Relativistic Heavy-Ion collisions: recent theoretical developments

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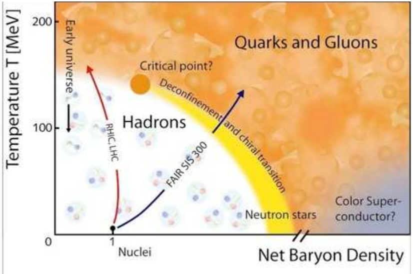
Heavy-ion collisions: exploring the QCD phase-diagram



QCD phases identified through the *order parameters*

- Polyakov loop $\langle L \rangle \sim e^{-\beta \Delta F_Q}$ energy cost to add an isolated color charge
- Chiral condensate $\langle \overline{q}q \rangle \sim$ effective mass of a "dressed" quark in a hadron

Region explored at LHC: *high-T/low-density* (early universe, $n_B/n_\gamma \approx 0.6 \cdot 10^{-9}$)

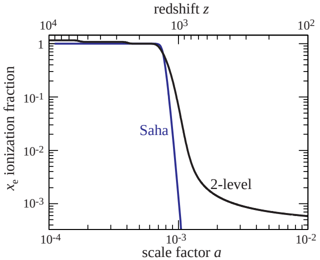
- From QGP (color deconfinement, chiral symmetry restored)
- to hadronic phase (confined, chiral symmetry breaking[1])

NB $\langle \overline{q}q \rangle \neq 0$ responsible for most of the baryonic mass of the universe: *only ∼35 MeV of the proton mass from* $m_{u/d} \neq 0$

[1] V. Koch, *Aspects of chiral symmetry*, Int.J.Mod.Phys. E6 (1997)

The QCD crossover: hadron vs atom formation

In the $\mu_B \to 0$ region the QCD transition is actually a *crossover*, i.e. a rapid but smooth change in the nature of the dominant charge (baryon, electric...) carriers, in analogy with the $e + p \leftrightarrow H + \gamma$ recombination in cosmology.



$$\frac{n_H}{n_p n_e} = \left(\frac{m_H}{m_p m_e}\frac{2\pi}{T}\right)^{3/2} \exp\left[\frac{m_p + m_e - m_H}{T}\right]$$

$$\approx \left(\frac{2\pi}{m_e T}\right)^{3/2} \exp\left[\frac{Q}{T}\right], \quad (Q = 13.6\ \mathrm{eV})$$

$X \equiv \frac{n_p}{n_p + n_H}$: ionization fraction (NB: $n_p = n_e$)

However they occur in *very different regimes*:

- One has $X = 0.5$ for $T_{\rm rec} = 0.323$ eV with $n_e^{\rm rec} \approx 0.122(n_B/n_\gamma)T_{\rm rec}^3$. This corresponds to a *Debye screening radius* of the electric interaction $r_D \equiv (T/n_e e^2)^{1/2} \approx 24\,\mathrm{cm} \gg a_0 \sim 10^{-10}\mathrm{m}$: atomic properties unaffected!
  Crossover occurs in a *dilute regime*
- In the QGP $m_D \equiv r_D^{-1} = gT(N_c/3 + N_f/6)^{1/2}$. At $T = 0.2$ GeV, for $\alpha_s = 0.3$, one has $r_D \approx 0.4\,\mathrm{fm} \sim r_h$: color interaction strongly modified!
  Crossover occurs in a *strongly interacting regime*

Active degrees of freedom around the QCD crossover

Lattice-QCD calculations (nowadays with *realistic quark masses*) allows one to calculate the cumulants of conserved charges (baryon number, eletric charge, strangeness) as well as of their product[2]

$$\langle X^m Y^n \rangle_c = \frac{\partial^{(m+n)}(\ln Z_{\rm QCD})}{\partial \hat{\mu}_X^m \partial \hat{\mu}_Y^n} \quad \text{with} \quad \hat{\mu}_i \equiv \mu_i/T,$$

where, considering the lowest orders, one has

$$\langle X^2 \rangle_c \equiv \langle \delta X^2 \rangle, \quad \langle X^3 \rangle_c \equiv \langle \delta X^3 \rangle, \quad \langle X^4 \rangle_c \equiv \langle \delta X^4 \rangle - 3\langle \delta X^2 \rangle, \quad \langle XY \rangle_c \equiv \langle \delta X \delta Y \rangle$$

Exploiting the fact that, at variance with hadrons, *all* quarks carry *fractional* baryon-number and electric charge, from the fluctuations of conserved charges and their correlations one can get information on the active degrees of freedom at a given temperature, i.e. whether they are hadrons (mesons and baryons) or deconfined quarks

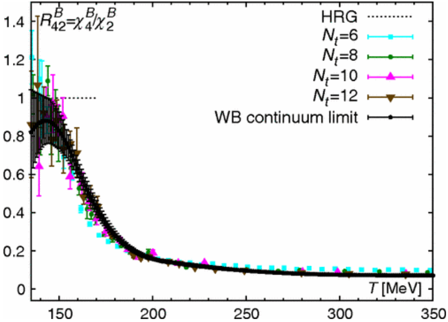
[2] M. Asakawa and M. Kitazawa, Prog.Part.Nucl.Phys. 90 (2016) 299

Active degrees of freedom around the QCD crossover

Fluctuations of *net* particle number (particles minus antiparticles) follow a Skellam distribution (difference of two Poissonian variables!). This provides a definite prediction for their cumulants:

$$\langle N^n \rangle_c = \langle N_{\rm part} \rangle + (-1)^n \langle N_{\rm antipart} \rangle \quad \longrightarrow \quad \frac{\langle N^{n+2m} \rangle_c}{\langle N^n \rangle_c} = 1$$

Having quarks baryon-number 1/3, while hadrons 0 or 1...



...in the hadron-gas phase

$$\frac{\langle B^{n+2m} \rangle_c}{\langle B^n \rangle_c} = 1$$

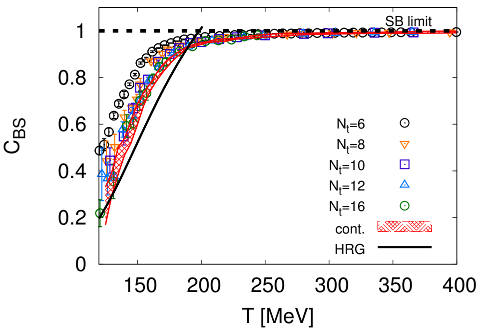
...in the QGP phase

$$\frac{\langle B^{n+2m} \rangle_c}{\langle B^n \rangle_c} = \frac{1}{9}$$

S. Borsanyi et al. PRL 113, 052301 (2014)

Strangeness around the QCD crossover

In the QGP phase strangeness is carried by s quarks, carrying also baryon number $B = 1/3$. In a HRG most of the strangeness is carried by kaons, for which $B = 0$; the lightest strange particle carrying baryon number $B = 1$ is the Λ. Correlation between strangeness and baryon-number fluctuations is a diagnostic tool of the active degrees of freedom!



