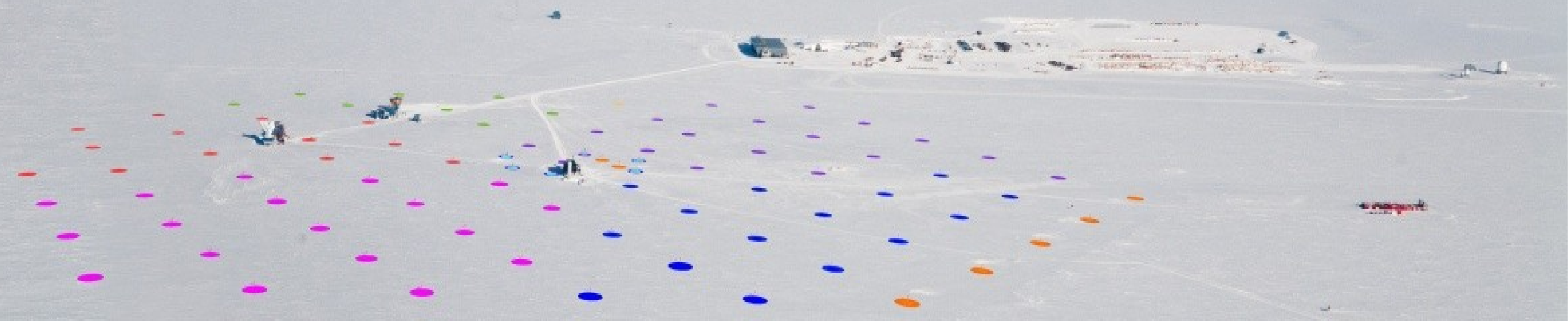
UNIVERSITÀ DEGLI STUDI DI TORINO
Scuola di Scienze della Natura
Corso di Laurea Triennale in Fisica

The attenuation length of muons in high-energy air showers measured by IceTop



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Laureando: *Rebecca Cerri*

Anno accademico: *2018/2019*

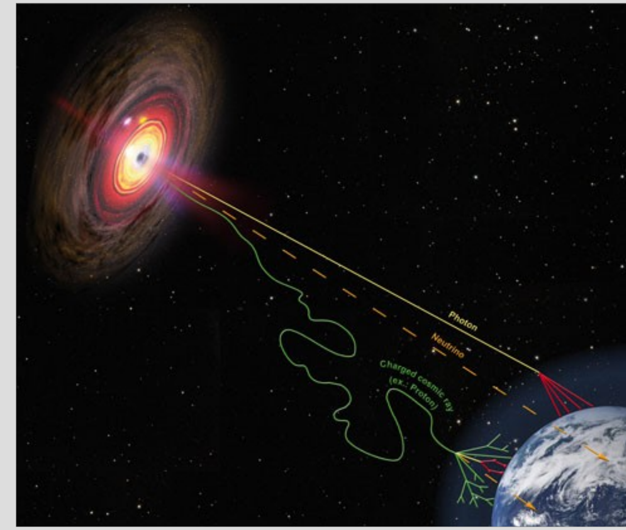
ERASMUS TRAINEESHIP

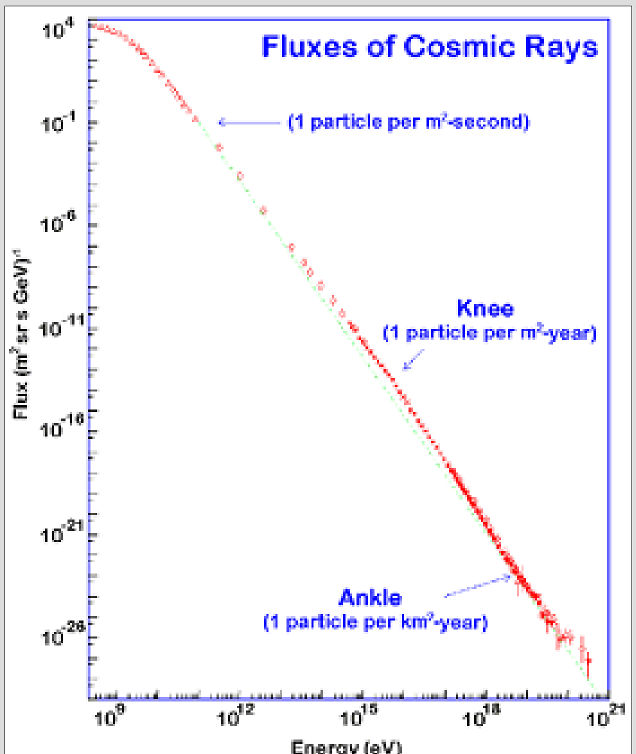
I had the opportunity to complete an Erasmus Traineeship of 2 months at the Karlsruhe Institute of Technology (KIT).
During this period I worked on my thesis project.

COSMIC RAYS

Cosmic rays are *nuclei of various chemical elements*, produced in astrophysical environments like supernovae, that propagate through galactic and inter-galactic space.

The iron nuclei and protons are the heaviest and lightest elements of the most abundant components of the cosmic radiation.





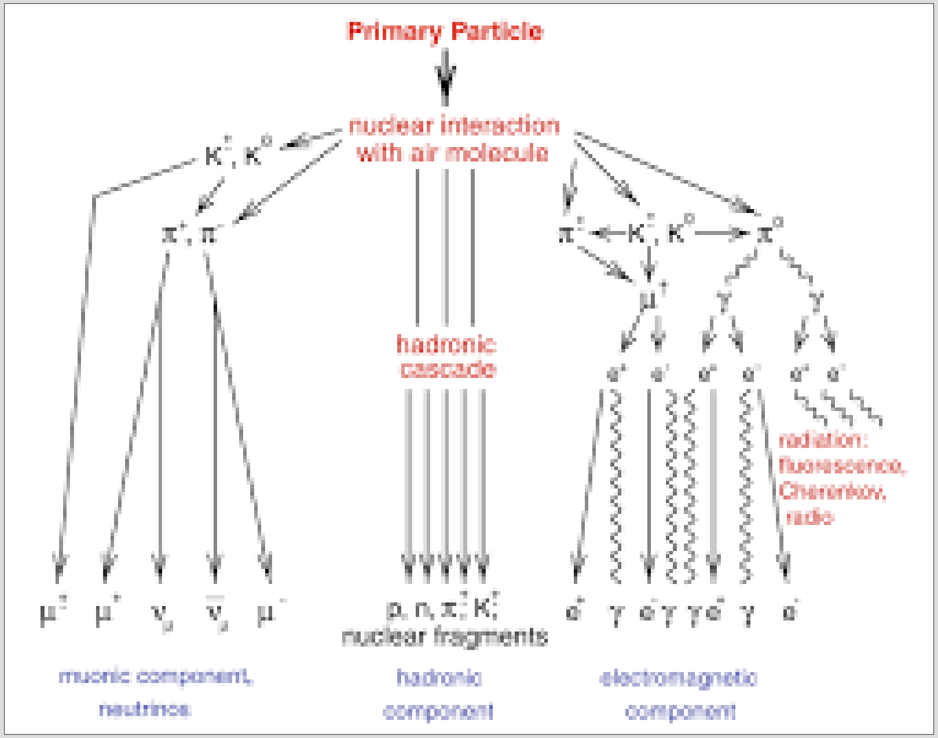
Before the knee we can have direct measurements with balloons or satellites. *After the knee we can only have indirect measurements from EAS (Extensive Air Shower).*

IceTop, the experiment on which this project is based, is used to measure the energy spectrum of cosmic ray primaries in the *range between 1 PeV and 1 EeV, therefore from the knee to close to the ankle.*

COSMIC RAYS

When a primary particle arrives in the atmosphere, it interacts with it creating an **Extensive Air Shower (EAS)**.

The shower is composed of a hadronic component in the central part and an electromagnetic component and a muonic component (and neutrinos) on the lateral part.

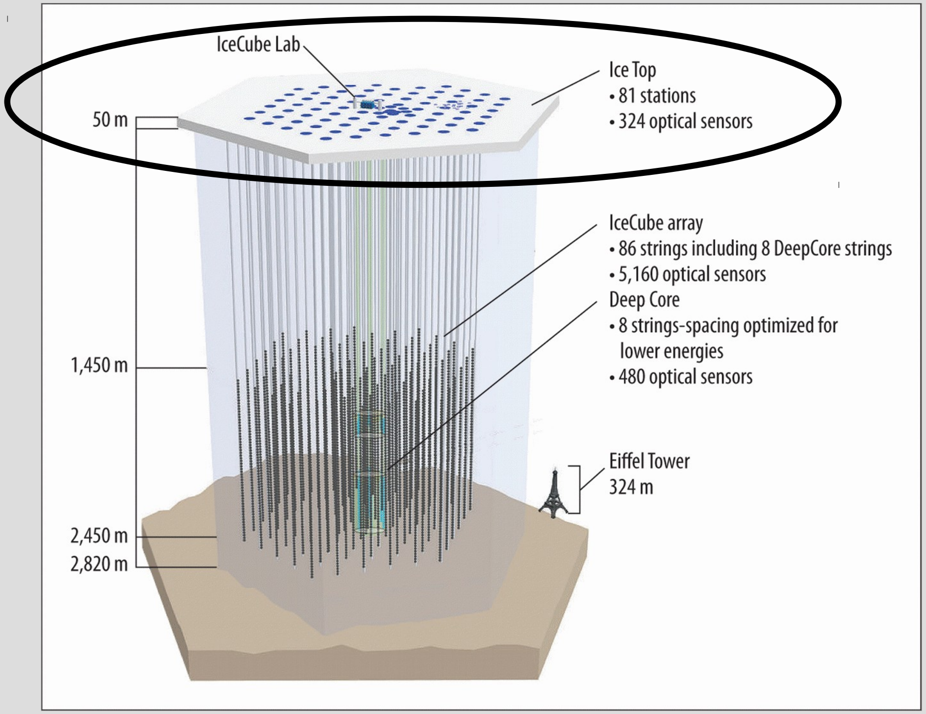


The shower has to be interpreted*:*
CORSIKA is a Monte Carlo code for detailed simulation of extensive air showers initiated by high energy cosmic ray particles. A hadronic interaction model (EPOS-LHC, SIBYLL 2.3 etc…) is used to describe the interaction of cosmic rays with the atmosphere.

IceTop

The IceCube Neutrino Observatory (IceCube) is a neutrino observatory constructed in Antarctica. **IceTop is the surface component of the IceCube detector** and it is used to measure the energy spectrum of cosmic ray primaries in the *range between 1.58 PeV and 1.26 EeV.*

Therefore IceTop is an air shower array consisting of **81 stations.**



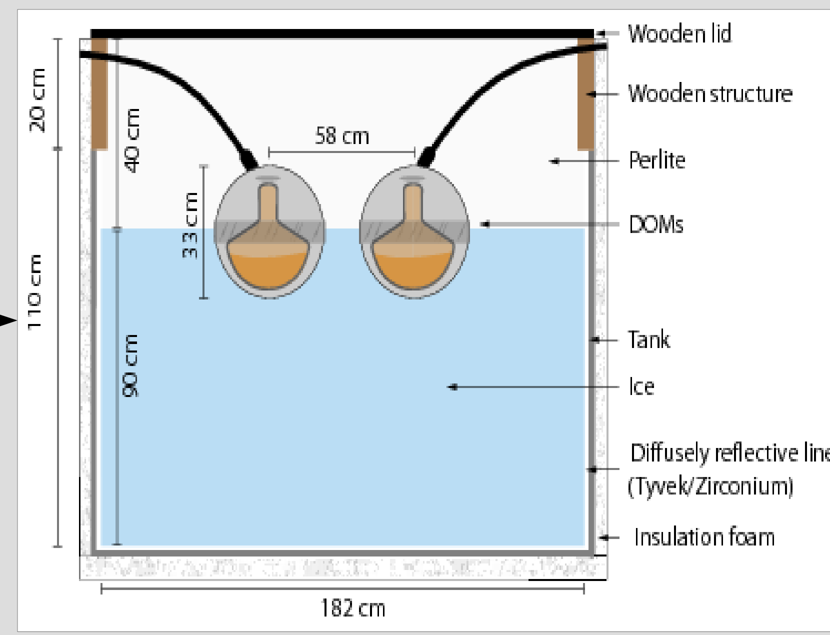
IceTop

Each *station* consists of two ice Cherenkov tanks separated by ten meters.
Each *tank* contains two Digital Optical Modules (DOMs) with a 10 inch photomultiplier tube (PMT) and electronics for signal processing and readout.

A **trigger** occurs when the signal in one of the DOMs in a tank has passed the discriminator threshold.

One station

Cross section of an IceTop tank



IceTop

The total charge collected at the PMT's anode, after digitization and baseline subtraction, **constitutes the tank's signal**.
The tanks register signals ranging from 0.2 to 1000 Vertical Equivalent Muons (VEM).

HLC signal
(Hard Local Coincidence signal)
→ When there are triggers in two neighboring tanks within a time window of 1 μs.

SLC signal
(Soft Local Coincidence signal)
→ When the partner tank within the station did not have a trigger.

SLC signals occur at large lateral distances, where the triggering probability is smallest.

Electrons and muons may give both types of signal.

AIM OF THE PROJECT

It is important **to validate the hadronic interaction models** (EPOS-LHC, SIBYLL 2.3 etc…).

Many research groups have already done this. However at KIT students have the opportunity to do it again.
Marta Bianciotto, a physics student, has worked on the hadronic interaction model SIBYLL 2.3.
In this project the model EPOS-LHC* was studied.

Therefore this dissertation will show the procedure and steps in this project and will compare the results of the two models.

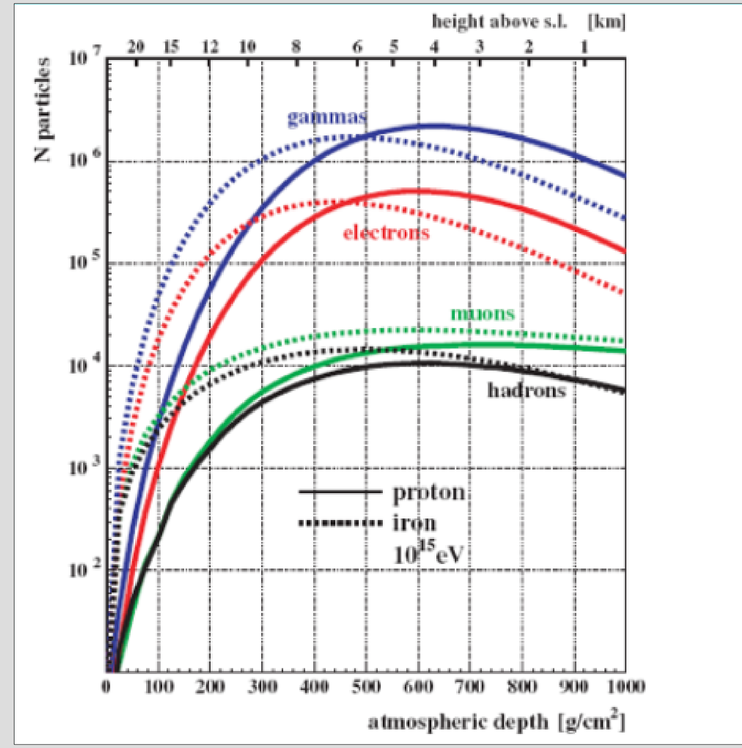
*EPOS is a sophisticated multiple scattering approach based on partons and Pomerons (parton ladders), with special emphasis on high parton densities.
EPOS-LHC is the latest version of this model fine tuned on the current results obtained at LHC.

AIM OF THE PROJECT

The particles that are produced in the shower are later reabsorbed by the atmosphere.

For example the image represents the development in the atmosphere of EAS produced by protons or iron nuclei at $E=10^{15}$ eV.

Therefore the number of these particles changes while the shower is developing in the atmosphere.
The attenuation length measures the rate of absorption of the particles.



To validate the model EPOS-LHC **the attenuation length of muons, $\alpha_\mu(r)$,** has been calculated.
This parameter is expressed in *g/cm²*, which means that this parameter *is the quantity of grams of atmosphere contained in a cm² that reduces the flow of muons of 1/e*.

AIM OF THE PROJECT

The relation between the muon density on the ground and the zenith angle of the primary particle can be expressed by:

$$\rho_\mu(r,\theta) = \rho^0_\mu(r) e^{-X_0 \sec\theta/\alpha_\mu(r)}$$

Where:

- X_0 is the depth of the atmosphere expressed in g/cm^2 and for IceTop X_0 = 692 g/ cm^2
- $\rho_{\mu,0}(r)$ is a normalization factor
- **$\alpha_\mu(r)$ is the attenuation length of muons.**

$\rho_{\mu,0}(r)$ and $\alpha_\mu(r)$ are two parameters of the fit and we need $\alpha_\mu(r)$.

PROCEDURE

Simulation data for proton and iron obtained using the model EPOS-LHC were provided.

1) The attenuation length of muons for proton and iron, ***α*, has been obtained from the simulation data.**

2) The attenuation length of muons, **α, has been obtained from the experimental data.**

3) These values have been compared.

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DEFINE DIFFERENT RANGE

In the showers many parameters and values depend on the energy, on the zenith angle and on the radial distance from the core.

Log10(Energy/GeV)

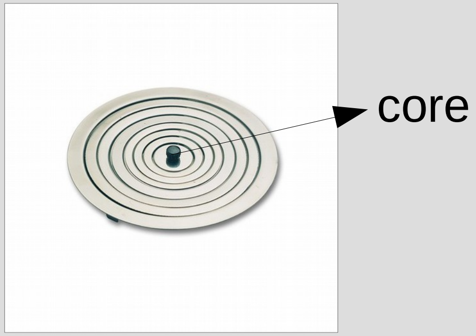
Energy range	Lower end	Upper end
1	6	6.5
2	6.5	7
3	7	7.5
4	7.5	8

Zenith angle

Angular range	Lower end [°]	Upper end [°]
1	0.0	17.51
2	17.51	25.19
3	25.19	31.41
4	31.41	37.0

Radial distance from the core

Radial distance range or ring	Lower end [m]	Upper end [m]
1	220	320
2	320	420
3	420	520
4	520	620
5	620	720
6	720	820
7	820	920

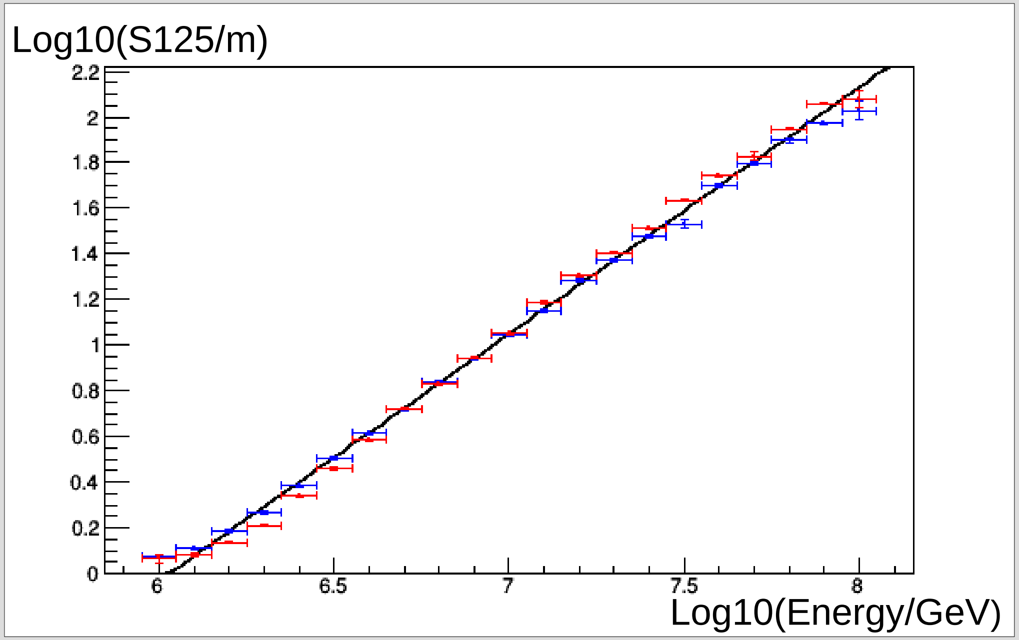


EXP. DATA – THE ENERGY ESTIMATION

The parameter S125 was used as an <u>estimator</u> of the energy of the experimental data.
S125 is a parameter of the shower and it represents the distance from the core where the muon density is approximately the same for every type of particle.

Therefore:
1) The simulation data for proton and iron have been taken
2) The relation between the energy and the parameter S125 has been studied
3) The relation obtained was applied to the experimental data

PROTON AND IRON, Energy 6-8, Angular range 1



y = p0 + p1*x

X = mean of the bin
ΔX = width of the bin

Y = mean of the distribution of the parameter s125 in that bin
ΔY = The standard deviation of the mean of the distribution of the s125 in that bin

EXP. DATA – THE ENERGY ESTIMATION

$y = p0 + p1*x$

	p0	δp0	p1	δp1	χ 2	N. Dof	Prob.
Angular range 1	-6.5	0.1	1.08	0.01	22.19	40	0.99
Angular range 2	-6.5	0.1	1.08	0.01	26.45	40	0.95
Angular range 3	-6.90	0.17	1.12	0.02	14.96	30	0.99
Angular range 4	-7.22	0.16	1.16	0.02	21.81	30	0.86

The values of the Log10(S125/m) corresponding to the Log10(Energy/GeV)=7 and the Log10(Energy/GeV)=7.5.

Log10(S125/m)	Log10(S125/m)
1.06	1.60
1.06	1.60
0.94	1.50
0.90	1.48

To obtain the attenuation length:

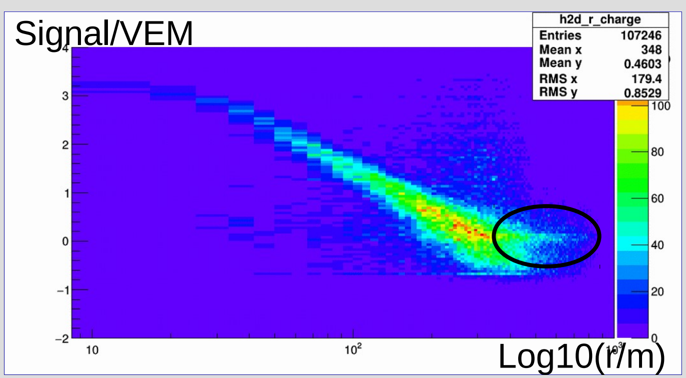
2-DIMESIONAL HISTOGRAMS (Radial distance from the core – Signal)

As explained earlier in this dissertation, *“signal”* means the charge signal produced by electrons or muons in the detector.

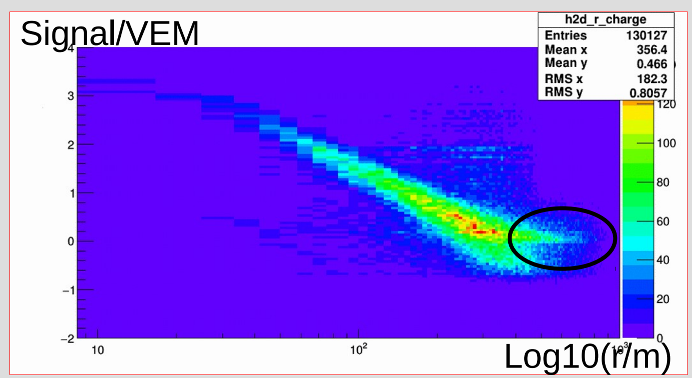
In these histograms the two types of signal have been included: <u>HLC and SLC</u>.

