I meccanismi delle glaciazioni: un approccio fisico

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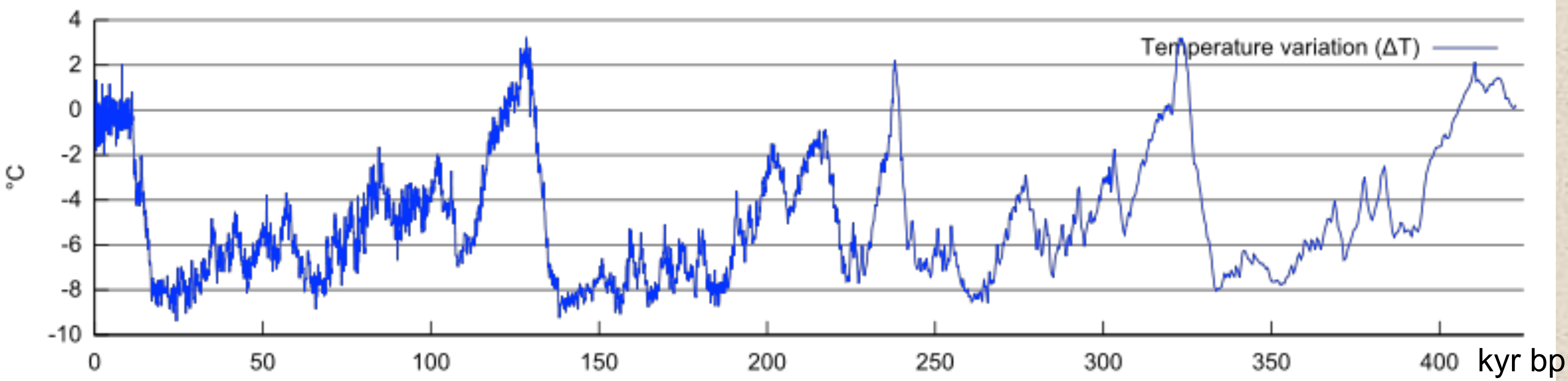
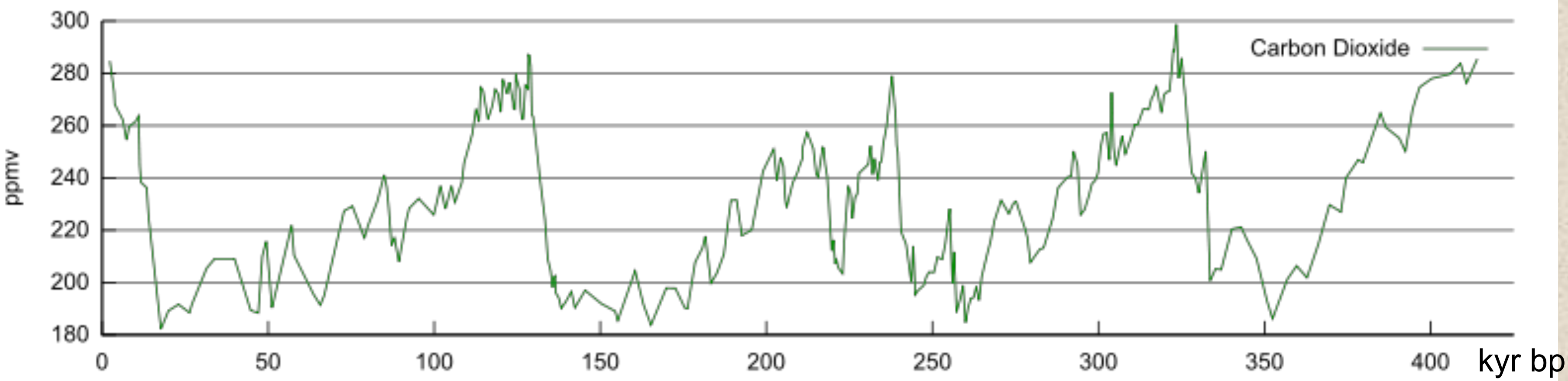
Temperature and CO_2 concentration from Vostok ice core data

Sawtooth temperature pattern:
increasingly colder glacial period,
then rapid return to interglacial