S. Borsanyi et al., JHEP 1201 (2012) 138

One evaluates the quantity ($\langle S \rangle = 0$)

$$C_{BS} \equiv -3\frac{\langle BS \rangle_c}{\langle S^2 \rangle_c} = -3\frac{\langle BS \rangle}{\langle S^2 \rangle}$$

In the QGP phase

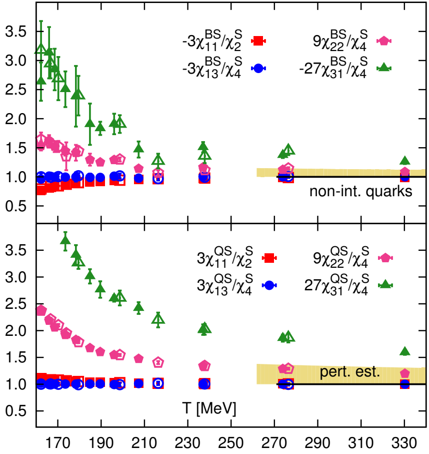
$$B = -(1/3)S \quad \longrightarrow \quad C_{BS} = 1$$

In the hadron-gas phase

$$C_{BS} = 3\frac{\langle \Lambda \rangle + \langle \overline{\Lambda} \rangle + \ldots + 3\langle \Omega^- \rangle + 3\langle \overline{\Omega}^+ \rangle}{\langle K^0 \rangle + \langle \overline{K}^0 \rangle + \ldots + 9\langle \Omega^- \rangle + 9\langle \overline{\Omega}^+ \rangle}$$

strongly dependent on temperature and very small at small temperature

Strangeness around the QCD crossover



A. Bazavov et al., PRL 111, 082301 (2013)

Ratios of higher-order generalized susceptibilities

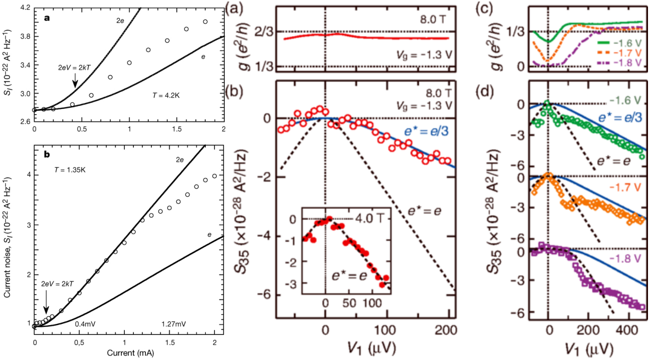
$$\chi_{mn}^{XY} \equiv \frac{\partial^{m+n}[P/T^4]}{\partial\hat{\mu}_X^m \partial\hat{\mu}_Y^n}$$

display a slower approach to a gas of weakly-interacting quarks.

- For $T_c \lesssim T \lesssim 2T_c$ strangeness non-trivially correlated with baryonic and electric charge: strongly-coupled nature of the QGP in this domain
- The possibility of a *flavour hierarchy* in the deconfinement transition was also suggested

Fluctuations and active degrees of freedom

Fluctuations are a very general tool to point-out the nature of *quasiparticle* excitations of a system. As an example, shot-noise measurement allows one to identify $e^* = 2e$ and $e^* = e/3$ charge-carriers in superconductivity and fractional quantum-Hall effect.



Electrons passing through a potential barrier in the time-interval Δt follow a Poisson distribution, so that

$$\langle N^n \rangle_c = \langle N \rangle \quad \longrightarrow \quad \frac{\langle Q^2 \rangle_c}{\langle Q \rangle} = \frac{q^2 \langle N^2 \rangle_c}{q \langle N \rangle} = q$$

From l-QCD susceptibilities to freeze-out parameters

If the experimental fluctuations of conserved charges (baryonic and electric) are of thermal origin, assuming that one is able to correct for non-thermal effects (efficiency, kinematic cuts, neutral particle...), by connecting the cumulants of their distributions with lattice-QCD results for generalized susceptibilities one should be able to estimate the chemical freeze-out parameters $T_{\rm fo}$ and $\mu_{\rm fo}$ (see F. Karsch, Central Eur. J. Phys. 10, 1234 (2012)).. In fact, although l-QCD results are available only for zero chemical potential, one can perform a Taylor expansion of the susceptibilities around $\mu_B=0$, e.g.

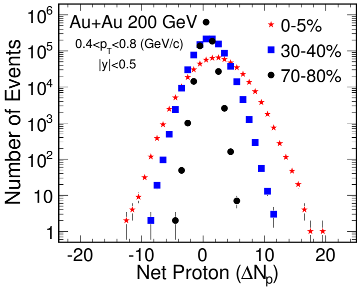
$$\chi^B_{2,\mu_B} = \chi^B_2 + \frac{1}{2}\chi^B_4\left(\frac{\mu_B}{T}\right)^2 + \dots \quad \chi^B_{1,\mu_B} = \chi^B_2\left(\frac{\mu_B}{T}\right) + \frac{1}{6}\chi^B_4\left(\frac{\mu_B}{T}\right)^3 + \dots$$

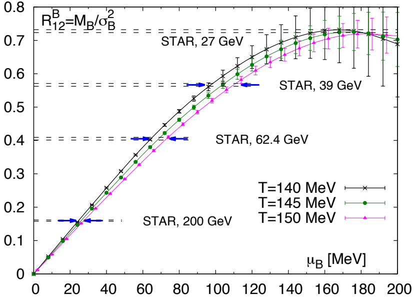
Considering the variance of the experimental baryon-number distribution one gets for instance

$$\frac{\langle B^2\rangle_c}{\langle B\rangle} = \frac{\chi^B_{2,\mu_B}}{\chi^B_{1,\mu_B}} = \frac{T}{\mu_B}\left[\frac{1+\frac{1}{2}(\chi^B_4/\chi^B_2)\left(\frac{\mu_B}{T}\right)^2+\dots}{1+\frac{1}{6}(\chi^B_4/\chi^B_2)\left(\frac{\mu_B}{T}\right)^2+\dots}\right],$$

allowing one to estimate μ_B/T.

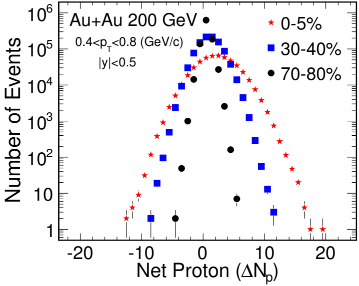
From l-QCD susceptibilities to freeze-out parameters

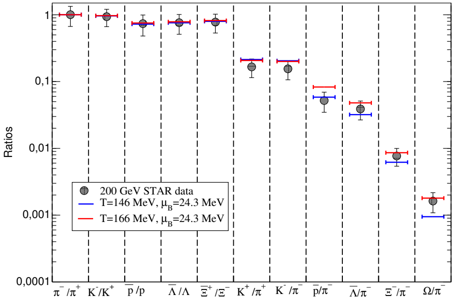




- FO parameters fixed to reproduce higher-order cumulants of electric-charge and baryon number (exp: net proton!) distributions

From l-QCD susceptibilities to freeze-out parameters





- FO parameters fixed to reproduce higher-order cumulants of electric-charge and baryon number (exp: net proton!) distributions
- Resulting FO temperature smaller than the one fixed by particle ratios, i.e. first-order cumulant (S. Borsanyi et al., PRL 113 (2014) 052301)
- Tension between proton and strange baryons: different freeze-out temperatures (R. Bellwied et al., PRL 111 (2013) 202302)?

