Brane world effective actions for D-brane with fluxes

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This talk is based on a work in progress: M. Bertolini (SISSA), M. B., A. Lerda (UPO), J.F. Morales (CERN) and R. Russo (CERN - Queen Mary), “Brane world effective actions for D-branes with fluxes”, to appear (soon!).

We also thank L. Gallot (Annecy) for collaboration at the initial stage.

Direct stringy derivation of (some parts of) the $\mathcal{N}=1$ effective action for the chiral matter in magnetized (or intersecting) D-brane models.

- Computation of the Kähler metric in the completely non-factorized (or oblique) case
- Conjecture about the correlators of non-abelian twist fields which enter the stringy Yukawa couplings in such oblique situations.
There is by now a very large literature about intersecting and magnetized brane worlds. The few references scattered on the slides are by no means meant to be exhaustive. I apologize for the many relevant ones which will be missing. (The reference list in the paper will be much longer)
Plan of the talk

1. Brane-worlds scenarios
2. D9 branes with general fluxes
3. Effective supersymmetric actions
4. The Kähler metric from strings
5. Relation to the Yukawa couplings
6. FI susy breaking from string diagrams
7. Conclusions and outlook
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Intersecting brane worlds

- Four-dimensional field theories with many “realistic” features arise from type IIA or B superstring models on suitable configurations of D-branes (and orientifolds).


In particular, intersecting brane worlds have received much attention recently:

- Type IIA on $\mathbb{R}^{1,3} \times T_6$ (more generally on a CY - not discussed here)
- D6 branes wrapping intersecting 3-cycles in $T_6$ support, on their non-compact world-volume, gauge groups and chiral matter (the latter are localized at the intersection points in the internal space)
- Consistency requirement: cancellation of RR tadpoles constrains the choice of 3-cycles.
Intersecting brane worlds

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      non-compact world-volume, gauge groups and chiral matter
      (the latter are localized at the intersection points in the internal
      space)
    - Consistency requirement: cancellation of RR tadpoles constrains the choice of 3-cycles.
Gauge groups and chiral matter from branes

- **Gauge groups** from multiple branes, bifundamental **chiral matter** from “twisted” strings, **replicas** from multiple intersections.

- **N.B.** The torus $\mathcal{T}_6$ is assumed to be factorized as $\mathcal{T}_2 \times \mathcal{T}_2 \times \mathcal{T}_2$. 

\[ X_5 \quad X_7 \quad X_9 \]

\[ X_4 \quad X_6 \quad X_8 \]
T-duality and magnetized branes

Upon T-duality (along one direction in each torus), IIA → IIB, and D6-branes intersecting on 3-cycles → D9 with magnetic fluxes

Strings connecting two D9 with different fluxes feel different b.c.’s at their two end-points. They are twisted.

The twists $\theta_i$ are determined from the quantized values of the fluxes

$$F_{MN}^{(\sigma)} = \frac{1}{2\pi} \frac{p_{MN}}{q_{MN}}$$

$p_{MN}$ = Chern class, $q_{MN}$ = wrapping of the D brane around the cycle $dX^M \wedge dX^N$. 
T-duality and magnetized branes

- Upon T-duality (along one direction in each torus), $\text{IIA} \rightarrow \text{IIB}$, and D6-branes intersecting on 3-cycles $\rightarrow$ D9 with magnetic fluxes

- If the torus is factorized as $\mathcal{T}_2 \times \mathcal{T}_2 \times \mathcal{T}_2$, fluxes respecting this factorization are matrices in $\text{so}(2) \oplus \text{so}(2) \oplus \text{so}(2)$
  Abelian situation: fluxes on different branes commute.

- General situation: fluxes on $\mathcal{T}_6$ represented by $\text{so}(6)$ matrices. Oblique case: fluxes on different branes do not commute.
  - Relevant in the context of the moduli stabilization problem
D9 branes with general fluxes
Boundary conditions on magnetized branes

- Bosonic part of the open string action:

\[ S_{\text{bos}} = -\frac{1}{4\pi\alpha'} \int d^2\xi \left[ \partial^\alpha x^M \partial_\alpha x^N G_{MN} + i\varepsilon^{\alpha\beta} \partial^\alpha x^M \partial_\beta x^N B_{MN} \right] - i \sum_{\sigma} q_\sigma \int_{C_\sigma} dx^M A_\sigma^M \]

- In presence of constant \( G, B \) and field-strengths \( F_\sigma \), the boundary conditions read

\[ \bar{\partial} x^M \bigg|_{\sigma=0,\pi} = (R_\sigma)^M_N \partial x^N \bigg|_{\sigma=0,\pi} \]

where the reflection matrix \( R_\sigma \) is given by

\[ R_\sigma = (G - F_\sigma)^{-1} (G + F_\sigma) \]

\[ F_\sigma = B + 2\pi\alpha' F_\sigma \]
Boundary conditions on magnetized branes

- Bosonic part of the open string action: 
  \( (x^M \text{ in the } T_6 \text{ directions, } \sigma = 0, \pi \text{ denotes the end-point}) \)

