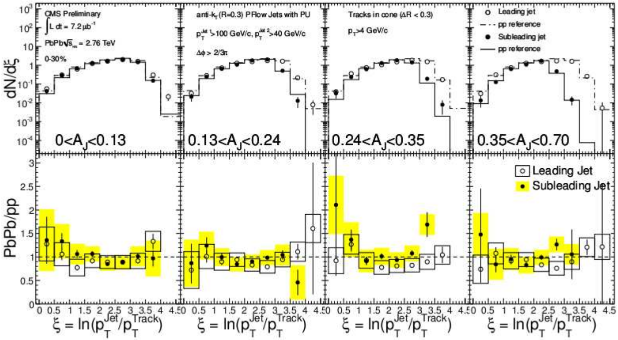

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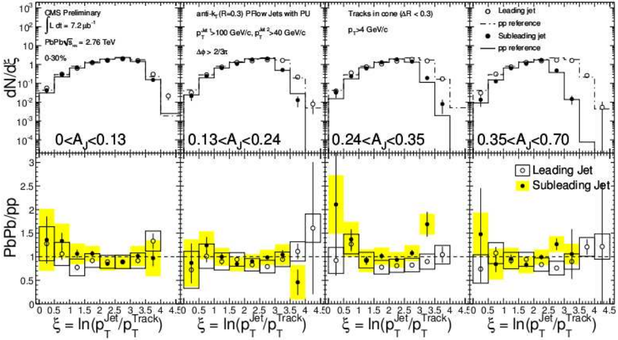
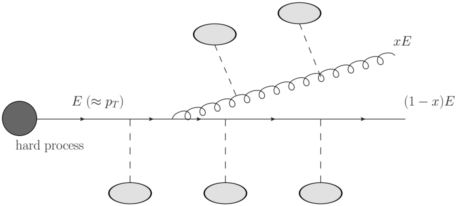


Hard component of jet-FF in AA not strongly modified wrt to pp. Data (for hard tracks!) compatible with vacuum-like fragmentation of jets with reduced energy

Physical interpretation of the data: energy-loss *at the parton level*!



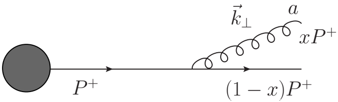
- Interaction of the high-p_T parton with the *color field of the medium* induces the radiation of (mostly) *soft* ($\omega \ll E$) and *collinear* ($k_\perp \ll \omega$) gluons;
- Radiated gluon can further re-scatter in the medium (cumulated $\mathbf{q}_\perp$ favor *decoherence* from the projectile).

The basic ingredients

- Vacuum-radiation spectrum;
- (Gunion-Bertsch) induced spectrum

Vacuum radiation by off-shell partons

A hard parton with $p_i \equiv \left[p_+, Q^2/2p_+, \mathbf{0}\right]$ loses its virtuality Q through gluon-radiation. In *light-cone coordinates*, with $p_\pm \equiv E \pm p_z/\sqrt{2}$:



$$k_g \equiv \left[xp_+, \frac{\mathbf{k}^2}{2xp_+}, \mathbf{k}\right]$$

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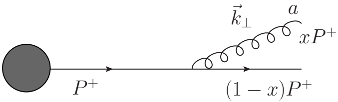

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- $k_\perp$ *vs* virtuality: $\mathbf{k}^2 = x\,(1-x)\,Q^2$;
- Radiation spectrum (our benchmark): IR and collinear divergent!

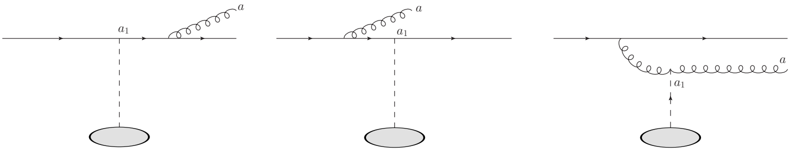
$$d\sigma^{\rm rad}_{\rm vac} = d\sigma^{\rm hard}\,\frac{\alpha_s}{\pi^2}\,C_R\,\frac{dk^+}{k^+}\,\frac{d\mathbf{k}}{\mathbf{k}^2}$$

- Time-scale (*formation time*) for gluon radiation:

$$\Delta t_{\rm rad} \sim Q^{-1}(E/Q) \sim 2\omega/\mathbf{k}^2 \quad (x \approx \omega/E)$$

Medium-induced radiation by on-shell partons

- On-shell partons propagating in a color field can radiated gluons.