Charmed degrees of freedom around deconfinement

QCD interactions don't change flavor, so that charm can be considered a *conserved charge* in heavy-ion collisions, as baryon number. Based on the fact that charm quarks have $B = 1/3$ while charmed hadrons have $B = 0, 1$, from the correlations of C and B fluctuations one can get information on the nature of the charm-carrying degrees of freedom, i.e. wheter they are mostly partonic or hadronic (C excess always associated to B excess in the partonic phase).

$$\chi_{kl}^{BC} = \left. \frac{\partial^{k+l}[P/T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_C^l} \right|_{\hat{\mu}_i = 0} \quad \text{where} \quad \hat{\mu}_i \equiv \mu_i / T$$

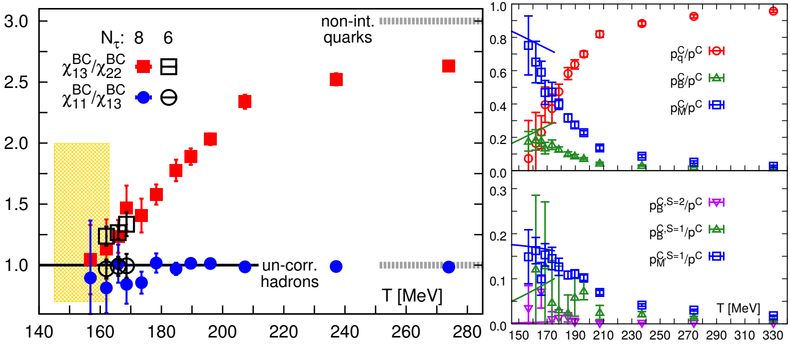
From the partial charm pressure (neglecting $C = 2, 3$ baryons)

$$P^C(T, \mu_C, \mu_M) = P_q^C(T) \cosh(\hat{\mu}_C + \hat{\mu}_B/3) + P_M^C(T) \cosh(\hat{\mu}_C) + P_B^C(T) \cosh(\hat{\mu}_C + \hat{\mu}_B)$$

One gets:

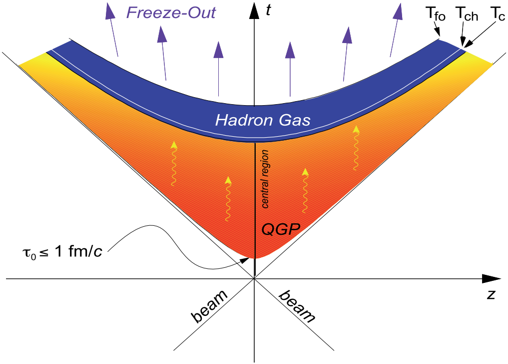
$$\frac{\chi_{mn}^{BC}}{\chi_{m+1,n-1}^{BC}} = B^{-1} \quad (1 \text{ for hadrons}, \; 3 \text{ for quarks})$$

Charmed degrees of freedom around deconfinement



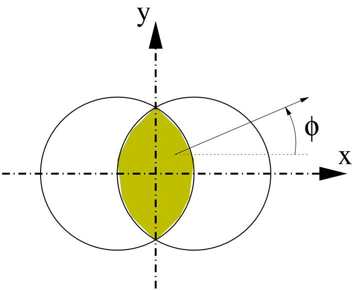
One can appreciate how, also for charm, the transition is a crossover and that slightly above T_c part of the charmed degrees of freedom are still/already hadronic states (see A. Bazavov et al., PLB 737 (2014) 210 and S. Mukherjee et al., PRD 93 (2016) 1, 014502)

Heavy-ion collisions: a cartoon of space-time evolution



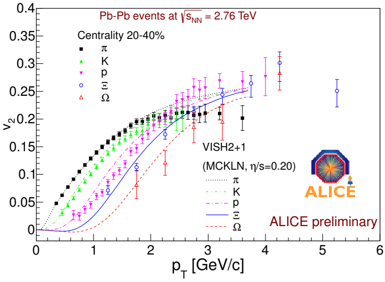
- Soft probes (low-p_T hadrons): collective behavior of the *medium*;
- Hard probes (high-p_T particles, heavy quarks, quarkonia): produced in *hard pQCD processes* in the initial stage, allow to perform a tomography of the medium

Hydrodynamic behavior: elliptic flow



- In *non-central collisions* particle emission is not azimuthally-symmetric!

Hydrodynamic behavior: elliptic flow



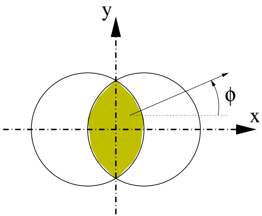
- In *non-central collisions* particle emission is not azimuthally-symmetric!
- The effect can be quantified through the *Fourier coefficient* v_2

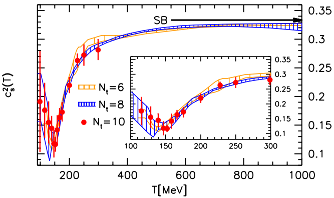
$$\frac{dN}{d\phi} = \frac{N_0}{2\pi}\left(1 + 2v_2\cos[2(\phi - \psi_{RP})] + \dots\right)$$

$$v_2 \equiv \langle \cos[2(\phi - \psi_{RP})] \rangle$$

- $v_2(p_T) \sim 0.2$ gives a modulation 1.4 vs 0.6 for in-plane vs out-of-plane particle emission!

Elliptic flow: physical interpretation



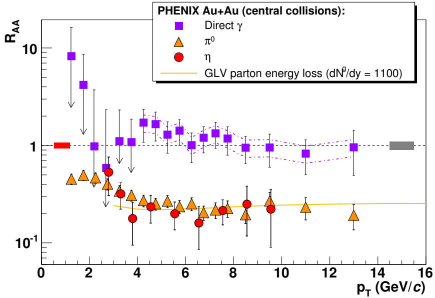


- Matter behaves like a fluid whose *expansion* is *driven by pressure gradients*

$$(\epsilon + P)\frac{dv^i}{dt} \underset{v \ll c}{=} -\frac{\partial P}{\partial x^i} \quad \text{(Euler equation)}$$

- Spatial anisotropy is converted into momentum anisotropy;
- At freeze-out *particles* are *mostly emitted along the reaction-plane*.
- It provides information on the EOS of the produced matter (Hadron Gas vs QGP) through the *speed of sound*: $\vec{\nabla} P = c_s^2 \vec{\nabla} \epsilon$

The medium is opaque: jet-quenching



Hard-photon $R_{AA} \approx 1$

- supports the Glauber picture (binary-collision scaling);
- entails that quenching of inclusive hadron spectra is a *final state effect* due to in-medium energy loss.

The *nuclear modification factor*

$$R_{AA} \equiv \frac{\left(dN^h/dp_T\right)^{AA}}{\langle N_{\rm coll}\rangle \left(dN^h/dp_T\right)^{pp}}$$

quantifies the suppression of high-p_T *hadron spectra*

Hydrodynamics and heavy-ion: recent theoretical achievements and phenomenological successes

- Development of a consistent relativistic formulation of hydrodynamic equations in the presence of *dissipative effects*; derivation of the universal lower bound $\eta/s = 1/4\pi$ for the viscosity to entropy-density ratio, in rough agreement with the data
- Study of higher flow-harmonics and event-by-event fluctuations
- Discovery of collective effects in small systems, such as high-multiplicity p-Pb and d-Au collisions (also p-p?)