Energy range 3, Angular range 1:

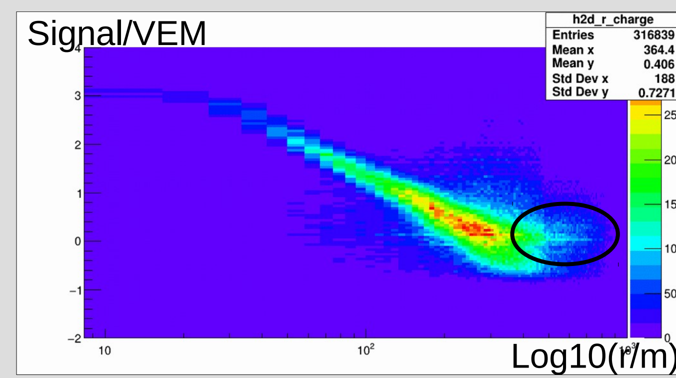
PROTON



IRON



EXP. DATA

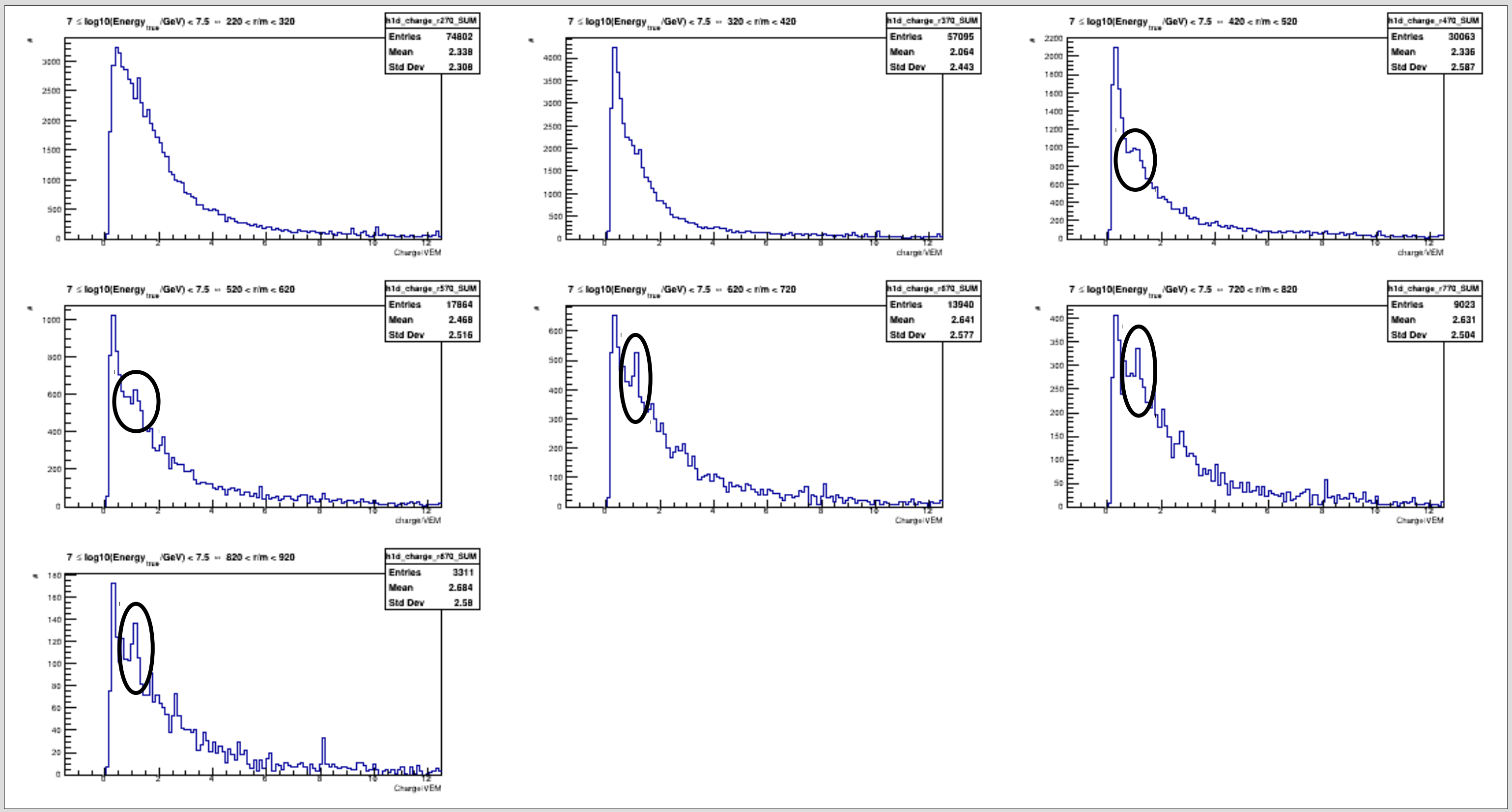


To obtain the attenuation length:

1-DIMESIONAL HISTOGRAMS (Signal)

Seven 1-dimensional histograms have been found from each 2-dimensional histogram with energy range 3.

EXP DATA: *Energy range 3, Angular range 1, each Radial distance range*

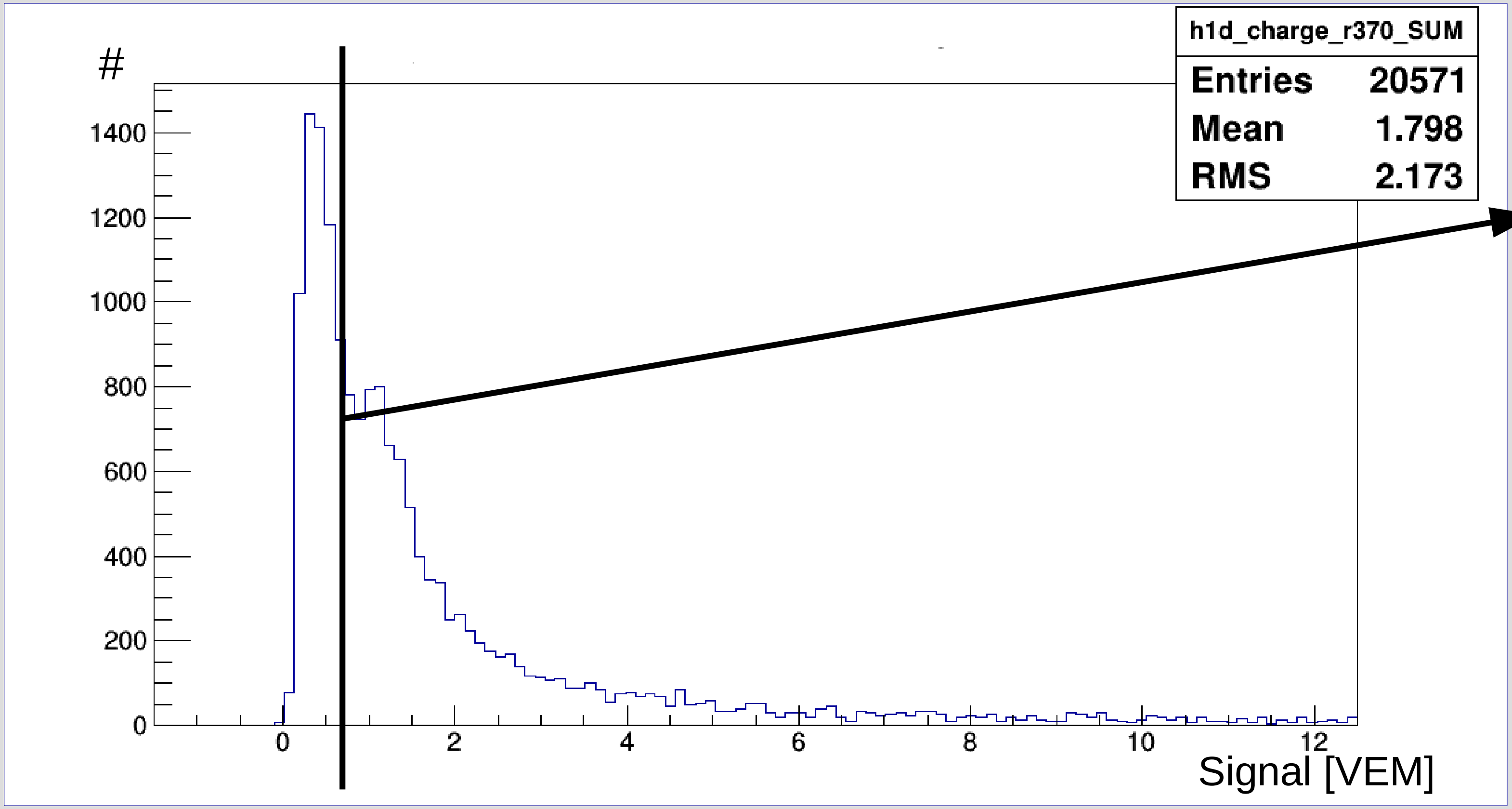


*X-axis: S*ignal [VEM]
Y-axis: Number of events

To obtain the attenuation length:

The *method to separate the signal produced by muons from the signal produced by electrons* has been decided.

PROTON: Energy range 3, Angular range 1, Radial distance range 2



0.7 VEM is the value of the separation.

0.7 VEM is twice the value of the signal where there is the peak of electrons.

To obtain the attenuation length:

It has been calculated *for each shower*:

- The sum of all signals produced by muons above 0.7 VEM *in each ring*.
- The number of the tanks *in each ring.*

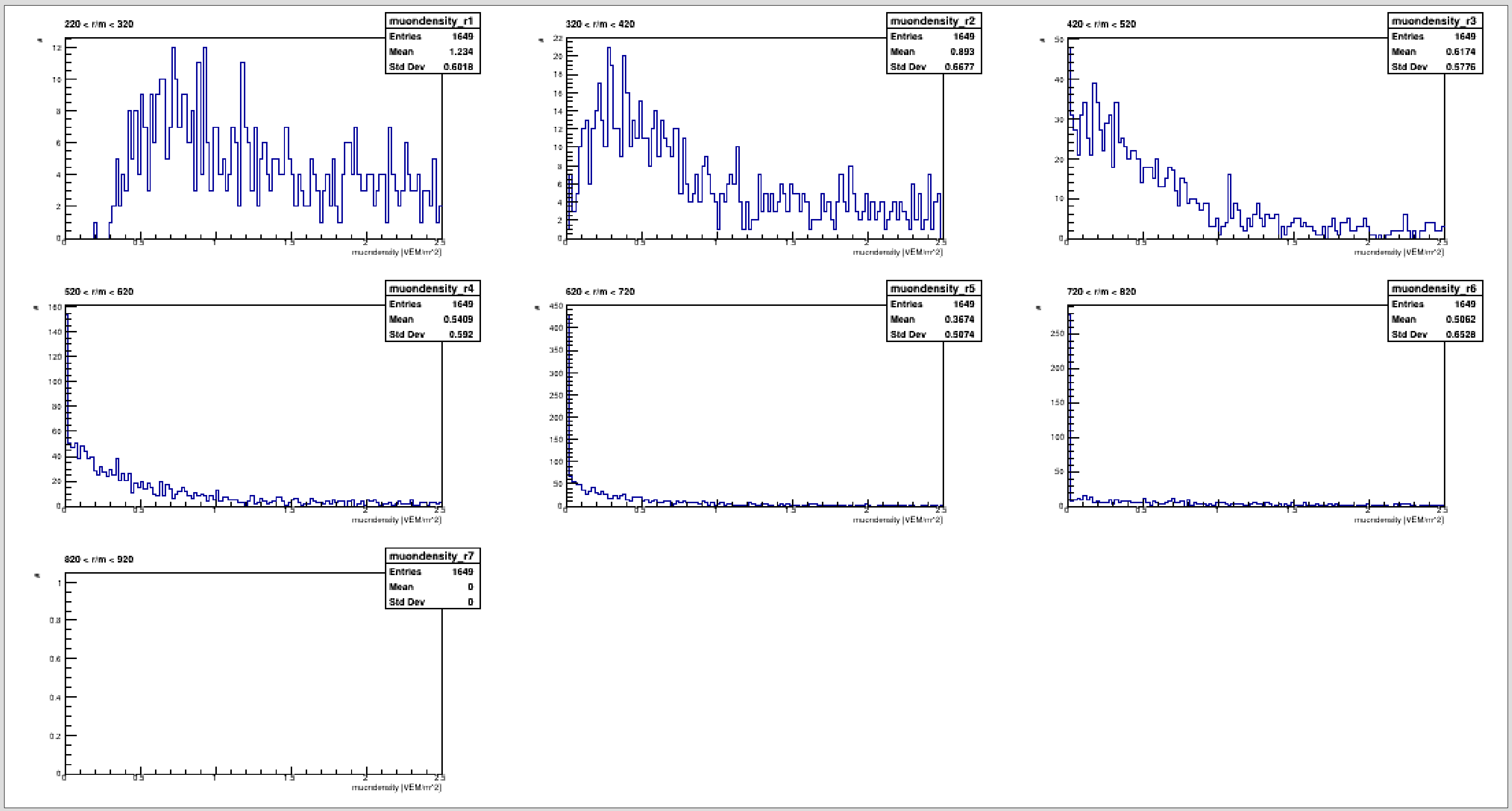
From these values the **muon density on the ground** *in each ring* has been obtained*:*

$$\text{Muon density} = \frac{\text{Sum of all signals produced by muons above 0.7 VEM}}{\text{Number of tanks * Area of one tank}}$$

To obtain the attenuation length:

1-DIMESIONAL HISTOGRAM (Muon density)

EXP. DATA: *Energy range 3, Angular range 1, each Radial distance range*

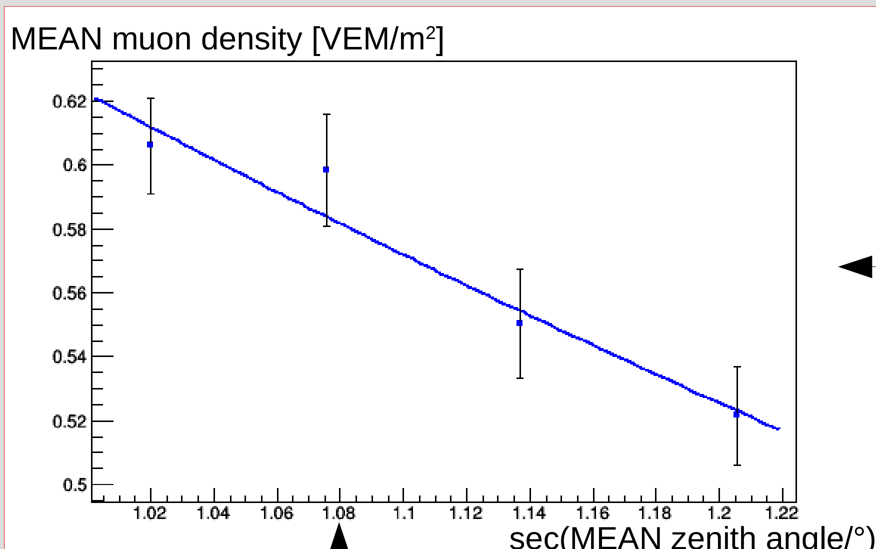


X-axis: Muon density [VEM/m^2], *Y-axis:* Number of events

To obtain the attenuation length:

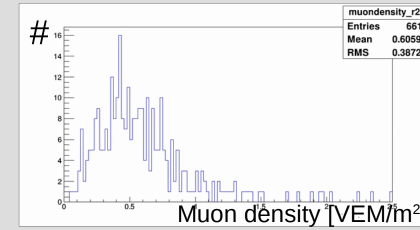
IRON, Energy range 3, Radial distance range 2

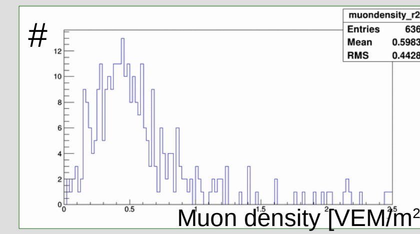
$$\rho_\mu(r,\theta) = \rho^0_\mu(r)e^{-X_0 \sec\theta/\alpha_\mu(r)}$$

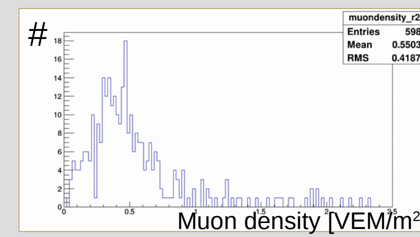


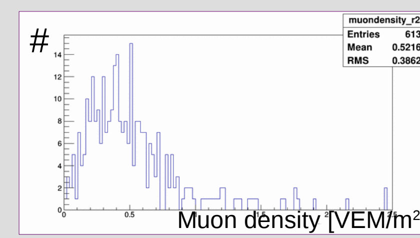
Y coordinates are the <u>mean</u> of these muon density histograms.

Y errors are the <u>deviation standard of the mean</u> of these histograms.

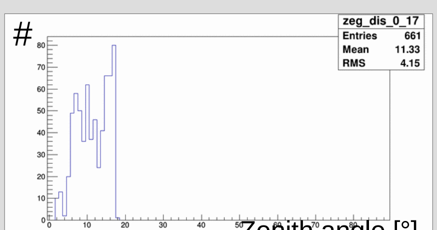


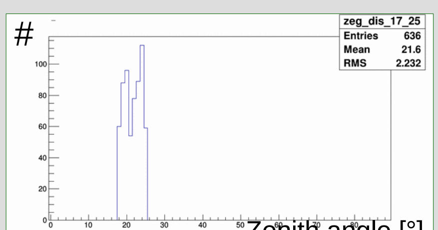


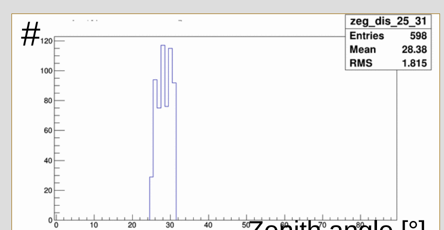


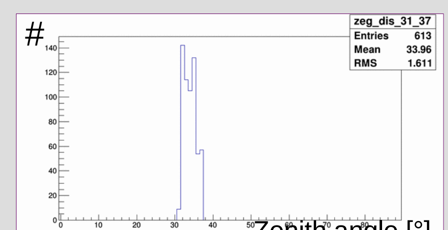


X coordinates are the <u>secant of the mean</u> of these zenith angle histograms.





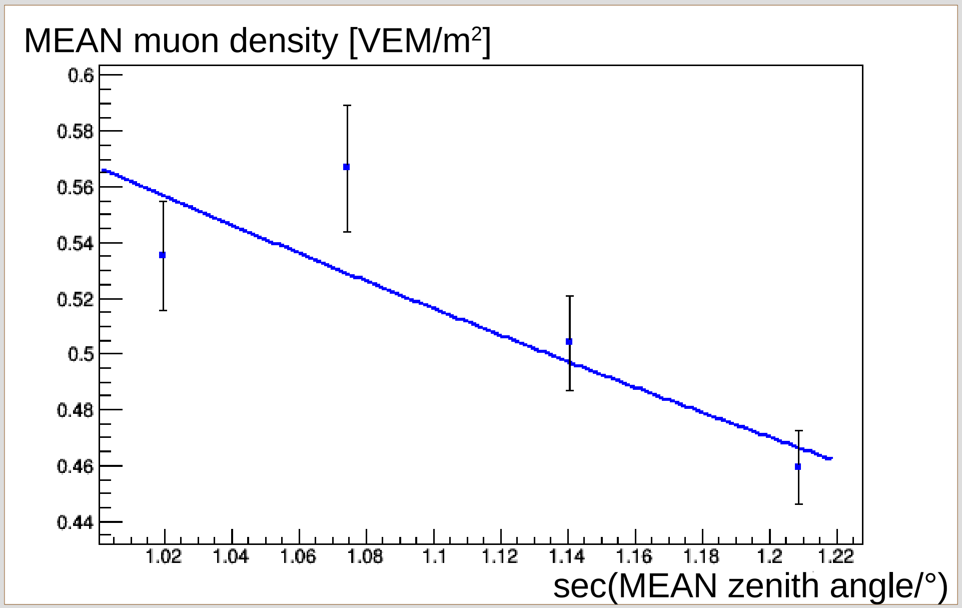




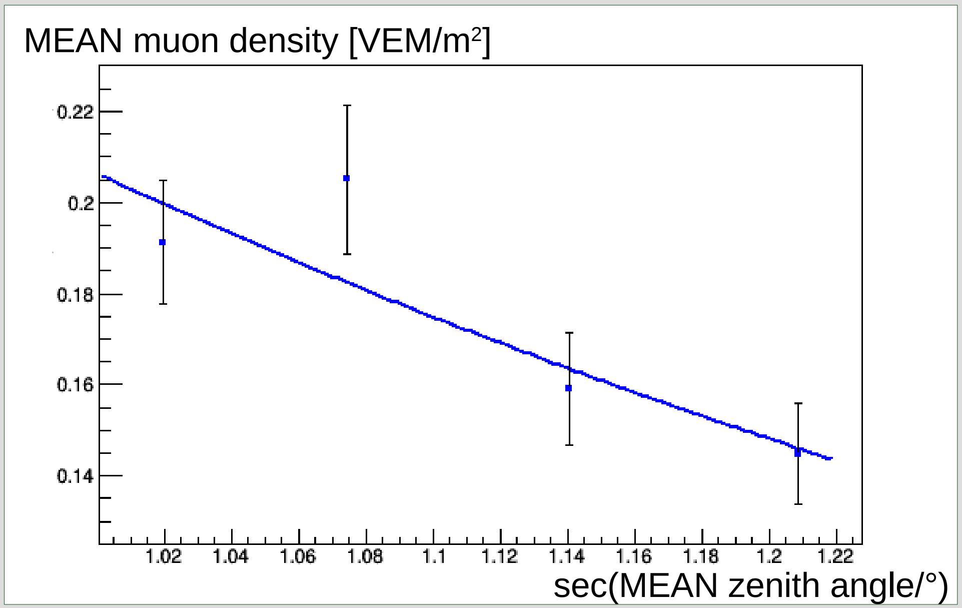
To obtain the attenuation length:

PROTON

Energy range 3, Radial distance range 2



Energy range 3, Radial distance range 5



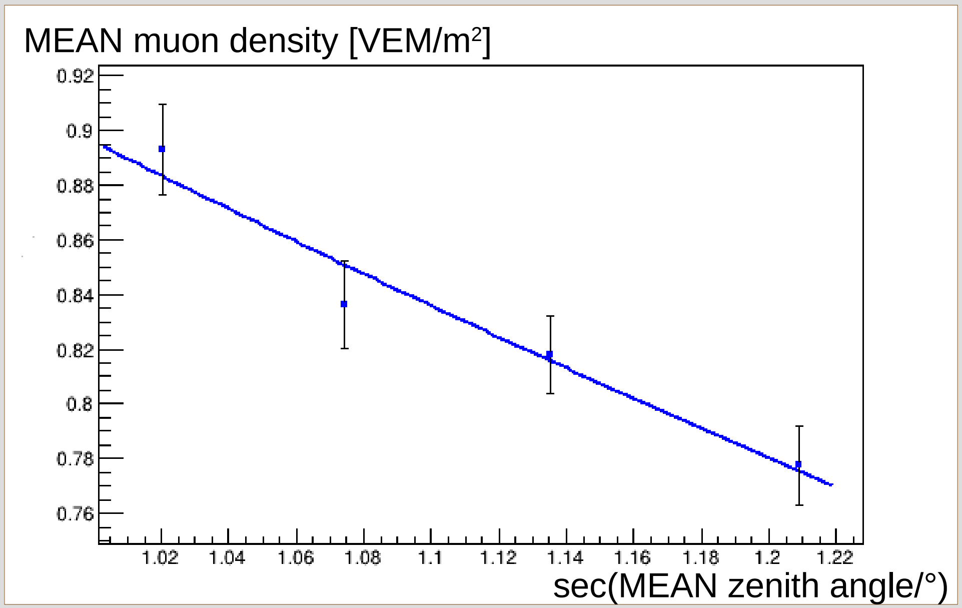
Range [m]	$P^0_\mu(r)$ [VEM/m^2]	$\delta P^0_\mu(r)$ [VEM/m^2]	$\alpha_\mu(r)$ [g/cm^2]	$\delta\alpha_\mu(r)$ [g/cm^2]	χ 2	N. of Dof.	Prob.
320-420	1.45	0.35	738	175	4.44	2	0.11
620-720	1.08	0.54	417	127	2.46	2	0.29

χ^2 test passed for both fits

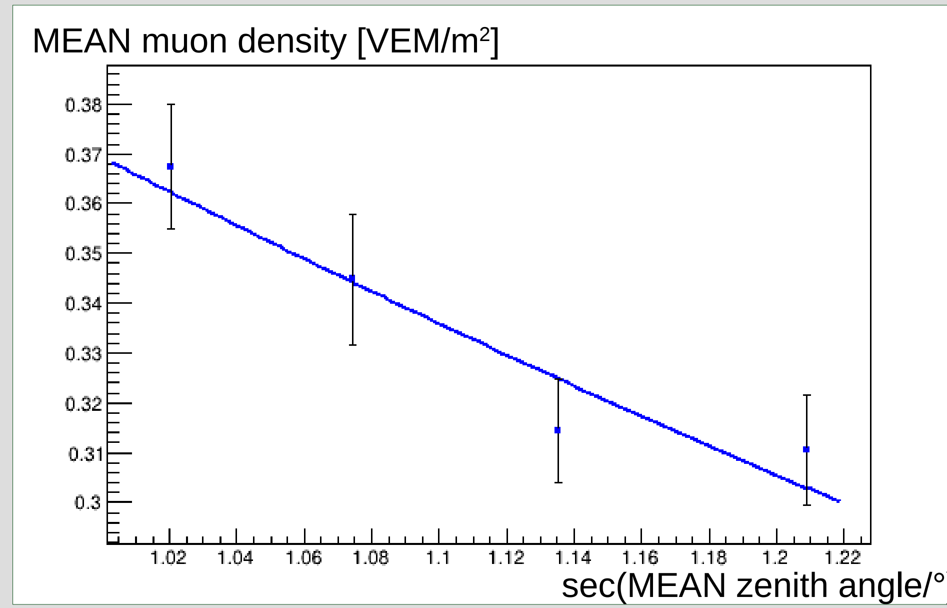
To obtain the attenuation length:

EXP. DATA

Energy range 3, Radial distance range 2



Energy range 3, Radial distance range 5



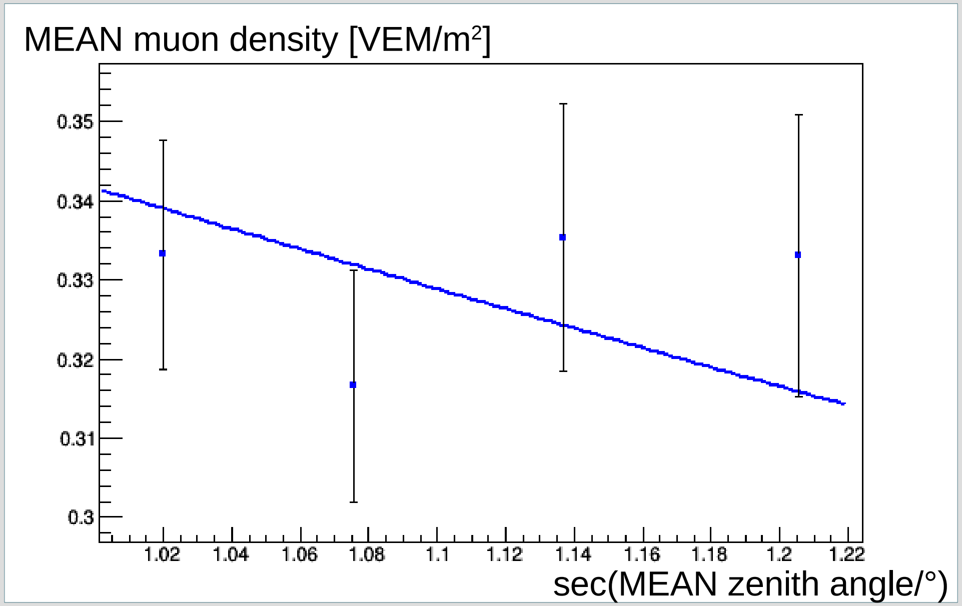
Range [m]	**$P^0_\mu(r)$** [VEM/m^2]	**$\delta P^0_\mu(r)$** [VEM/m^2]	**$\alpha_\mu(r)$** [g/cm^2]	**$\delta\alpha_\mu(r)$** [g/cm^2]	**χ 2**	**N. of Dof.**	**Prob.**
320-420	1.79	0.26	999	191	1.26	2	0.53
620-720	0.95	0.27	729	196	1.68	2	0.43

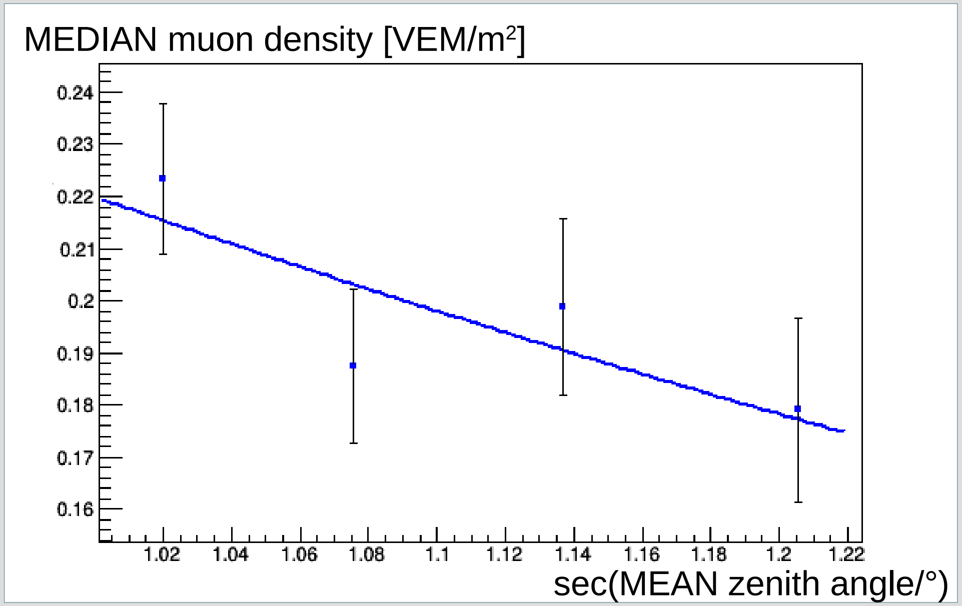
χ^2 test passed for both fits

To obtain the attenuation length:

IRON

MEAN	*MEDIAN*
Energy range 3, Radial distance range 4	Energy range 3, Radial distance range 4





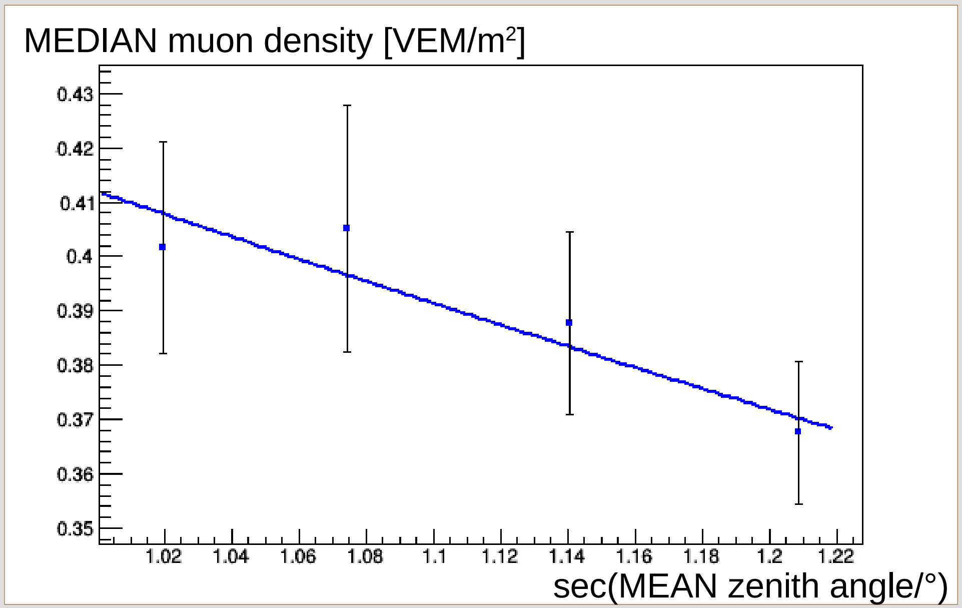
	Range [m]	$P^0_\mu(r)$ [VEM/m^2]	$\delta P^0_\mu(r)$ [VEM/m^2]	$\alpha_\mu(r)$ [g/cm^2]	$\delta\alpha_\mu(r)$ [g/cm^2]	χ 2	N. of Dof.	Prob.
MEAN	**520-620**	0.50	0.07	1817	104	2.61	2	0.27
MEDIAN	**520-620**	0.63	0.35	659	371	1.69	2	0.43

χ^2 test passed for both fits

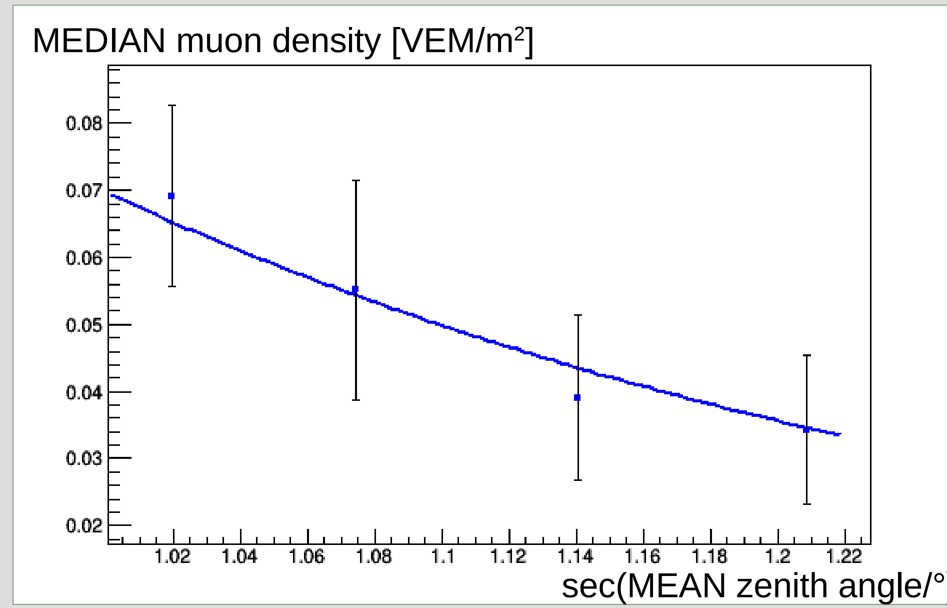
To obtain the attenuation length:

PROTON *MEDIAN*

Energy range 3, Radial distance range 2



Energy range 3, Radial distance range 5



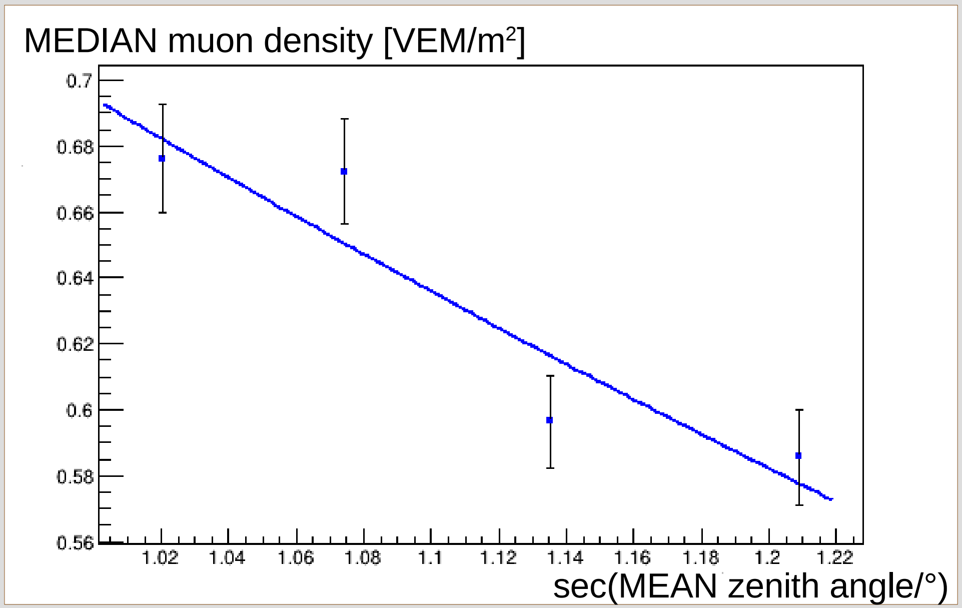
Range [m]	$P^0_\mu(r)$ [VEM/m^2]	$\delta P^0_\mu(r)$ [VEM/m^2]	$\alpha_\mu(r)$ [g/cm^2]	$\delta\alpha_\mu(r)$ [g/cm^2]	χ 2	N. of Dof.	Prob.
320-420	0.69	0.22	1351	688	0.35	2	0.84
620-720	2.00	1.35	206	7	0.21	2	0.90

χ^2 test passed for both fits

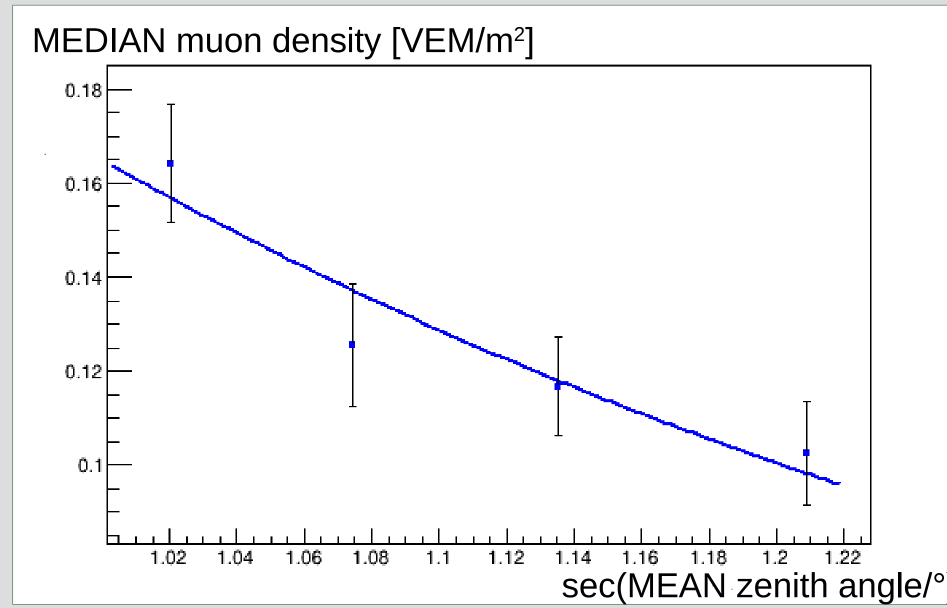
To obtain the attenuation length:

EXP. DATA *MEDIAN*

Energy range 3, Radial distance range 2



Energy range 3, Radial distance range 5



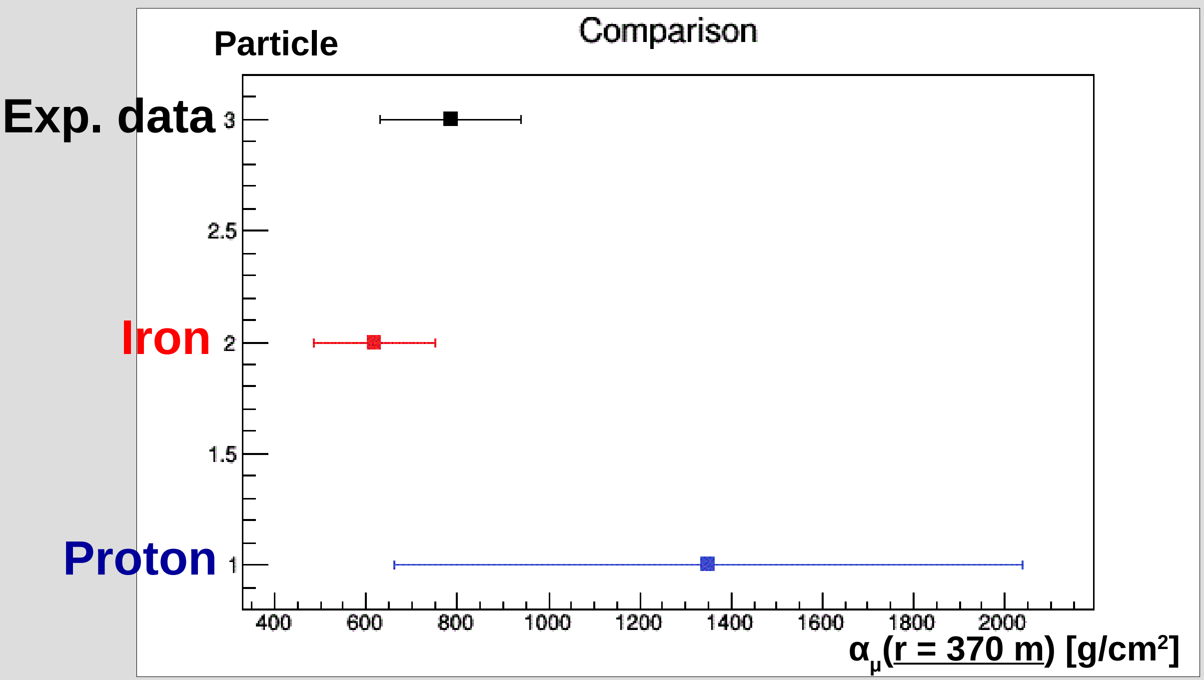
Range [m]	**$P^0_\mu(r)$** [VEM/m^2]	**$\delta P^0_\mu(r)$** [VEM/m^2]	**$\alpha_\mu(r)$** [g/cm^2]	**$\delta\alpha_\mu(r)$** [g/cm^2]	**χ 2**	**N. of Dof.**	**Prob.**
320-420	1.68	0.33	785	155	4.29	2	0.12
620-720	2.0	1.5	279	78	1.34	2	0.51

χ^2 test passed for both fits

COMPARISON (MEDIAN)

RADIAL DISTANCE RANGE 2 (320 m – 420 m)

	$\alpha_\mu(r)$ [g/cm^2]	$\delta\alpha_\mu(r)$ [g/cm^2]	χ 2	N. of Dof.	Prob.
Experimental data	785	155	4.29	2	0.12
Iron	618	133	0.51	2	0.77
Proton	1351	688	0.35	2	0.84

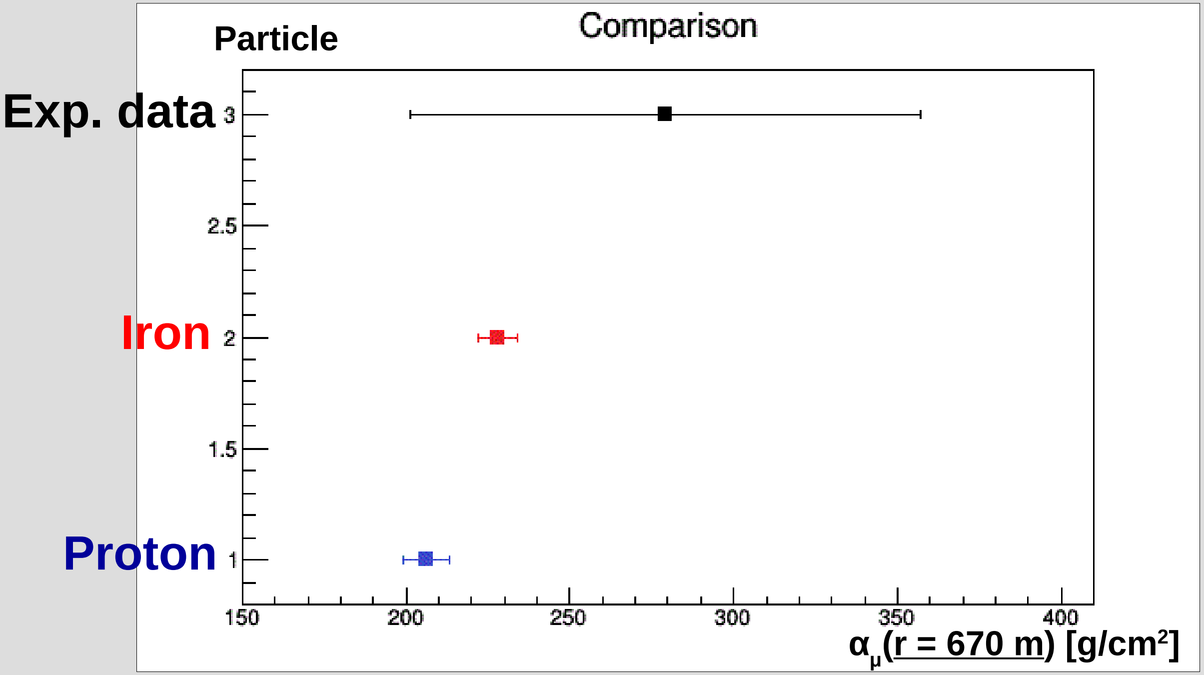


The model EPOS-LHC describes the experimental data. However, it must be taken into account that the error bar of proton is very big.

COMPARISON (MEDIAN)

RADIAL DISTANCE RANGE 5 (620 m – 720 m)

	$\alpha_\mu(r)$ [g/cm^2]	$\delta\alpha_\mu(r)$ [g/cm^2]	χ 2	N. of Dof.	Prob.
Experimental data	279	78	1.34	2	0.51
Iron	228	6	1.37	2	0.50
Proton	206	7	0.21	2	0.90



The model EPOS-LHC describes the experimental data. However, it must be taken into account the different width of the error bars.

Comparison SIBYLL 2.3 and EPOS LHC (MEAN)

It is very useful *to compare the results obtained with EPOS-LHC with the results obtained with another hadronic interaction model.* In fact, in this way it can be understood if the limits of this model depend on the model itself or on the cut of the charge signal produced by muons.
Marta Bianciotto, a former student, has worked on the model SIBYLL 2.3.

With proper attention *the $\alpha_{\mu}(r)$ values obtained from the simulation data of EPOS-LHC, the simulation data of SIBYLL 2.3 and the experimental data* have been compared:

- Marta Bianciotto did the fits using only the MEAN of muon density.
  Therefore, also for EPOS-LHC, the $\alpha_{\mu}(r)$ values obtained by using the MEAN of muon density in the fits, has been taken.

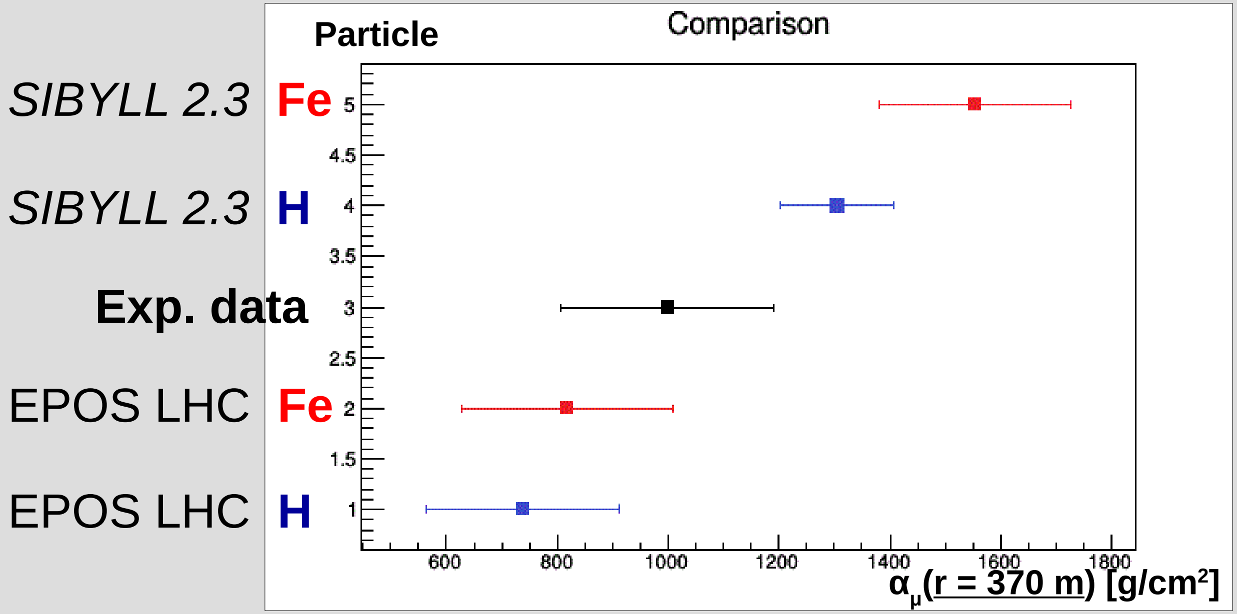
- The experimental data analysed in this project have been taken.
  This because the number of events of the experimental data taken in this project is bigger than the number of events of the experimental data taken in Marta's work.

Comparison SIBYLL 2.3 and EPOS LHC (MEAN)

Karlsruher Institut für Technologie

RADIAL DISTANCE RANGE 2 (320 m – 420 m)

	$\alpha_\mu(r)$ [g/cm^2]	$\delta\alpha_\mu(r)$ [g/cm^2]	χ 2	N. of Dof.
SIBYLL 2.3 **Fe**	1554	173	5.54	2
SIBYLL 2.3 **H**	1305	103	2.15	2
Exp. Data	999	191	1.26	2
EPOS LHC **Fe**	819	190	0.91	2
EPOS LHC **H**	738	175	4.44	2



EPOS-LHC:
The model describes the experimental data.