(a) (b) (c)

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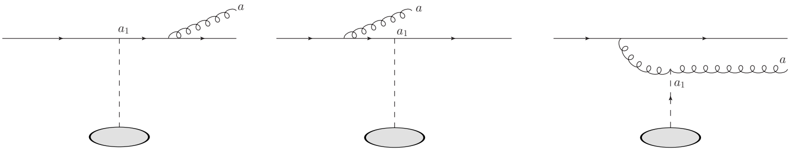

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- The single-inclusive gluon spectrum: the Gunion-Bertsch result

$$x\frac{dN_g^{\rm GB}}{dxd\mathbf{k}} = C_R\frac{\alpha_s}{\pi^2}\left(\frac{L}{\lambda_g^{\rm el}}\right)\left\langle[\mathbf{K}_0-\mathbf{K}_1]^2\right\rangle = C_R\frac{\alpha_s}{\pi^2}\left(\frac{L}{\lambda_g^{\rm el}}\right)\left\langle\frac{\mathbf{q}^2}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}\right\rangle$$

where C_R is the *color charge* of the hard parton and:

$$\mathbf{K}_0 \equiv \frac{\mathbf{k}}{\mathbf{k}^2}, \qquad \mathbf{K}_1 \equiv \frac{\mathbf{k}-\mathbf{q}}{(\mathbf{k}-\mathbf{q})^2} \qquad \text{and} \qquad \langle\dots\rangle \equiv \int d\mathbf{q}\frac{1}{\sigma^{\rm el}}\frac{d\sigma^{\rm el}}{d\mathbf{q}}$$

The induced spectrum: physical interpretation

$$\omega \frac{d\sigma^{\rm ind}}{d\omega d\mathbf{k}} = d\sigma^{\rm hard} C_R \frac{\alpha_s}{\pi^2} \left(\frac{L}{\lambda_g^{\rm el}} \right) \left\langle \left[(\mathbf{K}_0 - \mathbf{K}_1)^2 + \mathbf{K}_1^2 - \mathbf{K}_0^2 \right] \left(1 - \frac{\sin(\omega_1 L)}{\omega_1 L} \right) \right\rangle$$

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  The full radiation spectrum can be organized as

$$d\sigma^{\rm rad}=d\sigma^{\rm GB}+d\sigma^{\rm vac}_{\rm gain}+d\sigma^{\rm vac}_{\rm loss}$$

where

$$d\sigma^{\rm GB}=d\sigma^{\rm hard}C_R\frac{\alpha_s}{\pi^2}\left(L/\lambda_g^{\rm el}\right)\left\langle(\mathbf{K}_0-\mathbf{K}_1)^2\right\rangle(d\omega d\mathbf{k}/\omega)$$

$$d\sigma^{\rm vac}_{\rm gain}=d\sigma^{\rm hard}C_R\frac{\alpha_s}{\pi^2}\left(L/\lambda_g^{\rm el}\right)\langle\mathbf{K}_1^2\rangle(d\omega d\mathbf{k}/\omega)$$

$$d\sigma^{\rm vac}_{\rm loss}=\left(1-L/\lambda_g^{\rm el}\right)d\sigma^{\rm hard}C_R\frac{\alpha_s}{\pi^2}\mathbf{K}_0^2(d\omega d\mathbf{k}/\omega)$$

Average energy loss

Integrating the lost energy ω over the inclusive gluon spectrum:

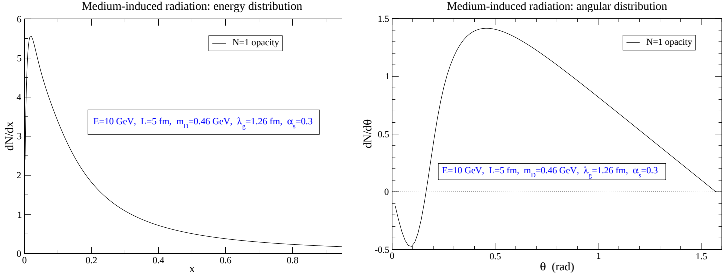
$$\langle \Delta E \rangle = \int d\omega \int d\mathbf{k}\, \omega \frac{dN_g^{\rm ind}}{d\omega d\mathbf{k}} \sim \frac{C_R \alpha_s}{4} \left(\frac{\mu_D^2}{\lambda_g^{\rm el}} \right) L^2 \ln \frac{E}{\mu_D}$$

- L^2 dependence on the medium-length;
- μ_D: Debye screening mass of color interaction $\sim$ *typical momentum exchanged in a collision*;
- $\mu_D^2/\lambda_g^{\rm el}$ often replaced by the *transport coefficient* $\hat{q}$, so that

$$\langle \Delta E \rangle \sim \alpha_s \hat{q} L^2$$

$\hat{q}$: average $q_\perp^2$ acquired per unit length

Numerical results



At variance with vacuum-radiation, medium induced spectrum

- Infrared safe (vanishing as $\omega \to 0$);
- Collinear safe (vanishing as $\theta \to 0$).

Depletion of gluon spectrum at small angles due to their rescattering in the medium!

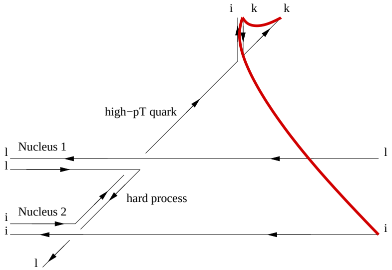
Medium-modification of color-flow for high-p_T probes

- I will mainly focus on leading-hadron spectra...
- ...but the effects may be relevant for more differential observables (e.g. jet-fragmentation pattern)

Essential ideas presented here in a $N = 1$ opacity calculation[3]

[3] A.B, J.G.Milhano and U.A. Wiedemann, J. Phys. G G38 (2011) 124118 and Phys. Rev. C85 (2012) 031901 + arXiv:1204.4342 [hep-ph]

Vacuum radiation: color flow (in large-N_c)



Final hadrons from the fragmentation of the Lund string (in red)

- First endpoint attached to the final quark fragment;
- Radiated gluon – *color connected* with the other daughter of the branching – belongs to the same string forming a kink on it;
- Second endpoint of the string here attached to the beam-remnant (very low p_T, very far in rapidity)

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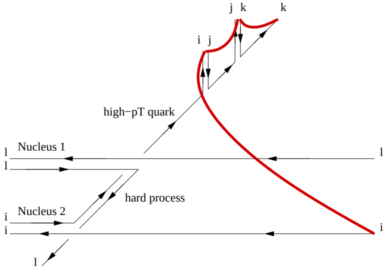

- Most of the radiated gluons in a shower remain color-connected with the projectile fragment;

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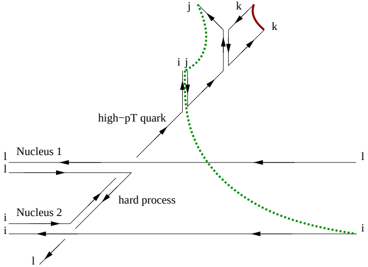


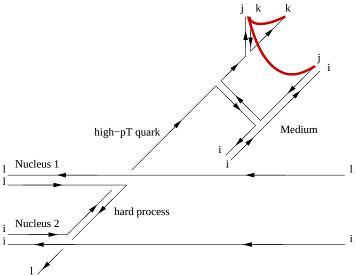
- Most of the radiated gluons in a shower remain color-connected with the projectile fragment;
- Only $g \to q\overline{q}$ splitting can break the color connection, BUT

$$P_{qg} \sim \left[z^2 + (1-z)^2\right] \quad vs \quad P_{gg} \sim \left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right]$$

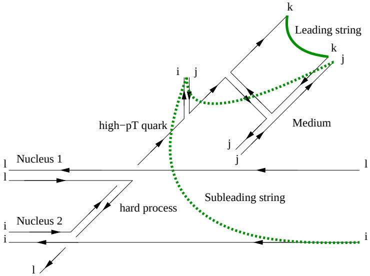
less likely: no soft (i.e. $z \to 1$) enhancement!