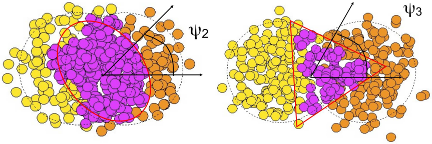
The QGP viscosity

From the comparison with the data one gets values for the *shear viscosity* close to the *universal lower bound* $\eta/s \approx 1/4\pi$ predicted by the AdS/CFT correspondence.
One can compare this with the values found for all the other known fluids:

fluid	P [Pa]	T [K]	η [Pa·s]	η/n [$\hbar$]	η/s [$\hbar/k_B$]
H_2O	$0.1{\cdot}10^6$	370	$2.9 \cdot 10^{-4}$	85	8.2
^4He	$0.1{\cdot}10^6$	2.0	$1.2 \cdot 10^{-6}$	0.5	1.9
H_2O	$22.6{\cdot}10^6$	650	$6.0 \cdot 10^{-5}$	32	2.0
^4He	$0.22{\cdot}10^6$	5.1	$1.7 \cdot 10^{-6}$	1.7	0.7
^6Li $(a=\infty)$	$12{\cdot}10^{-9}$	$23{\cdot}10^{-6}$	$\leq 1.7 \cdot 10^{-15}$	≤ 1	≤ 0.5
QGP	$88{\cdot}10^{33}$	$2{\cdot}10^{12}$	$\leq 5 \cdot 10^{11}$		≤ 0.4

leading to the conclusion that the QGP looks like the most *ideal fluid* ever observed

Event-by-event fluctuations



- Due to event-by-event fluctuations (e.g. of the nucleon positions) the initial density distribution is not smooth and can display *higher deformations*, each one with a *different azimuthal orientation* ψ_n.
- Higher harmonics ($m > 2$) contribute to the angular distribution

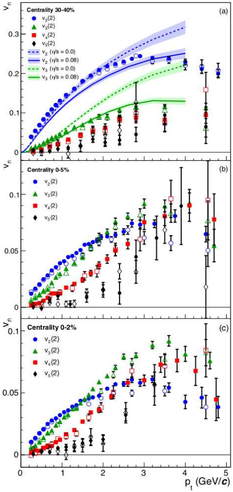
$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + 2\sum_m v_m \cos[m(\phi - \psi_m)]\right)$$

of the final hadrons, where *for each event*

$$v_m = \langle \cos[m(\phi - \psi_m)]\rangle \quad \text{and} \quad \psi_m = \frac{1}{m}\arctan\frac{\sum_i w_i \sin(m\phi_i)}{\sum_i w_i \cos(m\phi_i)}$$

The choice $w_i = p_T^i$ for the weights increase the resolution on ψ_m (one deals with a *finite number* of hadrons!)

Event-by-event fluctuations: experimental consequences



Fluctuating initial conditions give rise to[a]:

- Non-vanishing v_2 in central collisions;
- Odd harmonics (v_3 and v_5)

Hydro can reproduce also higher harmonics[b]

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[a] ALICE, Phys.Rev.Lett. 107 (2011) 032301
[b] B: Schenke *et al.*, PRC 85, 024901 (2012)

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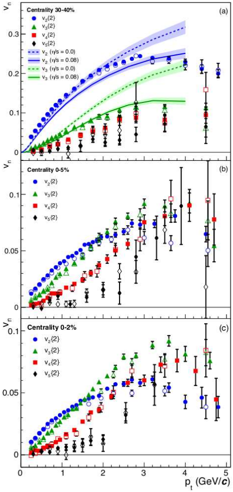

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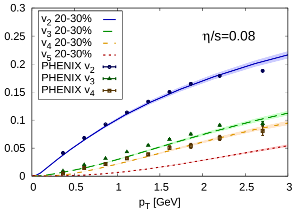

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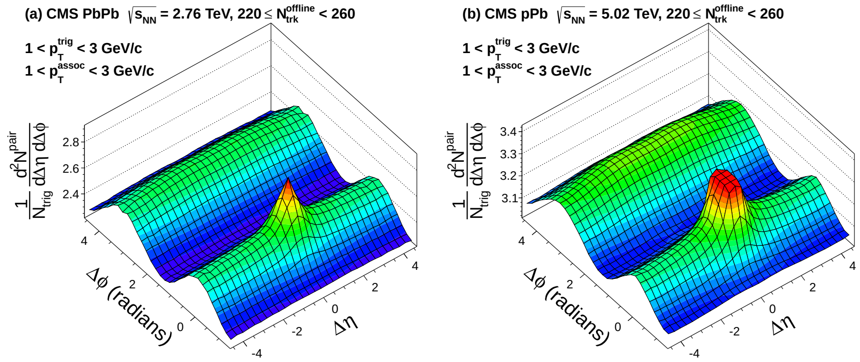
Hydrodynamic behavior in small systems?



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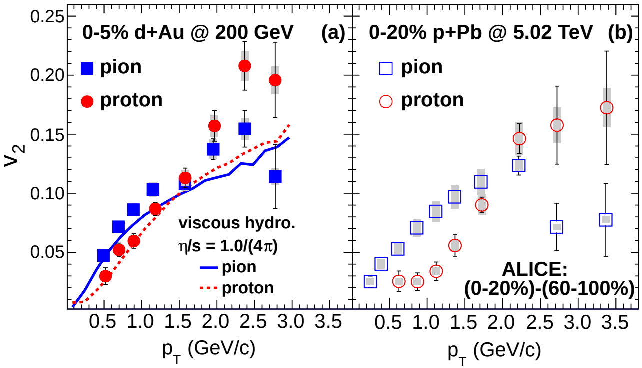

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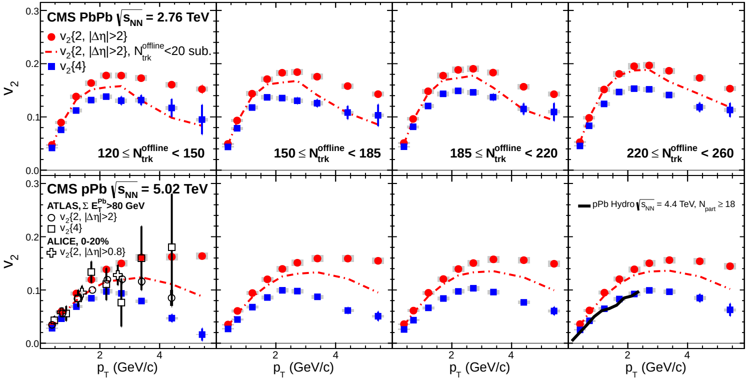

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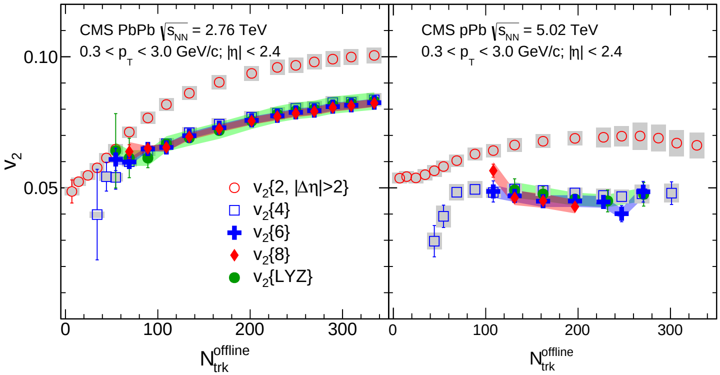