Strong correlation between CO_2 and T

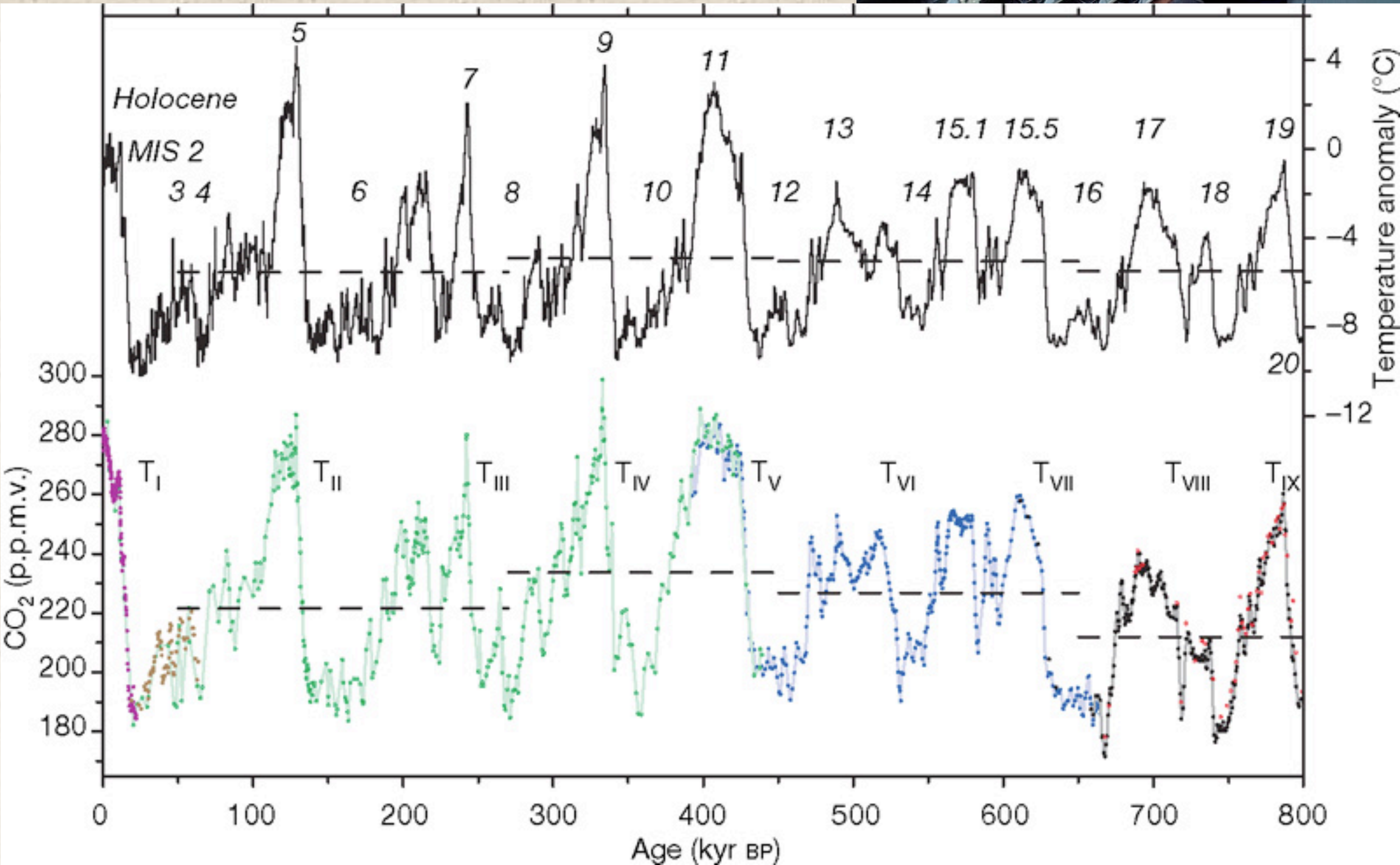
Vostok (1998) 3310 m = 420 kyr





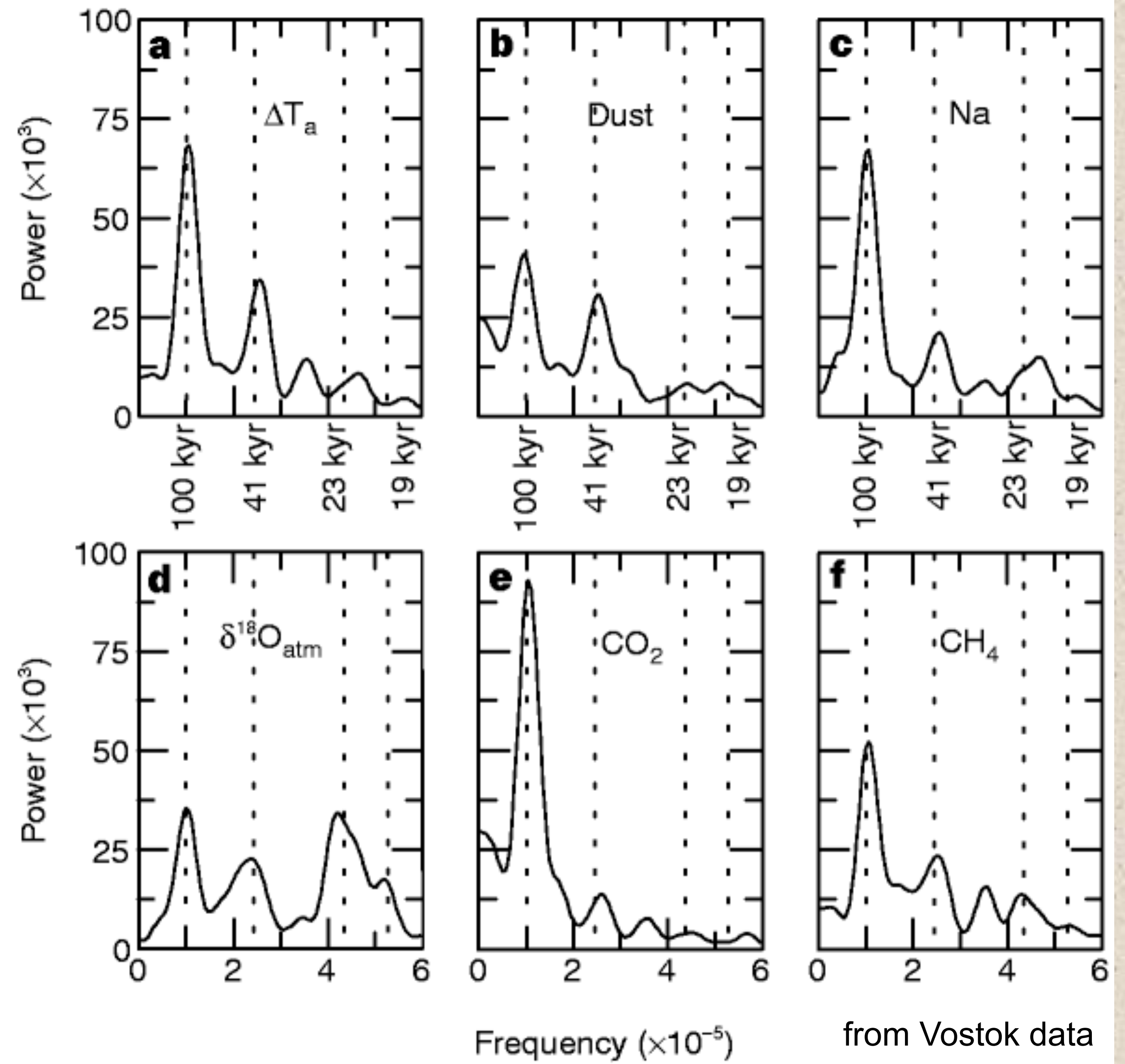
Moving in the past: Epica Dome C ice core

3270 m = 740 kyr (2004)



Periodic variation of recent climate

Power spectra of ice core data show characteristic periods of glacial-interglacial variations



from Vostok data

What forces the periodicity ?