Hadronization in the presence of medium-modified color flow

Hadronization à la PYTHIA

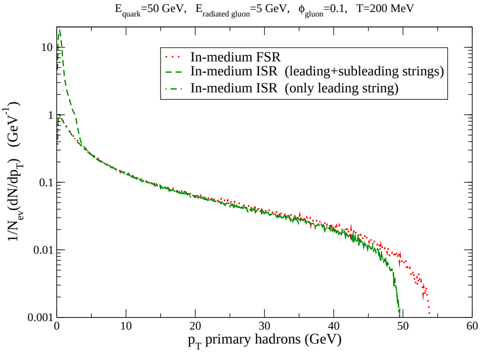


"Final State Radiation"
(gluon ∈ leading string)
Gluon contributes to leading hadron



"Initial State Radiation"
(gluon decohered: lost!)
Gluon contributes to *enhanced soft multiplicity* from subleading string

Fragmentation function



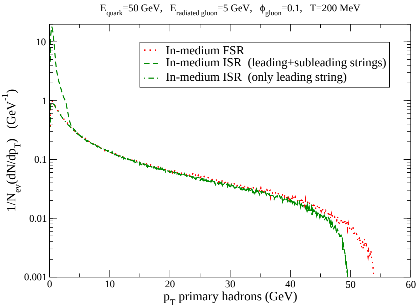
ISR characterized by:

- Depletion of hard tail of FF (gluon decohered!);
- Enhanced soft multiplicity from the subleading string

FF: higher order moments and hadron spectra

Starting from a steeply falling parton spectrum $\sim 1/p_T^n$ at the end of the shower evolution, single hadron spectrum sensitive to *higher moments* of FF:

$$dN^h/dp_T \sim \langle x^{n-1} \rangle / p_T^n$$



- Quenching of hard tail of FF affects higher moments: e.g.
  - FSR: $\langle x^6 \rangle \approx 0.078$;
  - ISR: $\langle x^6 \rangle_{\rm lead} \approx 0.052$

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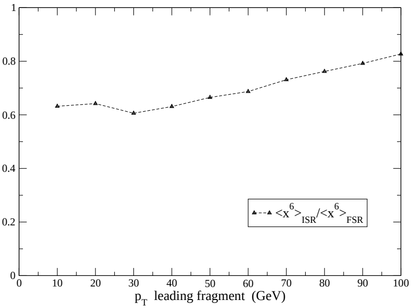

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  - FSR: $\langle x^6 \rangle \approx 0.078$;
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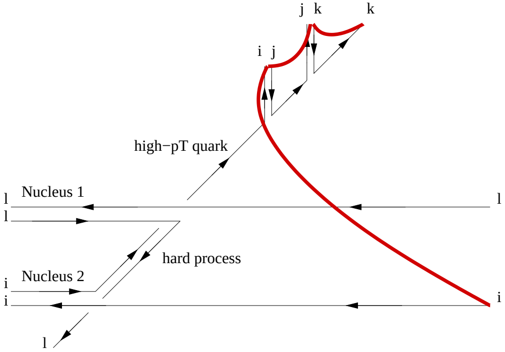
- Ratio of the two channels suggestive of the effect on the hadron spectrum

Relevance for info on medium properties

- Hadronization schemes developed to reproduce data from elementary collisions: a situation in which most of the radiated gluons are still color-connected with leading high-p_T fragment;



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$$\text{Parton Energy loss} \otimes \text{Vacuum Fragmentation}$$

