How (non-)linear is the hydrodynamics of heavy ion collisions?

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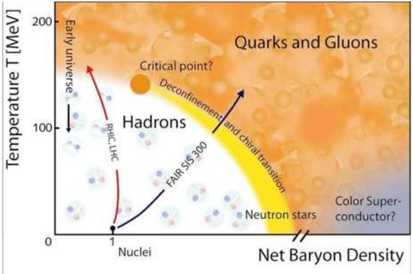
Outline

- Heavy-ion collisions: general introduction
- Phenomenological overview: soft observables and hydrodynamics
- Theory setup: Relativistic HydroDynamics (RHD)
- Linear and non-linear rensponse to initial fluctuations: a perturbative framework[1]

[1] Stefan Floerchinger, Urs Achim Wiedemann, A.B., Luca Del Zanna, Gabriele Inghirami and Valentina Rolando, arXiv:1312.5482 [hep-ph].

Heavy-ion collisions: exploring the QCD phase-diagram



QCD phases identified through the *order parameters*

- Polyakov loop $\langle L \rangle \sim e^{-\beta \Delta F_Q}$ energy cost to add an isolated color charge
- Chiral condensate $\langle \overline{q}q \rangle \sim$ effective mass of a "dressed" quark in a hadron

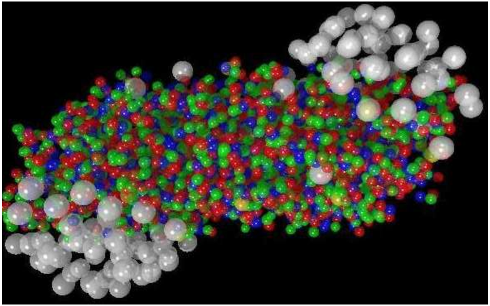
Region explored at LHC: *high-T/low-density* (early universe, $n_B/n_\gamma \sim 10^{-9}$)

- From QGP (color deconfinement, chiral symmetry restored)
- to hadronic phase (confined, chiral symmetry breaking[2])

NB $\langle \overline{q}q \rangle \neq 0$ responsible for most of the baryonic mass of the universe: *only $\sim$35 MeV of the proton mass from* $m_{u/d} \neq 0$

[2] V. Koch, *Aspects of chiral symmetry*, Int.J.Mod.Phys. E6 (1997)

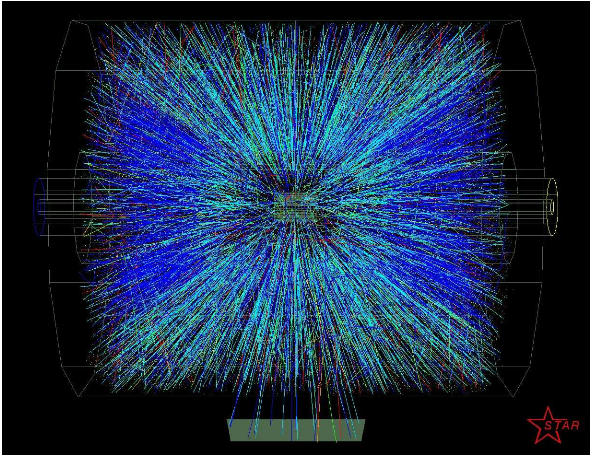
Heavy-ion collisions: a typical event



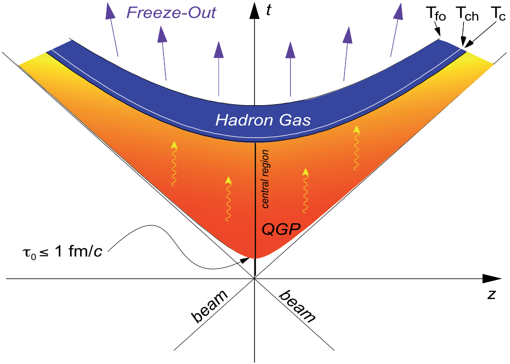
- Valence quarks of participant nucleons act as sources of strong color fields giving rise to *particle production*
- Spectator nucleons don't participate to the collision;

Almost all the energy and baryon number carried away by the remnants

Heavy-ion collisions: a typical event



Heavy-ion collisions: a cartoon of space-time evolution



- Soft probes (low-p_T hadrons): collective behavior of the *medium*;
- Hard probes (high-p_T particles, heavy quarks, quarkonia): produced in *hard pQCD processes* in the initial stage, allow to perform a tomography of the medium

Hydrodynamics and heavy-ion collisions

The success of hydrodynamics in describing particle spectra in heavy-ion collisions measured *at RHIC came as a surprise*!

- The general setup and its implications
- The main predictions
  - Radial flow
  - Elliptic flow
- Recent developments (fluctuating initial conditions)
  - Flow in central collisions
  - Higher flow harmonics
  - Event-by-event flow measurements
- What can we learn?
  - Equation of State (EOS) of the produced matter
  - Initial conditions
  - QGP viscosity

Hydrodynamics: the general setup

- Hydrodynamics is applicable in a situation in which $\lambda_{\mathrm{mfp}} \ll L$
- In this limit the behavior of the system is entirely governed by the *conservation laws*

$$\underbrace{\partial_\mu T^{\mu\nu} = 0}_{\text{four}-\text{momentum}}, \qquad \underbrace{\partial_\mu j_B^\mu = 0}_{\text{baryon number}},$$

  where

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu - P g^{\mu\nu}, \quad j_B^\mu = n_B u^\mu \quad \text{and} \quad u^\mu = \gamma(1, \vec{v})$$

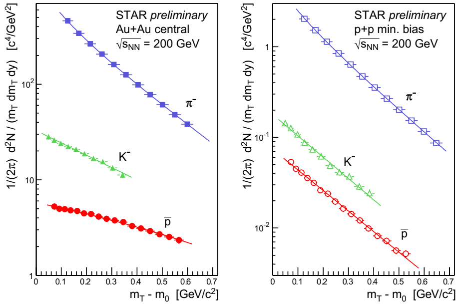
- Information on the medium is *entirely encoded* into the EOS

$$P = P(\epsilon)$$

- The transition from fluid to particles occurs at the *freeze-out hypersuface* Σ^{fo} (e.g. at $T = T_{\mathrm{fo}}$)

$$E(dN/d\vec{p}) = \int_{\Sigma^{\mathrm{fo}}} p^\mu d\Sigma_\mu \exp[-(p \cdot u)/T]$$

Hydro predictions: radial flow (I)



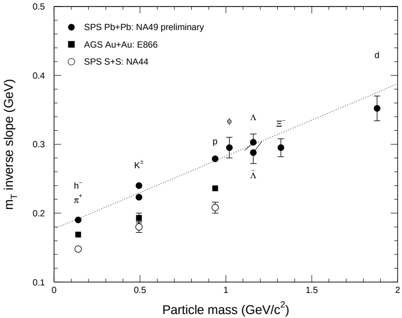
$$\frac{dN}{m_T dm_T} \sim e^{-m_T/\mathrm{T}_{\mathrm{slope}}} \equiv e^{-\sqrt{p_T^2+m^2}/\mathrm{T}_{\mathrm{slope}}}$$

- $T_{\mathrm{slope}}(\sim 167\,\mathrm{MeV})$ *universal* in pp collisions;
- T_{slope} *growing with m* in AA collisions: spectrum gets harder!