SIBYLL 2.3:

Gauss's Test

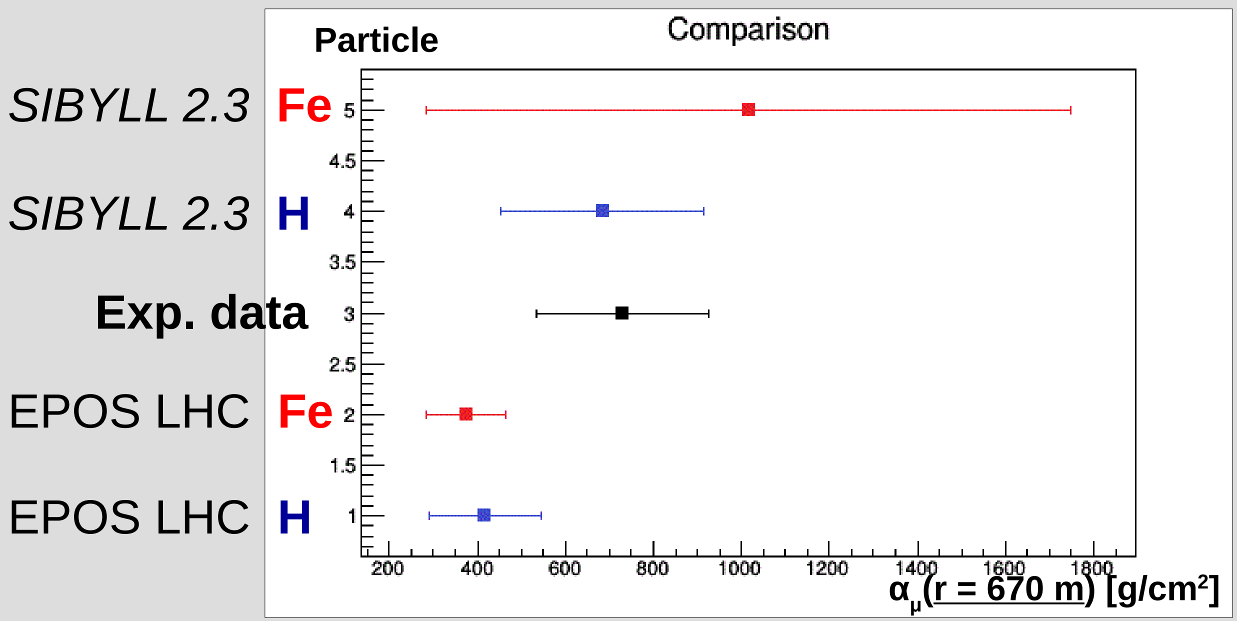
Proton and Exp. Data Z=1.41

Iron and Exp. Data Z=2.15

Comparison SIBYLL 2.3 and EPOS LHC (MEAN)

RADIAL DISTANCE RANGE 5 (620 m – 720 m)

	$\alpha_\mu(r)$ [g/cm^2]	$\delta\alpha_\mu(r)$ [g/cm^2]	χ 2	N. of Dof.
SIBYLL 2.3 **Fe**	1016	733	1.59	2
SIBYLL 2.3 **H**	684	232	0.54	2
Exp. Data	729	196	1.68	2
EPOS LHC **Fe**	375	91	3.41	2
EPOS LHC **H**	417	127	2.46	2



EPOS-LHC:
The model describes the experimental data.
Gauss's Test
Iron and Exp. Data: Z=1.64

SIBYLL 2.3:
The model describes the experimental data. However, it must be taken into account that the error bar of iron is very big.

CONCLUSIONS

The aim of the project was to validate the hadronic interaction model EPOS-LHC using the attenuation length of muons.

$\alpha_\mu(r)$ has been obtained for 5 radial distance range for proton, iron and experimental data.
These values have been obtained using the MEAN and the MEDIAN of the 1-dimensional histograms of muon density.
It has been noted that *the results using the MEDIAN are better, but there are still some problems*.

To understand if these problems are due to the model EPOS-LHC or to the cut of the signal produced by muons, the $\alpha_\mu(r)$ values obtained from EPOS-LHC and SIBYLL 2.3 have been compared.
It has become clear that *the limits found in this project are most probably due to the cut of the signal, independent from the model*. This is because almost all of the problems are found in both models.

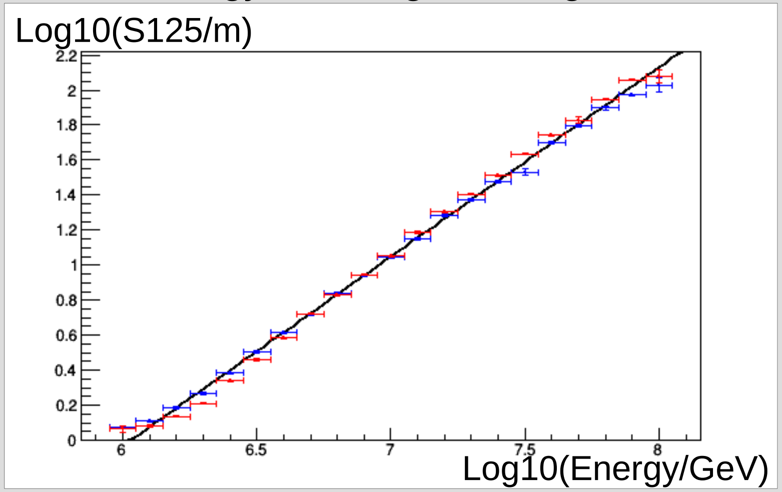
The question that remains is to understand the value of the signal where the cut is applied.

THANK YOU!

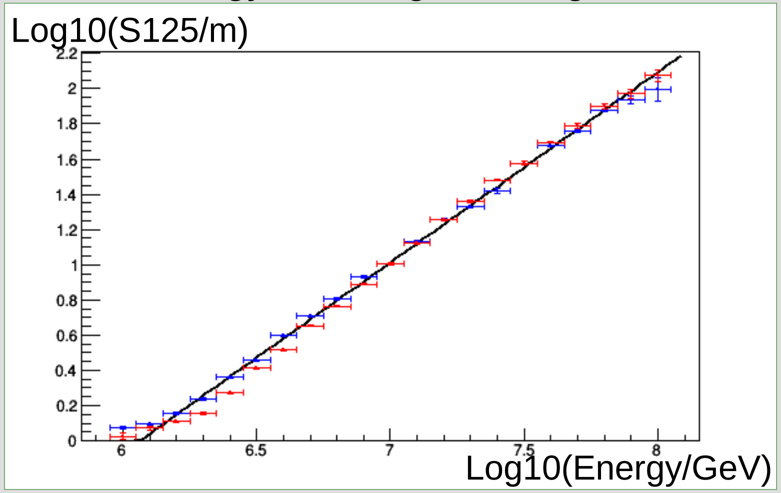
EXP. DATA – THE ENERGY ESTIMATION

PROTON and IRON

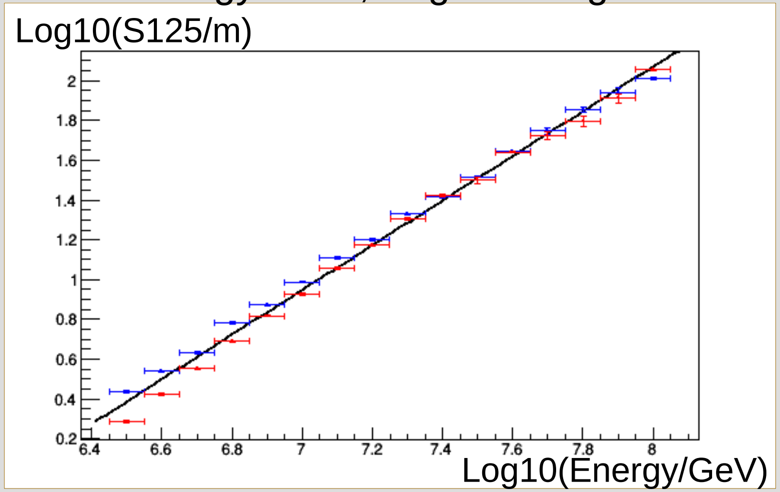
Energy 6-8, Angular range 1



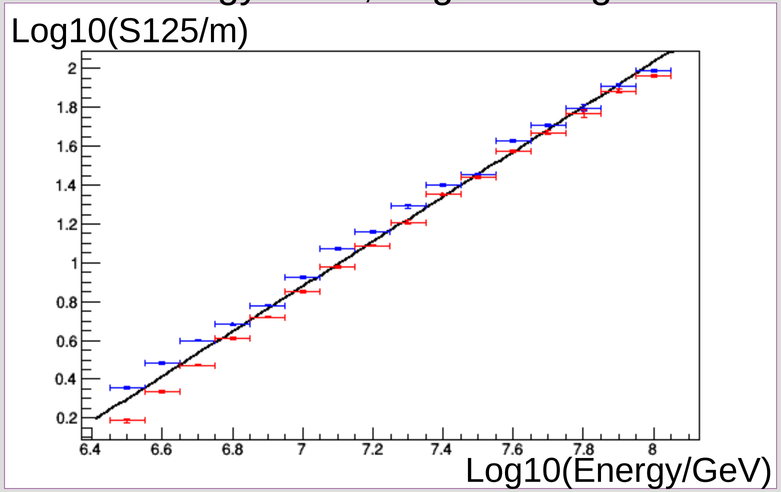
Energy 6-8, Angular range 2



Energy 6.5-8, Angular range 3



Energy 6.5-8, Angular range 4



y = p0 + p1*x

X = mean of the bin
ΔX = width of the bin

Y = mean of the distribution of the parameter s125 in that bin
ΔY = The standard deviation of the mean of the distribution of the s125 in that bin

The last two fits (angular range 3 and 4) start from the energy 6.5.

EXP. DATA – THE ENERGY ESTIMATION

PROTON and IRON

The values of Log10(s125/m) corresponding to the Log10(Energy/GeV)=7 and the Log10(Energy/GeV)=7.5.

y = p0 + p1*x

	p0	δp0	p1	δp1	χ 2	N. Dof	Prob.
Angular range 1	-6.5	0.1	1.08	0.01	22.19	40	0.99
Angular range 2	-6.5	0.1	1.08	0.01	26.45	40	0.95
Angular range 3	-6.90	0.17	1.12	0.02	14.96	30	0.99
Angular range 4	-7.22	0.16	1.16	0.02	21.81	30	0.86

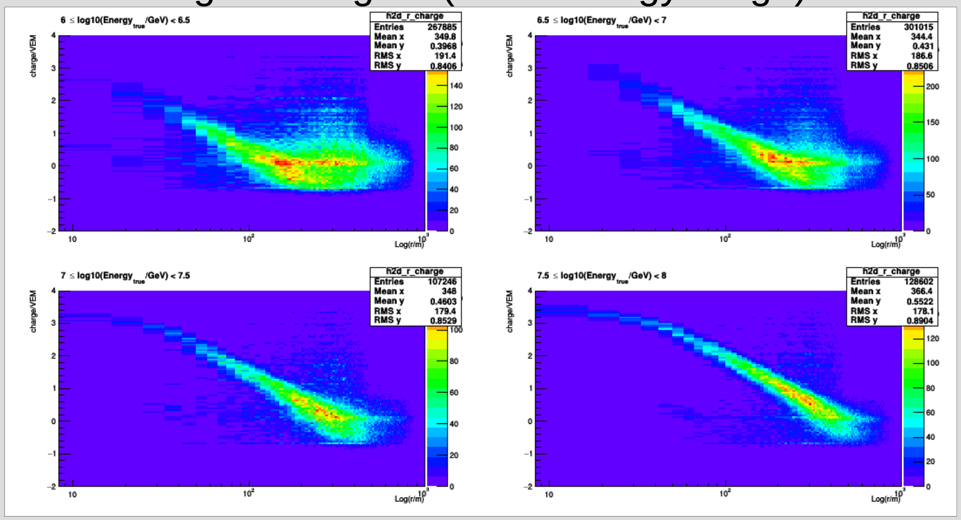
→

Log10(S125/m)	Log10(S125/m)
1.06	1.60
1.06	1.60
0.94	1.50
0.90	1.48

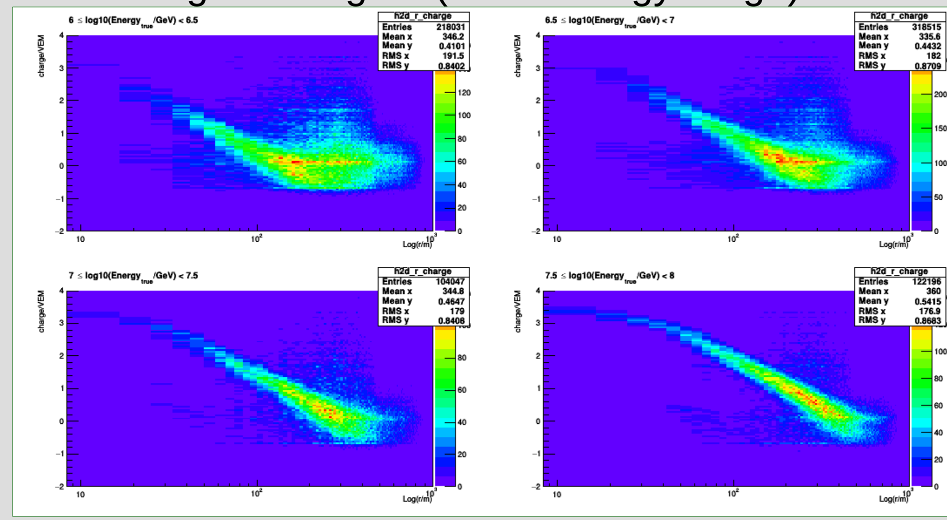
2-DIMENSIONAL HISTOGRAMS
(Radial distance from the core – Signal)

PROTON

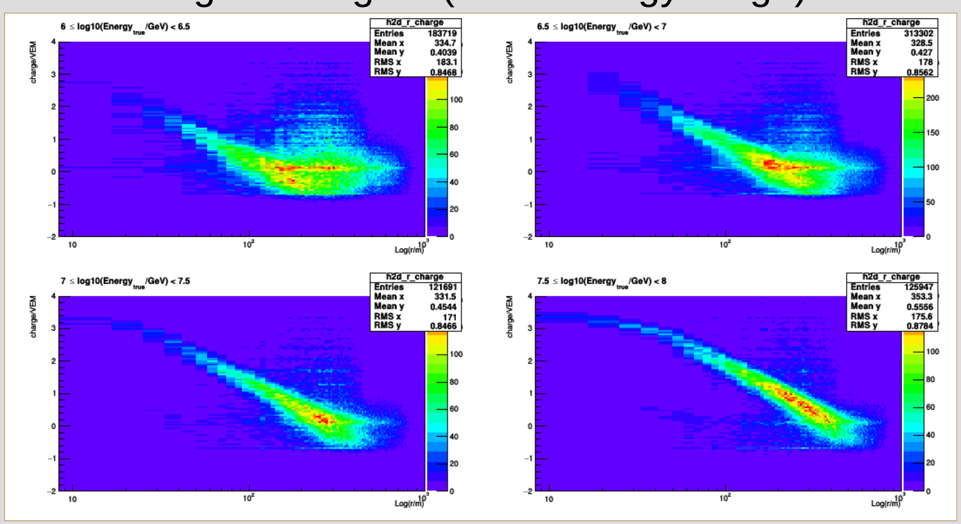
Angular range 1 (each Energy range):



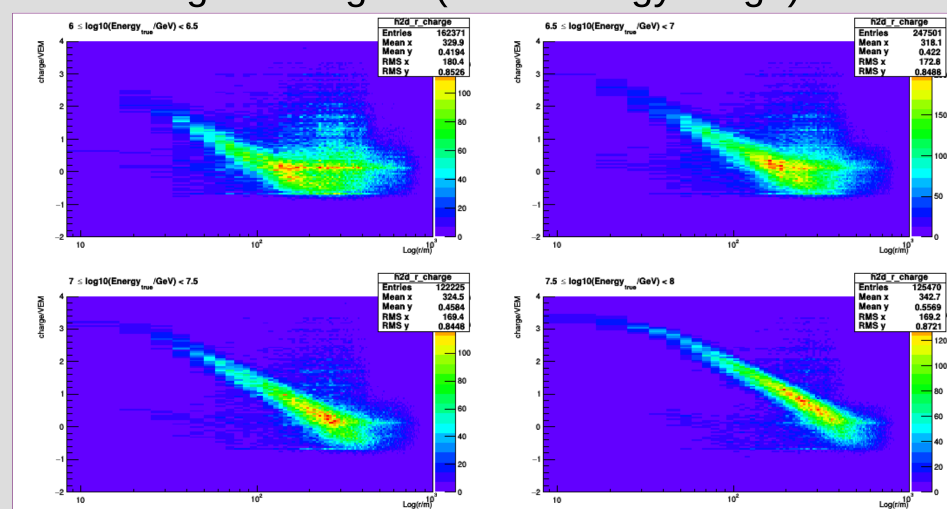
Angular range 2 (each Energy range):



Angular range 3 (each Energy range):



Angular range 4 (each Energy range):

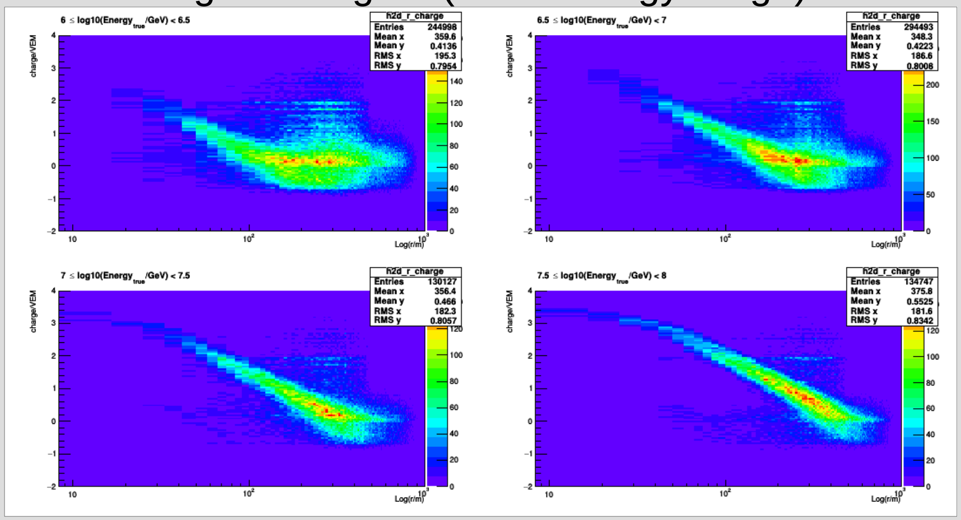


X-axis: Log10(r/m), *Y-axis:* Signal/VEM

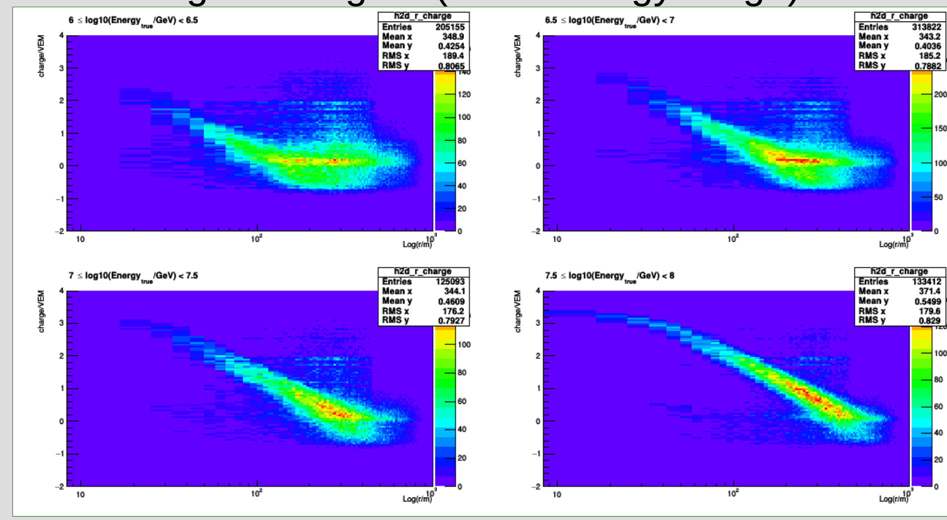
2-DIMENSIONAL HISTOGRAMS
(Radial distance from the core – Signal)

IRON

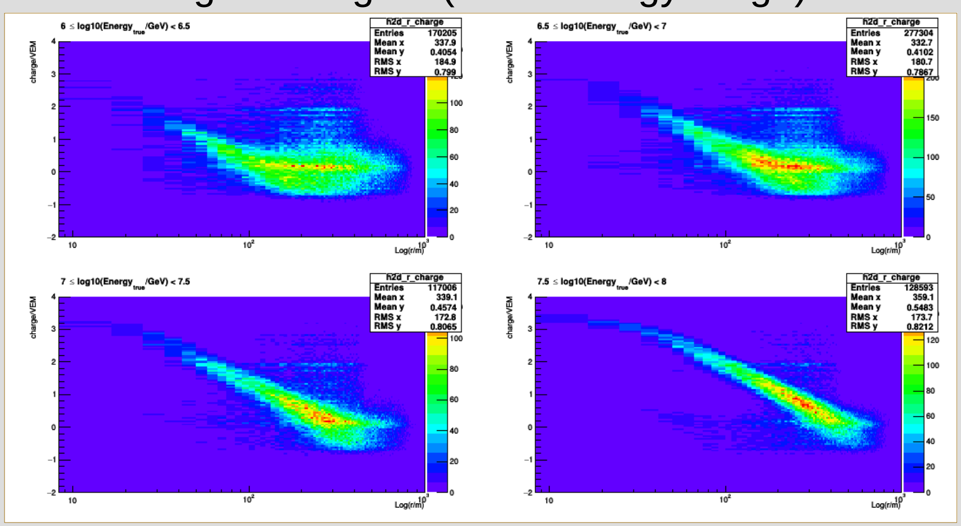
Angular range 1 (each Energy range):



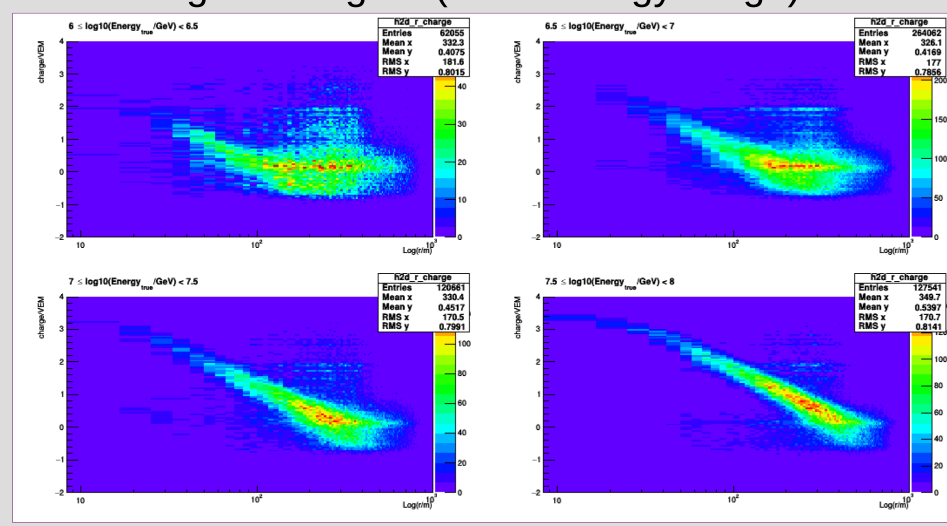
Angular range 2 (each Energy range):



Angular range 3 (each Energy range):



Angular range 4 (each Energy range):

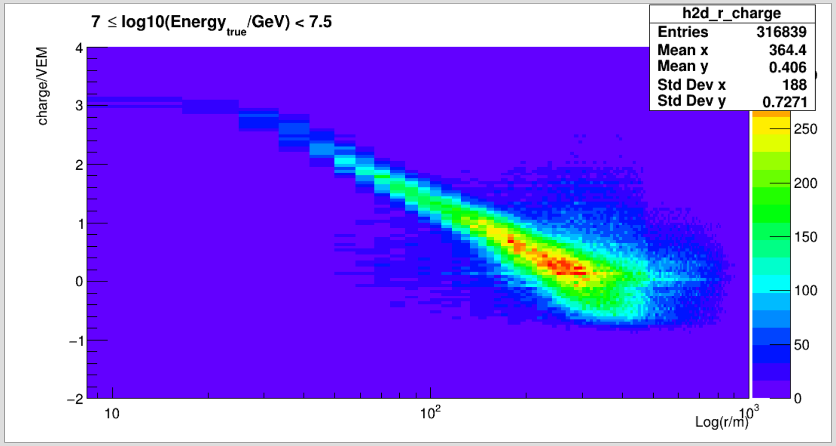


X-axis: Log10(r/m), *Y-axis:* Signal/VEM

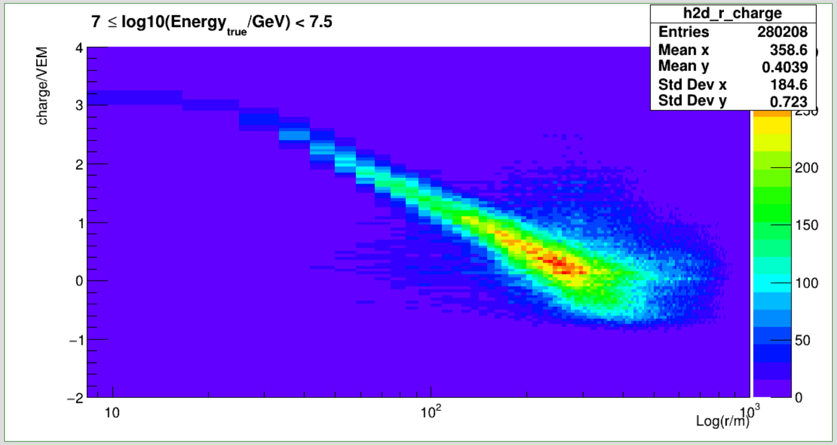
2-DIMENSIONAL HISTOGRAMS
(Radial distance from the core – Signal)