M. Milankovitch in 1941 proposed that glacial period are forced by astronomical variations of earth orbit.

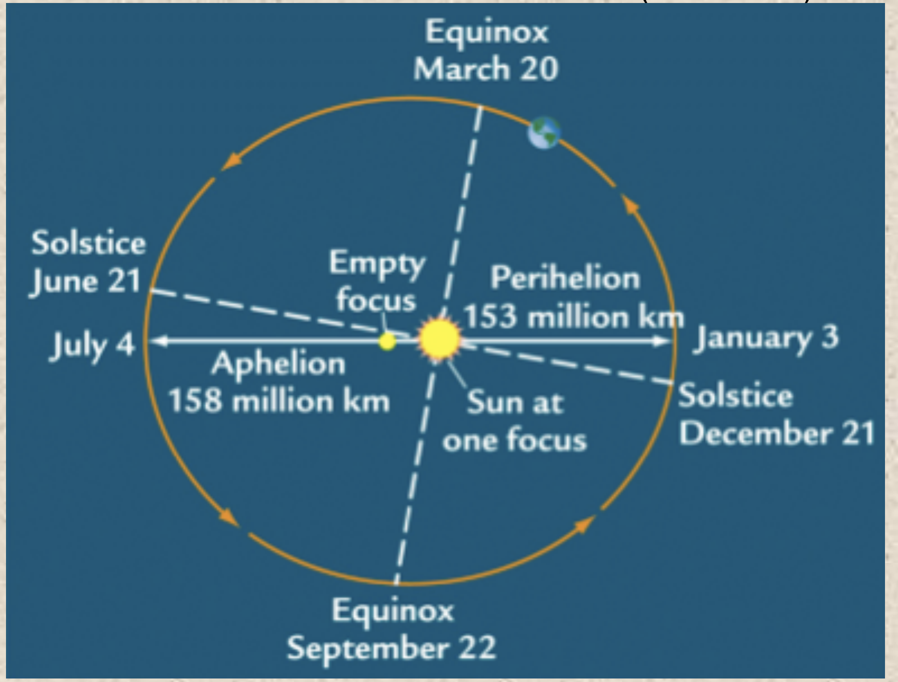
Pioneers: J.Adhemar (~1840), J.Croll (~1860)

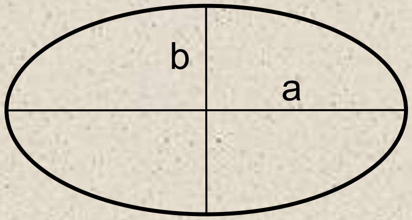
Milutin Milankovitch
(1879-1958)

Eccentricity

The eccentricity of the earth orbit changes between e≈0.005 to e≈0.061 (present e=0.0167)

Periods of eccentricity variations:
100 ky and 413 ky





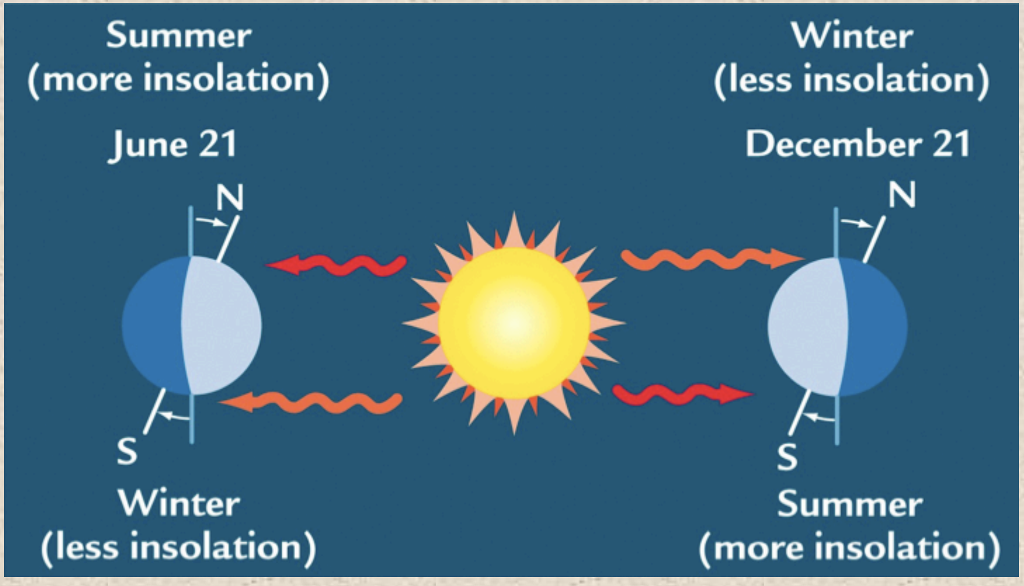
$$e = \frac{\left(a^2 - b^2\right)^{1/2}}{a}$$

Obliquity

The tilt of earth axis changes between θ=22.1° and θ=24.5° with period T=41 ky

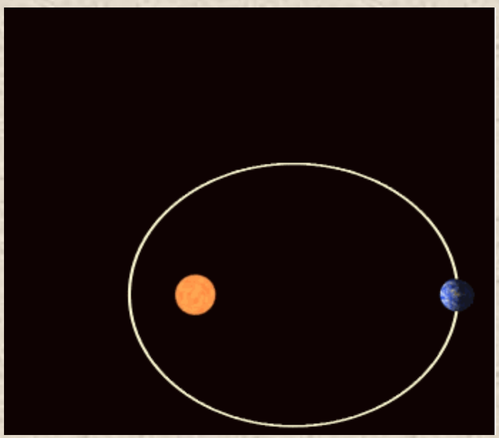
Present obliquity: θ=23.5°

Obliquity is proportional to the strength of the seasons, especially at high latitudes