Hydro predictions: radial flow (II)

Physical interpretation:

Thermal emission on top of a collective flow



$$\frac{1}{2} m \langle \mathbf{v}_\perp^2 \rangle = \frac{1}{2} m \left\langle \left(\mathbf{v}_{\perp th} + \mathbf{v}_{\perp flow} \right)^2 \right\rangle = \frac{1}{2} m \langle \mathbf{v}_{\perp th}^2 \rangle + \frac{1}{2} m \mathbf{v}_{\perp flow}^2$$

$$\Longrightarrow \quad T_{\rm slope} = T_{\rm fo} + \frac{1}{2} m \mathbf{v}_{\perp flow}^2$$

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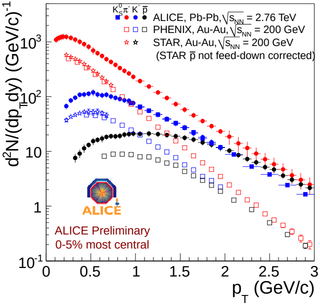

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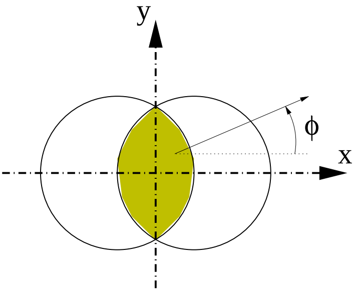
Radial flow gets larger going from RHIC to LHC!

Hydro predictions: elliptic flow



- In *non-central collisions* particle emission is not azimuthally-symmetric!

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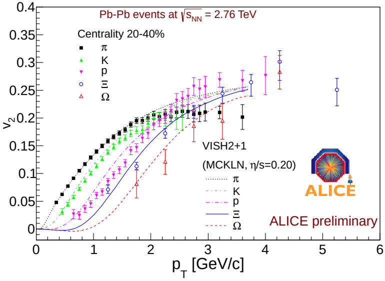


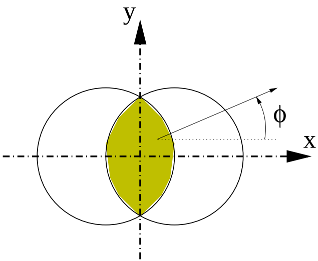
- In *non-central collisions* particle emission is not azimuthally-symmetric!
- The effect can be quantified through the *Fourier coefficient* v_2

$$\frac{dN}{d\phi} = \frac{N_0}{2\pi}\left(1 + 2v_2\cos[2(\phi - \psi_{RP})] + \dots\right)$$

$$v_2 \equiv \langle \cos[2(\phi - \psi_{RP})] \rangle$$

- $v_2(p_T) \sim 0.2$ gives a modulation 1.4 vs 0.6 for in-plane vs out-of-plane particle emission!

Elliptic flow: physical interpretation



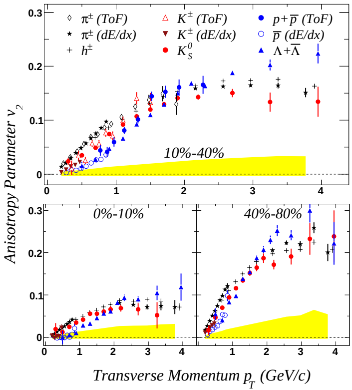
- Matter behaves like a fluid whose *expansion* is *driven by pressure gradients*

$$(\epsilon + P)\frac{dv^i}{dt} \underset{v \ll c}{=} -\frac{\partial P}{\partial x^i} \quad \text{(Euler equation)}$$

- Spatial anisotropy is converted into momentum anisotropy;
- At freeze-out particles are mostly emitted along the reaction-plane.

Elliptic flow: mass ordering

The mass ordering of v_2 is a direct consequence of the hydro expansion



- Particles emitted according to a thermal distribution $\sim\exp[-p\cdot u(x)/T_{\rm fo}]$ in the local rest-frame of the fluid-cell;
- Parametrizing the fluid velocity as

$$u^\mu \equiv \gamma_\perp(\cosh Y, \mathbf{u}_\perp, \sinh Y),$$

one gets $(v_z \equiv \tanh Y = z/t)$

$$p\cdot u = \gamma_\perp[m_\perp \cosh(y-Y) - \mathbf{p}_\perp\cdot\mathbf{u}_\perp]$$

- Dependence on m_T at the basis of *mass ordering* at fixed p_T

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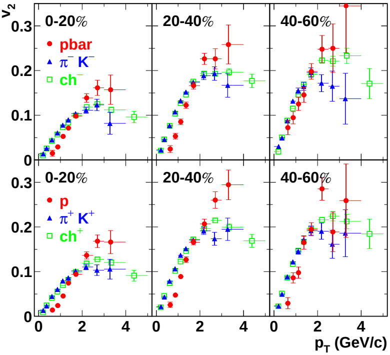

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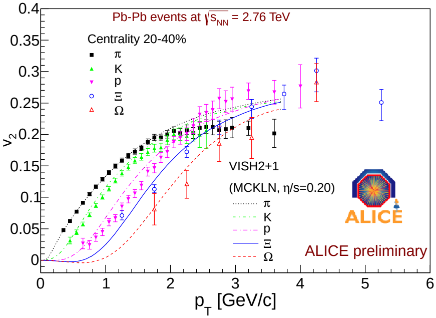

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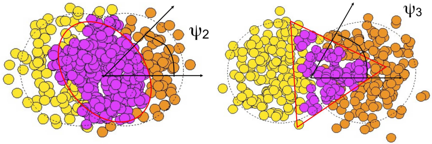
Event by event fluctuations



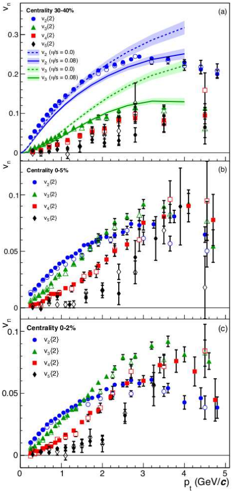
- Due to event-by-event fluctuations (e.g. of the nucleon positions) the initial density distribution is not smooth and can display *higher deformations*, each one with a *different azimuthal orientation*.
- Higher harmonics ($m > 2$) contribute to the angular distribution