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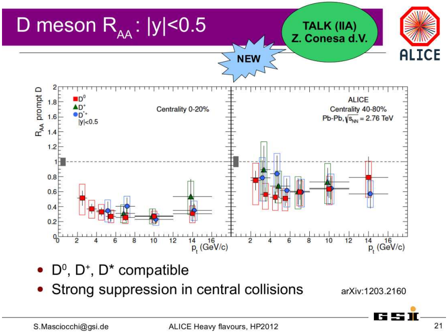
- Color-decoherence of radiated gluon might contribute to reproduce the observed high-p_T suppression with milder values of the medium transport coefficients (e.g. $\hat{q}$).

Heavy-flavor

Experimental findings



- Sizeable *suppression* of D meson spectra;

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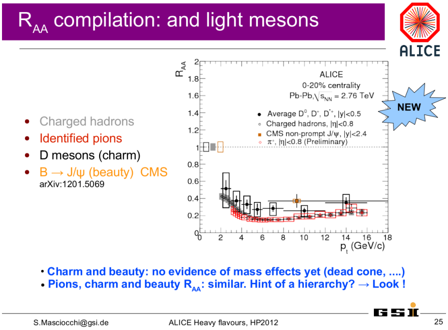
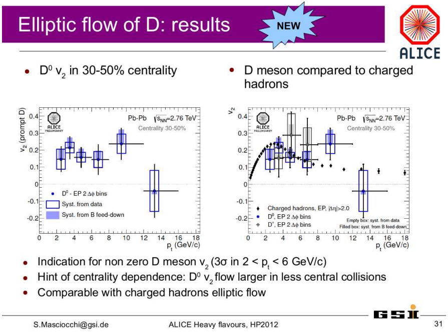


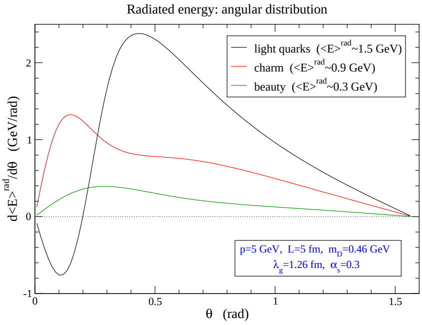
- Sizeable *suppression* of D meson spectra;
- Important *suppression* also of J/ψ from B decays;

Experimental findings



- Sizeable *suppression* of D meson spectra;
- Important *suppression* also of J/ψ from B decays;
- D mesons seem to *follow the collective flow of light hadrons*

Some challenges posed by experimental data



- Color charge: C_F vs C_A;
- Mass effect: radiation from b strongly suppressed;
- Reconsidering the importance of collisional energy loss?

A possible tool to study the heavy-quark dynamics in the QGP: the relativistic Langevin equation

- Trivial extensions of jet-quenching calculations to the massive case simply describe the energy-loss of heavy quarks, which remain *external probes* crossing the medium;
- The Langevin equation allows to follow the *relaxation to thermal equilibrium*.[4]

[4] W.M. Alberico *et al.*, EPJC 71, 1666 and J.Phys.G G38 (2011) 124144

Update of the HQ momentum in the plasma: the recipe

$$\frac{\Delta p^i}{\Delta t} = -\underbrace{\eta_D(p)p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}} \quad ,$$

with the properties of the noise encoded in

$$\langle \xi^i(\mathbf{p}_t)\xi^j(\mathbf{p}_{t'})\rangle = b^{ij}(\mathbf{p}_t)\frac{\delta_{tt'}}{\Delta t} \qquad b^{ij}(\mathbf{p}) \equiv \kappa_L(p)\hat{p}^i\hat{p}^j + \kappa_T(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)$$

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Transport coefficients to calculate:

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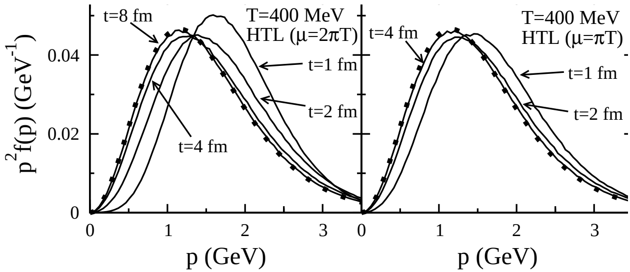
- *Friction* term (dependent on the discretization scheme!)