EXP. DATA

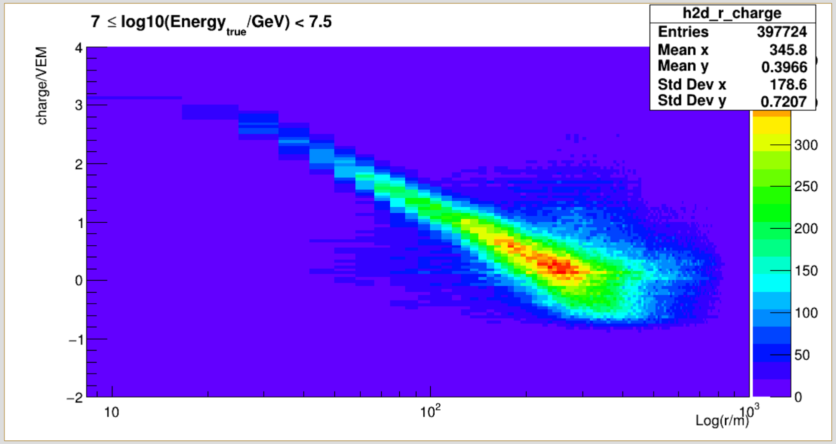
Energy range 3, Angular range 1



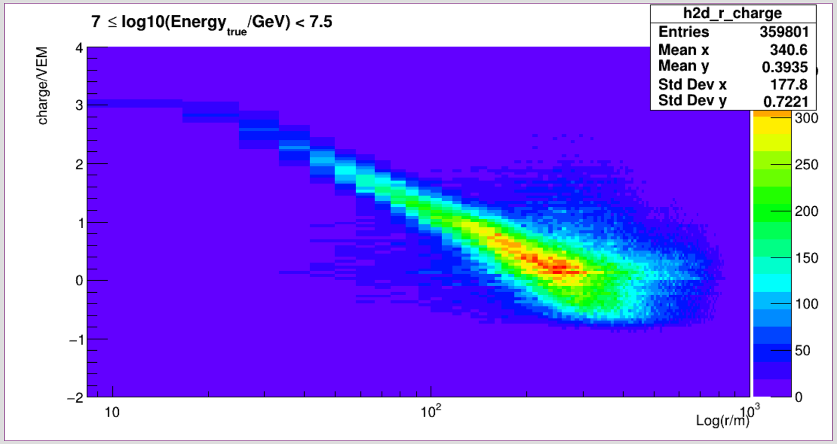
Energy range 3, Angular range 2



Energy range 3, Angular range 3



Energy range 3, Angular range 4

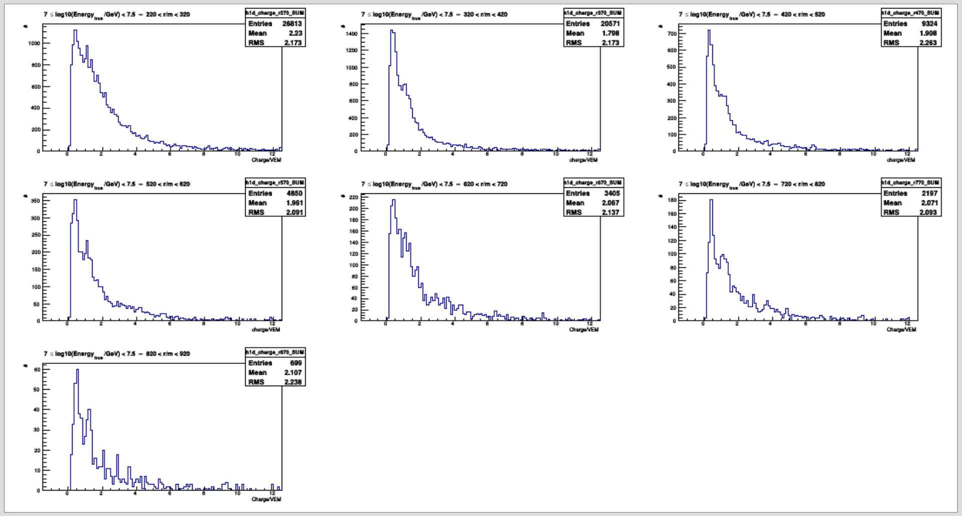


X-axis: Log10(r/m), *Y-axis:* Signal/VEM

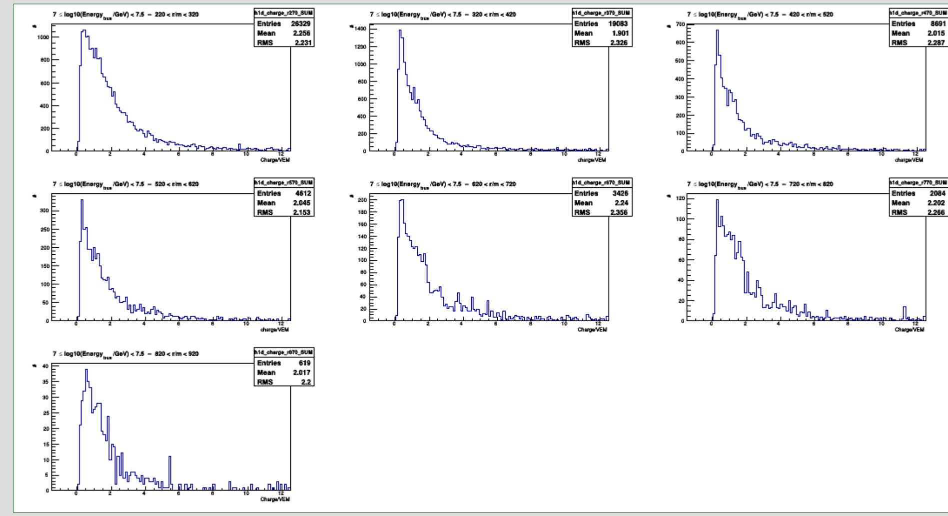
1-DIMENSIONAL HISTOGRAMS
(Signal)

PROTON

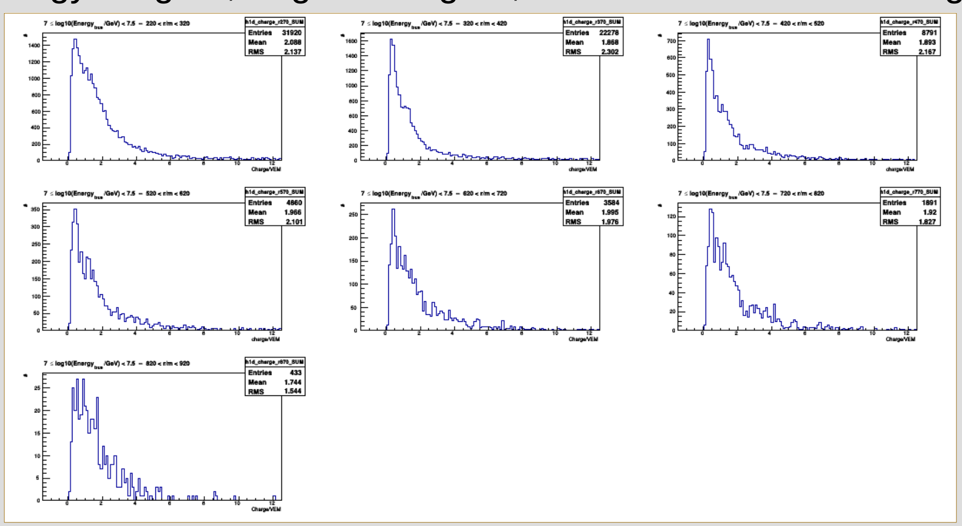
Energy range 3, Angular range 1, each Radial distance range:



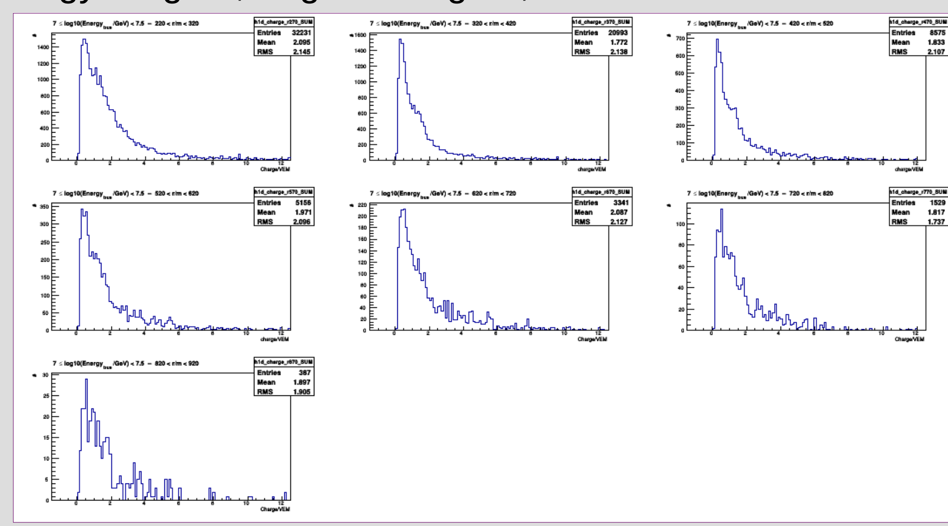
Energy range 3, Angular range 2, each Radial distance range:



Energy range 3, Angular range 3, each Radial distance range:



Energy range 3, Angular range 4, each Radial distance range:

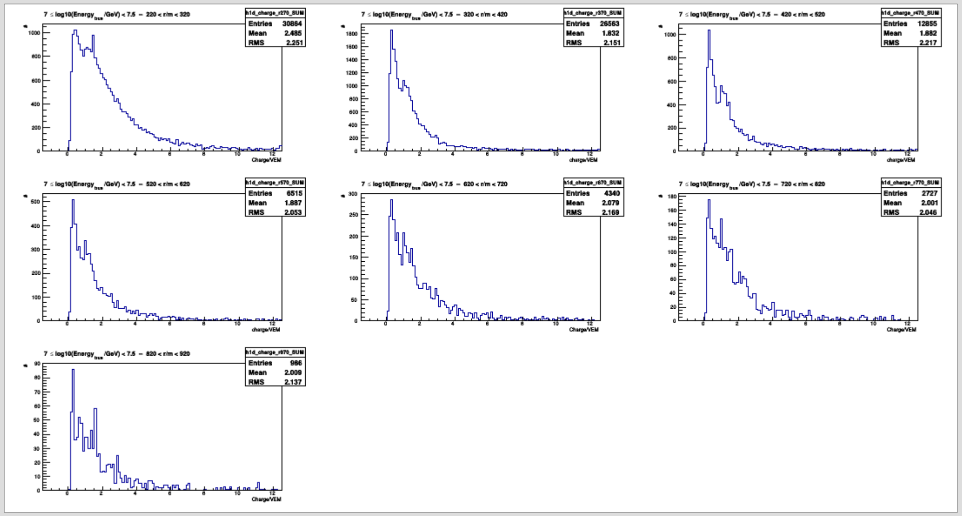


X-axis: Signal [VEM], *Y-axis:* Number of events

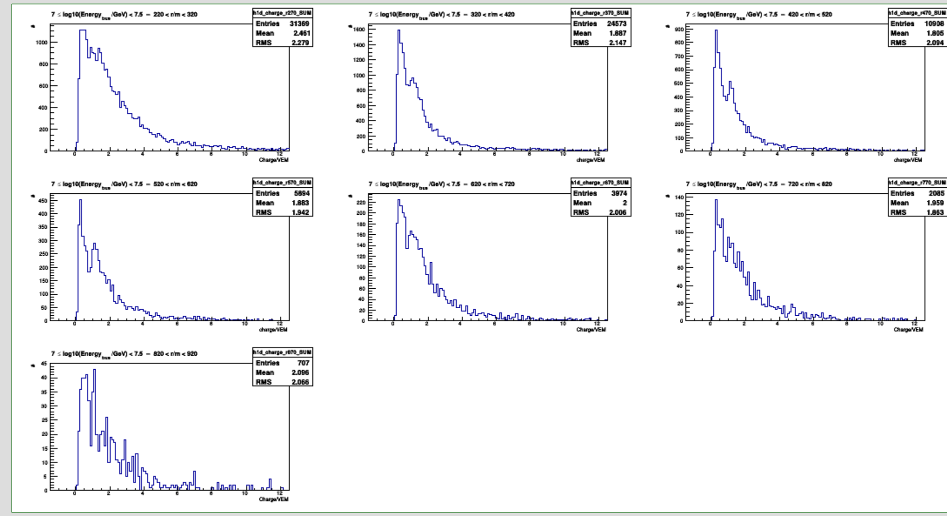
1-DIMENSIONAL HISTOGRAMS
(Signal)

IRON

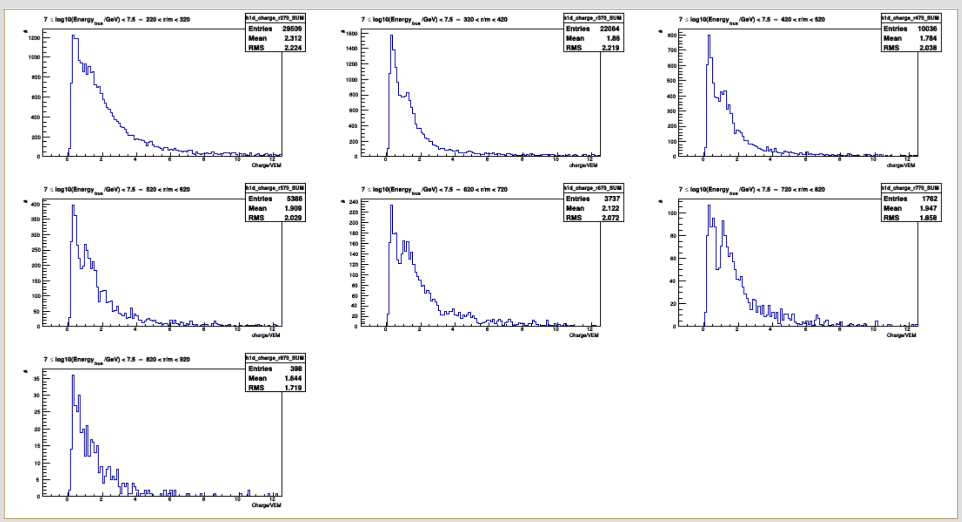
Energy range 3, Angular range 1, each Radial distance range:



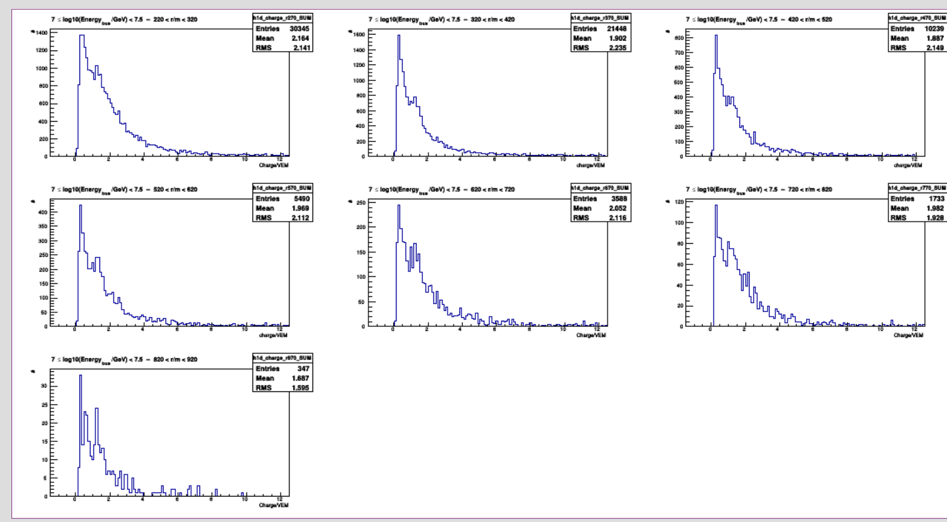
Energy range 3, Angular range 2, each Radial distance range:



Energy range 3, Angular range 3, each Radial distance range:



Energy range 3, Angular range 4, each Radial distance range:

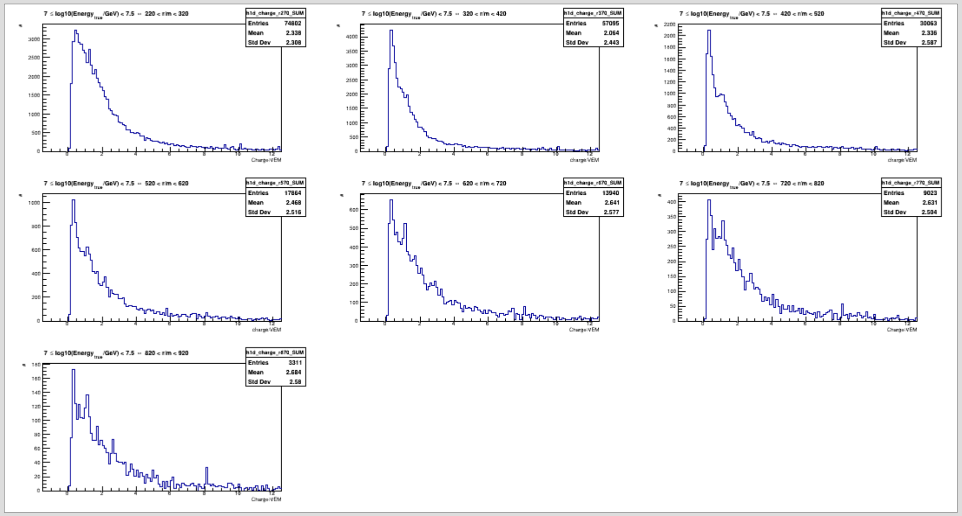


X-axis: Signal [VEM], *Y-axis:* Number of events

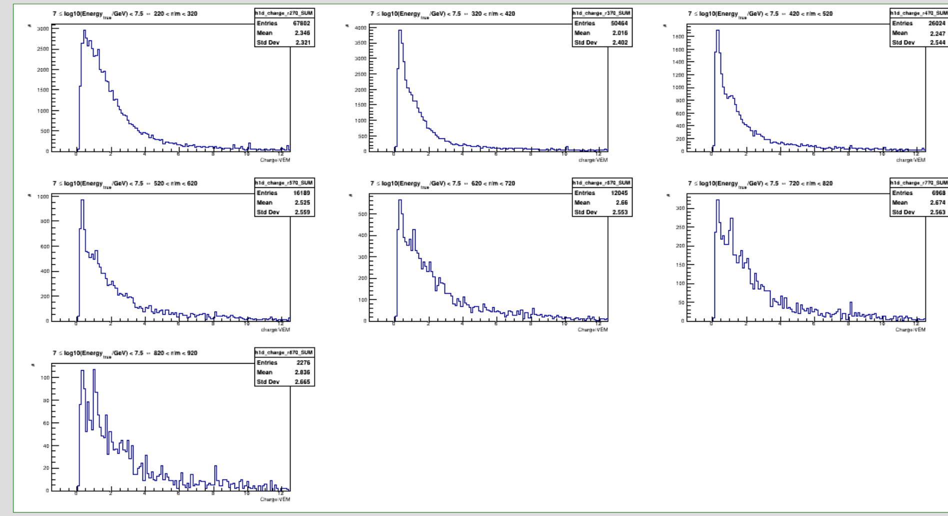
1-DIMENSIONAL HISTOGRAMS
(Signal)

EXP. DATA

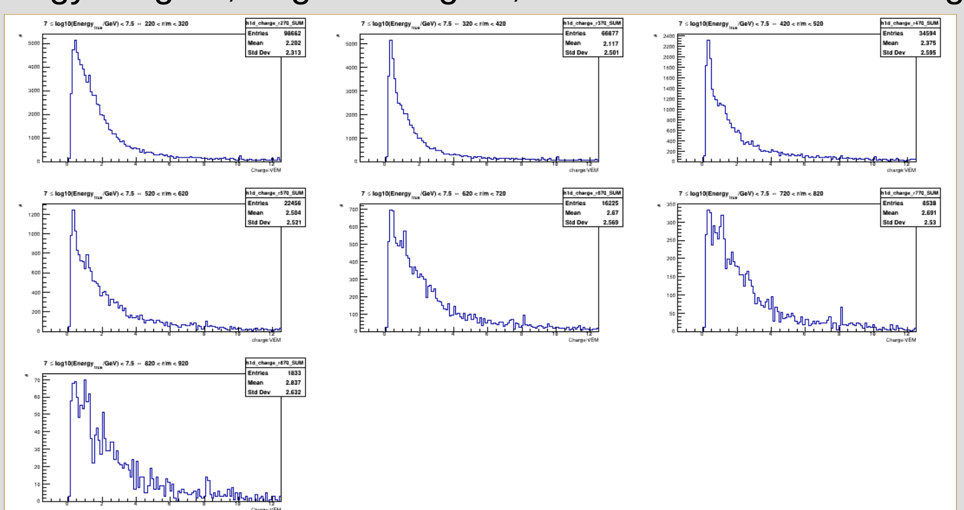
Energy range 3, Angular range 1, each Radial distance range:



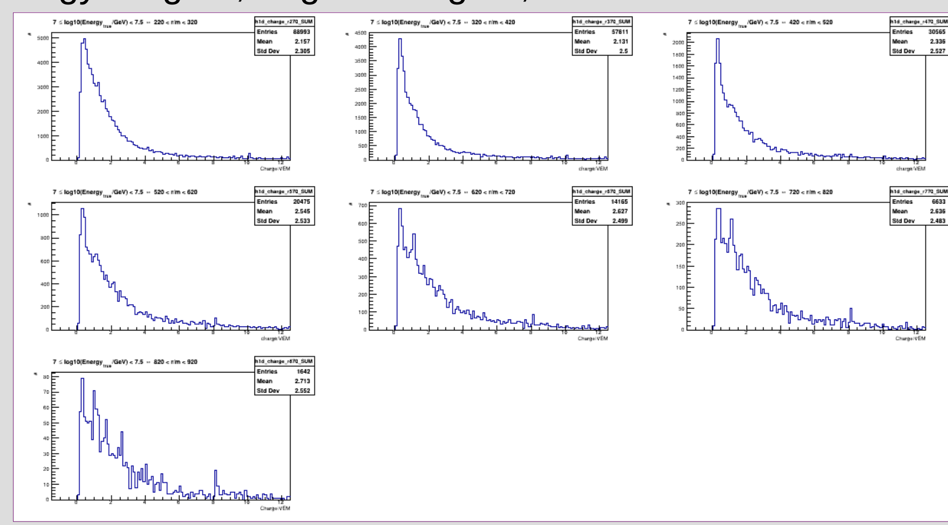
Energy range 3, Angular range 2, each Radial distance range:



Energy range 3, Angular range 3, each Radial distance range:



Energy range 3, Angular range 4, each Radial distance range:

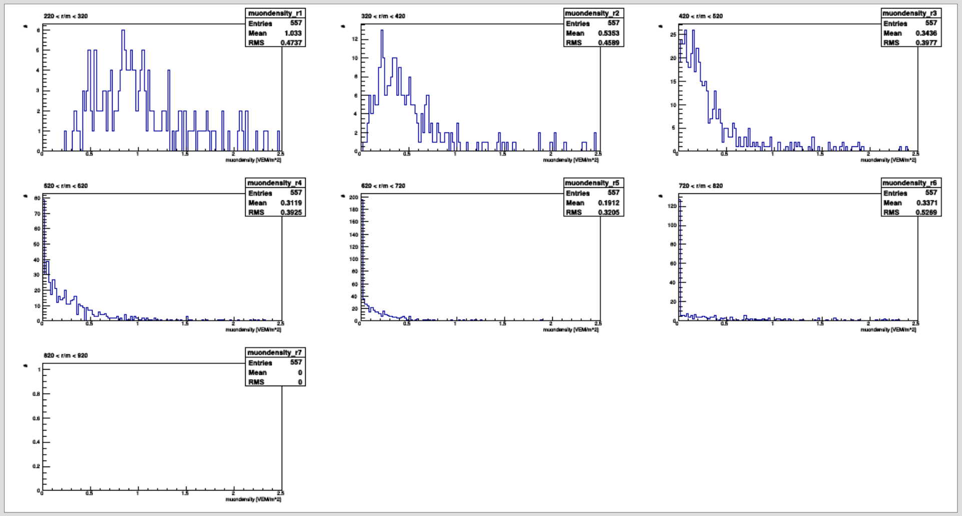


X-axis: Signal [VEM], *Y-axis:* Number of events

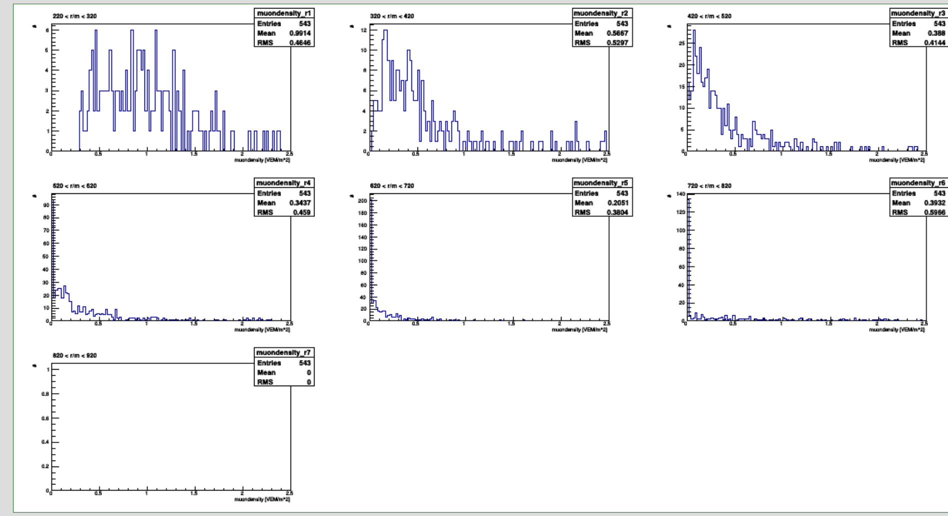
1-DIMENSIONAL HISTOGRAMS
(Muon density)

PROTON

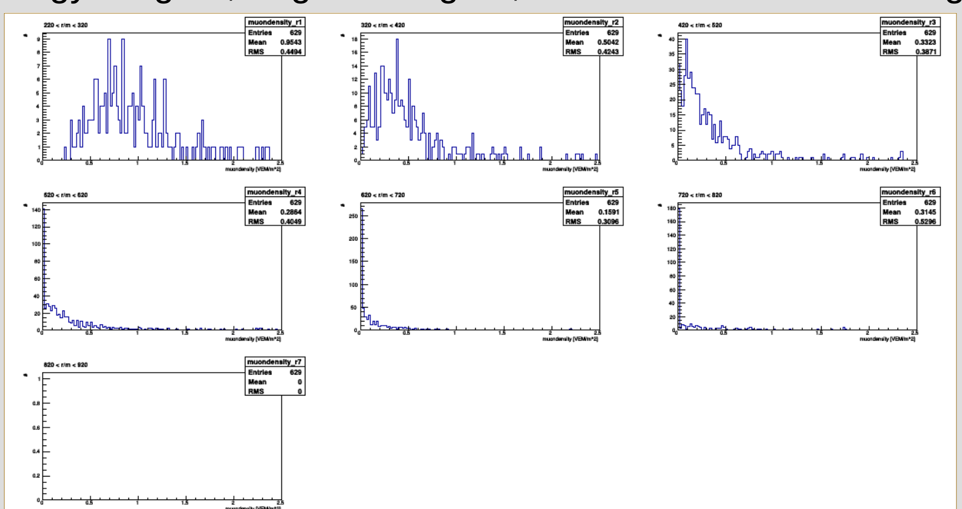
Energy range 3, Angular range 1, each Radial distance range:



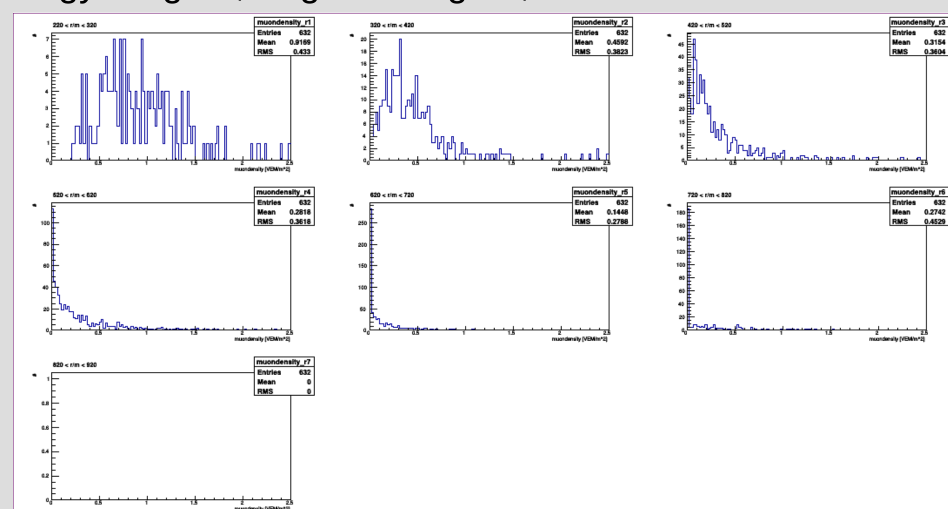
Energy range 3, Angular range 2, each Radial distance range:



Energy range 3, Angular range 3, each Radial distance range:



Energy range 3, Angular range 4, each Radial distance range:

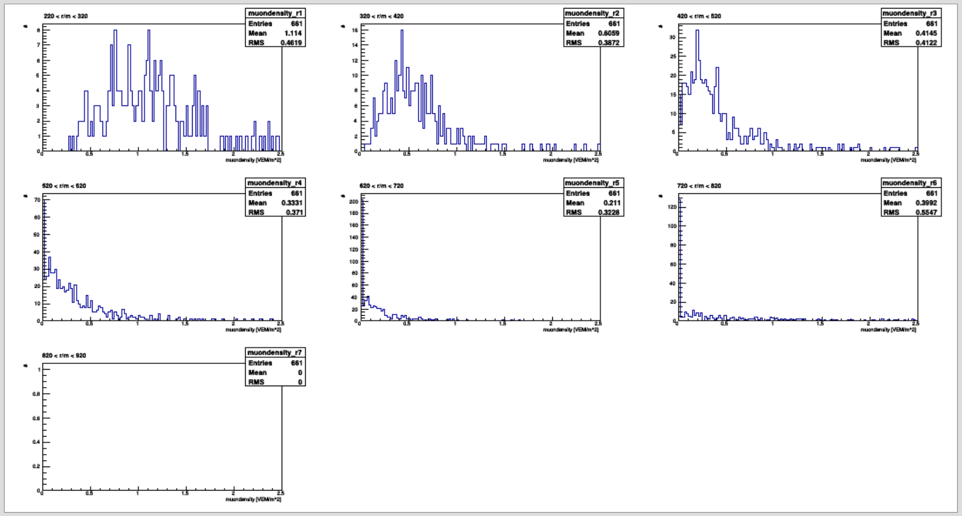


X-axis: Muon density [VEM/m^2], *Y-axis:* Number of events

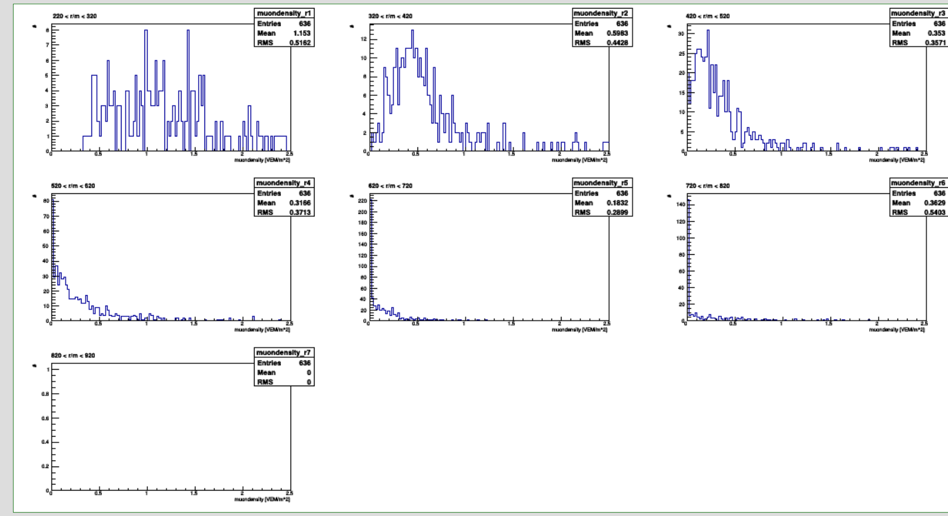
1-DIMENSIONAL HISTOGRAMS
(Muon density)

IRON

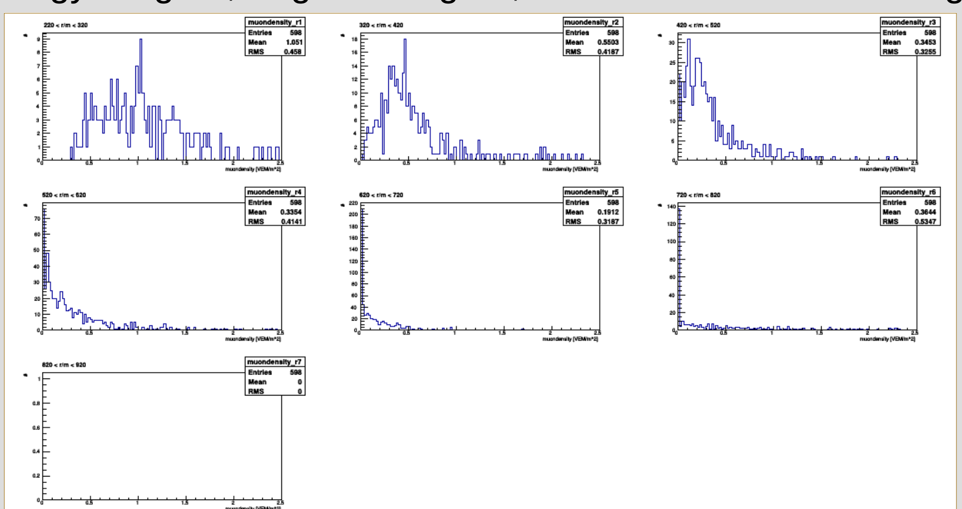
Energy range 3, Angular range 1, each Radial distance range:



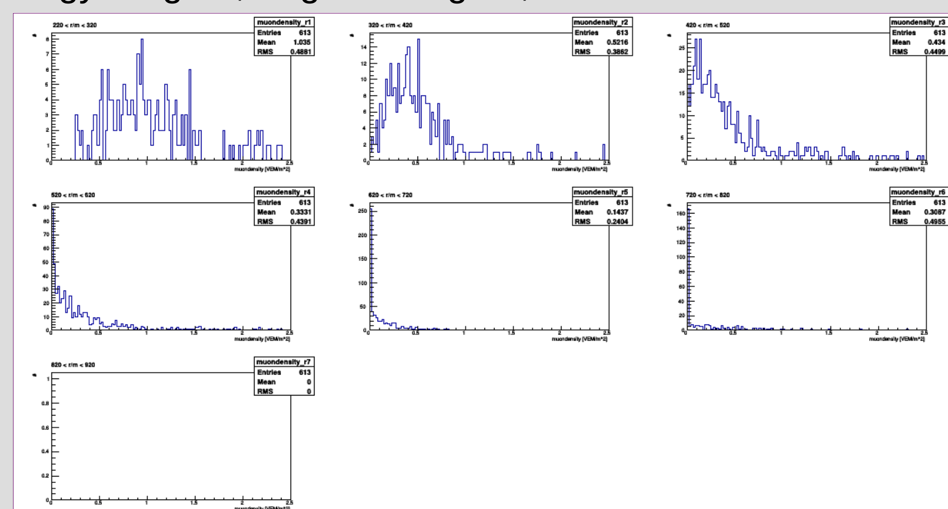
Energy range 3, Angular range 2, each Radial distance range:



Energy range 3, Angular range 3, each Radial distance range:



Energy range 3, Angular range 4, each Radial distance range:



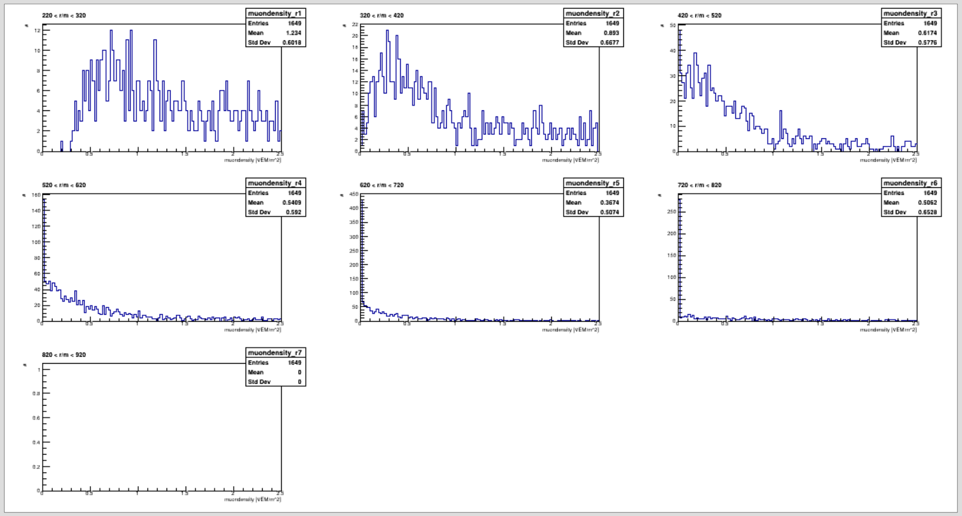
X-axis: Muon density [VEM/m^2], *Y-axis:* Number of events

1-DIMENSIONAL HISTOGRAMS
(Muon density)

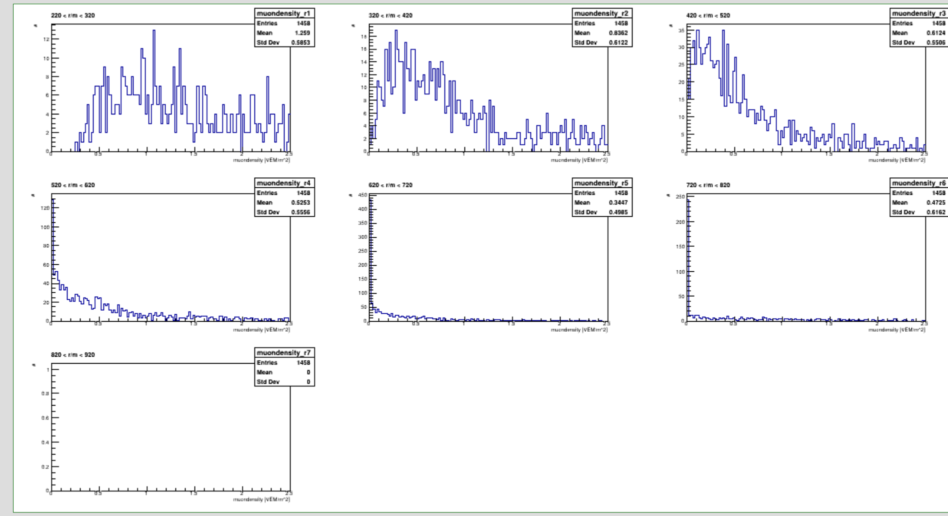
Karlsruher Institut für Technologie

EXP. DATA

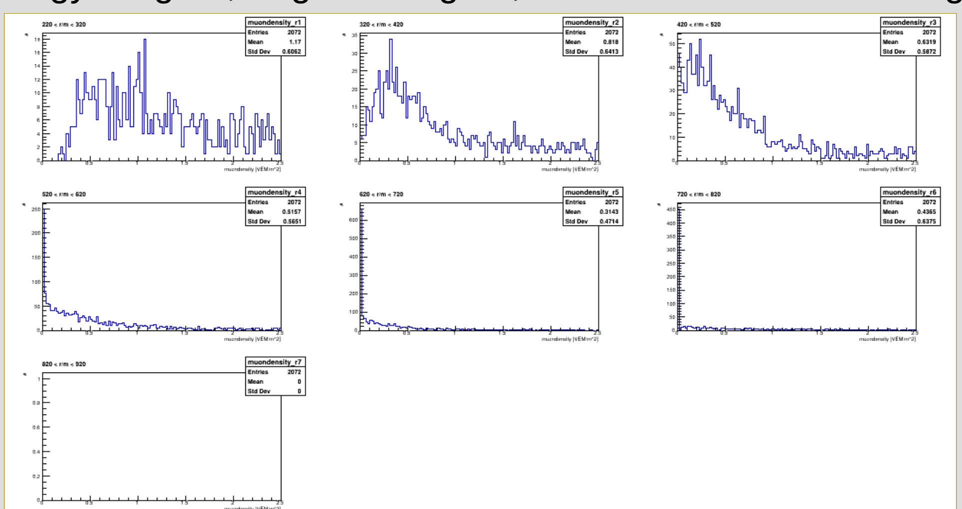
Energy range 3, Angular range 1, each Radial distance range:



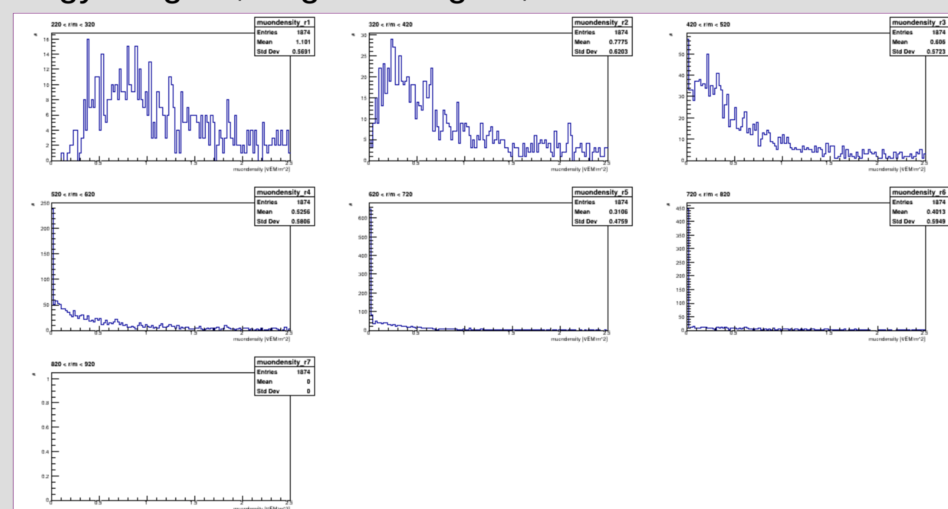
Energy range 3, Angular range 2, each Radial distance range:



Energy range 3, Angular range 3, each Radial distance range:



Energy range 3, Angular range 4, each Radial distance range:

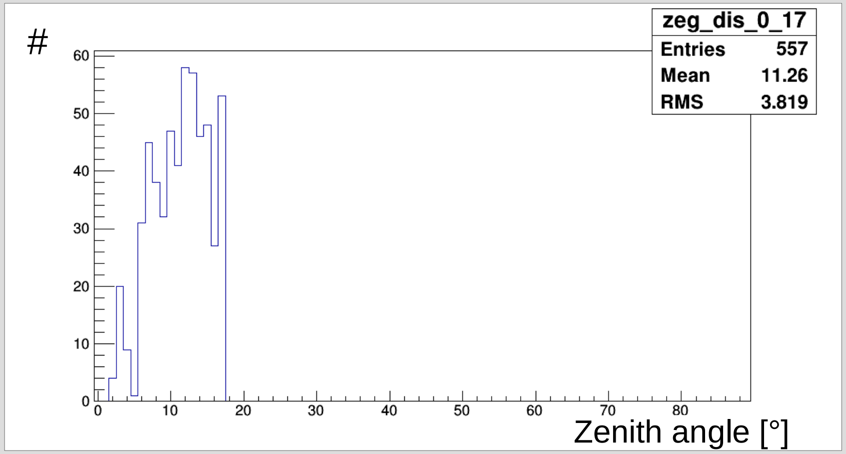


X-axis: Muon density [VEM/m²], *Y-axis:* Number of events

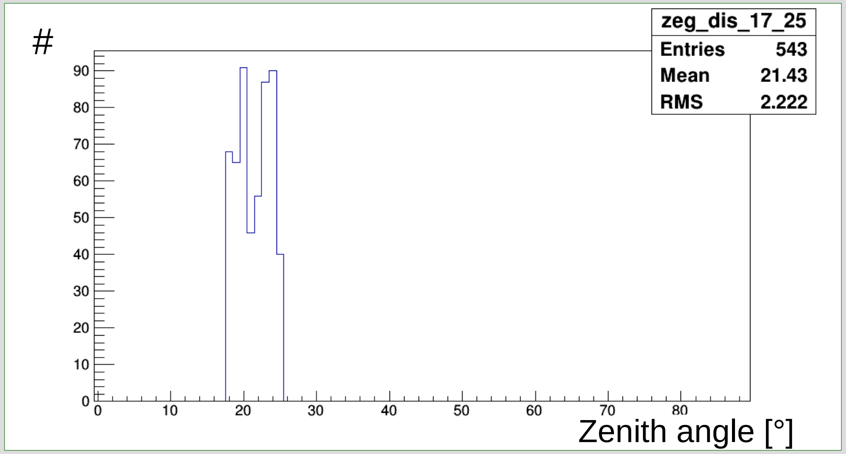
1-DIMENSIONAL HISTOGRAMS
(Zenith angle)

PROTON

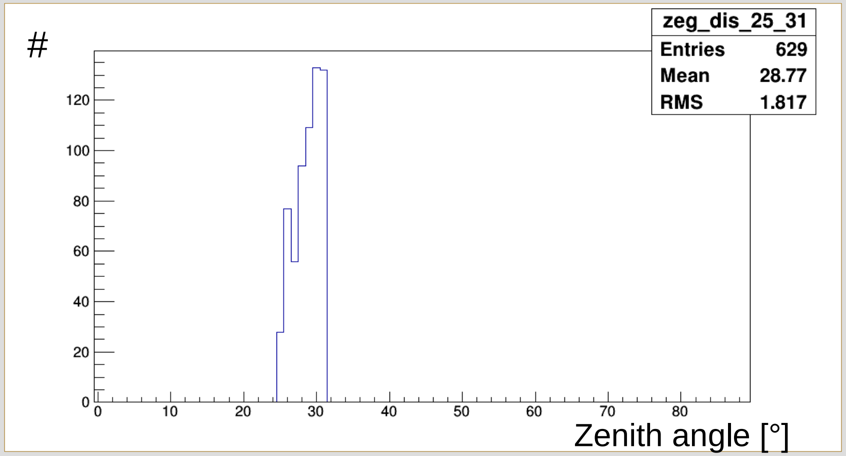
Energy range 3, Angular range 1



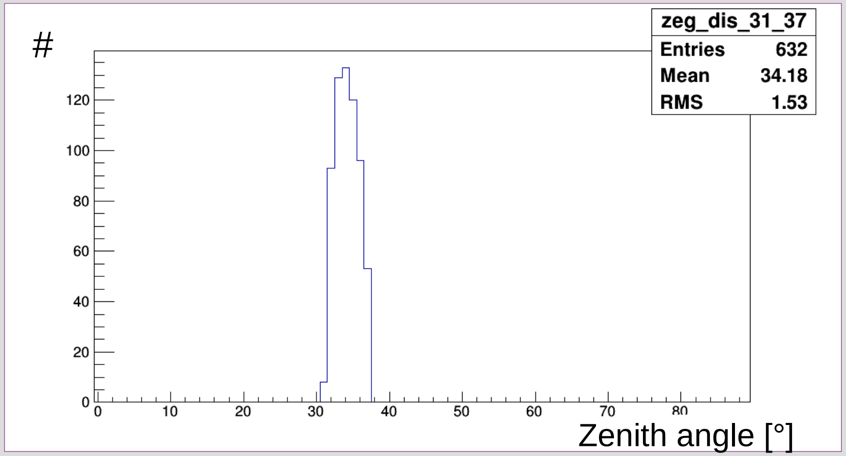
Energy range 3, Angular range 2



Energy range 3, Angular range 3



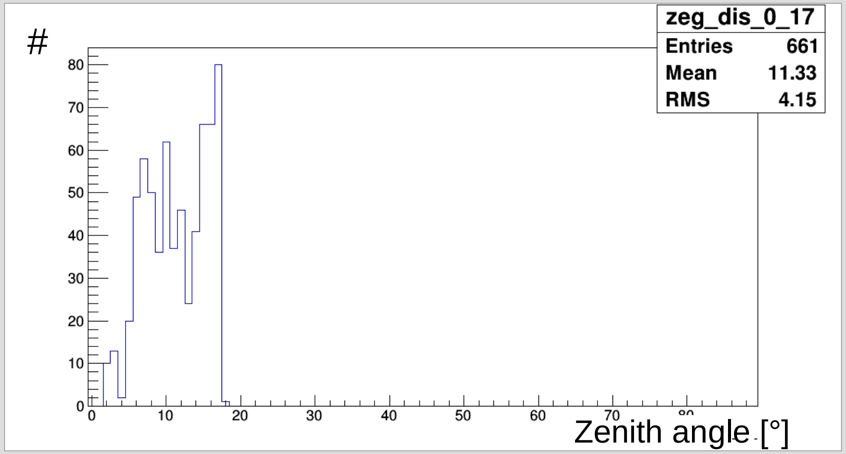
Energy range 3, Angular range 4



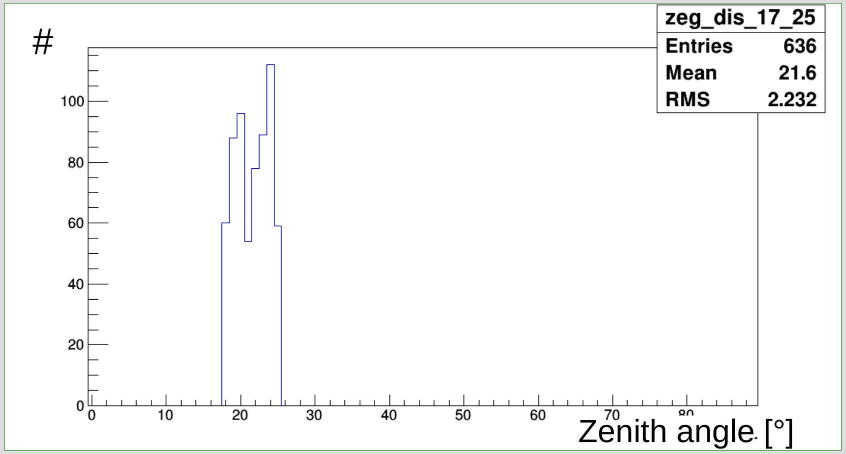
1-DIMENSIONAL HISTOGRAMS
(Zenith angle)