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + 2\sum_m v_m \cos[m(\phi - \psi_m)]\right)$$

of the final hadrons, where *for each event*,

$$v_m = \langle\cos[m(\phi - \psi_m)]\rangle \quad \text{and} \quad \psi_m = \frac{1}{m}\arctan\frac{\sum_i p_T^i \sin(m\phi_i)}{\sum_i p_T^i \cos(m\phi_i)}$$

Event-by-event fluctuations: experimental consequences



Fluctuating initial conditions give rise to[a]:

- Non-vanishing v_2 in central collisions;
- Odd harmonics (v_3 and v_5)

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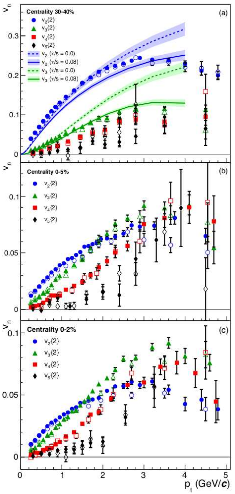

Fluctuating initial conditions give rise to[a]:

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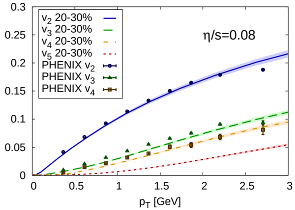
Hydro can reproduce also higher harmonics[b]



[a]ALICE, Phys.Rev.Lett. 107 (2011) 032301
[b]B: Schenke *et al.*, PRC 85, 024901 (2012)

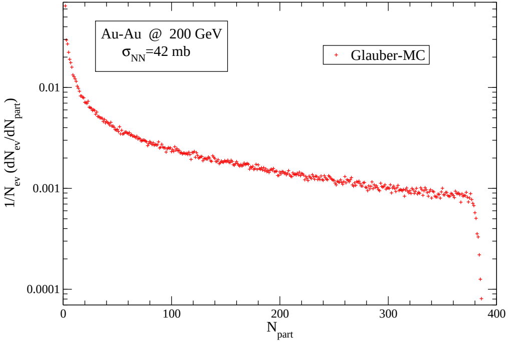
Modelling the initial conditions: Glauber-MC approach

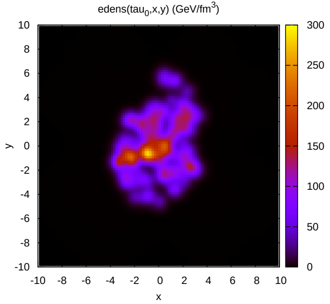
- Generate $N_{\rm conf}$ configurations, each configuration obtained extracting from a Woods-Saxon distribution
  - the coordinates of the A nucleons of nucleus A;
  - the coordinates of the B nucleons of nucleus B.
- For each configuration re-write the nucleon coordinates wrt the center-of-mass of each nucleus;
- Given a configuration, extract a possible impact parameter from the distribution $dP = 2\pi bdb$, with $b < b_{\rm max} = 20$ fm;
- Nucleons i and j collide if $(x_i - x_j)^2 + (y_i - y_j)^2 < \sigma_{\rm NN}/\pi$
  - *If* at least one collision occurs...keep b and store the info;
  - *Else* extract a different b and repeat.
- Final events can be organized in *centrality classes* according to $N_{\rm part}$ (or $N_{\rm coll}$ or a combination of the two).

Glauber-MC initial conditions: results

Taking a smeared energy-density distribution around each participant

$$\epsilon(x, y, \tau_0) = \frac{K}{2\pi\sigma^2} \sum_{i=1}^{N_{\rm part}} \exp\left[-\frac{(x - x_i)^2 + (y - y_i)^2}{2\sigma^2}\right]$$





Characterizing the initial conditions

For each event the initial density distribution can be characterized in terms of *complex eccentricity coeffients*

$$\epsilon_{n,m} e^{im\Psi_{n,m}} \equiv -\frac{\int d\vec{r}\, r^n e^{im\phi} \epsilon(\vec{r},\tau_0)}{\int d\vec{r}\, r^n \epsilon(\vec{r},\tau_0)} \equiv -\frac{\{r^n \cos(m\phi)\} + i\{r^n \sin(m\phi)\}}{\{r^n\}}$$

whose orientation and modulus are given by

$$\Psi_{n,m} = \frac{1}{m} \operatorname{atan2}\left(-\{r^n \sin(m\phi)\}, -\{r^n \cos(m\phi)\}\right)$$

and

$$\epsilon_{n,m} = \frac{\sqrt{\{r_\perp^n \cos(m\phi)\}^2 + \{r_\perp^n \sin(m\phi)\}^2}}{\{r_\perp^n\}} = -\frac{\{r^n \cos[m(\phi - \Psi_{n,m})]\}}{\{r^n\}}$$

Connecting initial conditions to hadron spectra

Hydrodynamics is expected to propagate the *initial* eccentricity of the density distribution into the *final* azimuthal anisotropy of hadron spectra

- Averages: $\langle \epsilon_{n,m} \rangle \longrightarrow \langle v_m \rangle$
- Probability distributions: $P(\epsilon_{n,m}) \longrightarrow P(v_m)$
- Correlations, e.g. $\langle \epsilon_{n,m} \epsilon_{n',m'} \rangle \longrightarrow \langle v_m v_{m'} \rangle$

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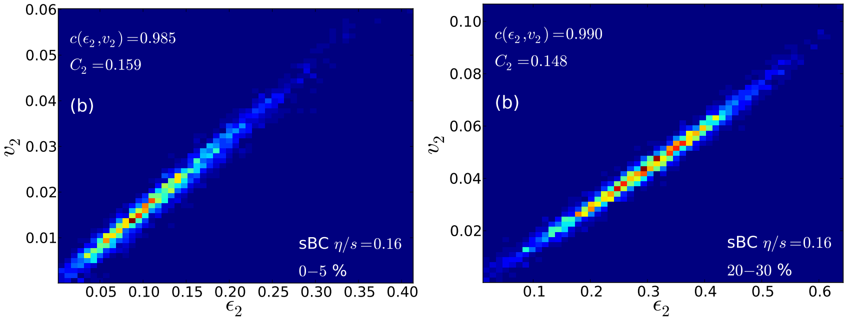
Basic question

To what extent $v_m \sim \epsilon_{n,m}$ and $\psi_m \sim \Psi_{n,m}$?
in particular with realistic initial conditions involving several modes, which can give rise to non-linear effect...