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  \]
The above b.c.’s can be solved in terms of a **holomorphic, multivalued** field $X^M(z)$ defined all over the complex $z$ plane (**doubling trick**):

$$X^M(e^{2\pi i}z) = R^M_N X^N(z), \quad R = R^{-1}_\pi R_0$$

- Both $R_0$ and $R_\pi$, and hence $R$, preserve the metric: $^t R G R = G$
- We can go to a complex basis $Z = (Z^i, \bar{Z}^\dagger) = \mathcal{E} X$, where

$$R \equiv \mathcal{E} R \mathcal{E}^{-1} = \text{diag}(e^{2i\pi \theta_1}, \ldots, e^{2i\pi \theta_d}, e^{-2i\pi \theta_1}, \ldots, e^{-2i\pi \theta_d})$$

for $0 \leq \theta_i < 1$ ($d = 3$ in our case).
Twisted world-sheet fields

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for $0 \leq \theta_i < 1$ ($d = 3$ in our case).
The open string basis

The open string complex, multivalued, fields $Z^i(z)$, and the corresponding w.s fermions $\Psi^i(z)$, have mode expansions shifted by $\theta_i$.

The $\theta_i$ play exactly the same role as the angles between intersecting D6. They represent the 3 "open string moduli" which determine the open string CFT properties.

The vacuum $|\theta\rangle$ is created by bosonic and fermionic twist fields

$$|\theta\rangle = \lim_{z \to 0} \prod_{i=1}^{d} \sigma_{\theta_i}(z) s_{\theta_i}(z) |0\rangle$$

The physical vertices contain (excited) twist fields
Dependence of the twists on the closed moduli

The $d$ open string twists $\theta_i$ depend on the $4d^2$ closed string parameters $G_{MN}$ and $B_{MN}$ and on the quantized fluxes $F_{0,\pi}^{MN}$ (or on the wrapping numbers for the intersecting branes).
Dependence of the twists on the closed moduli

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- For intersecting D-branes, the $\theta_i$ depend on the moduli describing the shape of the torus:

$$\tan(\pi \theta) = \frac{U_2 n}{m + U_1 n}$$
Dependence of the twists on the closed moduli

- The $d$ open string twists $\theta_i$ depend on the $4d^2$ closed string parameters $G_{MN}$ and $B_{MN}$ and on the quantized fluxes $F_{0,\pi}^{MN}$ (or on the wrapping numbers for the intersecting branes).

- For general magnetized branes, from their definition as eigenvalues of the monodromy $R$ we obtain

$$2\pi i \frac{\partial \theta_i}{\partial m} = \frac{1}{2} \left( \mathcal{E} G^{-1} \frac{\partial(G - B)}{\partial m} \left[ R_\pi - R_0 \right] \mathcal{E}^{-1} \right)_i$$

$$- \frac{1}{2} \left( \mathcal{E} \left[ R_\pi^{-1} - R_0^{-1} \right] G^{-1} \frac{\partial(G + B)}{\partial m} \mathcal{E}^{-1} \right)_i$$

where $m$ is a generic closed string modulus, built out of $G$ and $B$. 
Dependence of the twists on the closed moduli

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- For general magnetized branes, from their definition as eigenvalues of the monodromy $R$ we obtain:

$$
2\pi i \frac{\partial \theta_i}{\partial m} = \frac{1}{2} \left( \mathcal{E} \left( G - B \right) \frac{\partial}{\partial m} + \left[ R_\pi - R_0 \right] \mathcal{E}^{-1} \right)_{ii} - \frac{1}{2} \left( \mathcal{E} \left[ R_\pi^{-1} - R_0^{-1} \right] G^{-1} \frac{\partial}{\partial m} \mathcal{E}^{-1} \right)_{ii}
$$

where $m$ is a generic closed string modulus, built out of $G$ and $B$.

- Applies to general toroidal configurations with any $G$ and $B$, and to generic (i.e. non-abelian) fluxes $F_\sigma$.
Dependence of the twists on the closed moduli

- The $d$ open string twists $\theta_i$ depend on the $4d^2$ closed string parameters $G_{MN}$ and $B_{MN}$ and on the quantized fluxes $F_{0,\pi}^{MN}$ (or on the wrapping numbers for the intersecting branes).

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$$2\pi i \frac{\partial \theta_i}{\partial m} = \frac{1}{2} \left( \mathcal{E} \left( G^{-1} \frac{\partial (G - B)}{\partial m} \right) [R\pi - R_0] \mathcal{E}^{-1} \right)_{ii}$$

$$- \frac{1}{2} \left( \mathcal{E} \left[ R^{-1}_\pi - R_0^{-1} \right] G^{-1} \frac{\partial (G + B)}{\partial m} \mathcal{E}^{-1} \right)_{ii}$$

where $m$ is a generic closed string modulus, built out of $G$ and $B$.

- Crucial formula to reconstruct the Kähler metric for the twisted scalars from mixed open/closed amplitudes.
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In the factorized case, and upon $T$-duality, reproduces the dependence of the angles just described.
Effective supersymmetric actions
Supersymmetric brane-worlds?