$$\eta_D^{\rm Ito}(p) = \frac{\kappa_L(p)}{2TE_p} - \frac{1}{E_p^2}\left[(1-v^2)\frac{\partial \kappa_L(p)}{\partial v^2} + \frac{d-1}{2}\frac{\kappa_L(p)-\kappa_T(p)}{v^2}\right]$$

fixed in order to insure approach to equilibrium (Einstein relation): Langevin ⇔ Fokker Planck with steady solution $\exp(-E_p/T)$

In a static medium...



For $t \gg 1/\eta_D$ one approaches a relativistic Maxwell-Jüttner distribution[5]

$$f_{\rm MJ}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T\, K_2(M/T)}, \qquad \text{with} \quad \int d^3p\, f_{\rm MJ}(p) = 1$$

(Test with a sample of c quarks with $p_0 = 2$ GeV/c)

[5] A.B., A. De Pace, W.M. Alberico and A. Molinari, NPA 831, 59 (2009)

In an expanding fluid...

The fields $u^\mu(x)$ and $T(x)$ are taken from the output of two longitudinally boost-invariant ("Hubble-law" longitudinal expansion $v_z = z/t$)

$$x^\mu = (\tau \cosh \eta, \mathbf{r}_\perp, \tau \sinh \eta) \quad \text{with} \quad \tau \equiv \sqrt{t^2 - z^2}$$

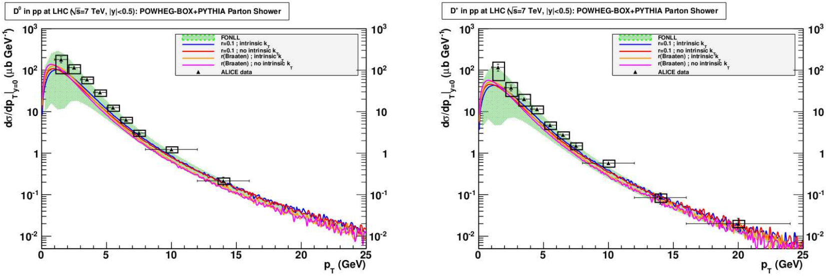
$$u^\mu = \bar{\gamma}_\perp (\cosh \eta, \bar{\mathbf{v}}_\perp, \sinh \eta) \quad \text{with} \quad \bar{\gamma} \equiv \frac{1}{\sqrt{1 - \bar{\mathbf{v}}_\perp^2}}$$

hydro codes[6].

- $u^\mu(x)$ used to perform the update each time in the fluid rest-frame;
- $T(x)$ allows to fix at each step the value of the transport coefficients.

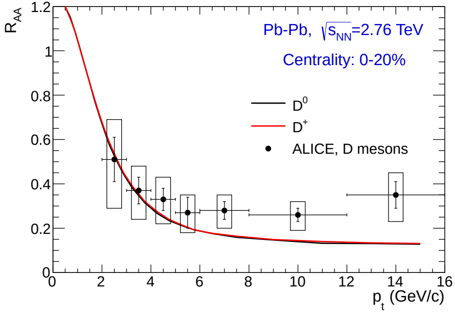
[6] P.F. Kolb, J. Sollfrank and U. Heinz, Phys. Rev. C **62** (2000) 054909
P. Romatschke and U.Romatschke, Phys. Rev. Lett. **99** (2007) 172301

Numerical results: spectra in p-p



Hard production in elementary p-p collisions generated with POWHEG + PYTHIA PS: nice agreement with FONLL outcome and ALICE results

Numerical results: spectra in Pb-Pb



In Pb-Pb collisions c and b quarks are then propagated inside the medium through the Langevin equation[7]

[7] M. Monteno talk at "Hard Probes 2012"