Flow vs eccentricity



(Results from Niemi, Denicol, Holopainen and Huovinen, Phys.Rev.C 87 (2013) 054901)

- Strong correlation for the $n = 2, 3$ harmonics

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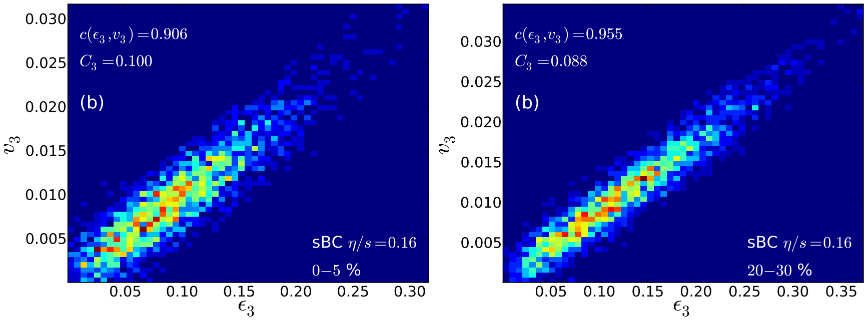

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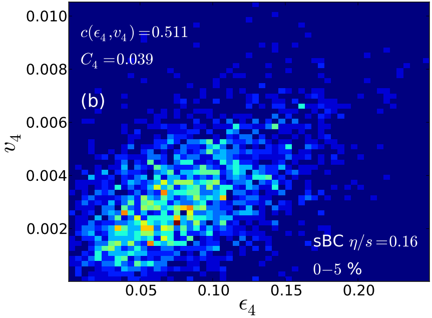

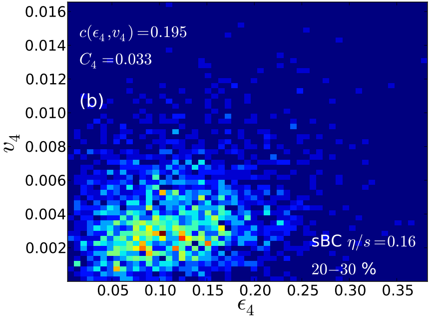

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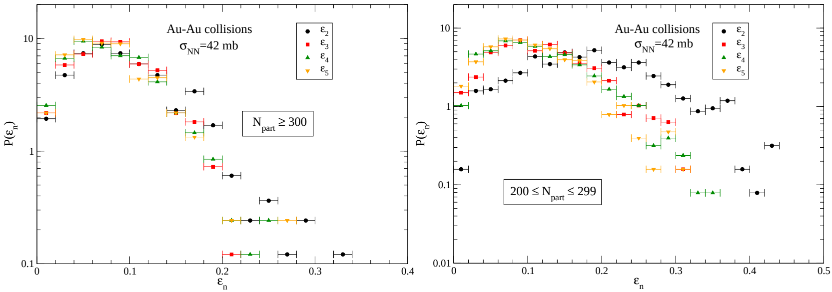
- Strong correlation for the $n = 2, 3$ harmonics
- Mild correlation for the $n = 4$ harmonic only in central events.

$P(\epsilon_n)$ vs $P(v_n)$



Event-by-event flow measurements allow to connect probability distribution of

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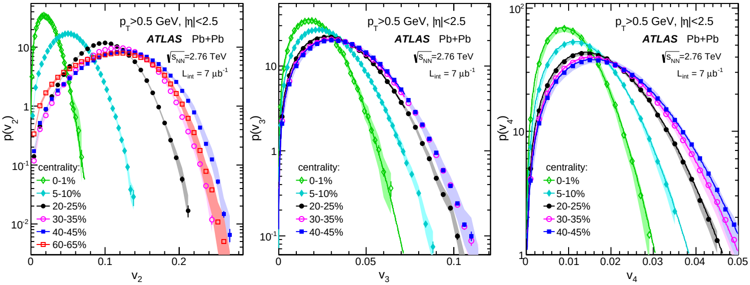

Event-by-event flow measurements allow to connect probability distribution of

- initial fluctuations ($\epsilon_m \equiv \epsilon_{2,m}$)
- different flow harmonics (ATLAS coll., JHEP 1311 (2013) 183)

allowing one to put constraints on the initial state.

Relativistic hydrodynamics

Relativistic hydrodynamics: the ideal case

In the absence of non-vanishing conserved charges ($n_B = 0$), the evolution of an *ideal fluid* is completely described by the *conservation of the ideal energy-momentum tensor*:

$$\partial_\mu T^{\mu\nu} = 0, \quad \text{where} \quad T^{\mu\nu} = T^{\mu\nu}_{\text{eq}} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu}$$

It is convenient to project the above equations

- along the fluid velocity ($u_\nu \partial_\mu T^{\mu\nu} = 0$)

$$D\epsilon = -(\epsilon + P)\Theta, \quad (\text{with } D \equiv u^\mu \partial_\mu \text{ and } \Theta \equiv \partial_\mu u^\mu)$$

- and perpendicularly to it ($\Delta_{\alpha\nu}\partial_\mu T^{\mu\nu} = 0$, with $\Delta_{\alpha\nu} \equiv g_{\alpha\nu} - u_\alpha u_\nu$)

$$(\epsilon + P)Du^\alpha = \nabla^\alpha P \quad (\text{with } \nabla^\alpha \equiv \Delta^{\alpha\mu}\partial_\mu),$$

  which is the *relativistic* version of the *Euler equation* (fluid acceleration driven by pressure gradients)

Viscous hydrodynamics

Better flow measurements required the introduction of *viscous corrections* to the energy-momentum tensor in order to reproduce the data:

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \Pi^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \pi^{\mu\nu} - \Pi\Delta^{\mu\nu},$$

where we have isolated the *traceless* ($\pi^\mu_\mu = 0$) shear viscous tensor $\pi^{\mu\nu}$. The condition $u_\mu\Pi^{\mu\nu}=u_\mu\pi^{\mu\nu}=0$ (Landau frame) defines the fluid velocity

$$u_\mu u_\nu T^{\mu\nu} = u_\mu u_\nu T^{\mu\nu}_{\rm eq} = \epsilon \quad (\overline{T}^{00} = \overline{T}^{00}_{\rm eq} = \epsilon \text{ in the LRF})$$

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- Projecting along u_ν:

$$D\epsilon = -(\epsilon + P + \Pi)\Theta + \pi^{\mu\nu}\nabla_{<\mu}u_{\nu>},$$

after replacing $\nabla_\mu u_\nu \longrightarrow \nabla_{<\mu}u_{\nu>} \equiv \frac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \frac{1}{3}\Delta_{\mu\nu}\Theta$

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- Projecting along $\Delta_{\alpha\nu}$:

$$(\epsilon + P + \Pi)Du^\alpha = \nabla^\alpha(P + \Pi) - \Delta^\alpha_\nu \partial_\mu \pi^{\mu\nu}$$

Fixing the viscous tensor: first order formalism

A way to fix the viscous tensor is through the 2^{nd} law of thermodynamics, imposing $\partial_\mu s^\mu \geq 0$.