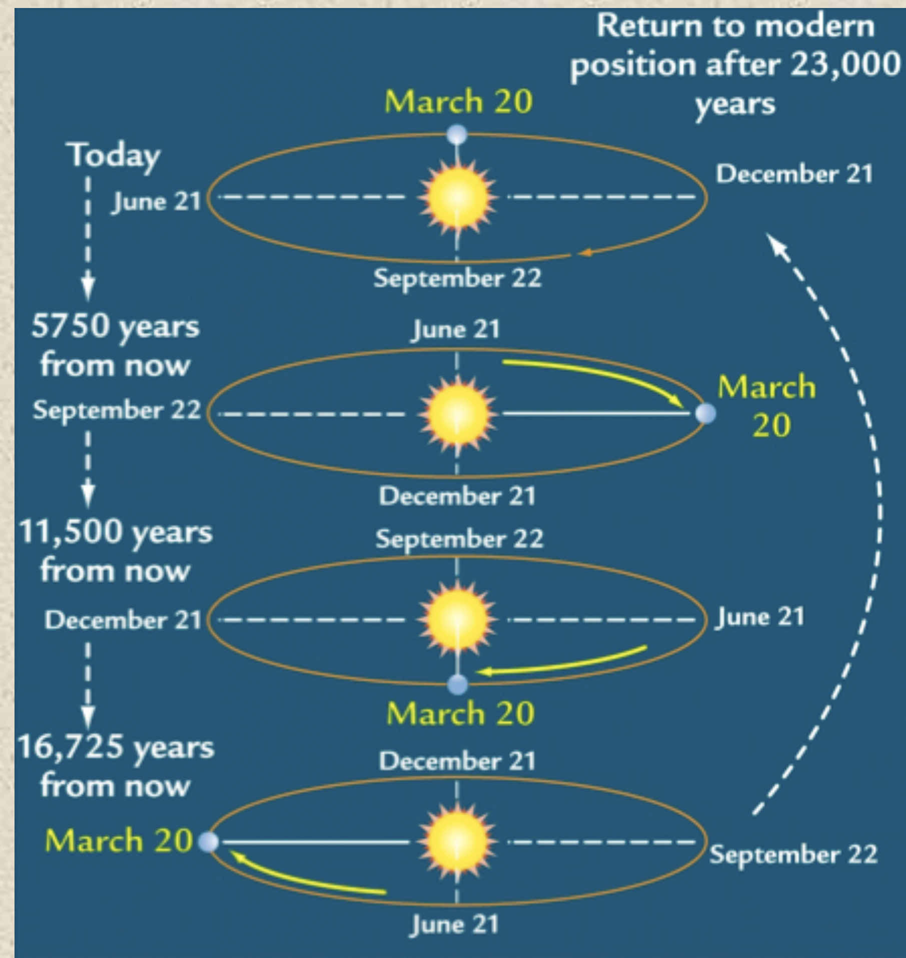


Precession of the equinoxes

The combined effect of the axial precession (T=26 ky) and orbital precession causes the precession of the equinoxes (and solstices) with period T=23 ky



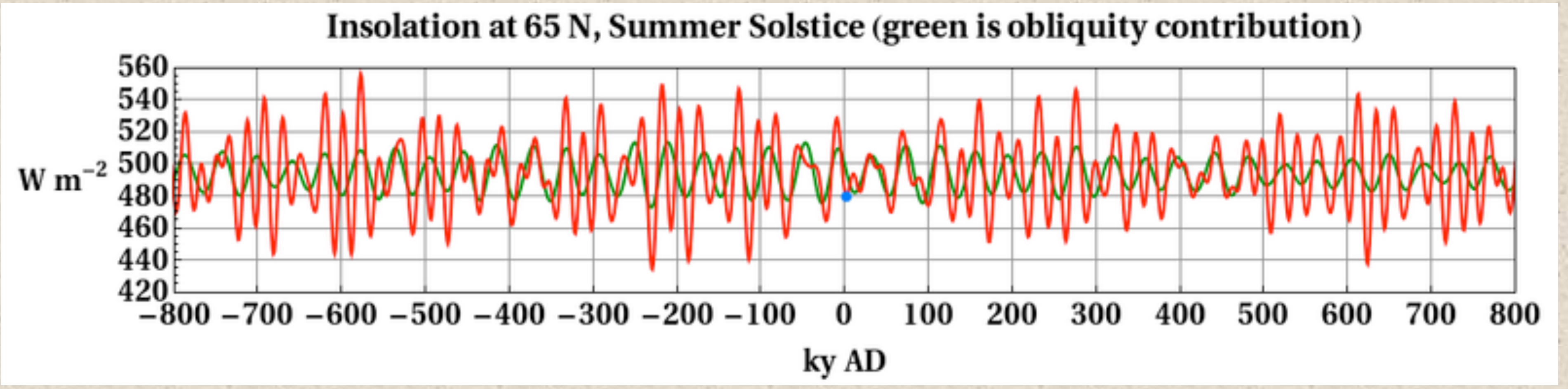
orbital precession



Insolation

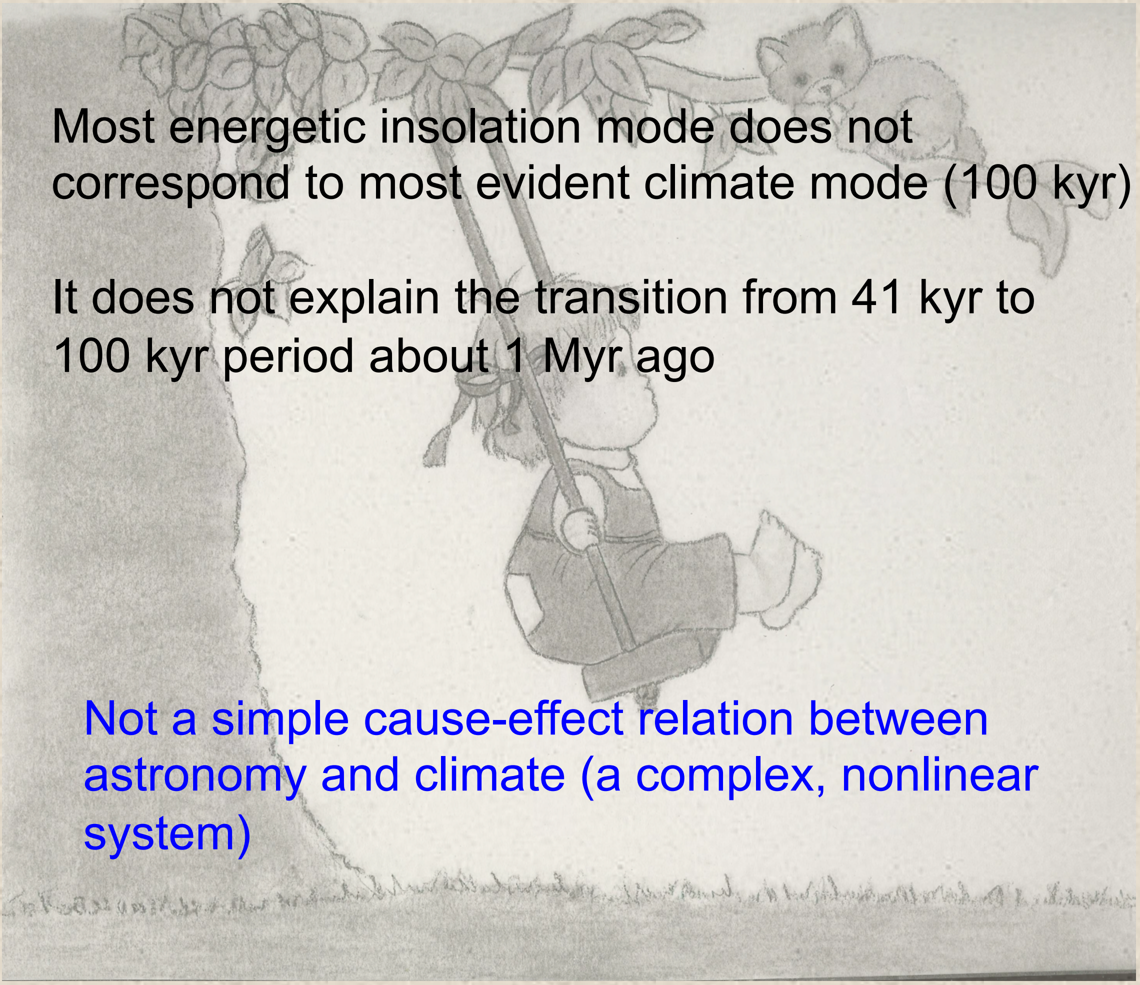
The combination of obliquity (T=41 ky) and precession (23 ky, modulated by eccentricity periods T=100 ky and 413 ky) determines the insolation at a given latitude and season.