- Simplest models with standard-model-like features break all susy.
- Preserving some susy requires some tuning, in the closed and in the open string sector.
- In the closed, bulk sector:
  - $\mathcal{T}_6$ compact $\rightarrow$ cancel RR tadpoles
  - cancel NS-NS tadpoles for susy $\rightarrow$ orientifolds;
- In the open sector, i.e. on the branes:
  - Susy generically broken for the open strings connecting two different D-branes: angles $\theta_i$ $\rightarrow$ twists in the CFT $\rightarrow$ mass split between $R$ and $NS$ spectrum
  - Susy (partially) preserved for particular values of the twists
Supersymmetric configurations

- The SUSY preserved on the twisted strings can be described in the space of the \( \theta_i \)'s, which we take in \([0, 1)\).

- For \( \theta_1 = \theta_2 = \theta_3 = 0 \), \( \mathcal{N} = 4 \) susy spectrum (like for strings between parallel branes in flat space).
Supersymmetric configurations

- The SUSY preserved on the twisted strings can be described in the space of the $\theta_i$'s, which we take in $[0, 1)$.

- When one $\theta$ vanishes, we get an $\mathcal{N} = 2$ hyper-multiplet:
  - two massless scalars from NS
  - two massless fermions from R sector
Supersymmetric configurations

- The SUSY preserved on the twisted strings can be described in the space of the $\theta_i$'s, which we take in $[0, 1)$.

- On the faces, e.g., for $\sum_{j \neq i} \theta_j - \theta_i = 0$ (which we will write as $\sum_j \varepsilon_{j(i)} \theta_j = 0$) we have $\mathcal{N} = 1$ chiral multiplets $\Phi^i$
  - one massless scalar $\phi^i$ from NS
  - one chiral fermion $\chi^i$ from R sector

- Preserved susy charge on the w.s.:

\[
Q_\alpha = \frac{1}{2\pi i} \oint dz e^{-\phi/2} S_\alpha e^{\frac{i}{2} \sum_j \varepsilon_{j(i)} \varphi^j (z)}
\]
Supersymmetric configurations

The SUSY preserved on the twisted strings can be described in the space of the $\theta_i$'s, which we take in $[0,1)$.

In the interior of the tetrahedron, we still have a chiral massless fermion from R sector, but only massive scalars.
Supersymmetric configurations

- The SUSY preserved on the twisted strings can be described in the space of the $\theta_i$'s, which we take in $[0, 1)$.

- Outside the tetrahedron, the scalars would become tachyonic.
Supersymmetric configurations

The SUSY preserved on the twisted strings can be described in the space of the $\theta_i$'s, which we take in $[0, 1)$.

We will consider spontaneously broken $\mathcal{N} = 1$ by taking $\theta$'s close to a face:

$$\theta_i = \theta_i^{(0)} + 2\alpha' \delta_i , \quad \sum_j \varepsilon_{j(i)} \theta_j^{(0)} = 0$$

with $\theta_i^{(0)}$ and $\delta_i$ fixed in the limit $\alpha' \to 0$. 
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- The scalar $\phi^i$ gets a mass $M^2 = \frac{1}{2\alpha'} \sum_j \varepsilon_j(i) \theta_j = \sum_j \varepsilon_j(i) \delta_j$
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- Amounts to spontaneous susy breaking à la FI from v.e.v.'s of the auxiliary fields $D$. We’ll describe later it later at the string level.
Effective action in the N=1 case

- The l.e.e.a is an $\mathcal{N} = 1$ SUGRA coupled with gauged matter coming from different sectors:
  - from the closed string sector, upon usual $\mathcal{T}_6$ compactification.
    - For instance, $6^2$ moduli $m$ from NS-NS bkg fields $G_{MN}, B_{MN}$ describing the stringy shape of the $\mathcal{T}_6$.
  - from the open string sector, gauge + matter fields living on the D-branes.
    - In particular, chiral multiplets $\Phi^i$ (“twisted” matter) from strings stretching between different D-branes (localized at their intersections)

- $\mathcal{N} = 1$ l.e.e.a for open string modes determined by moduli-dependent functions:
  - Kähler metric for the chiral mult. (non-holomorphic in the action)
  - Complexified gauge coupling function and superpotential, (holomorphic)
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  - Complexified gauge coupling function and superpotential, (holomorphic)
Regarding the moduli as fixed, the Kähler potential for the twisted chiral matter will be of the form

\[ K = K_{\phi_i \phi_i}(m) \bar{\phi}^i \phi^i + O(\phi^4) \]

(easy to check that there’s no mixing between \( \phi^i \) and \( \phi^j \) with \( i \neq j \) in our cases).