IRON

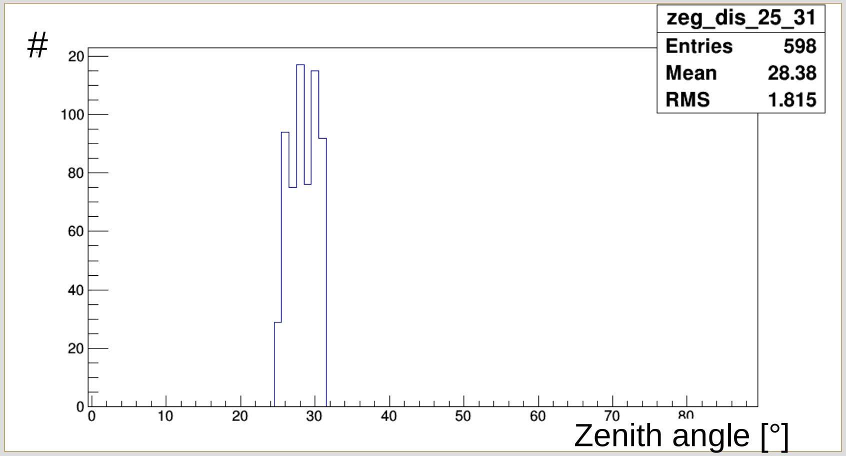
Energy range 3, Angular range 1



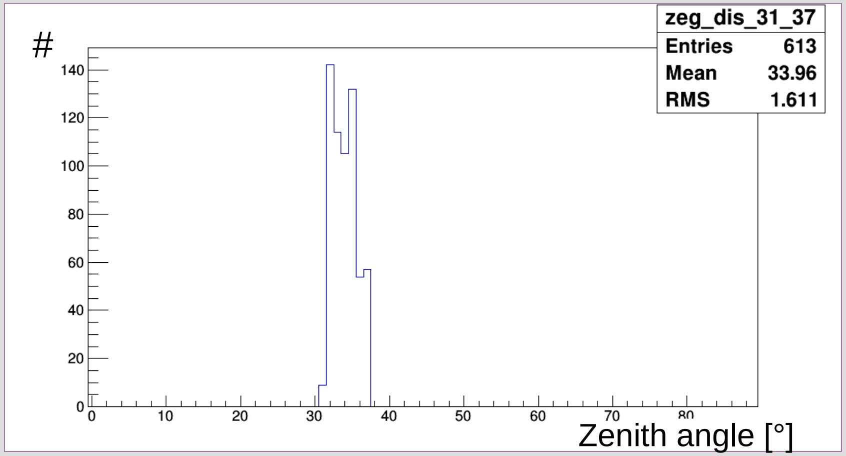
Energy range 3, Angular range 2



Energy range 3, Angular range 3



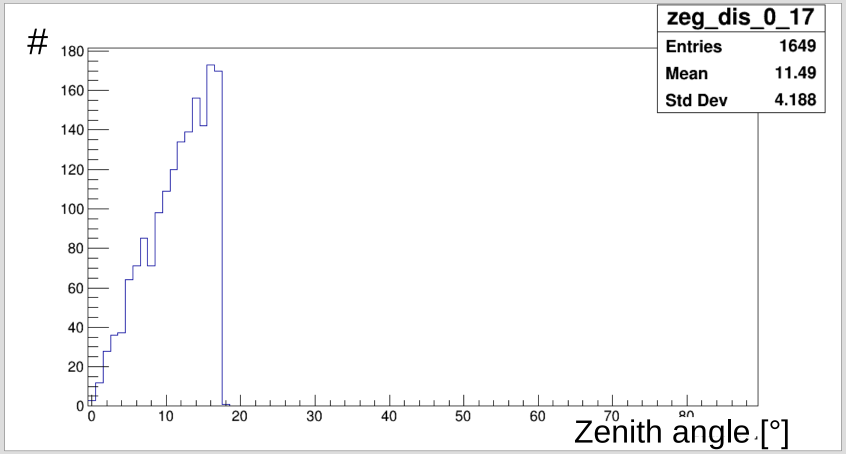
Energy range 3, Angular range 4



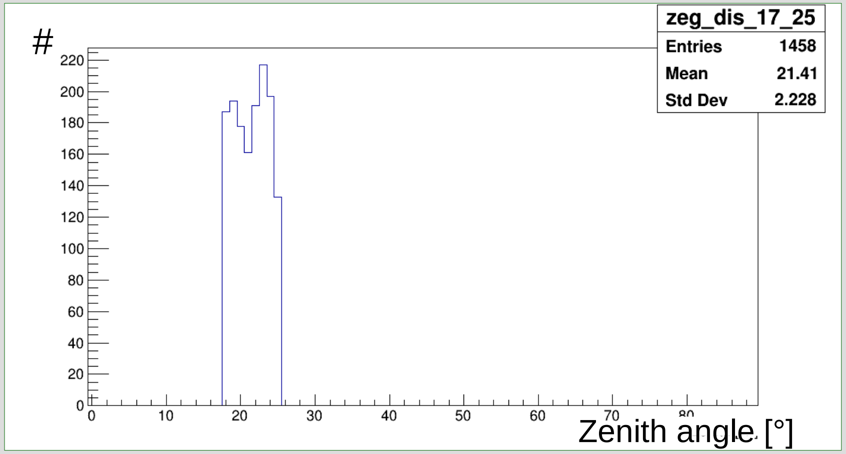
1-DIMENSIONAL HISTOGRAMS
(Zenith angle)

EXP. DATA

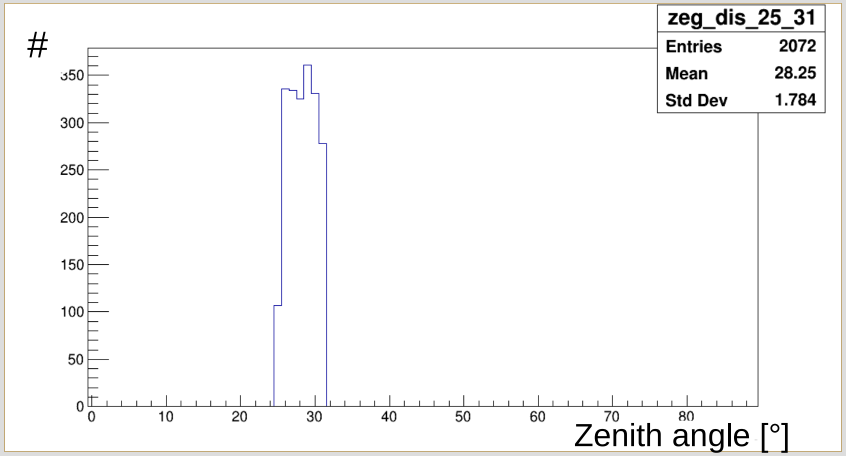
Energy range 3, Angular range 1



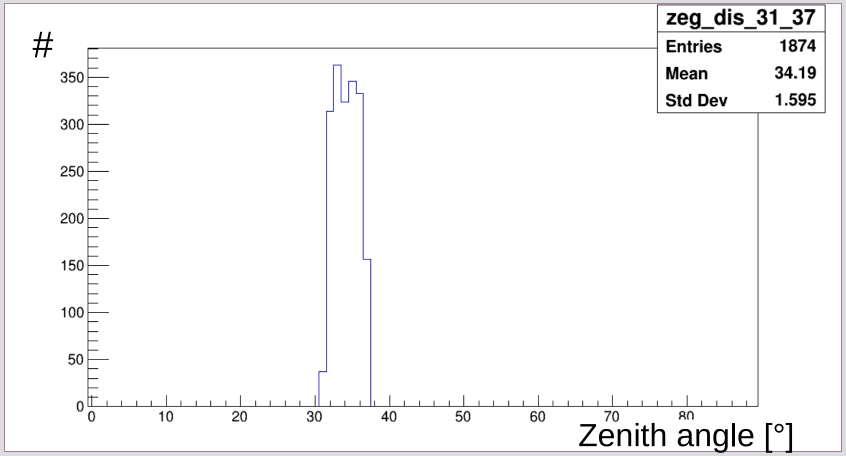
Energy range 3, Angular range 2



Energy range 3, Angular range 3



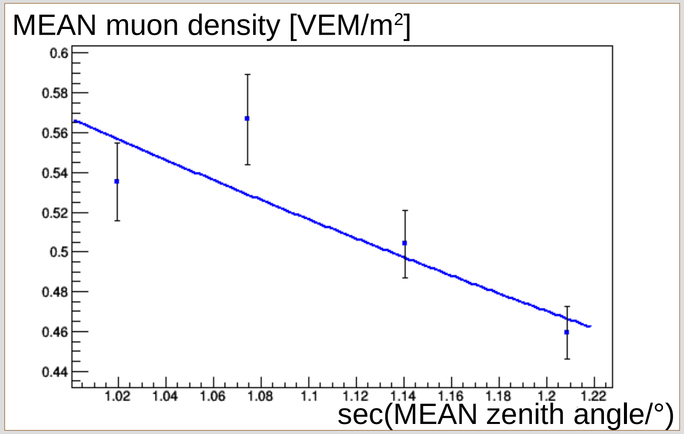
Energy range 3, Angular range 4



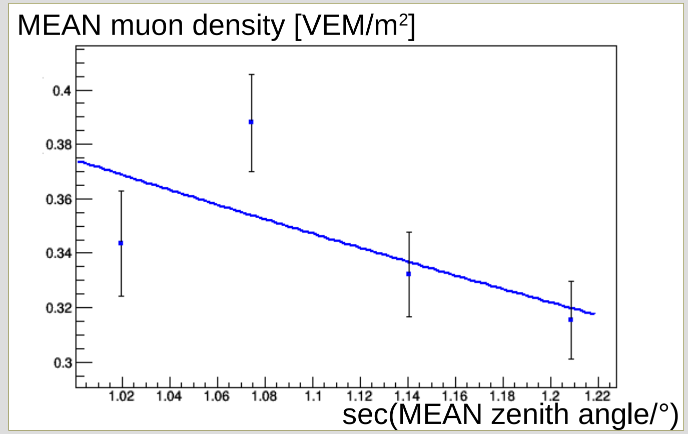
THE FITS (MEAN)

PROTON

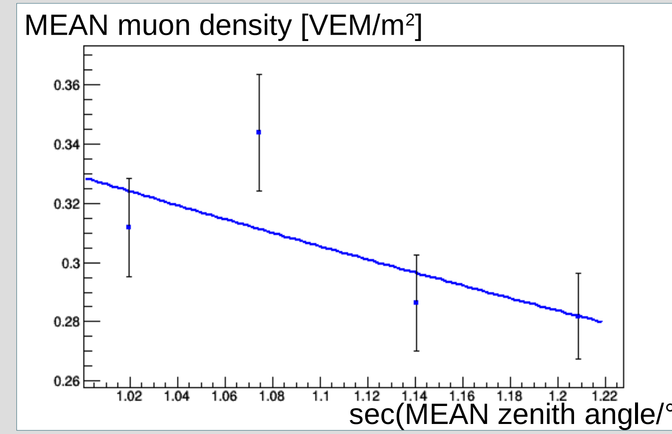
Energy range 3, Radial distance range 2



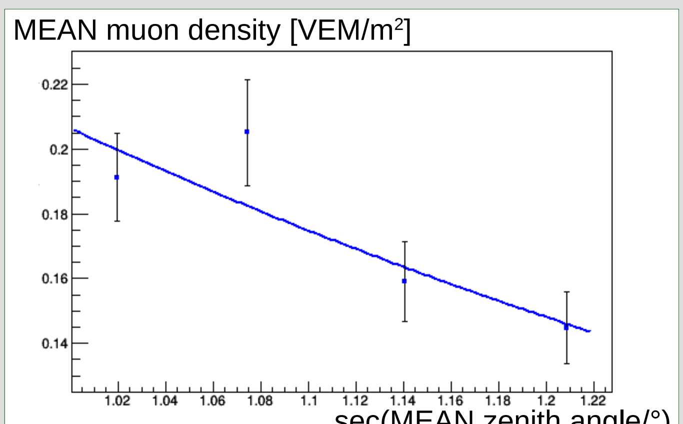
Energy range 3, Radial distance range 3



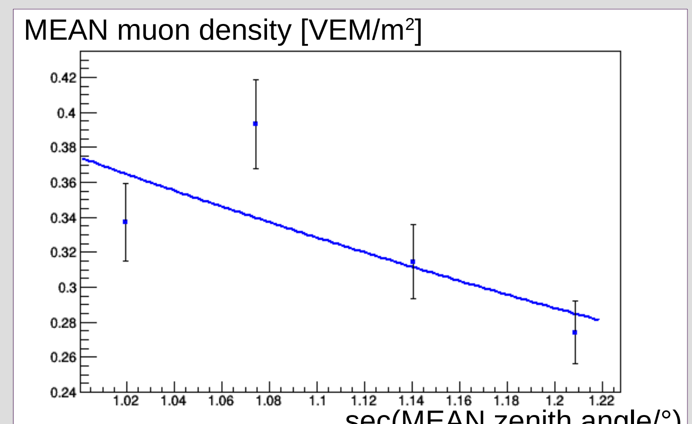
Energy range 3, Radial distance range 4



Energy range 3, Radial distance range 5



Energy range 3, Radial distance range 6



THE FITS (MEAN)

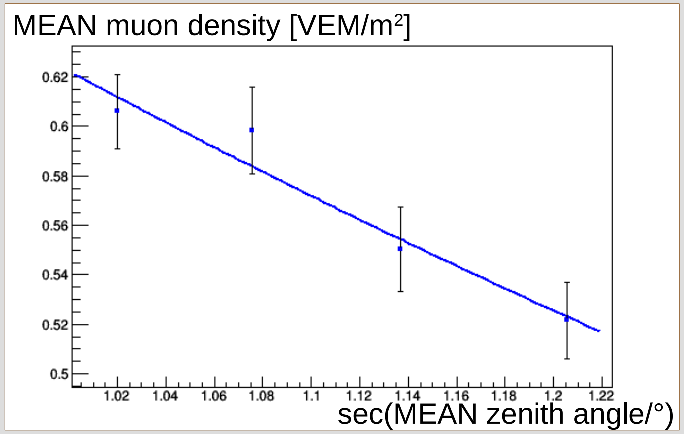
PROTON

Range [m]	$P^0_\mu(r)$ [VEM/m^2]	$\delta P^0_\mu(r)$ [VEM/m^2]	$\alpha_\mu(r)$ [g/cm^2]	$\delta\alpha_\mu(r)$ [g/cm^2]	χ 2	N. of Dof.	Prob.
320-420	1.45	0.35	738	175	4.44	2	0.11
420-520	0.80	0.29	917	397	5.53	2	0.06
520-620	0.69	0.26	939	445	3.64	2	0.16
620-720	1.08	0.54	417	127	2.46	2	0.29
720-820	1.39	0.58	528	174	6.29	2	0.04

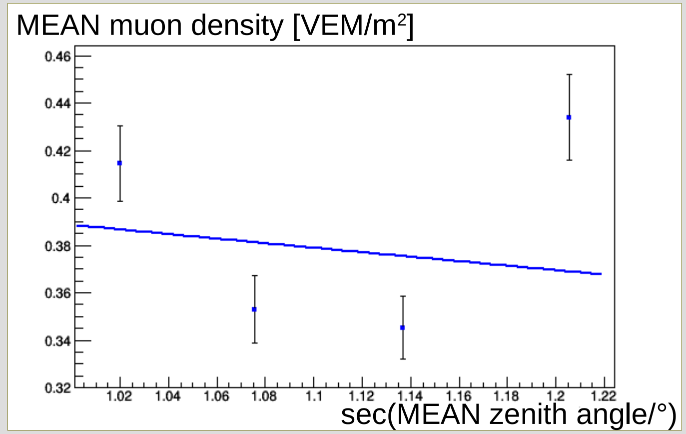
THE FITS (MEAN)

IRON

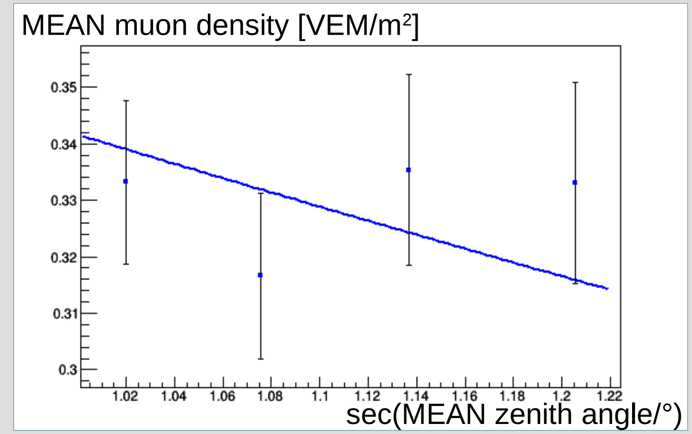
Energy range 3, Radial distance range 2



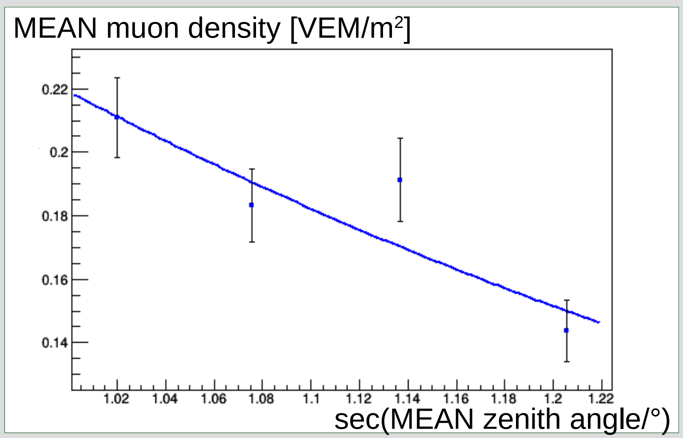
Energy range 3, Radial distance range 3



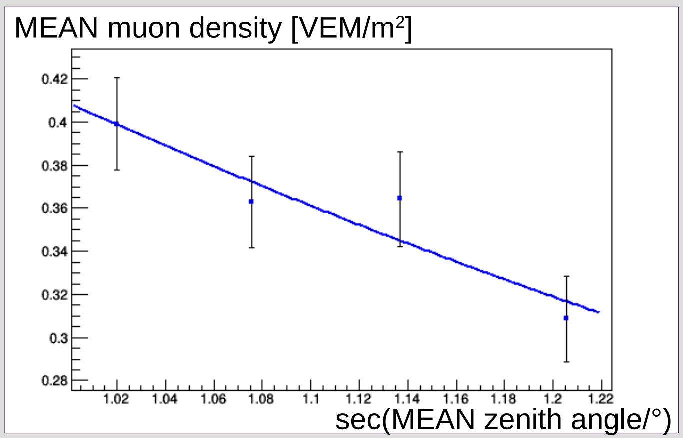
Energy range 3, Radial distance range 4



Energy range 3, Radial distance range 5



Energy range 3, Radial distance range 6



THE FITS (MEAN)

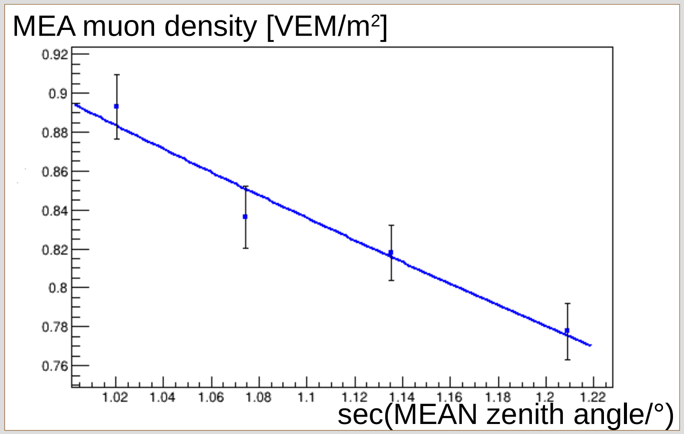
IRON

Range [m]	$P^0_\mu(r)$ [VEM/m^2]	$\delta P^0_\mu(r)$ [VEM/m^2]	$\alpha_\mu(r)$ [g/cm^2]	$\delta\alpha_\mu(r)$ [g/cm^2]	χ 2	N. of Dof.	Prob.
320-420	1.45	0.31	819	190	0.91	2	0.64
420-520	0.50	0.08	2745	196	24.92	2	0.000004
520-620	0.50	0.07	1817	104	2.61	2	0.27
620-720	1.39	0.59	375	91	3.41	2	0.18
720-820	1.42	0.57	557	187	1.13	2	0.57

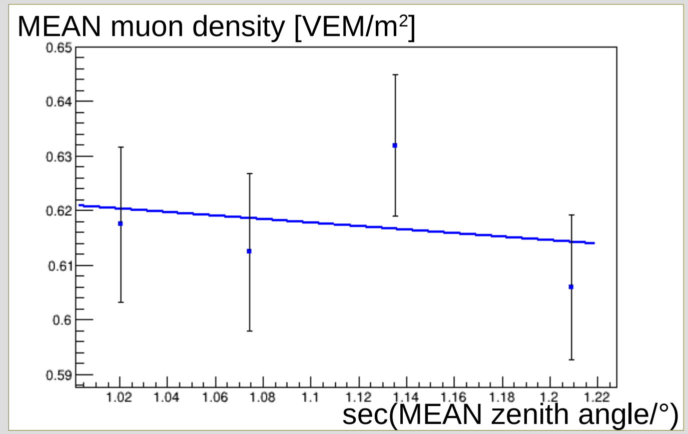
THE FITS (MEAN)