Astronomical modulation of insolation is particularly important at high latitude.



A.L.Berger (1978), J.Laskar (1990)

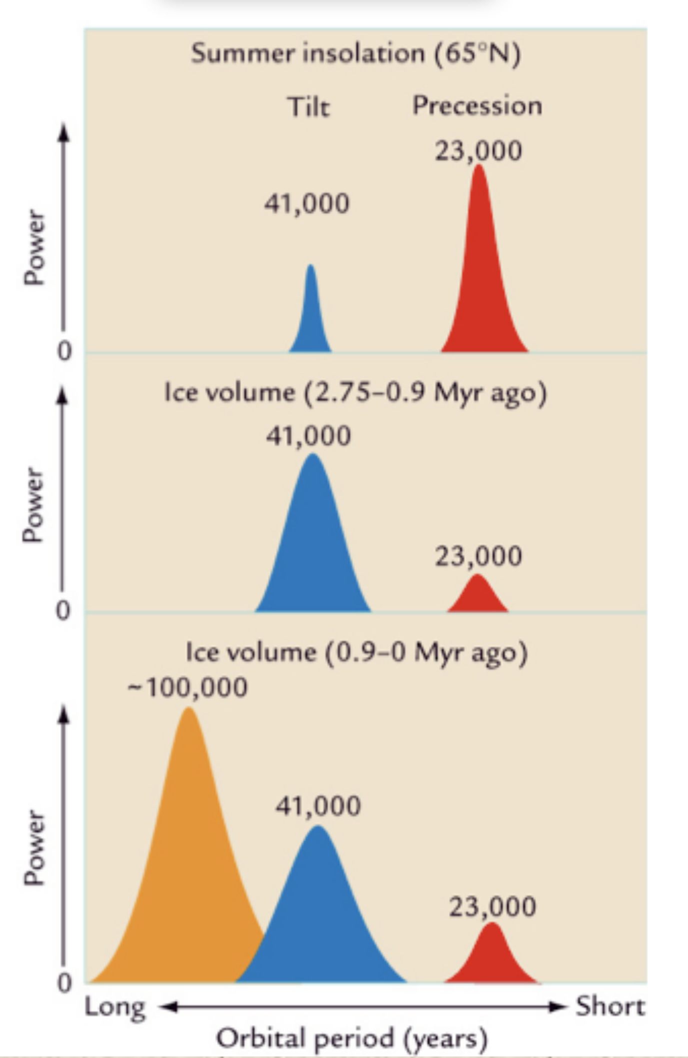
Astronomical driving: not so simple



Most energetic insolation mode does not correspond to most evident climate mode (100 kyr)

It does not explain the transition from 41 kyr to 100 kyr period about 1 Myr ago

Not a simple cause-effect relation between astronomy and climate (a complex, nonlinear system)



- nonlinear coupling with higher frequency down to 100 kyr
- pacemaker for internal CO_2 cycle @ 100 kyr

Astronomical driving: a pacemaker for glacial cycles

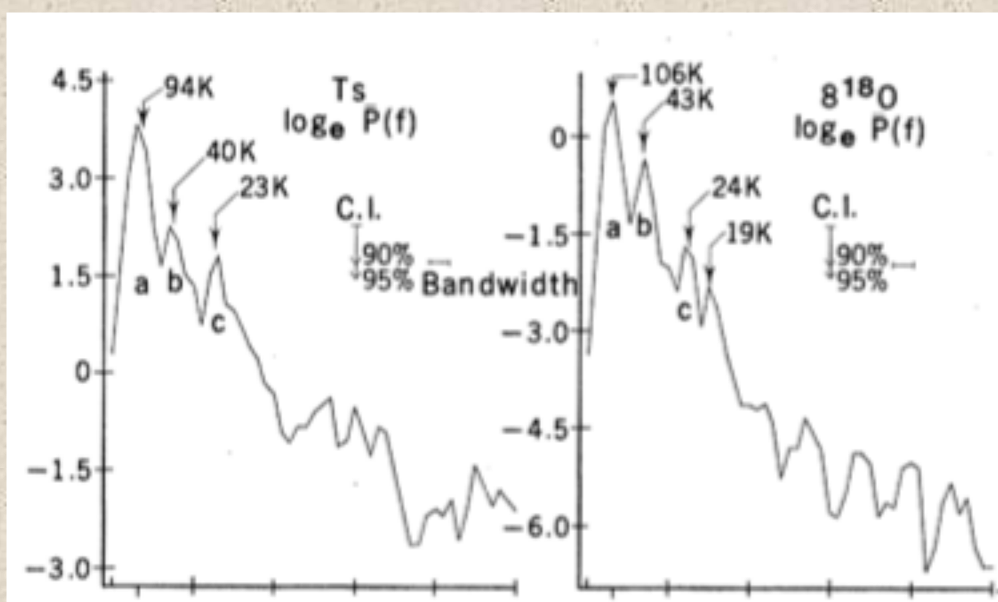
CLIMAP oceanic sediment cores
show clear peaks in the spectra of $\delta^{18}O$, SST T_s
coreresponding to astronomical periods