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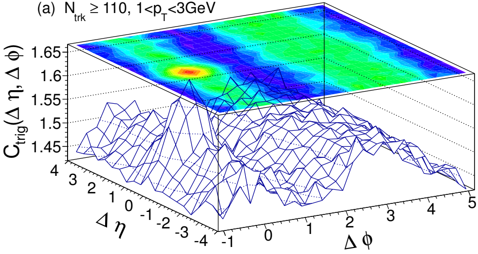

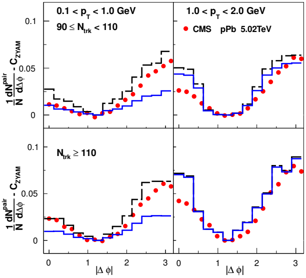

Hydrodynamic calculations (P. Bozek and W. Broniowski, PLB 718 (2013), 1557) are able to reproduce experimental data. Double ridge ($\Delta\phi \approx 0, \pi$) natural consequence of

- Approximate invariance of the initial condition for longitudinal boosts, which do not change the transverse energy-density distribution and the orientation of the event-plane ψ_2
- Hydrodynamic evolution of the medium, tending to emit particles along the direction of ψ_2, due to the larger pressure gradient

Heavy flavour: recent (and less recent) theoretical and phenomenological developments

- Transport calculations: conceptual setup
- Heavy-flavour transport coefficients: lQCD results
- In-medium hadronization and recombination
- Heavy-flavour observables in small systems

Transport theory: the Boltzmann equation

Time evolution of HQ phase-space distribution $f_Q(t,\mathbf{x},\mathbf{p})$[3]:

$$\frac{d}{dt}f_Q(t,\mathbf{x},\mathbf{p}) = C[f_Q]$$

- Total derivative along particle trajectory

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{x}} + \mathbf{F}\frac{\partial}{\partial \mathbf{p}}$$

Neglecting $\mathbf{x}$-dependence and mean fields: $\partial_t f_Q(t,\mathbf{p}) = C[f_Q]$

- Collision integral:

$$C[f_Q] = \int d\mathbf{k}[\underbrace{w(\mathbf{p}+\mathbf{k},\mathbf{k})f_Q(\mathbf{p}+\mathbf{k})}_{\text{gain term}} - \underbrace{w(\mathbf{p},\mathbf{k})f_Q(\mathbf{p})}_{\text{loss term}}]$$

$w(\mathbf{p},\mathbf{k})$: HQ transition rate $\mathbf{p} \to \mathbf{p}-\mathbf{k}$

[3] For results based on BE see e.g. BAMPS papers and Catania-group studies

From Boltzmann to Fokker-Planck

Expanding the collision integral for *small momentum exchange*[4] (Landau)

$$C[f_Q] \approx \int d\mathbf{k} \left[k^i \frac{\partial}{\partial p^i} + \frac{1}{2} k^i k^j \frac{\partial^2}{\partial p^i \partial p^j} \right] [w(\mathbf{p}, \mathbf{k}) f_Q(t, \mathbf{p})]$$

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The Boltzmann equation reduces to the *Fokker-Planck equation*

$$\frac{\partial}{\partial t} f_Q(t, \mathbf{p}) = \frac{\partial}{\partial p^i} \left\{ A^i(\mathbf{p}) f_Q(t, \mathbf{p}) + \frac{\partial}{\partial p^j} [B^{ij}(\mathbf{p}) f_Q(t, \mathbf{p})] \right\}$$

where (verify!)

$$A^i(\mathbf{p}) = \int d\mathbf{k}\, k^i w(\mathbf{p}, \mathbf{k}) \longrightarrow \underbrace{A^i(\mathbf{p}) = A(p)\, p^i}_{\text{friction}}$$

$$B^{ij}(\mathbf{p}) = \frac{1}{2} \int d\mathbf{k}\, k^i k^j w(\mathbf{p}, \mathbf{k}) \longrightarrow \underbrace{B^{ij}(\mathbf{p}) = \hat{p}^i \hat{p}^j B_0(p) + (\delta^{ij} - \hat{p}^i \hat{p}^j) B_1(p)}_{\text{momentum broadening}}$$

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Problem reduced to the *evaluation of three transport coefficients*

[4] B. Svetitsky, PRD 37, 2484 (1988)

The relativistic Langevin equation

The Fokker-Planck equation can be recast into a form suitable to follow the dynamics of each individual quark: the Langevin equation

$$\frac{\Delta p^i}{\Delta t} = -\underbrace{\eta_D(p)p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}} \ ,$$

with the properties of the noise encoded in

$$\langle \xi^i(\mathbf{p}_t)\xi^j(\mathbf{p}_{t'})\rangle = b^{ij}(\mathbf{p}_t)\frac{\delta_{tt'}}{\Delta t} \qquad b^{ij}(\mathbf{p}) \equiv \kappa_\parallel(p)\hat{p}^i\hat{p}^j + \kappa_\perp(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)$$

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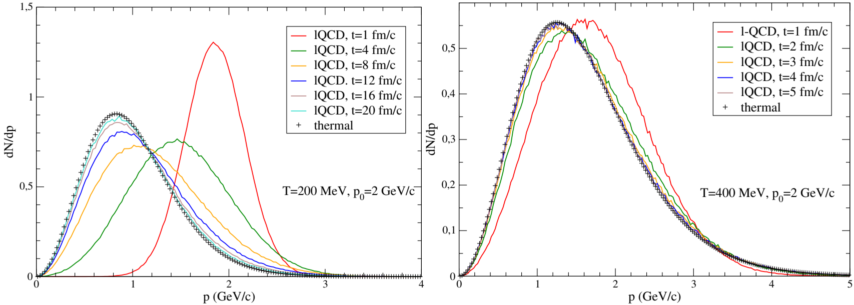
Transport coefficients to calculate:

- *Momentum diffusion* $\kappa_\perp \equiv \frac{1}{2} \frac{\langle \Delta p_\perp^2 \rangle}{\Delta t}$ and $\kappa_\parallel \equiv \frac{\langle \Delta p_\parallel^2 \rangle}{\Delta t}$;
- *Friction* term (dependent on the discretization scheme!)

$$\eta_D{}^{\text{Ito}}(p) = \frac{\kappa_\parallel(p)}{2TE_p} - \frac{1}{E_p^2} \left[(1 - v^2) \frac{\partial \kappa_\parallel(p)}{\partial v^2} + \frac{d-1}{2} \frac{\kappa_\parallel(p) - \kappa_\perp(p)}{v^2} \right]$$

fixed in order to assure approach to equilibrium (Einstein relation):

A first check: thermalization in a static medium



(Sample of c quarks with $p_0 = 2$ GeV/c and l-QCD transport coefficients)
For $t \gg 1/\eta_D$ one approaches a relativistic Maxwell-Jüttner distribution

$$f_{\rm MJ}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T\, K_2(M/T)}, \qquad \text{with} \quad \int d^3p\, f_{\rm MJ}(p) = 1$$

The larger κ ($\kappa \sim T^3$), the faster the approach to thermalization.

