Self-attraction effect and correction on three absolute gravimeters

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Absolute gravimeters:

- MPG-2: Germany, prototype instrument;
- FG5-238: U.S.A., Micro-g LaCoste Incorporation, applicable for FG5-2xx of similar geometry (thanks to F. Greco, A. Pistorio → see talk);
- IMGC-02: Italy, prototype instrument, rise-and-fall motion.
Self-Attraction Effect

How the gravity field is influenced by the mass of... the gravimeter?
... a mug of coffee?
... a (fat) operator?

Three main parts simulated:

- **read-out electronic case**: SAE from this source is negligible, e.g. a mass of 200 kg placed at 1 m produces a SAE < 0.1 \( \mu \text{Gal} \);
- **measuring system**: supporting tripod, seismometer or super-spring system with its basis, several detectors and the interferometer;
- **launch system**: the vacuum chamber with the dropping mechanism and its basis with all the accessories.

For MPG-2 and FG5-238 an average contribution of \((0.1 \pm 0.1) \mu \text{Gal}^{1}\) due to the **co-falling system** has been added.

\[1 \mu \text{Gal} = 10^{-8} \text{ m/s}^2\]
Finite Element Method simulation

A mesh adapted to the geometry of the absolute gravimeter has been chosen to simulate the main parts, using the COMSOL® Multiphysics software, via similar approach used for the cold atomic gravimeter.

G.D’Agostino, S. Merlet, A. Landragin, F. Pereiera don Santos (2011)

example: FG5-238
Self Attraction Effect for the MPG-2

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Self Attraction Effect for the FG5-238

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![Graph showing self-attraction effect](Image)

- Self Attraction Effect [μGal] along the Z [m] axis.
- The graph illustrates the effect of self-attraction on the FG5-238 system.
- Peak and trough points are highlighted to indicate areas of interest.

Additional notes on the graph are not provided in the image.
Self-attraction effect & correction

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The SAE is approximately linear, so it can be treated in addition to the conventional vertical gradient $\gamma$:

$$g(z) = (g_0 + \gamma z) + (g_0 \mathrm{SAE} + \gamma \mathrm{SAE} z). \tag{1}$$

The $g_m$ is obtained as mean-weighted value of the free-fall acceleration $\rightarrow$ it is referred to the effective measurement height $h_{\text{eff}}$ and we have:

$$g_m = g(h_{\text{eff}}) = (g_0 + \gamma h_{\text{eff}}) + (g_0 \mathrm{SAE} + \gamma \mathrm{SAE} h_{\text{eff}}). \tag{2}$$

The Self Attraction Correction (SAC) is:

$$\Delta g_{\text{SAC}} = -(g_0 \mathrm{SAE} + \gamma \mathrm{SAE} h_{\text{eff}}). \tag{3}$$
The observed SAE is linearly approximated: $g_{0\text{SAE}}$ and $\gamma_{\text{SAE}}$ are computed. The SAC is computed by the interpolation to the $h_{\text{eff}}$ (or $z_b$).
MPG2:

- linear interpolation of the SAE to the level \( z = h_{\text{eff}} \);
- \( \text{SAC} = -1.69 \, \mu\text{Gal} \) (with 0.1 \( \mu\text{Gal} \) → co-falling system).

FG5-238:

- gravity gradient \( \gamma \) as external known quantity → transformation of the linear model automatically shifts the mean-weighted measurement result from \( z = h_{\text{eff}} \) to the start of measurement interval \( H \) with \( \Delta g_{\gamma} = -\gamma h_{\text{eff}} \);
- \( h_{\text{eff}} \) not depending on constant gravity gradient → correction valid for results at \( z = h_{\text{eff}} \) or \( z = 0 \);
- \( \text{SAC} = -1.23 \, \mu\text{Gal} \) (with 0.1 \( \mu\text{Gal} \) → co-falling system).