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$$Ts = \epsilon + P \quad \text{and} \quad T\, ds = d\epsilon$$

one gets

$$\partial_\mu s^\mu = u^\mu \partial_\mu s + s\, \partial_\mu u^\mu = \frac{1}{T}[D\epsilon + (\epsilon + P)\Theta] \geq 0$$

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$$D\epsilon = -(\epsilon + P + \Pi)\Theta + \pi^{\mu\nu}\nabla_{<\mu}u_{\nu>},$$

one gets

$$\partial_\mu s^\mu = \frac{1}{T}\left[-\Pi\Theta + \pi^{\mu\nu}\nabla_{<\mu}u_{\nu>}\right] \geq 0$$

which is identically satisfied if (relativistic Navier Stokes result)

$$\Pi = -\zeta\Theta \quad \text{and} \quad \pi^{\mu\nu} = 2\eta\nabla^{<\mu}u^{\nu>},$$

where ζ and η are the bulk and shear *viscosity coefficients*.

Relativistic causal theory: second order formalism

The naive relativistic generalization of the Navier Stokes equations violates causality! This pathology can be cured *including viscous corrections into the entropy current*, of second order in the gradients:

$$s^\mu = s^\mu_{\mathrm{eq}} + Q^\mu = s u^\mu - \left(\beta_0 \Pi^2 + \beta_2 \pi_{\alpha\beta}\pi^{\alpha\beta}\right) \frac{u^\mu}{2T}$$

One gets then $(Df \equiv \dot{f})$:

$$T\partial_\mu s^\mu = \Pi \left[-\Theta - \beta_0 \dot{\Pi} - T\Pi\, \partial_\mu(\beta_0 u^\mu/2T)\right] + \pi^{\alpha\beta}\left[\nabla_{<\alpha} u_{\beta>} - \beta_2 \dot{\pi}_{\alpha\beta} - T\pi_{\alpha\beta}\, \partial_\mu(\beta_2 u^\mu/2T)\right] \geq 0,$$

which is satisfied if $\Pi \approx \zeta[-\Theta - \beta_0\dot{\Pi}]$ and $\pi_{\alpha\beta} \approx 2\eta[\nabla_{<\alpha} u_{\beta>} - \beta_2 \dot{\pi}_{\alpha\beta}]$.

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which is satisfied if $\Pi \approx \zeta[-\Theta - \beta_0\dot{\Pi}]$ and $\pi_{\alpha\beta} \approx 2\eta[\nabla_{<\alpha}u_{\beta>} - \beta_2\dot{\pi}_{\alpha\beta}]$. One has then to evolve also the components of the viscous tensor (6 independent equations, due to $u_\mu\pi^{\mu\nu} = 0$ and $\pi^\mu_\mu = 0$)

$$\dot{\Pi} \approx -\frac{1}{\zeta\beta_0}[\Pi + \zeta\Theta] \quad \text{and} \quad \dot{\pi}_{\alpha\beta} \approx -\frac{1}{2\eta\beta_2}[\pi_{\alpha\beta} - 2\eta\nabla_{<\alpha}u_{\beta>}],$$

whera $\tau_\Pi \equiv \zeta\beta_0$ and $\tau_\pi \equiv 2\eta\beta_2$ play the role of *relaxation times*.

Numerical implementation: the ECHO-QGP code

We employed for our numerical studies the ECHO-QGP code

- Some references...
  - An *italian project* (MIUR & INFN): L. Del Zanna, V. Chandra, G. Inghirami, V. Rolando, A. Beraudo, A. De Pace, G. Pagliara, A. Drago and F.Becattini: Eur.Phys.J. C73 (2013) 2524
  - based on the astrophysical code ECHO: L. Del Zanna *et al.*, (2007) Astron.Astrophys.,473,11
  - ECHO-QGP webpage: http://www.astro.unifi.it/echo-qgp/
- The main features:
  - Possibility to run both with Cartesian and Bjorken ($\tau \equiv \sqrt{t^2 - z^2}$, $\eta \equiv \frac{1}{2} \ln \frac{t+z}{t-z}$) coordinates,
  - both in (2+1)D and in (3+1)D;
  - in the ideal or viscous case;
  - with any EOS and initial condition supplied by the user

Mode-by-mode hydrodynamics

- Original proposal presented in Phys.Rev. C88 (2013) 044906 (Stefan Floerchinger and U.A. Wiedemann)
- Results obtained through full RHD calculations presented in arXiv:1312.5482 [hep-ph] and displayed in this talk

Mode-by-mode hydrodynamics: the general idea

- *For each event* the system is initialized via a full set of hydrodynamic fields on a τ_0-hypersurface (w being the enthalpy density):

$$h_i(\tau_0, r, \varphi, \eta) = \left(w, u^r, u^\phi, u^\eta, \Pi, \pi^{\mu\nu}\right)$$

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- *For each event* one can express h_i in terms of a smooth background $h_i^{\rm BG}$, obtained averaging over a large sample of events, and a fluctuating term $\tilde{h}_i$. One can write for instance:

$$w = w_{\rm BG}(1+\tilde{w}), \quad u^r = u^r_{\rm BG} + \frac{1}{\sqrt{2}}(\tilde{u}^- + \tilde{u}^+), \quad u^\phi = \frac{i}{\sqrt{2}r}(\tilde{u}^- - \tilde{u}^+)$$

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- We propose the following expansion for the evolution of $\tilde{h}_i(\tau)$ on top of the evolved backgroud $\{h_j^{\rm BG}(\tau)\}$:

$$\tilde{h}_i(\tau, r, \varphi) = \int_{r',\varphi'} \mathcal{G}_{ij}(\tau, \tau_0, r, r', \varphi - \varphi')\, \tilde{h}_j(\tau_0, r', \varphi')$$
$$+\frac{1}{2}\int_{r',r'',\varphi',\varphi''} \mathcal{H}_{ijk}(\tau, \tau_0, r, r', r'', \varphi-\varphi', \varphi-\varphi'')\, \tilde{h}_j(\tau_0, r', \varphi')\, \tilde{h}_k(\tau_0, r'', \varphi'') + \mathcal{O}(\tilde{h}^3)$$

Mode-by-mode hydrodynamics: the strategy

From the *exact numerical solution* (with ECHO-QGP)