Lattice-QCD transport coefficients: setup

A non-perturbative estimate of HF transport coefficient in the QGP can be extracted from lattice-QCD simulations.
One consider the non-relativistic limit of the Langevin equation:

$$\frac{dp^i}{dt} = -\eta_D p^i + \xi^i(t), \quad \text{with} \quad \langle \xi^i(t) \xi^j(t') \rangle = \delta^{ij} \delta(t - t') \kappa$$

Hence, in the $p \to 0$ limit:

$$\kappa = \frac{1}{3} \int_{-\infty}^{+\infty} dt \langle \xi^i(t) \xi^i(0) \rangle_{\rm HQ} \approx \frac{1}{3} \int_{-\infty}^{+\infty} dt \underbrace{\langle F^i(t) F^i(0) \rangle_{\rm HQ}}_{\equiv D^>(t)},$$

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In the static limit the force is due to the color-electric field:

$$\mathbf{F}(t) = g\int d\mathbf{x} Q^\dagger(t,\mathbf{x}) t^a Q(t,\mathbf{x}) \mathbf{E}^a(t,\mathbf{x})$$

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In a thermal ensemble $\sigma(\omega) \equiv D^>(\omega) - D^<(\omega) = (1-e^{-\beta\omega})D^>(\omega)$ and

$$\kappa \equiv \lim_{\omega\to 0} \frac{D^>(\omega)}{3} = \lim_{\omega\to 0} \frac{1}{3}\frac{\sigma(\omega)}{1-e^{-\beta\omega}} \underset{\omega\to 0}{\sim} \frac{1}{3}\frac{T}{\omega}\sigma(\omega)$$

Lattice-QCD transport coefficients: results

The spectral function $\sigma(\omega)$ has to be reconstructed starting from the *euclidean electric-field correlator*

$$D_E(\tau) = -\frac{\langle \mathrm{Re}\,\mathrm{Tr}[U(\beta,\tau)gE^i(\tau,\mathbf{0})U(\tau,0)gE^i(0,\mathbf{0})]\rangle}{\langle \mathrm{Re}\,\mathrm{Tr}[U(\beta,0)]\rangle}$$

inverting the "Laplace-like" transform

$$D_E(\tau) = \int_0^{+\infty} \frac{d\omega}{2\pi} \frac{\cosh[\omega(\tau-\beta/2)]}{\sinh(\beta\omega/2)} \sigma(\omega)$$

NB ill-posed problem! Thousands of parameters ($\sigma(\omega_i)$) to fix against a limited set ($\ll 10^2$) of data ($D_E(\tau_i)$). Bayesian techniques or χ^2-fitting based on some prior information or ansatz are employed

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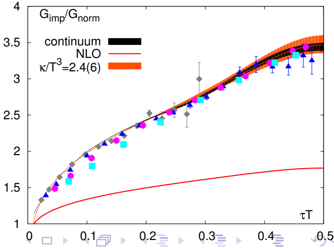
$\sim$3-5 times larger then the perturbative result (W.M. Alberico *et al*, EPJC 73 (2013) 2481). Challenge: approaching the continuum limit in full QCD (i.e. with dynamical quarks)!



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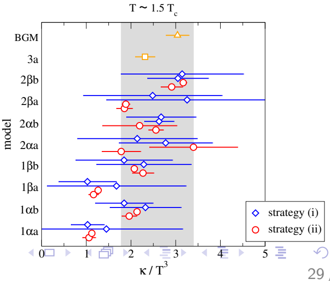

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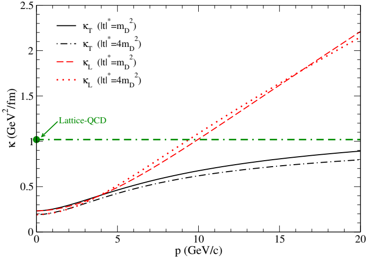

The Langevin/FP approach: a critical perspective

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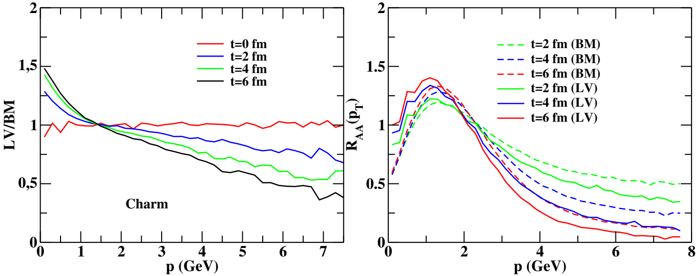

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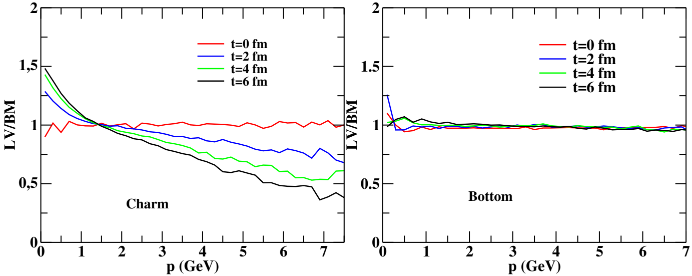


For beauty on the other hand Langevin≡Boltzmann!

From quarks to hadrons

In the presence of a medium, rather then fragmenting like in the vacuum (e.g. $c \to cg \to c\overline{q}q$), HQ's can hadronize by recombining with light thermal partons from the medium. This has been implemented in several ways in the literature:

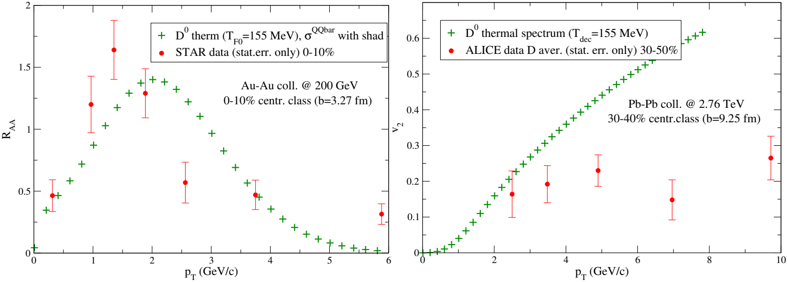
- $2 \to 1$ coalescence of partons close in phase-space: $Q + \overline{q} \to M$
- String formation: $Q + \overline{q} \to \text{string} \to \text{hadrons}$
- Resonance formation/decay $Q + \overline{q} \to M^{\star} \to Q + \overline{q}$

In-medium hadronization may affect the R_{AA} and v_2 of final D-mesons due to the *collective (radial and elliptic) flow* of light quarks.
Furthermore, it can change the HF hadrochemistry, leading for instance to and enhanced productions of strange particles (D_s) and baryons (Λ_c): no need to excite heavy $s\overline{s}$ or diquark-antidiquark pairs from the vacuum as in elementary collisions, a lot of thermal partons available nearby!
Selected results will be shown in the following.