10 December 1976, Volume 194, Number 4270 SCIENCE

Variations in the Earth's Orbit: Pacemaker of the Ice Ages

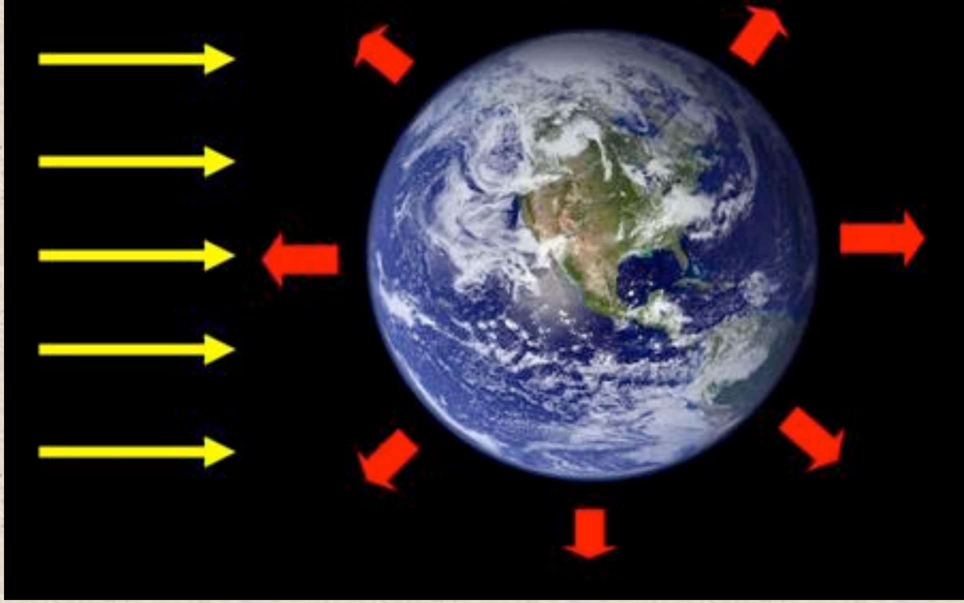
For 500,000 years, major climatic changes have followed variations in obliquity and precession.

J. D. Hays, John Imbrie, N. J. Shackleton



Milankovitch hypothesis become a theory

A 0-dimensional climate model



Energy balance for the earth

$$C\frac{dT}{dt} = P_{in}(T,t) - P_{out}(T) + \sqrt{2D}\eta(t)$$

$P_{in}(t)$ is the power coming from the sun

$$P_{in}(T,t) = \pi R_T^2 S\left(1+\varepsilon\cos(\omega t)\right)\left(1-a(T)\right)$$

solar constant $S = 1370\ \mathrm{Wm^{-2}}$ (outside the atmosphere)
$\varepsilon\cos(\omega t)$ represents the (small) Milankovich modulation
$a(T)$ is the albedo

$P_{out}(T)$ is the power emitted by the earth:

$$P_{out}(T) = 4\pi R_T^2 \sigma T^4$$

blackbody radiation according to Stefan-Boltzmann law with $\sigma = 5.67\ 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$

$\sqrt{2D}\eta(t)$ represents fast, internal dynamics (meteorology, ocean-atmos interactions)

0-order analysis of the 0-dimensional model

For simplicity we neglect albedo, Milankovich modulation and meteorology

$$C\frac{dT}{dt} = \pi R_T^2\left(S - 4\sigma T^4\right)$$

In equilibrium condition (dT/dt=0) we obtain

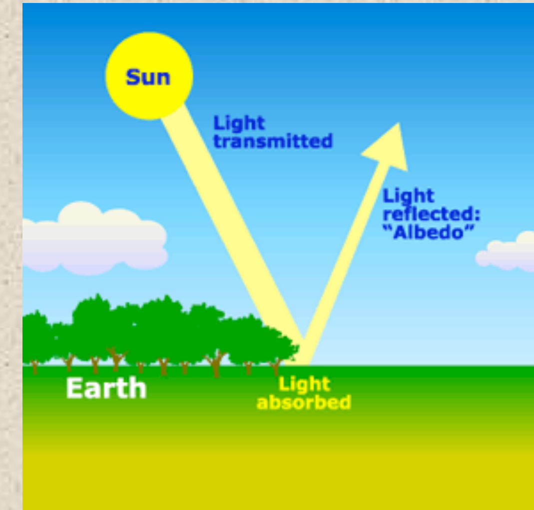
$$T_{eq} = \left(\frac{S}{4\sigma}\right)^{1/4} \simeq 279\ \text{K} \simeq 5.6^o C$$

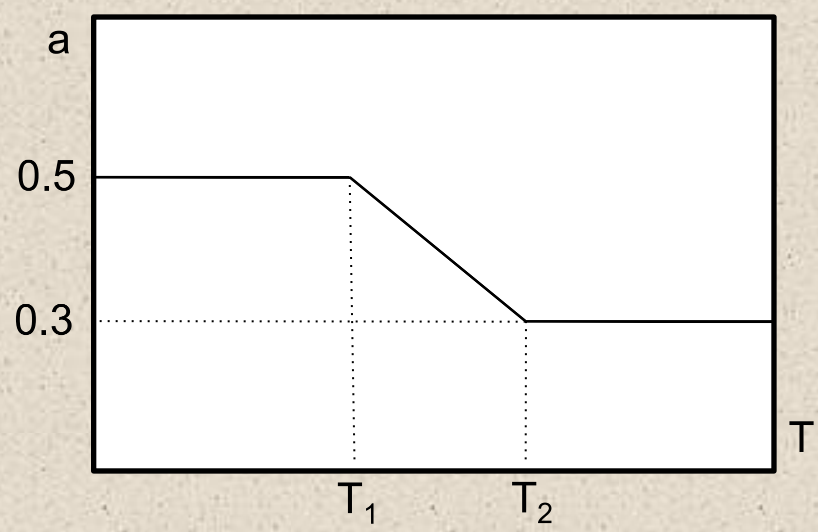
Not so bad !

But lower than actual value (about 14°C), mainly because atmosphere is neglected.

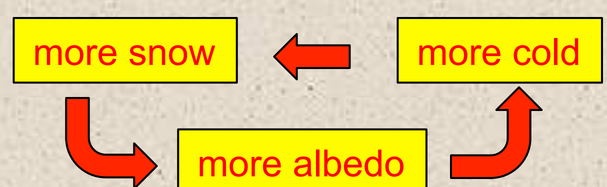
Albedo

The albedo $a(T)$ is a reflection coefficient $0 \leq a \leq 1$:
the present average value of the earth is $a \approx 0.3$ and
it depends on the quantity of ice (and thereofore on T).