This corresponds to a lagrangian kinetic term

\[ \mathcal{L} = -K_{\phi_i \phi_i}(m)(\partial_\mu \bar{\phi}^i \partial^\mu \phi^i + M^2 \bar{\phi}^i \phi^i) \]

The dependence of the "metric" \( K_{\phi_i \phi_i} \) on the closed string moduli \( m \) can be determined from mixed open/closed amplitudes.
The Kähler metric from strings
Mixed amplitudes and the Kähler metric

Let $V_m$ be the closed string NS-NS vertex for the modulus $m$. The amplitude

$$A_{\phi^i \phi^i m} \sim \langle V_{\bar{\phi}^i} V_m V_{\phi^i} \rangle$$

is related to the derivative w.r.t. $m$ of the scalar kinetic term. [Lust et al., 2004]

String amplitudes would give canonical kinetic terms, so

$$V_{\phi^i} \to \sqrt{K_{\phi^i \phi^i}} V_{\phi^i}, \quad V_{\bar{\phi}^i} \to \sqrt{K_{\bar{\phi}^i \phi^i}} V_{\bar{\phi}^i}$$

We have then

$$A_{\phi^i \phi^i m} = i K^{-1}_{\phi^i \phi^i} \frac{\partial}{\partial m} \frac{\partial}{\partial \phi^i} \frac{\partial}{\partial \bar{\phi}^i} \mathcal{L} = i K^{-1}_{\phi^i \phi^i} \frac{\partial}{\partial m} \left[ K_{\phi^i \phi^i} \left( k_1 k_2 - M^2 \right) \right]$$
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$$A_{\phi^i \bar{\phi}^i m} \sim \langle V_{\bar{\phi}^i} V_m V_{\phi^i} \rangle$$

is related to the derivative w.r.t. $m$ of the scalar kinetic term. [Lust et al., 2004]

String amplitudes would give canonical kinetic terms, so

$$V_{\phi^i} \rightarrow \sqrt{K_{\bar{\phi}^i \phi^i}} V_{\phi^i}, \quad V_{\bar{\phi}^i} \rightarrow \sqrt{K_{\bar{\phi}^i \phi^i}} V_{\bar{\phi}^i}$$

We have then

$$A_{\phi^i \bar{\phi}^i m} = i K^{-1}_{\bar{\phi}^i \phi^i} \frac{\partial}{\partial m} \frac{\partial}{\partial \phi^i} \frac{\partial}{\partial \bar{\phi}^i} \mathcal{L} = i K^{-1}_{\phi^i \bar{\phi}^i} \frac{\partial}{\partial m} \left[ K_{\phi^i \bar{\phi}^i} \left( k_1 k_2 - M^2 \right) \right]$$
Closed string moduli vertices

The vertex for the insertion of a generic modulus $m$ reads

$$V_m(z, \bar{z}) = \frac{\partial}{\partial m} (G - B)_{MN} V^M_L(z) V^N_R(\bar{z})$$

where

$$V^M_L(z) = \left[ \partial X^M_L(z) + i (k_L \cdot \psi_L) \psi^M(z) \right] e^{i k_L \cdot X_L(z)} ,$$

$$V^N_R(\bar{z}) = \left[ \partial X^N_R(\bar{z}) + i (k_R \cdot \psi_R) \psi^N(\bar{z}) \right] e^{i k_R \cdot X_R(\bar{z})}$$
The form of the amplitude

The amplitude $A_{\bar{\phi} \phi' m}$ reads

$$A_{\bar{\phi} \phi' m} = \left[ \frac{\partial}{\partial m} (G - B) \cdot R_0 \right]_{MN} \mathcal{E}^M_a \mathcal{E}^N_b A^{ab}$$
The form of the amplitude

The amplitude $A_{\phi^i_\bar{\phi}^i_m}$ reads

$$A_{\phi^i_\bar{\phi}^i_m} = \left[ \frac{\partial}{\partial m} (G - B) \cdot R_0 \right]_{MN} E^M_a E^N_b \partial^{ab}$$

Impose the boundary identification $V^M_R(\bar{z}; k_R) = R^M_0 N V^N_L(\bar{z}; k_R)$
The amplitude $A_{\phi^i \phi^i m}$ reads

$$A_{\phi^i \phi^i m} = \left[ \frac{\partial}{\partial m} (G - B) \cdot R_0 \right]_{MN} \mathcal{E}^M_a \mathcal{E}^N_b A^{ab}$$

Switch to the open string complex basis $Z^a = \mathcal{E}^a_M X^M$
The form of the amplitude

The amplitude $A_{\phi_i \phi'_m}$ reads

$$A_{\phi_i \phi'_m} = \left[ \frac{\partial}{\partial m} (G - B) \cdot R_0 \right]_{MN} \mathcal{E}^M_a \mathcal{E}^N_b A^{ab}$$

The matrix $A^{ab}$ is the CFT correlator

$$A^{ab} = \frac{e^{-\pi i \alpha' s/2}}{8 \pi \alpha'^2} \langle V_{\phi'} V^a_L V^b_L V_{\phi'} \rangle$$
The form of the amplitude

The amplitude $A_{\phi^i \phi^i m}$ reads

$$A_{\phi^i \phi^i m} = \left[ \frac{\partial}{\partial m} (G - B) \cdot R_0 \right]_{MN} \mathcal{E}^M_a \mathcal{E}^N_b A^{ab}$$