Full kinetic equilibrium: expectations vs data

In the case in which transport coefficients are so strong to make charmed particles reach full kinetic equilibrium, they would flow with the medium, eventually decoupling from a freeze-out hypersurface

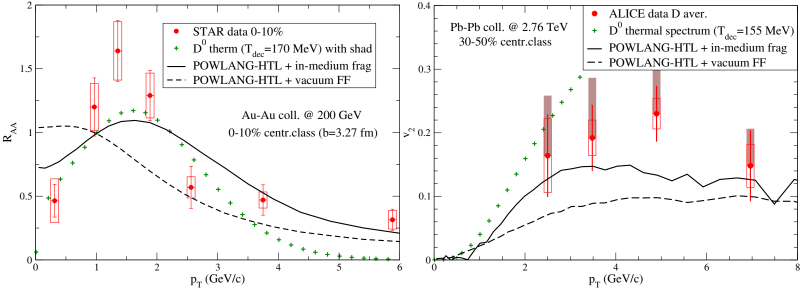
$$E(dN/d\vec{p}) = \int_{\Sigma_{\rm dec}} p^\mu d\Sigma_\mu \exp[-p\cdot u_{\rm fluid}/T_{\rm dec}]$$



The radial flow of the medium would boost particles from low to moderate p_T, while at higher p_T particles would be thermally suppressed: this would lead to a bump in the R_{AA}. The flow anisotropy translates into a sizable v_2.

From quarks to hadrons: effect on R_{AA} and v_2

Experimental data display a peak in the R_{AA} and a sizable v_2 one would like to interpret as a signal of *charm radial flow and thermalization* (green crosses: full thermal equilibrium, decoupling from FO hypersurface)



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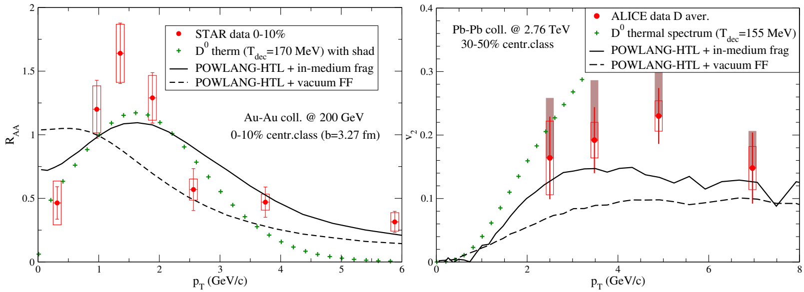
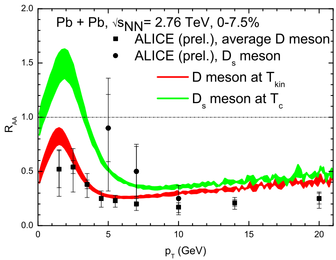


However, *comparing transport results with/without the boost* due to u^{μ}_{fluid}, at least part of the effect might be due to the radial and elliptic flow of the light partons from the medium picked-up at hadronization (POWLANG results A.B. et al., in EPJC 75 (2015) 3, 121).

In-medium hadronization and change in HF hadrochemistry

The abundance of strange quarks in the plasma can lead e.g. to an enhanced production of D_s mesons wrt p-p collisions via $c + \bar{s} \to D_s$



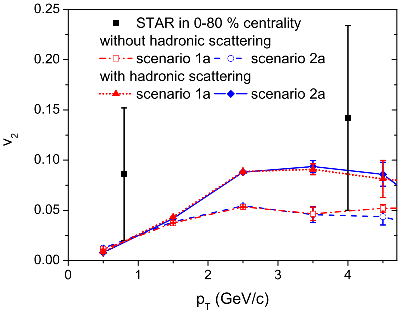
ALICE data for D and D_s mesons (A. Barbano for the ALICE Collaboration, J.Phys.Conf.Ser. 668 (2016) no.1, 012040) compared with TAMU-model predictions (M- He et al., PLB 735 (2014) 445)

Langevin transport simulation in the QGP + hadronization modeled via

$$\left(\partial_t + \vec{v} \cdot \vec{\nabla}\right) F_M(t, \vec{x}, \vec{p}) = -\underbrace{(\Gamma/\gamma_p) F_M(t, \vec{x}, \vec{p})}_{M \to Q + \bar{q}} + \underbrace{\beta(t, \vec{x}, \vec{p})}_{Q + \bar{q} \to M}$$

$$\text{with} \quad \sigma(s) = \frac{4\pi}{k^2} \frac{(\Gamma m)^2}{(s - m^2)^2 + (\Gamma m)^2}$$

Room for hadronic rescattering?



- Although characterized by smaller values of the temperature and hence of the transport coefficients, in the late hadronic stage of the evolution the fireball is characterized by the maximum elliptic flow
- Including rescattering in the hadronic phase in transport models enhances the elliptic flow (see e.g. T. Song et al., PRC 92 (2015) 1, 014910)

HF in small systems: event-by-event hydrodynamics

Event-by-event fluctuations (e.g. in the nucleon positions) modeled by Glauber-MC calculation leads to an initial *eccentricity* (responsible for a non-vanishing elliptic flow)

$$s(\mathbf{x}) = \frac{K}{2\pi\sigma^2}\sum_{i=1}^{N_{\rm coll}} \exp\left[-\frac{(\mathbf{x}-\mathbf{x}_i)^2}{2\sigma^2}\right] \quad \longrightarrow \quad \epsilon_2 = \frac{\sqrt{\{y^2-x^2\}^2+4\{xy\}^2}}{\{x^2+y^2\}}$$

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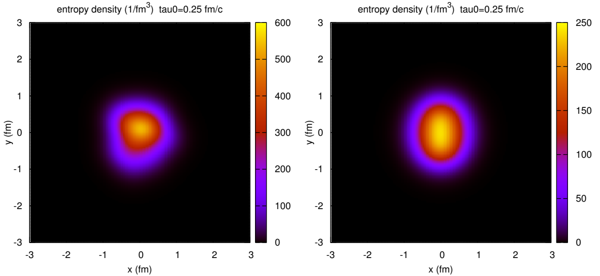
One can consider an *average background* obtained summing all the events of a given centrality class rotated by the *event-plane* angle ψ_2



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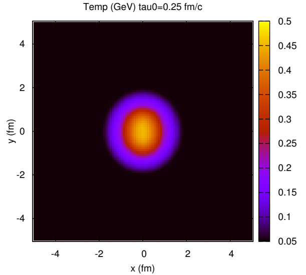
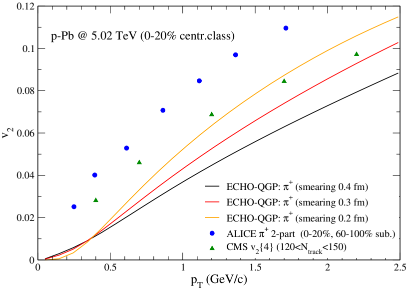
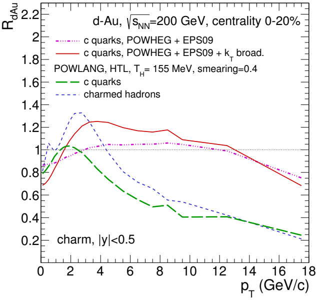