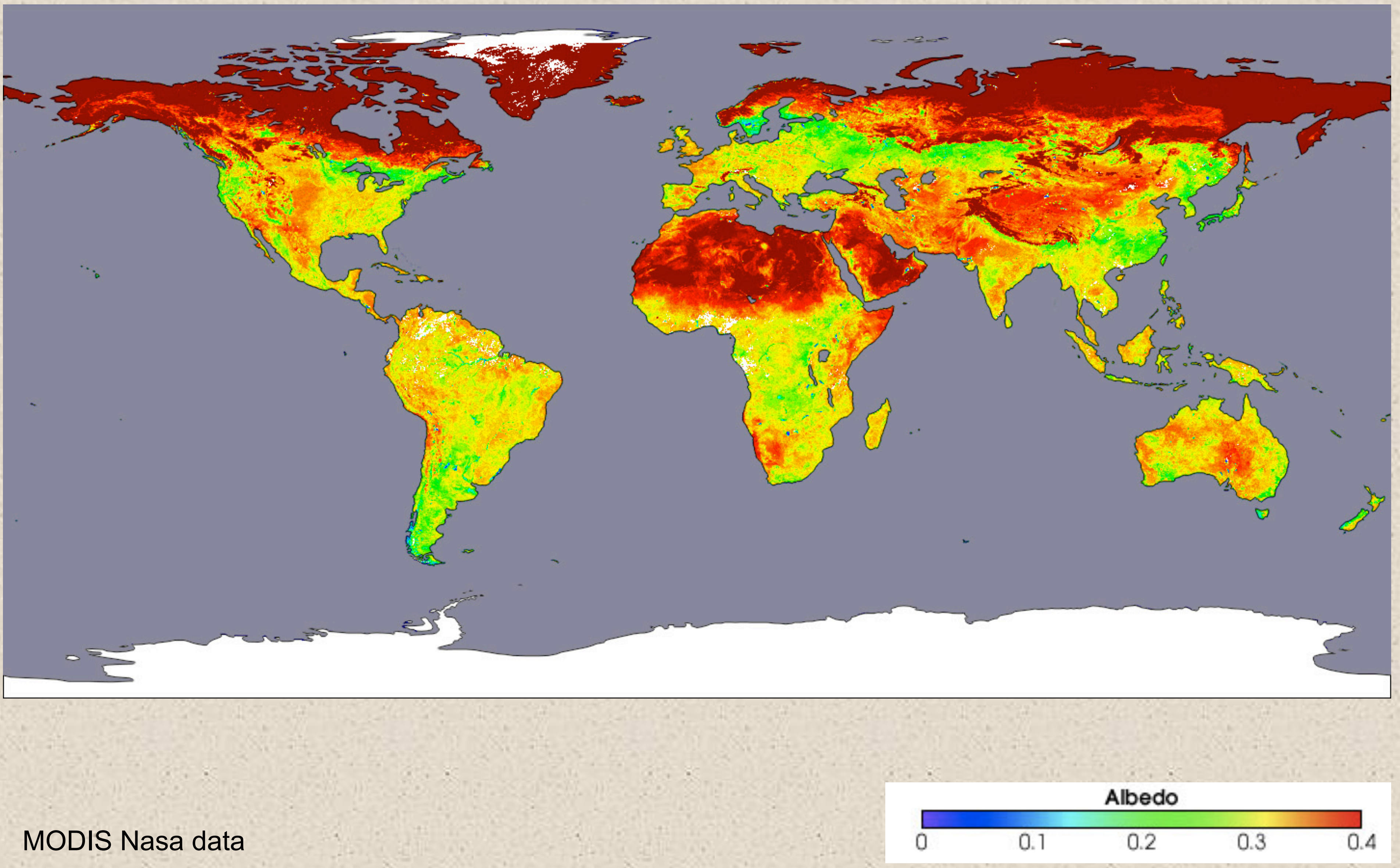




The mean albedo depends on temperature
this generate a positive feedback loop
in the earth energy balance



Albedo distribution (april 2002)

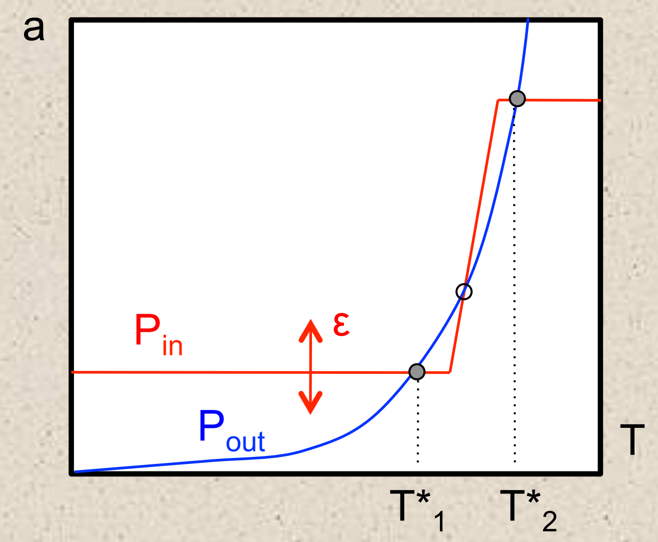


MODIS Nasa data

The role of albedo: multiple equilibria

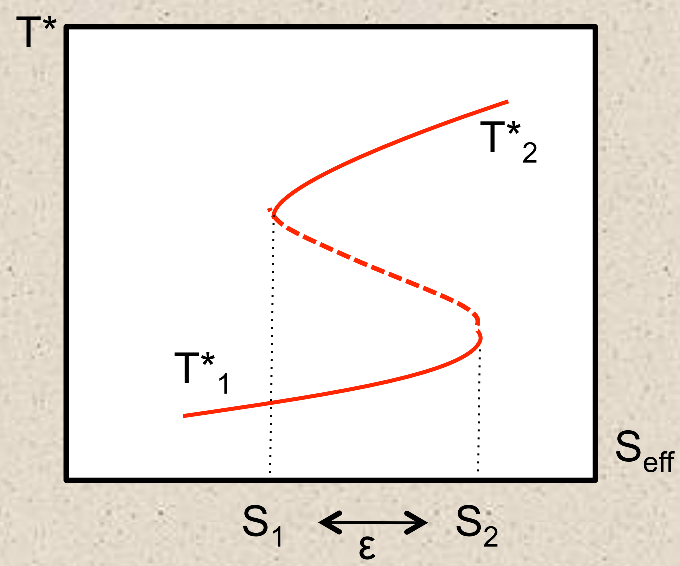
In presence of the albedo the energy balance model becomes

$$c\frac{dT}{dt} = \frac{S}{4}\left(1+\varepsilon\cos(\omega t)\right)\left(1-a(T)\right) - \sigma T^4$$