EXP. DATA

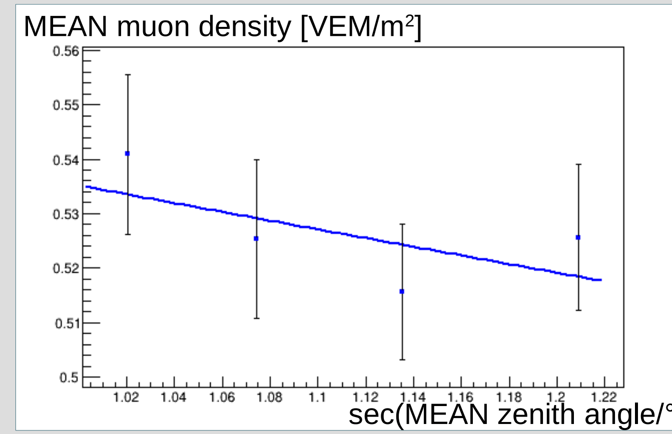
Energy range 3, Radial distance range 2



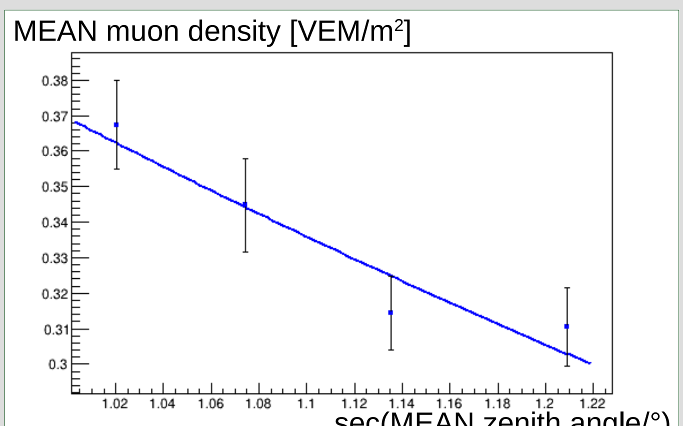
Energy range 3, Radial distance range 3



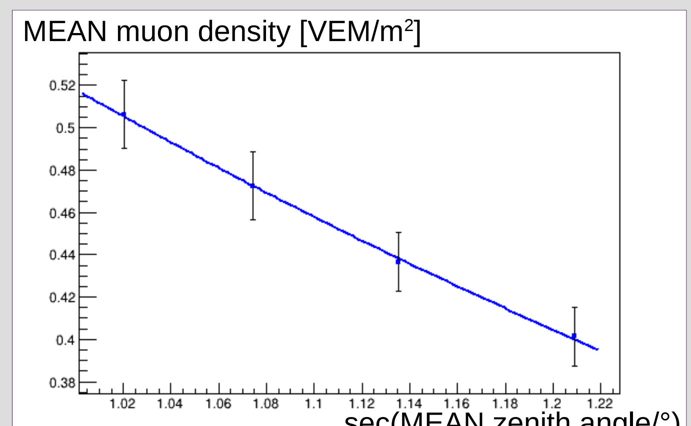
Energy range 3, Radial distance range 4



Energy range 3, Radial distance range 5



Energy range 3, Radial distance range 6



THE FITS (MEAN)

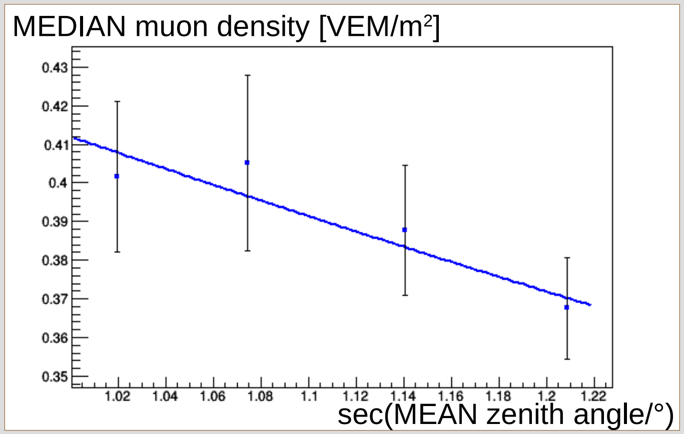
EXP. DATA

Range [m]	$P^0_\mu(r)$ [VEM/m^2]	$\delta P^0_\mu(r)$ [VEM/m^2]	$\alpha_\mu(r)$ [g/cm^2]	$\delta\alpha_\mu(r)$ [g/cm^2]	χ 2	N. of Dof.	Prob.
320-420	1.79	0.26	999	191	1.26	2	0.53
420-520	0.65	0.11	13271	39033	2.02	2	0.36
520-620	0.62	0.13	4555	5662	1.09	2	0.58
620-720	0.95	0.27	729	196	1.68	2	0.43
720-820	1.79	0.47	559	107	0.03	2	0.99

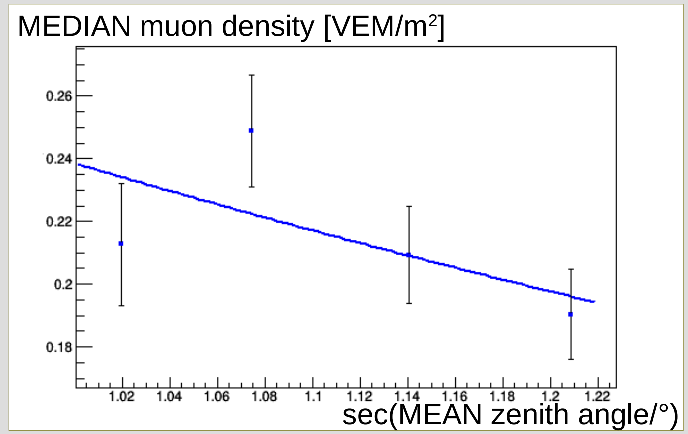
THE FITS (MEDIAN)

PROTON

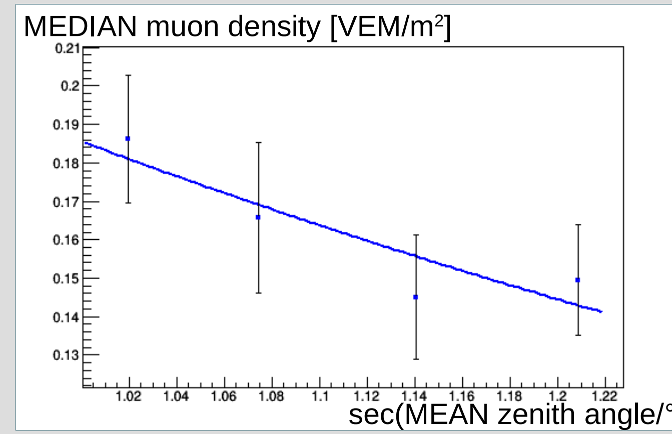
Energy range 3, Radial distance range 2



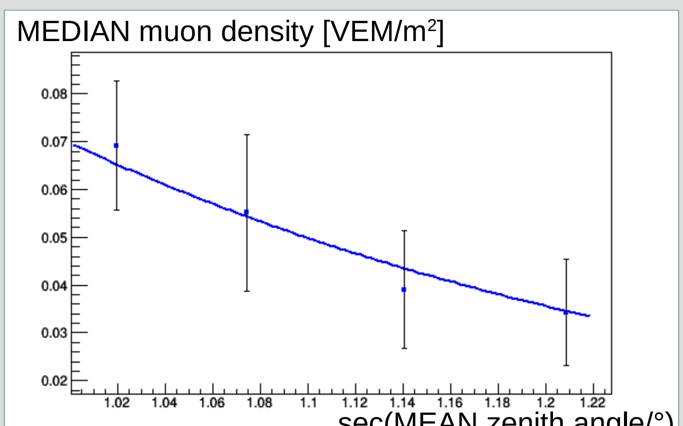
Energy range 3, Radial distance range 3



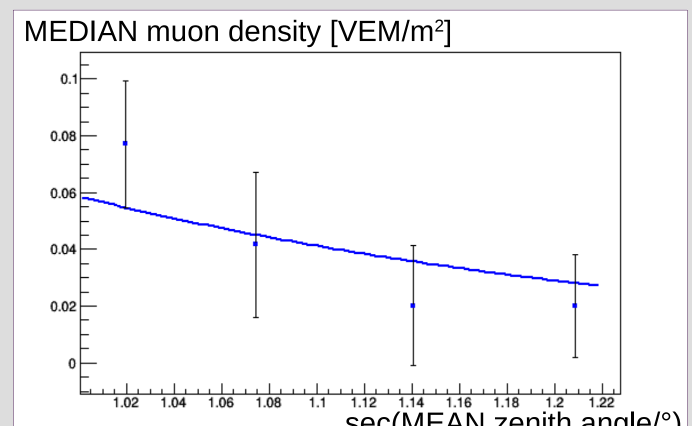
Energy range 3, Radial distance range 4



Energy range 3, Radial distance range 5



Energy range 3, Radial distance range 6



THE FITS (MEDIAN)

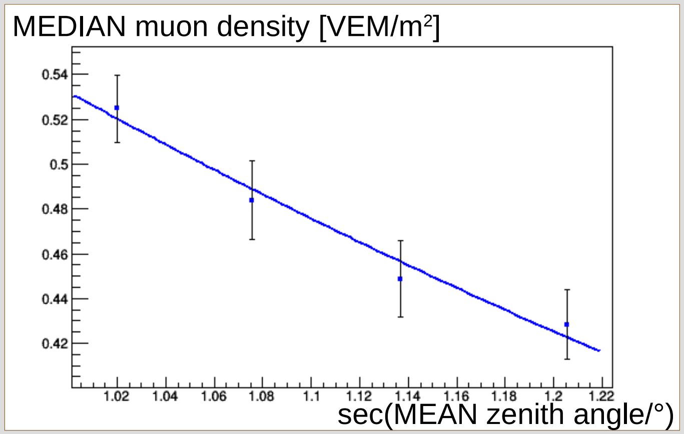
PROTON

Range [m]	$P^0_\mu(r)$ [VEM/m^2]	$\delta P^0_\mu(r)$ [VEM/m^2]	$\alpha_\mu(r)$ [g/cm^2]	$\delta\alpha_\mu(r)$ [g/cm^2]	χ 2	N. of Dof.	Prob.
320-420	0.69	0.22	1351	688	0.35	2	0.84
420-520	0.61	0.31	738	405	3.60	2	0.17
520-620	0.65	1.12	554	302	0.76	2	0.68
620-720	2.00	1.35	206	7	0.21	2	0.90
720-820	2.0	1.5	196	13	1.75	2	0.42

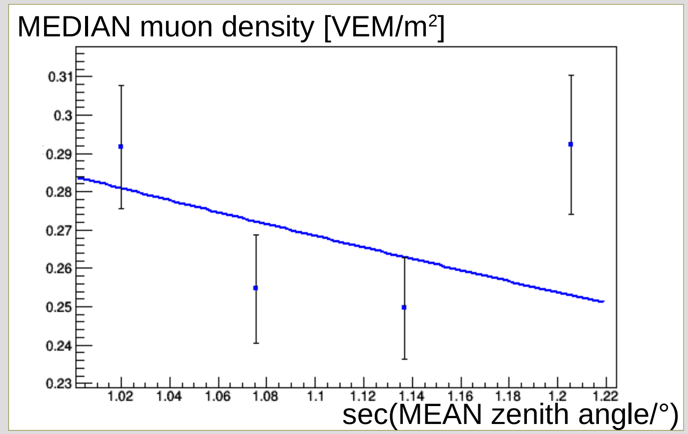
THE FITS (MEDIAN)

IRON

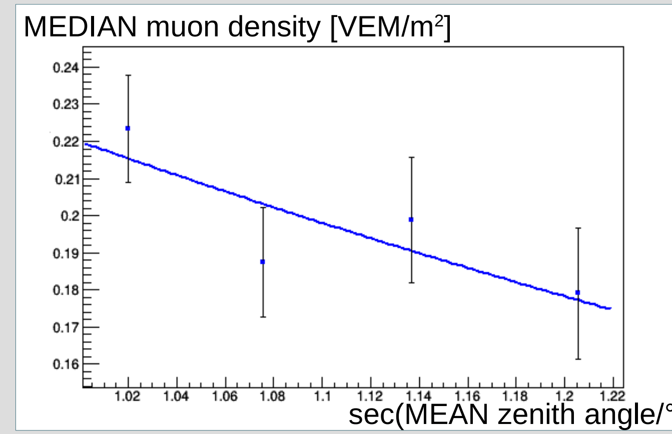
Energy range 3, Radial distance range 2



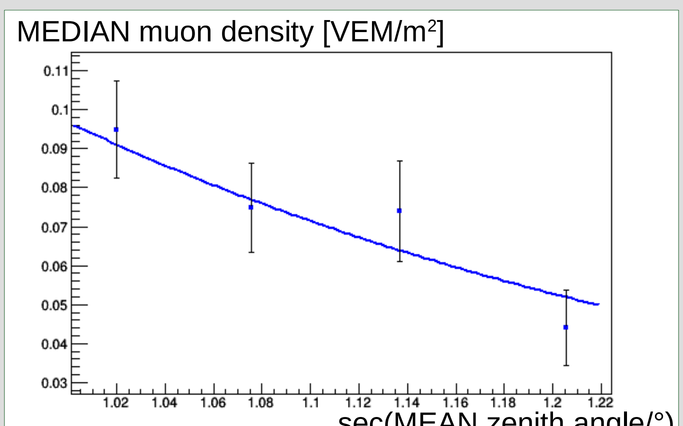
Energy range 3, Radial distance range 3



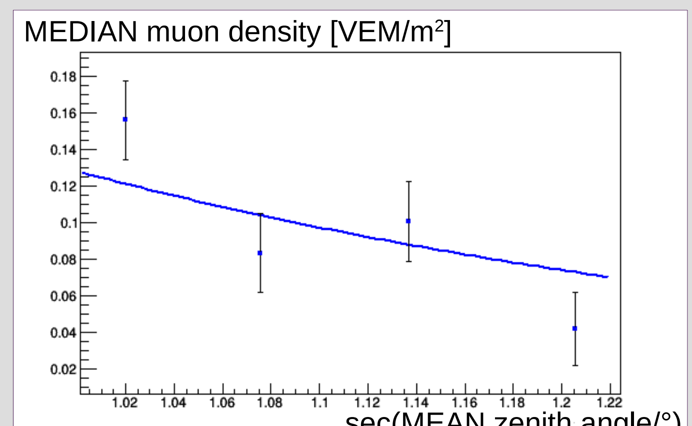
Energy range 3, Radial distance range 4



Energy range 3, Radial distance range 5



Energy range 3, Radial distance range 6



THE FITS (MEDIAN)

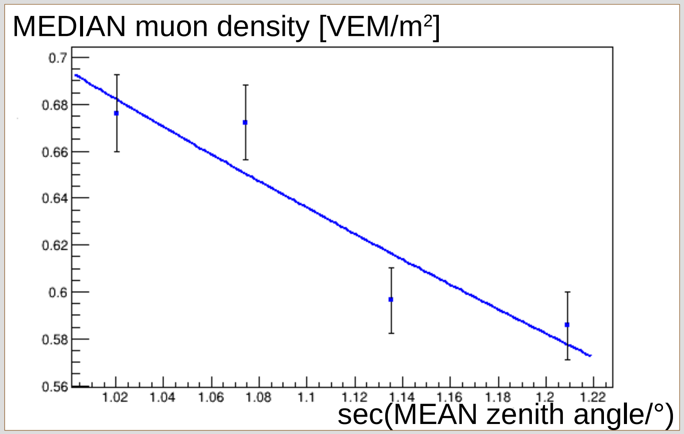
IRON

Range [m]	$P^0_\mu(r)$ [VEM/m^2]	$\delta P^0_\mu(r)$ [VEM/m^2]	$\alpha_\mu(r)$ [g/cm^2]	$\delta\alpha_\mu(r)$ [g/cm^2]	χ 2	N. of Dof.	Prob.
320-420	1.6	0.4	618	133	0.51	2	0.77
420-520	0.50	0.13	1224	55	7.68	2	0.02
520-620	0.63	0.35	659	371	1.69	2	0.43
620-720	2.00	1.43	228	6	1.37	2	0.50
720-820	2.00	1.14	252	9	6.32	2	0.04

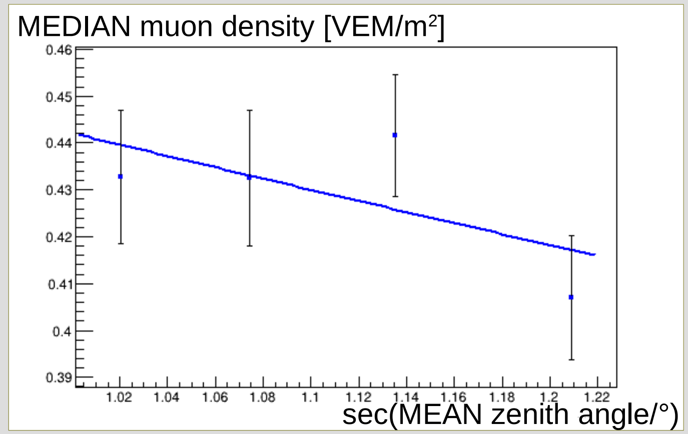
THE FITS (MEDIAN)

EXP. DATA

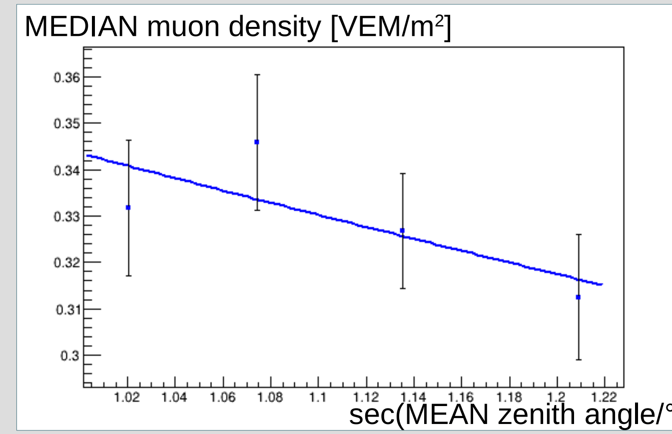
Energy range 3, Radial distance range 2



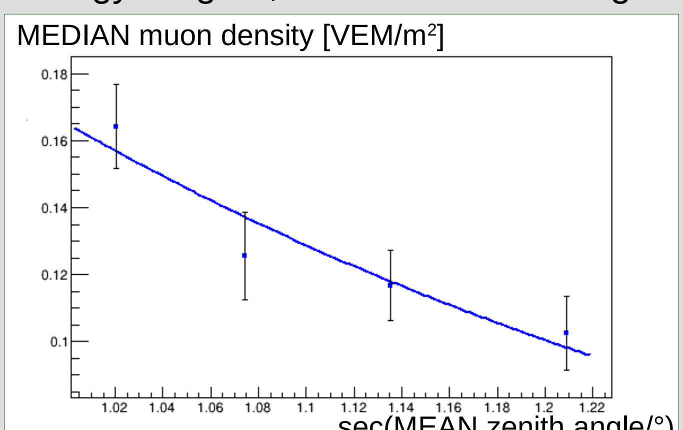
Energy range 3, Radial distance range 3



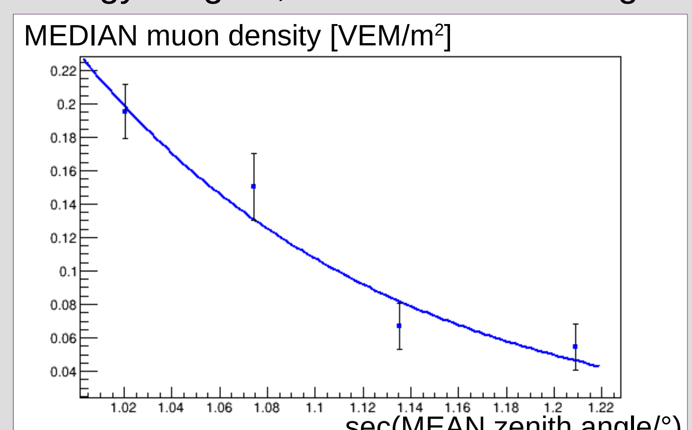
Energy range 3, Radial distance range 4



Energy range 3, Radial distance range 5



Energy range 3, Radial distance range 6



THE FITS (MEDIAN)

EXP. DATA

Range [m]	$P^0_\mu(r)$ [VEM/m^2]	$\delta P^0_\mu(r)$ [VEM/m^2]	$\alpha_\mu(r)$ [g/cm^2]	$\delta\alpha_\mu(r)$ [g/cm^2]	χ 2	N. of Dof.	Prob.
320-420	1.68	0.33	785	155	4.29	2	0.12
420-520	0.58	0.15	2494	2030	2.32	2	0.31
520-620	0.51	0.17	1753	1317	1.18	2	0.55
620-720	2.0	1.5	279	78	1.34	2	0.51
720-820	507	656	90	14	2.41	2	0.30

OTHER COMPARISONS (MEDIAN)

RADIAL DISTANCE RANGE 1 (220 m – 320 m)

The fits have not been done due to the electron contamination.

OTHER COMPARISONS (MEDIAN)

RADIAL DISTANCE RANGE 3 (420 m – 520 m)

	$\alpha_\rho(r)$ [g/cm^2]	$\delta\alpha_\rho(r)$ [g/cm^2]	χ 2	N. of Dof.	Prob.
Experimental data	2494	2030	2.32	2	0.31
Iron	1224	55	7.68	2	0.02
Proton	738	405	3.60	2	0.17

Even if it is not possible to compare these $\alpha_\rho(r)$ values, due to the fact that the iron fit didn't pass the χ^2 test, it can been concluded that the model EPOS-LHC does not describe the experimental data, because the points in the iron fit do not follow the correct trade.

Moreover, despite the fact that the fit for proton and for the experimental data passed the χ^2 test, there are still problems.

OTHER COMPARISONS (MEDIAN)

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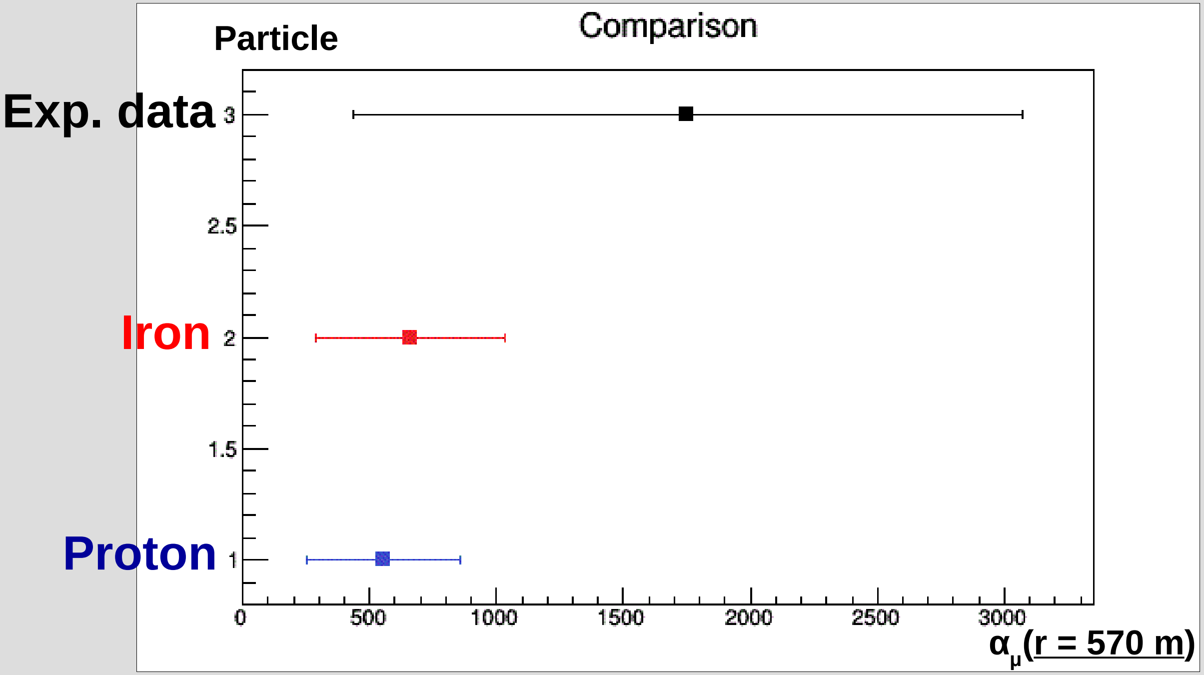
RADIAL DISTANCE RANGE 4 (520 m – 620 m)

	$\alpha_\rho(r)$ [g/cm^2]	$\delta\alpha_\rho(r)$ [g/cm^2]	χ 2	N. of Dof.	Prob.
Experimental data	1753	1317	1.18	2	0.55
Iron	659	371	1.69	2	0.43
Proton	554	302	0.76	2	0.68



The model EPOS-LHC describes the experimental data. However, it must be taken into account that the error bar of the experimantal data is very big.

OTHER COMPARISONS (MEDIAN)

RADIAL DISTANCE RANGE 6 (720 m – 820 m)

	$\alpha_\rho(r)$ [g/cm^2]	$\delta\alpha_\rho(r)$ [g/cm^2]	χ 2	N. of Dof.	Prob.
Experimental data	90	14	2.41	2	0.30
Iron	252	9	6.32	2	0.04
Proton	196	13	1.75	2	0.42

Even if it is not possible to compare these $\alpha_\rho(r)$ values, due to the fact that the iron fit didn't pass the χ^2 test, it can been concluded that the model EPOS-LHC does not describe the experimental data, because the points in the iron fit do not follow the correct trade.

OTHER COMPARISONS (MEDIAN)

RADIAL DISTANCE RANGE 7 (820 m – 920 m)

The fits have not been done because the muon density histograms are empty.