- both for the average background $\{h_i^{\rm BG}(\tau_0)\} \underset{\text{full hydro}}{\longrightarrow} \{h_i^{\rm BG}(\tau)\}$
- and for fluctuating initial conditions $\{h_i(\tau_0)\} \underset{\text{full hydro}}{\longrightarrow} \{h_i(\tau)\}$

we will show that the expansion

$$\tilde{h}_i(\tau, r, \varphi) = \int_{r', \varphi'} \mathcal{G}_{ij}(\tau, \tau_0, r, r', \varphi - \varphi')\, \tilde{h}_j(\tau_0, r', \varphi')$$

$$+\frac{1}{2}\int_{r', r'', \varphi', \varphi''} \mathcal{H}_{ijk}(\tau, \tau_0, r, r', r'', \varphi - \varphi', \varphi - \varphi'')\, \tilde{h}_j(\tau_0, r', \varphi')\, \tilde{h}_k(\tau_0, r'', \varphi'') + \mathcal{O}(\tilde{h}^3)$$

actually holds and in particular that

- the dominant response to initial fluctuations is (in most cases) linear
- non-linearities (*important in some cases*) can be consistently interpreted as higher-order corrections within our perturbative expansion and quantitatively reproduced

Mode-by-mode hydrodynamics: density perturbations

We will focus on the hydrodynamic propagation of initial *fluctuating density distributions*, parametrized in terms of the coefficients $\tilde{w}_l^{(m)}$ of a Fourier-Bessel expansion ($k_l^{(m)} \equiv z_l^{(m)}/R$, with $z_l^{(m)}$ the l^{th}-zero of J_m)

$$w(\tau_0, r, \varphi) = w_{BG}(\tau_0, r)\left(1 + \sum_{m=-\infty}^{\infty} \tilde{w}^{(m)}(\tau_0, r) e^{im\varphi}\right), \quad \tilde{w}^{(m)}(\tau_0, r) = \sum_{l=1}^{\infty} \tilde{w}_l^{(m)} J_m\left(k_l^{(m)} r\right)$$

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The goal: understanding which of the initial Fourier modes in the expansion of $\tilde{w}(\tau_0, r, \varphi)$ contribute to the various azimuthal harmonics

$$\tilde{w}^{(m)}(\tau, r) \equiv \int_0^{2\pi} d\varphi e^{-im\varphi} \tilde{w}(\tau, r, \varphi)$$

at a later time, by analyzing the full RHD outcomes by ECHO-QGP for

$$w_{\mathrm{BG}}(\tau_0) \longrightarrow w_{\mathrm{BG}}(\tau) \quad \text{and} \quad w(\tau_0) \longrightarrow w(\tau) \equiv w_{\mathrm{BG}}(\tau)\left[1 + \tilde{w}(\tau)\right]$$

Propagation and interaction of different Fourier modes

From the perturbative expansion for the hydrodynamic fluctuations and performing a *Fourier decomposition of the response functions*

$$\mathcal{G}(\tau, \tau_0, r, r', \Delta\varphi) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} e^{im\Delta\varphi} \mathcal{G}^{(m)}(\tau, \tau_0, r, r')$$

$$\mathcal{H}(\tau, \tau_0, r, r', r'', \Delta\varphi', \Delta\varphi'') = \frac{1}{(2\pi)^2} \sum_{m', m''=-\infty}^{\infty} e^{i(m'\Delta\varphi' + m''\Delta\varphi'')} \mathcal{H}^{(m', m'')}(\tau, \tau_0, r, r', r'')$$

one obtains for the m^{th} harmonic of the enthalpy fluctuations

$$\tilde{w}^{(m)}(\tau, r) = \int_{r'} \mathcal{G}^{(m)}(\tau, \tau_0, r, r')\, \tilde{w}^{(m)}(\tau_0, r')$$

$$+\frac{1}{2} \int_{r', r''} \frac{1}{2\pi} \sum_{m', m''} \delta_{m, m'+m''} \mathcal{H}^{(m', m'')}(\tau, \tau_0, r, r', r'')\, \tilde{w}^{(m')}(\tau_0, r')\, \tilde{w}^{(m'')}(\tau_0, r'') + \ldots$$

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A single m-mode at τ_0 can give rise at time τ to:

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A single m-mode at τ_0 can give rise at time τ to:

- an m-mode at linear order

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$$+\frac{1}{2}\int_{r',r''}\frac{1}{2\pi}\sum_{m',m''}\delta_{m,m'+m''}\mathcal{H}^{(m',m'')}(\tau,\tau_0,r,r',r'')\,\tilde{w}^{(m')}(\tau_0,r')\,\tilde{w}^{(m'')}(\tau_0,r'')+\dots$$

A single m-mode at τ_0 can give rise at time τ to:

- an m-mode at linear order
- a 0 and $2m$-mode at quadratic order

Propagation and interaction of different Fourier modes

From the perturbative expansion for the hydrodynamic fluctuations and performing a *Fourier decomposition of the response functions*

$$\mathcal{G}(\tau,\tau_0,r,r',\Delta\varphi)=\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}e^{im\Delta\varphi}\mathcal{G}^{(m)}(\tau,\tau_0,r,r')$$

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A single m-mode at τ_0 can give rise at time τ to:

- an m-mode at linear order
- a 0 and $2m$-mode at quadratic order
- a $3m$-mode and corrections to the m-mode at cubic order...

Setting the initial conditions

The fluctuations being real sets the constraint $\tilde{w}_l^{(-m)} = (-1)^m (\tilde{w}_l^{(m)})^*$.
Parametrizing the weights as $\tilde{w}_l^{(m)} = |\tilde{w}_l^{(m)}| e^{-im\psi_l^{(m)}}$ allows one to recast the expansion for the initial enthalpy density into the form (reminiscent of the *harmonic decomposition of* azimuthal *single-particle distributions*)

$$w(\tau_0, r, \varphi) = w_{BG}(\tau_0, r) \left(1 + \sum_{l=1}^{\infty} \tilde{w}_l^{(0)} J_0\left(k_l^{(0)} r\right) + 2 \sum_{m=1}^{\infty} \sum_{l=1}^{\infty} |\tilde{w}_l^{(m)}| \cos\left[m(\varphi - \psi_l^{(m)})\right] J_m\left(k_l^{(m)} r\right)\right),$$

which will be then *evolved via the full hydrodynamic equations*.

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which will be then *evolved via the full hydrodynamic equations*.