The matrix $A^{ab}$ is the CFT correlator

$$A^{ab} = \frac{e^{-\pi \alpha' s/2}}{8 \pi \alpha'^2} \langle V_{\phi^i} V_L^a V_L^b V_{\phi^i} \rangle$$

Overall normalization
The form of the amplitude

The amplitude $A_{\phi^i \phi'^m}$ reads

$$A_{\phi^i \phi'^m} = \left[ \frac{\partial}{\partial m} (G - B) \cdot R_0 \right]_{MN} \mathcal{E}^M a \mathcal{E}^N b A^{ab}$$

The matrix $A^{ab}$ is the CFT correlator

$$A^{ab} = \frac{e^{-\pi i \alpha' s/2}}{8\pi \alpha'²} \langle V_{\phi^i} V_L^a V_L^b V_{\phi'^i} \rangle$$

- Cocycle to put off-shell in a controlled way the closed string vertex

$$s = (k_1 + k_2)^2 = (k_L + k_R)^2$$

$$= 2(k_1 \cdot k_2 - M^2) = 2k_L \cdot k_R$$
The form of the amplitude

- The amplitude $A_{\bar{\phi}' \phi'^m}$ reads

$$A_{\bar{\phi}' \phi'^m} = \left[ \frac{\partial}{\partial m} (G - B) \cdot R_0 \right]_{MN} \mathcal{E}^M_a \mathcal{E}^N_b A^{ab}$$

- The matrix $A^{ab}$ is the CFT correlator

$$A^{ab} = \frac{e^{-\pi \alpha' s/2}}{8\pi \alpha'^2} \langle V_{\bar{\phi}^i} V^a_L V^b_L V_{\phi'^i} \rangle$$

- Vertices in the open string complex basis $Z^a$
The CFT correlator

- It is easy to see that the correlator $A^{ab}$ has the matrix form

$$A \equiv \begin{pmatrix} 0 & A_j \delta^{ij} \\ \bar{A}_j \delta^{ij} & 0 \end{pmatrix}, \quad \text{with} \quad A_j = \frac{e^{-\pi i \alpha' s/2}}{8 \pi \alpha'^2} \langle V_{\phi}^j V_L^i \bar{V}_L^j V_{\phi}^i \rangle$$

- Now we must:
  - insert the explicit form of the vertices $V_{\phi}^j(x_1)$ and $V_{\phi}^i(x_2)$
  - integrate their positions $x_{1,2}$ over the real axis and the position $z$ of the closed vertex $V_L^j(z)$ over the upper half plane, up to $SL(2, \mathbb{R})$

- We get

$$A_j = \frac{i \varepsilon_j(i)}{4\pi \alpha'} e^{i \pi \theta_j} \sin \left[ \pi (\theta_j + \frac{\alpha' s}{2}) \right] \frac{\Gamma(\alpha' s + 1)\Gamma(1 - \theta_j - \frac{\alpha' s}{2})}{\Gamma(1 - \theta_j + \frac{\alpha' s}{2})}$$

$$= \frac{i \varepsilon_j(i)}{4\pi \alpha'} e^{i \pi \theta_j} \sin(\pi \theta_j)(1 - \frac{1}{2} \alpha' s \rho_j) + O(\alpha' s^2)$$

- We have defined $\rho_j = \psi(1 - \theta_j) + \psi(\theta_j) + 2 \gamma_E$
The CFT correlator

It is easy to see that the correlator $A^{ab}$ has the matrix form

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Now we must:

- insert the explicit form of the vertices $V_{\bar{j}}(x_1)$ and $V_{\phi}(x_2)$
- integrate their positions $x_{1,2}$ over the real axis and the position $z$ of the closed vertex $V_L^j(z)$ over the upper half plane, up to $SL(2, \mathbb{R})$

We get

$$A_j = \frac{i \varepsilon_j(i)}{4 \pi \alpha'} e^{i \pi \theta_j} \sin \left[ \pi (\theta_j + \alpha' s/2) \right] \frac{\Gamma(\alpha' s + 1) \Gamma(1 - \theta_j - \alpha' s/2)}{\Gamma(1 - \theta_j + \alpha' s/2)}$$

$$= \frac{i \varepsilon_j(i)}{4 \pi \alpha'} e^{i \pi \theta_j} \sin(\pi \theta_j)(1 - \frac{1}{2} \alpha' s \rho_j) + O(\alpha' s^2)$$

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- We get

$$A_j = \frac{i \epsilon_j(i)}{4\pi \alpha'} e^{i\pi \theta_j} \sin \left[ \pi \left( \theta_j + \alpha' s/2 \right) \right] \frac{\Gamma(\alpha' s + 1)\Gamma(1 - \theta_j - \alpha' s/2)}{\Gamma(1 - \theta_j + \alpha' s/2)}$$

$$= \frac{i \epsilon_j(i)}{4\pi \alpha'} e^{i\pi \theta_j} \sin(\pi \theta_j)\left(1 - \frac{1}{2} \alpha' s \rho_j \right) + O \left( \alpha' s^2 \right)$$

- We have defined $\rho_j = \psi(1 - \theta_j) + \psi(\theta_j) + 2\gamma_E$
The CFT correlator