IMGC-02:

- non-linear model → a posteriori correction is not possible
- an estimation of the SAC can be given by \( \Delta g_{\text{SAC}} = -[g_{0\text{SAE}} + \gamma (z_{b} - h_{0})] \), where \( z_{b} \simeq H/5.8 \);
- \( \text{SAC} = +0.61 \, \mu\text{Gal} \).
Uncertainty evaluation

Uncertainty budget of the SAC

- **incomplete modelling**: incomplete knowledge of density and geometry of the parts $\rightarrow 0.1 \, \mu\text{Gal}$;

- **FEM simulation**: discrepancy FEM simulation / mathematical model, SAE dependent $\rightarrow 0.1 \, \mu\text{Gal}$ (overestimated);

- **SAE non-linearity**: maximal SAE deviations from straight lines $\rightarrow 0.13 \, \mu\text{Gal}$ (MPG-2), $0.15 \, \mu\text{Gal}$ (FG5-238), $0.06 \, \mu\text{Gal}$ (IMGC-02) (overestimated);

- **co-falling system**: average effect estimated $\rightarrow 0.1 \, \mu\text{Gal}$ (MPG-2, FG5-238).
The final results are as follow:

<table>
<thead>
<tr>
<th></th>
<th>MPG-2</th>
<th>FG5-238</th>
<th>IMGC-02</th>
</tr>
</thead>
<tbody>
<tr>
<td>measurement range $H$ [m]</td>
<td>0.094</td>
<td>0.192</td>
<td>0.174×2</td>
</tr>
<tr>
<td>effective height $h_{eff}$ or $z_b$ [m]</td>
<td>0.038</td>
<td>0.074</td>
<td>0.029</td>
</tr>
<tr>
<td>SAC [$\mu$Gal]</td>
<td>−1.7</td>
<td>−1.2</td>
<td>+0.6</td>
</tr>
<tr>
<td>$u_{SAC}$ [$\mu$Gal]</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

E. Biolcati, S. Svitlov, A. Germak (2012)

The computed corrections are consistent with other independent estimates (values in $\mu$Gal):

<table>
<thead>
<tr>
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<th>MPG-2</th>
<th>FG5-238</th>
<th>IMGC-02</th>
</tr>
</thead>
<tbody>
<tr>
<td>E. Biolcati, S. Svitlov, A. Germak (2012)</td>
<td>−1.7±0.2</td>
<td>−1.2±0.2</td>
<td>+0.6±0.1</td>
</tr>
<tr>
<td>L. Robertson (1996)</td>
<td>-</td>
<td>−1.4±0.1</td>
<td>-</td>
</tr>
<tr>
<td>V. Palinkas, J. Liard, Z. Jiang (2012)</td>
<td>−1.4±0.1</td>
<td>−1.1±0.2</td>
<td>-</td>
</tr>
</tbody>
</table>
Conclusion

The developed procedure can be applied to other AGs, knowing peculiar features as geometry, mass values and fit model.

MPG2:
- a SAC of \((-1.7 \pm 0.2) \mu\text{Gal}\) can be applied at \(h_{\text{eff}}\);
- value > declared uncertainty of 0.5 \(\mu\text{Gal}\) → previous results can be corrected.

FG5-238:
- a SAC of \((-1.2 \pm 0.2) \mu\text{Gal}\) can be applied at top of drop or \(h_{\text{eff}}\);
- @ICAG2005 a contribution of 0.1 \(\mu\text{Gal}\) was considered → all previous absolute gravity measurement results (not only during the comparisons) can be corrected.

IMGC-02:
- a SAC of \((+0.6 \pm 0.1) \mu\text{Gal}\) can be considered, but not a posteriori applied;
- value negligible w.r.t. the declared uncertainty of 4.3 \(\mu\text{Gal}\) @ICAG2005.

@ICAG2009 the applied corrections for MPG-2 and FG5-238 are consistent with current report.

Dedicated paper: E. Biolcati, S. Svitlov, A. Germak, Metrologia 49 (2012) 560-566
For the future... be aware of where you put your masses!

Thanks for your attention.
Backup slides
geometry draw

- regular and homogeneous $\rightarrow$ actual values
- complex or inhomogeneous $\rightarrow$ equivalent mass and volume

mesh implementation

equation solving

Validation: gravitational field simulated and calculated for two spheres of different density and volume $\rightarrow$ 89.9% of the residuals are below 0.1 $\mu$Gal.