In the following we will study the evolution of a selected set of Fourier-Bessel modes, exploring for the weights $\tilde{w}_l^{(m)}$ the typical range of values provided by a sample of Glauber-MC initial conditions

Initialization with a single-mode

We start considering the evolution of a single $(m=2, l=1)$ mode on top of an average background

$$w(\tau_0, \vec{r}) = w_{BG}(\tau_0, r) \left[1 + 2|\tilde{w}_1^{(2)}| J_2 \left(k_1^{(2)} r\right) \cos\left(2(\varphi - \psi_1^{(2)})\right)\right]$$

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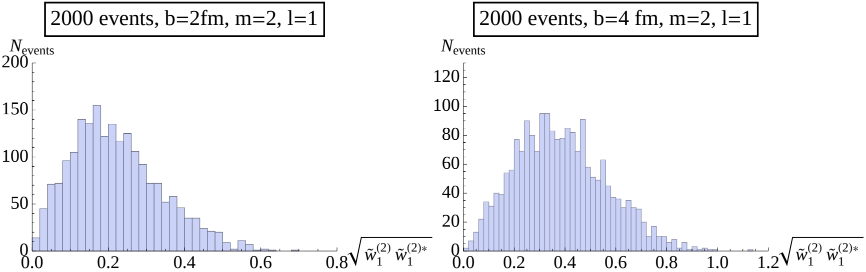
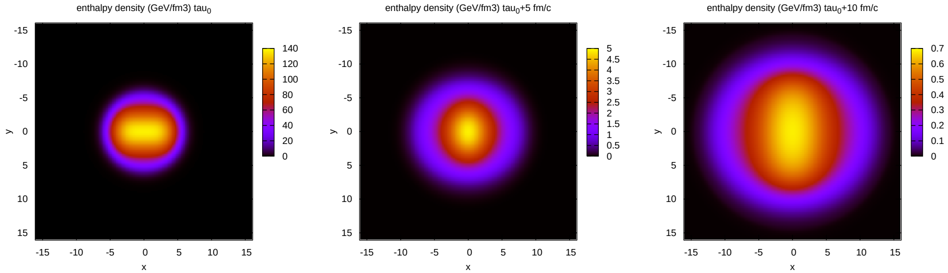
We will explore values of $|w_1^{(2)}|$ typical of *central collisions*

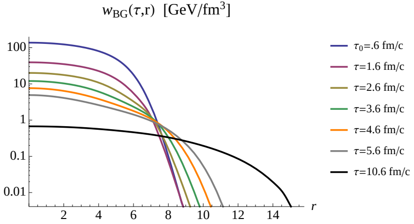


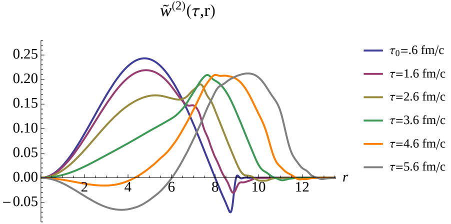
Single-mode (linear) evolution



- We evolve an initial condition with $w_1^{(2)} = 0.5$ at $\tau = 0.6$ fm/c, with $\eta/s = 0.08$

Single-mode (linear) evolution





- We evolve an initial condition with $w_1^{(2)} = 0.5$ at $\tau = 0.6$ fm/c, with $\eta/s = 0.08$
- After subtracting the background one can follow the evolution of the $m = 2$ mode

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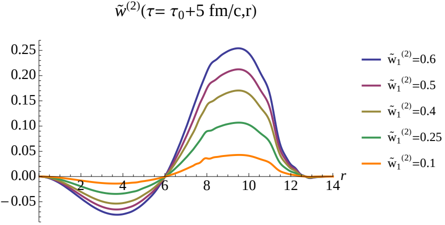
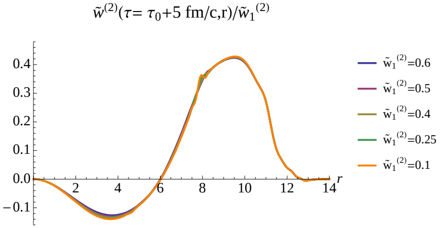
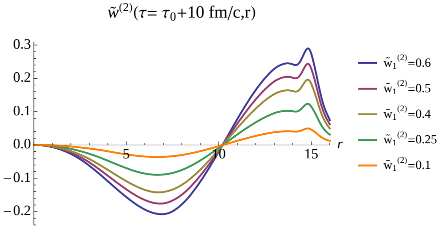
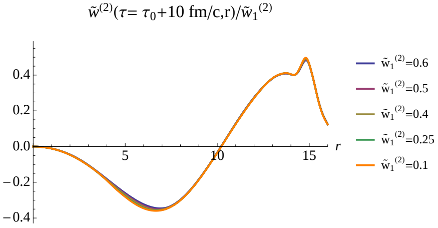




- We evolve an initial condition with $w_1^{(2)} = 0.5$ at $\tau = 0.6$ fm/c, with $\eta/s = 0.08$
- After subtracting the background one can follow the evolution of the $m = 2$ mode
- Varying the weight of the initial perturbation and rescaling the result by $w_1^{(2)}$ one can verify that *the evolution is to very good accuracy linear*,

Single-mode (linear) evolution





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- After subtracting the background one can follow the evolution of the $m = 2$ mode
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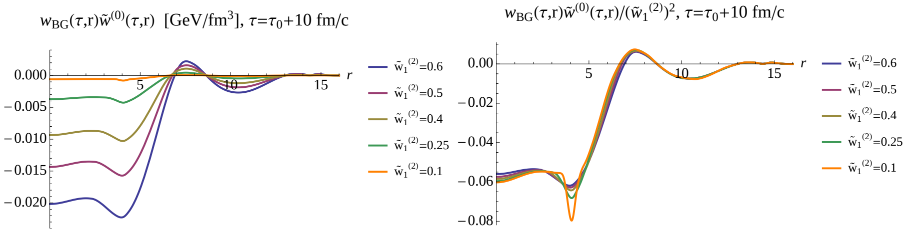
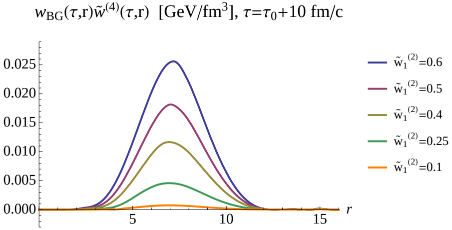
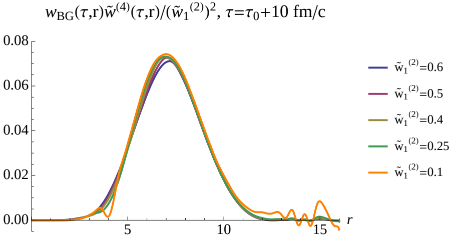
Single-mode evolution: non-linear effects