- It is easy to see that the correlator $A^{ab}$ has the matrix form

$$A \equiv \begin{pmatrix} 0 & A j \delta^{ij} \\ \bar{A} j \delta^{ij} & 0 \end{pmatrix}, \quad \text{with} \quad A j = \frac{e^{-\pi i \alpha' s/2}}{8\pi \alpha'^2} \langle V_{\bar{\phi} i} V_{L}^{j} \bar{V}_{L}^{j} V_{\phi}^{i} \rangle$$

- Now we must:
  - insert the explicit form of the vertices $V_{\bar{\phi} i}(x_1)$ and $V_{\phi}^{i}(x_2)$
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- We get

$$A j = \frac{i \varepsilon j(i)}{4\pi \alpha'} e^{i\pi \theta j} \sin \left[ \pi (\theta j + \alpha' s/2) \right] \frac{\Gamma(\alpha' s + 1)\Gamma(1 - \theta j - \alpha' s/2)}{\Gamma(1 - \theta j + \alpha' s/2)}$$

$$= \frac{i \varepsilon j(i)}{4\pi \alpha'} e^{i\pi \theta j} \sin(\pi \theta j)(1 - \frac{1}{2} \alpha' s \rho j) + O \left( \alpha' s^2 \right)$$

- We have defined $\rho j = \psi(1 - \theta j) + \psi(\theta j) + 2\gamma E$
The result for the amplitude

Altogether, one can write (up to 2-derivative terms, i.e. up to $s^2$) the correlator $A^{ab}$ in matrix form as

$$A = \frac{1}{2} G^{-1} (R^{-1} - 1) H, \quad H = i \begin{pmatrix} h_j & 0 \\ 0 & -h_j \end{pmatrix}$$

with

$$h_j = \frac{\varepsilon j(i)}{4\pi \alpha'} \left( 1 - \frac{1}{2} \alpha' s \rho_j \right) = \frac{1}{2\pi} K^{-1}_{\phi^i \phi^i} \frac{\partial}{\partial \theta^j} K_{\phi^i \phi^i} (k_1 \cdot k_2 - M^2)$$

and

$$K_{\phi^i \phi^i} = e^{2\gamma_E \alpha' M^2} \sqrt{\frac{\Gamma(1 - \theta_i)}{\Gamma(\theta_i)}} \prod_{k \neq i} \sqrt{\frac{\Gamma(\theta_k)}{\Gamma(1 - \theta_k)}}$$
The result for the amplitude

Altogether, one can write (up to 2-derivative terms, i.e. up to $s^2$) the correlator $A^{ab}$ in matrix form as

$$A = \frac{1}{2} G^{-1} (R^{-1} - 1) \mathcal{H} , \quad \mathcal{H} = i \begin{pmatrix} h_j & 0 \\ 0 & -h_j \end{pmatrix}$$

with

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and

$$K_{\bar{\phi}^i \phi^i} = e^{2\gamma E \alpha' M^2} \sqrt{\frac{\Gamma(1 - \theta_i)}{\Gamma(\theta_i)}} \prod_{k \neq i} \sqrt{\frac{\Gamma(\theta_k)}{\Gamma(1 - \theta_k)}}$$

We used the kinematics $s = 2(k_1 \cdot k_2 - M^2)$, the dependence of $M^2$ on $\theta_j$ and the fact that $\psi(x) = \frac{d \ln \Gamma(x)}{dx}$.
The result for the amplitude

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$$A = \frac{1}{2} G^{-1} (R^{-1} - 1) \mathcal{H}, \quad \mathcal{H} = i \begin{pmatrix} h_j & 0 \\ 0 & -h_j \end{pmatrix}$$

with

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$$K_{\phi^i \phi^i} = e^{2\gamma_E \alpha' M^2} \sqrt{\frac{\Gamma(1 - \theta_i)}{\Gamma(\theta_i)}} \prod_{k \neq i} \sqrt{\frac{\Gamma(\theta_k)}{\Gamma(1 - \theta_k)}}$$

- The exponential term goes to 1 in the field theory limit
The magic of the result

- Substituting into the expression of the correlator $A_{\bar{\phi}^i \phi^i m}$ we get after some algebra

$$A_{\bar{\phi}^i \phi^i m} = \frac{1}{2} \mathcal{E} G^{-1} \frac{\partial}{\partial m} (G - B) (R_\pi - R_0) \mathcal{E}^{-1} |^j_j h_j - h.c.$$  

- Comparing with the expression of the dependence of the twists $\theta_i$ from the moduli $m$ we can write

$$A_{\bar{\phi}^i \phi^i m} = 2\pi \frac{\partial \theta_j}{\partial m} h_j = K_{\bar{\phi}^i \phi^i}^{-1} \frac{\partial \theta_j}{\partial m} \frac{\partial}{\partial \theta_j} K_{\bar{\phi}^i \phi^i} (k_1 \cdot k_2 - M^2)$$

- This is the expression we expected if $K_{\bar{\phi}^i \phi^i}$ really is the Kähler metric
Summarizing, in the field theory limit the expression of the Kähler metric $K_{\bar{\phi}^i \phi^i}$ for the scalar $\phi^i$ depends on the moduli only through the open string twists

$$\theta_i^{(0)} = \lim_{\alpha' \to 0} \theta_i$$

in an $\mathcal{N} = 1$ configuration. Explicitly,

$$K_{\bar{\phi}^i \phi^i} = \sqrt{\frac{\Gamma(1 - \theta_i^{(0)})}{\Gamma(\theta_i^{(0)})}} \prod_{k \neq i} \sqrt{\frac{\Gamma(\theta_k^{(0)})}{\Gamma(1 - \theta_k^{(0)})}}$$

This holds for a general toroidal compactification, and with arbitrary magnetic fluxes, also non-commuting.