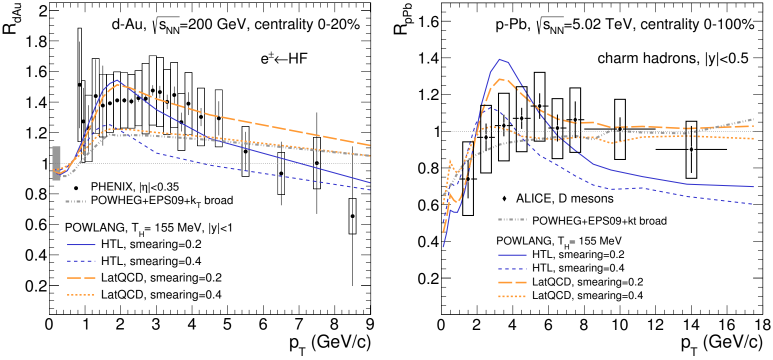
HF in small systems: Initial and Final-State effects



The final result comes from the interplay of initial and final-state effects:

- nPDF's (shadowing and anti-shadowing)
- k_T-broadening in nuclear-matter
- energy-loss in the hot-medium
- in-medium hadronization via recombination

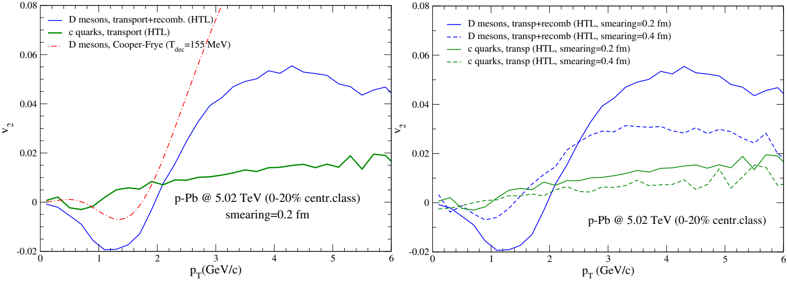
HF in small systems: transport-model predictions



We display our predictions, with different initializations (source smearing) and transport coefficients (HTL vs lQCD), compared to

- HF-electron R_{dAu} by PHENIX at RHIC (left panel)
- D-mesons R_{pPb} by ALICE at the LHC (right panel)

HF in small systems: non-vanishing elliptic flow?



We also predict a non-vanishing v_2 of charmed hadrons, arising mainly from the elliptic flow inherited from the light thermal partons

A window on topological aspects of QFT: the Chiral Magnetic Effect

In non-central high-energy nuclear collision huge magnetic fields $B \sim 10^{15}$ T are present during the first instants

- CME: conceptual setup[5]
- CME in condensed matter[6]
- CME in heavy-ion collisions: how to detect it?
- The necessity of a reliable decription of B+QGP evolution: RMHD

[5] D.E. Kharzeev et al. Prog.Part.Nucl.Phys. 88 (2016) 1
[6] Q. Li et al., Nature Phys. 12 (2016) 550

CME: $U_A(1)$ symmetry and quantum anomaly

La *massless* QCD Lagrangian is invariant under the $U_A(1)$ transformation

$$q \longrightarrow e^{-i\alpha\gamma^5} q, \quad \overline{q} \longrightarrow \overline{q} e^{-i\alpha\gamma^5} \quad (\text{since } \{\gamma^\mu, \gamma^5\} = 0)$$

rotating *by opposite angles* R and L components of the quark fields ($\gamma^5 q_{R/L} = \pm q_{R/L}$).
The symmetry *would be* associated to the conservation of the axial charge

$$Q_A = \int d^3x\, q^\dagger(x)\gamma^5 q(x) = \int d^3x\, [q_R^\dagger(x) q_R(x) - q_L^\dagger(x) q_L(x)] = N_R - N_L,$$

i.e. to the number of right-handed minus left-handed quarks.

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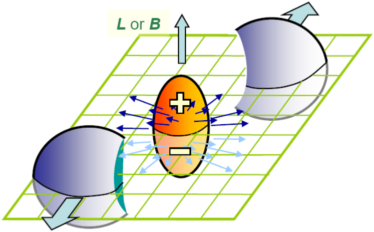
However, although being a symmetry of the *classical* QCD action, $U_A(1)$ is not a symmetry of the theory, being broken by quantum fluctuations:

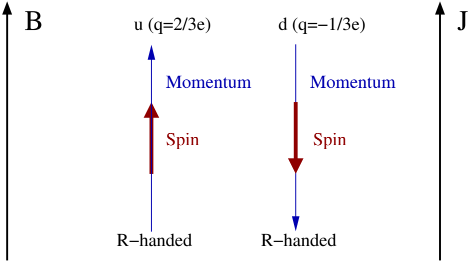
$$\frac{d}{dt}(N_R - N_L) = -N_f \frac{g^2}{16\pi^2} \int d^3x \, \frac{1}{2} \epsilon^{\alpha\beta\mu\nu} F^a_{\mu\nu} F^a_{\alpha\beta}$$

$$\equiv -N_f \frac{g^2}{16\pi^2} \int d^3x \, \widetilde{F}^{\alpha\beta,a} F^a_{\alpha\beta} \neq 0$$

Non-trivial topological configurations of the colour field can lead, event by event, to an excess of quarks of a given chirality (QCD anomaly)

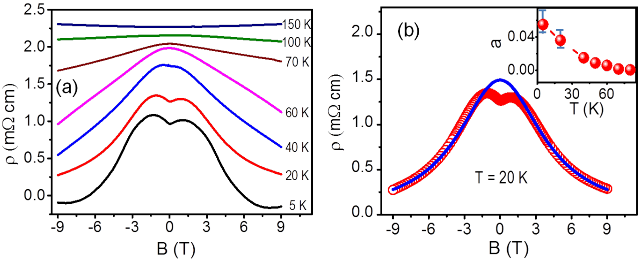
CME: the role of the magnetic field





- Huge magnetic field in the direction orthogonal to the reaction plane
- Spin of u/d quarks aligned/anti-aligned with $\vec{B}$
- Event-by-event, $U_A(1)$ anomaly leads to an excess of right or left-handed quarks
- For massless quarks chirality≡helicity $\longrightarrow$ if $N_R > N_L$ one has an excess of u-quarks moving upwards and d-quarks moving downwards: an electric current $\vec{J} \equiv \sigma_5 \vec{B}$ develops

CME in condensed matter



The discovery of Dirac semimetals opened the possibility of studying chiral fermions in condensed matter. Chiral imbalance induced by $\vec{E}||\vec{B}$, representing a non-trivial topological configuration ($\vec{E}\cdot\vec{B} \sim \widetilde{F}_{\mu\nu}F^{\mu\nu}$). Evolution of chiral charge-density (τ_V relaxation time for chirality-flip):

$$\frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2\hbar^2 c}\vec{E}\cdot\vec{B} - \frac{\rho_5}{\tau_V} \quad \longrightarrow \quad \rho_5 \underset{t\gg\tau_V}{\sim} \frac{e^2\tau_V}{4\pi^2\hbar^2 c}\vec{E}\cdot\vec{B}$$

$$\text{From} \quad \rho_5 \sim \mu_5\left(T^2 + \frac{\mu^2}{\pi^2}\right) \quad \text{and} \quad \vec{J}_{\rm CME} = \frac{e^2}{2\pi^2}\mu_5\vec{B} \quad \text{one gets}$$

$$J^i_{\rm CME} \equiv \sigma^{ij}_{\rm CME}E^j \quad \longrightarrow \quad \sigma^{zz}_{\rm CME} \sim B^2 \quad \text{(see figure)}$$