- A $m = 0$ mode arises at quadratic order in $w_1^{(2)}$ from $\delta_{0,2-2}$

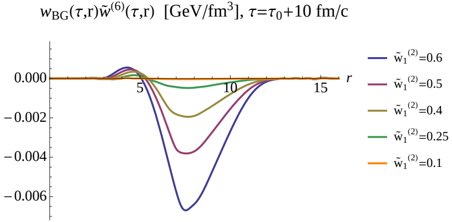
Single-mode evolution: non-linear effects

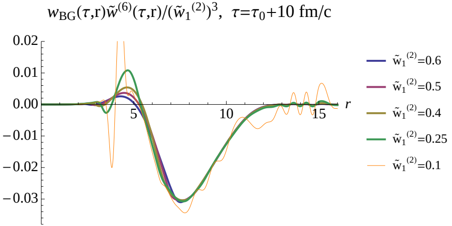




- A $m=0$ mode arises at quadratic order in $w_1^{(2)}$ from $\delta_{0,2-2}$
- A $m=4$ mode arises at quadratic order in $w_1^{(2)}$ from $\delta_{4,2+2}$

Single-mode evolution: non-linear effects





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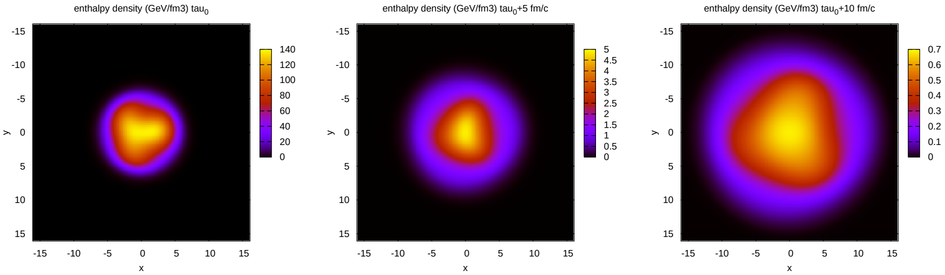
- A $m=4$ mode arises at quadratic order in $w_1^{(2)}$ from $\delta_{4,2+2}$
- A $m=6$ mode arises at cubic order in $w_1^{(2)}$ from $\delta_{6,2+2+2}$

Interaction between different modes

We evolve and initial condition containing two modes, $(m=2, l=2)$ and $(m=3, l=1)$, with all possible combinations of weights $|\tilde{w}_2^{(2)}|=0.1, 0.25$ and $|\tilde{w}_1^{(3)}|=0.1, 0.25$ and phases $\psi_2^{(2)}=0$ and $\psi_1^{(3)}=-0.2$.



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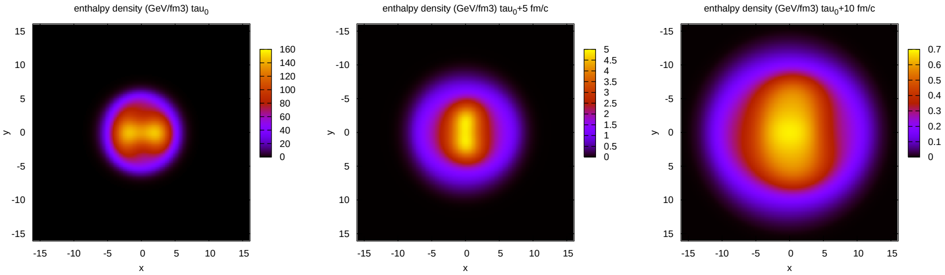

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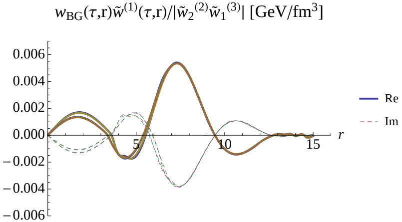

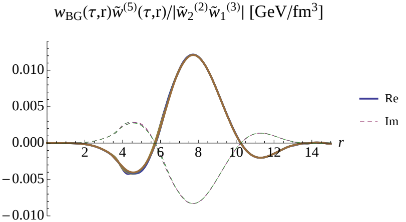

$m=1$ $(\delta_{1,3-2})$ and $m=5$ $(\delta_{5,3+2})$ harmonics arise from the interference of the two initial modes

- They display the expected scaling behavior

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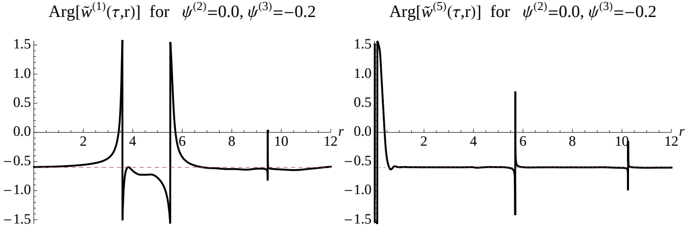

$m=1$ $(\delta_{1,3-2})$ and $m=5$ $(\delta_{5,3+2})$ harmonics arise from the interference of the two initial modes

- They display the expected scaling behavior
- Their phases are consistent with the expectation $3\psi_1^{(3)}$

Relevance for realistic initial conditions

Embed a single $(m = 2, l = 1)$ mode $(\tilde{w}_1^{(2)} = 0.5)$

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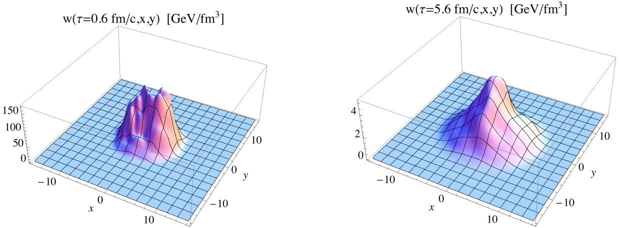

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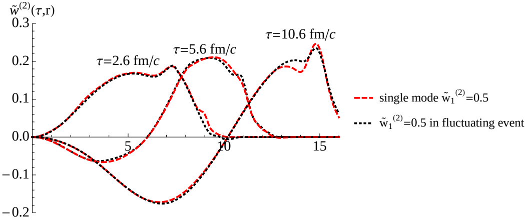

The assumption of a predominantly linear response on top of a suitably chosen background is applicable for realistic initial conditions that display *strong fluctuations*

Conclusions

We have provides evidence *from full numerical solutions* that the hydrodynamical evolution of initial density fluctuations in heavy ion collisions can be understood order-by-order in a perturbative series in deviations from a smooth and azimuthally symmetric background solution

- to leading linear order, modes with different azimuthal wave numbers do not mix
- deviations from a linear response to the initial fluctuations can be quantitatively understood as quadratic and higher order corrections.

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We plan to perform a more systematic study in the future

- investigating the role of viscosity
- extending the analysis to a wider set of centrality classes