Generalizes [Lust et al., 2004]
Relation to the Yukawa couplings
Stringy expression of the Yukawa couplings

In the stringy description, Yukawa couplings have the form $Y_{ijk} = A_{ijk} \mathcal{W}_{ijk}$, where

- $\mathcal{W}_{ijk} = \text{classical contribution from extended world-sheets bordered by the intersecting branes}$. [Cremades et al. 2003],[Abel-Owen, 2003],...
  - Multiple intersections $\rightarrow$ families
  - different minimal world-sheets $\rightarrow$ exponential hierarchy of couplings
  - have counterparts in magnetized brane worlds [Cremades et al., 2004]

- $A_{ijk} = \text{quantum fluctuations given by the correlator of the twisted vertices located at the intersections}$. [Cvetic-Papadimitriou, 2003],...
Yukawa couplings and $N=1$ superpotential

- In $\mathcal{N} = 1$ susy, the Yukawa couplings arise from the superpotential

$$\int d^2\theta \, W(\Phi^i) + \text{c.c.} \rightarrow \dots + \frac{\partial W}{\partial \phi^i \partial \phi^j} \chi^i \chi^j + \text{h.c.}$$

For $W = W_{ijk} \Phi^i \Phi^j \Phi^k$, the $W_{ijk}$ are the Yukawa couplings in the basis where the kinetic terms are determined by the Kähler potential $K$.

- When realized in string compactifications, non-renormalization property: $W$ gets no perturbative $\alpha'$ corrections

- In the brane-world context, we identify therefore the $W_{ijk}$ as the classical world-sheet instanton contributions:

$$W_{ijk} = \mathcal{W}_{ijk}$$
Yukawa couplings and $N=1$ superpotential

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  \[
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  \]

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$[\text{GSW, vol. 2}], [\text{Burgess et al, 2005}], \ldots$

In the brane-world context, we identify therefore the $W_{ijk}$ as the classical world-sheet instanton contributions:

$$W_{ijk} = \mathcal{W}_{ijk}$$
Kähler metric and quantum Yukawas

- The \( \mathcal{N} = 1 \) holomorphic couplings \( W_{ijk} \) are related to the physical ones, \( Y_{ijk} \) (the ones provided by the string computation) by rescaling the fields \( \phi^i, \chi^j, \chi^k \) to give them canonical kinetic terms.

- One has thus

\[
Y_{ijk} = (K_{\bar{\phi}^i \phi^i} K_{\bar{\phi}^j \phi^j} K_{\bar{\phi}^k \phi^k})^{-1/2} W_{ijk}
\]

- We had already found

\[
Y_{ijk} = A_{ijk} W_{ijk}
\]

- Hence, the amplitude \( A_{ijk} \) for the three twisted vertices should be factorizable into

\[
A_{ijk} = (K_{\bar{\phi}^i \phi^i} K_{\bar{\phi}^j \phi^j} K_{\bar{\phi}^k \phi^k})^{-1/2}
\]
The abelian case

- In the case of a factorized torus with commuting angles (or for D-branes at angles) the direct computation of the string amplitude $A_{ijk}$ is possible.

- It involves in particular the correlator of three bosonic twist fields on the torus which are simultaneously expressible in terms of twist angles $\{\theta_i\}, \{\nu_i\}, \{\lambda_i\}$.

- This correlator is computable by factorization of the 4-twist amplitude, and its dependence on the three sets of angles factorizes.

- In the end, one indeed finds

$$A_{ijk} = (K^-_{\phi_i\phi_i} K^-_{\phi_j\phi_j} K^-_{\phi_k\phi_k})^{-1/2}$$

in agreement with the non-renormalization theorem.

[Cvetic-Papadimitriou, 2003, Lust et al., 2004]
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in agreement with the non-renormalization theorem.
The non-abelian case?

- We have considered the general case in which the reflection matrices at the various boundaries do not commute, and shown that the Kähler metric remains the same.

- Hence the monodromy matrices $R_{\theta, \nu, \lambda}$ induced by the three twist operators cannot, in general, be simultaneously diagonalized.

- We have thus to deal with ("non-abelian twist fields"), whose 3-point CFT correlators are not known. Their computation represents a challenge.

- The non-renormalization theorem, however, suggests that the correlator still factorizes and depends on the three sets $\{\theta_i\}, \{\nu_i\}, \{\lambda_i\}$ of monodromy eigenvalues.
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- The non-renormalization theorem, however, suggests that the correlator still factorizes and depends on the three sets $\theta_i$, $\nu_i$, $\lambda_i$ of monodromy eigenvalues.
FI susy breaking from string diagrams
The mass of the scalars and the FI mechanism

- When the $\theta_i$ are close to $\mathcal{N} = 1$ values: $\theta_i = \theta_i^0 + 2\pi \alpha' \delta_i$, with $\sum_j \varepsilon_{j(i)} \theta_i^0 = 0$, the twisted scalar $\phi^i$ acquires a mass

$$M^2 = \frac{1}{2\pi \alpha'} \sum_j \varepsilon_{j(i)} \theta_i = \sum_j \varepsilon_{j(i)} \delta_i$$

- This susy breaking arises as a FI process involving the auxiliary fields $D$ in the (untwisted) gauge multiplets

- The twisted fields transform in the bi-fundamental

- We expect by susy a coupling to the auxiliary fields $D_\pi$, $D_0$ of the gauge multiplets:

$$(D_\pi - D_0) \overline{\phi}^i \phi^i$$
The vertex describing the auxiliary field $D$ w.r.t. to the preserved susy is

$$V_D \propto \sum_j \varepsilon_{j(i)} \bar{\psi}^i \psi^i$$

These diagrams account for the interaction term

$$(D_{\pi} - D_0) \bar{\phi}^i \phi^i$$
The VEV of the auxiliary fields

- The auxiliary field $D$ gets a vev $\langle D \rangle$ in presence of NS-NS background

This diagram computes the derivative $\partial_m \langle D \rangle$ w.r.t. a generic NS-NS modulus $m$:

$$\partial_m \langle D \rangle = \langle V_m V_D \rangle$$

$$= \frac{1}{4\pi\alpha'} \frac{\partial}{\partial m} (G - B)_{MN} \langle V^M_L V^N_R V_D \rangle$$

- Boundary reflection: $V^N_R = R^N_P V^N_L$. Go to the complex basis $\psi^i$, get a simple correlator. Finally

$$\partial_m \langle D \rangle = -\frac{1}{4\pi\alpha'} \sum_{i=1}^3 \mathcal{E} G^{-1} \partial(G - B) \frac{\partial m}{\partial m} R \mathcal{E}^{-1} \bigg|_{ii} - \text{h.c.}$$
The induced mass term for the twisted scalars

- The coupling to the $D$ fields induces a mass term for $\phi^i$

\[
M^2 \bar{\phi}^i \phi^i = (\langle D_\pi \rangle - \langle D_0 \rangle) \bar{\phi}^i \phi^i
\]

- From the above direct string computation we find

\[
\frac{\partial M^2}{\partial m} = \frac{\partial}{\partial m} \langle D_\pi - D_0 \rangle
\]

\[
= -\frac{1}{4\pi\alpha'} \sum_i \mathcal{E} G^{-1} \frac{\partial (G - B)}{\partial m} (R_\pi - R_0) \mathcal{E}^{-1} \bigg|_{ii} - \text{h.c.}
\] (1)

We reconstruct the Jacobian $\frac{\partial \theta_j}{\partial m}$

- We get thus

\[
\frac{\partial M^2}{\partial m} = \frac{1}{2\alpha'} \frac{\partial}{\partial m} \sum_j \mathcal{E} j(i) \theta_j
\] (2)
What about the F auxiliary fields?

- The stringy vertex for the untwisted auxiliary fields $F$

\[ V_{F(i)} \propto \sum_j \epsilon_{ijk} \psi^j \psi^k \]

- Notice the difference w.r.t. the $D$ vertex

- Gets a v.e.v. $\langle F(i) \rangle$ from the interaction with the NS-NS moduli $m$ similarly to the $D$ field

  - However, it is non-zero only when the reflection matrix $R$ has $(2,0)$ components in the complex basis $\psi^i$

- The $F(i)$ have no trilinear coupling to $\bar{\phi}^i, \phi^i$ so its v.e.v. does not give a mass to $\phi^i$. 
Conclusions and outlook
We discuss the derivation of the $\mathcal{N} = 1$ effective action for the chiral matter arising from twisted open strings in magnetized/intersecting brane worlds directly from string diagrams. We extend the derivation of the Kähler metric to the general case:

- compactification on a non-factorized $T^6$, with any $G_{MN}$, $B_{MN}$;
- oblique magnetic fluxes on the branes;
- susy breaking à la FI inducing a mass term for the scalars.

The connection to the Yukawa couplings provided by the non-renormalization of the superpotential leads to a conjecture about correlators of non-abelian twist fields.
Outlook

- The most pressing task:
  - ... finish the paper!
- Investigate the CFT of non-abelian twist fields
- The dependence from RR closed backgrounds
Outlook

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  ▶  ... finish the paper!

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Conclusions and outlook

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Some references
A few ref.s on magnetized and intersecting branes


Some reviews on IBW’s


A few ref.s on Yukawas in brane-worlds


A few other ref.s (mixed amplitudes, oblique fluxes, ...) 