Three fixed points (graphical evaluation):
T^*_1 and T^*_2 stable and one unstable
which depend on the astronomical modulation ε

Bifurcation diagram

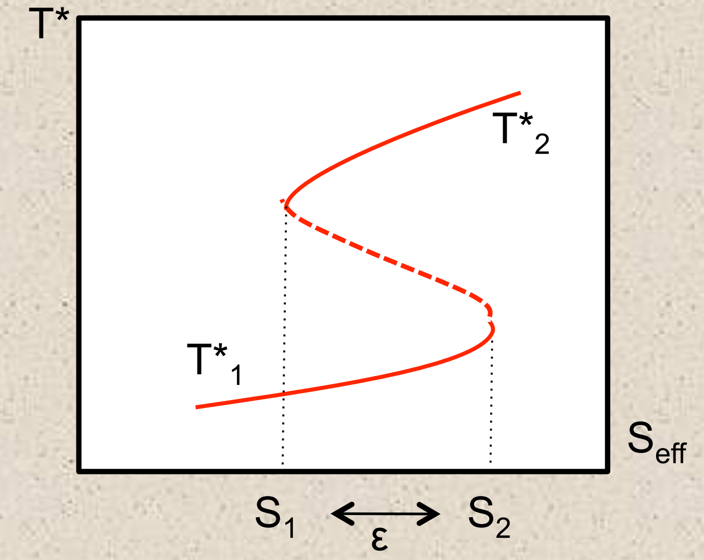


The number and the stability of fixed points change with the effective input

$$S_{eff} = \frac{S}{4}\left(1+\varepsilon\cos(\omega t)\right)$$

Double well potential

The bifurcation diagram suggests a simple dynamical model based on a double well potential



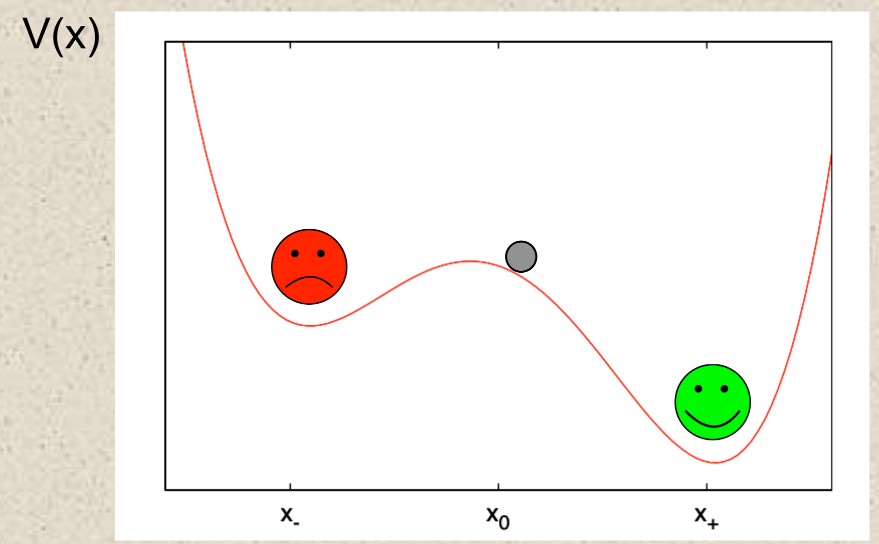
$$c\frac{dT}{dt} = \frac{S}{4}\left(1+\varepsilon\cos(\omega t)\right)\left(1-a(T)\right) - \sigma T^4 + \sqrt{2D}\eta(t)$$

⇓

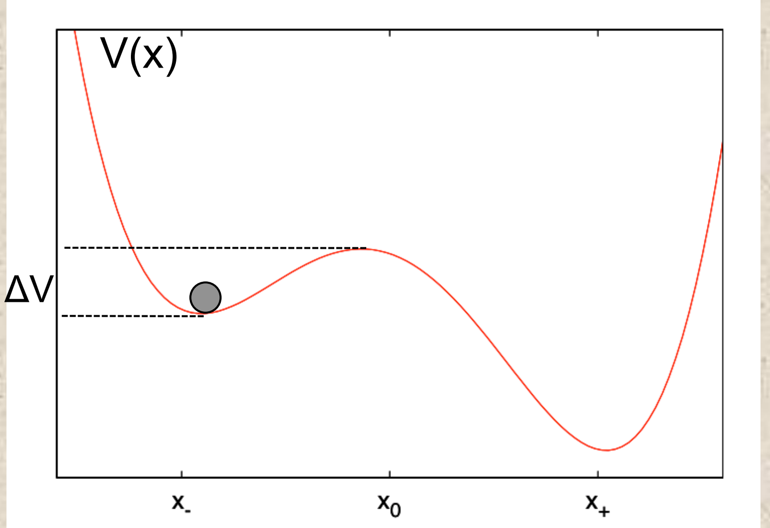
$$\frac{dx}{dt} = -\frac{dV(x,t)}{dx} + \sqrt{2D}\eta(t)$$

with a modulated double well potential, e.g. $V(x,t) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + \varepsilon x\cos(\omega t)$

How can the system jump from the glacial well to the interglacial well ?



The role of noise: Arrhenius formula



Particle moving in a potential field with noise

$$\frac{dx}{dt} = -\frac{dV(x,t)}{dx} + \sqrt{2D}\eta(t)$$

stochastic differential equation

Probability to jump the barrier between the two wells is given by [Kramers, 1940]

$$p = p_0 e^{-\frac{\Delta V}{D}}$$

Universal:
weakly depends (p_0) on the form of V(x): only ΔV is important

if D -> 0 then p -> 0 for any ΔV: no deterministic jump

if ΔV -> 0 then p -> 1 for any D: no barrier

Chemical reaction rates increase with temperature
(double every 10°C)

Svante Arrhenius, 1889
(greenhouse effect,1896)

Stochastic resonance / 1

R.Benzi, G.Parisi, A.Sutera and A.Vulpiani, *Tellus* **34** (1982)
C.Nicolis, *Tellus* **34** (1982)

Milankovich cycles modulates the depth of the two wells
Meteorological noise provides the jumping mechanism

$$\frac{dx}{dt} = -\frac{dV(x,t)}{dx} + \sqrt{2D}\eta(t)$$

Given a potential barrier ΔV and a noise amplitude D the mean jumping time (inverse probabillity) is given by the Arrhenius formula

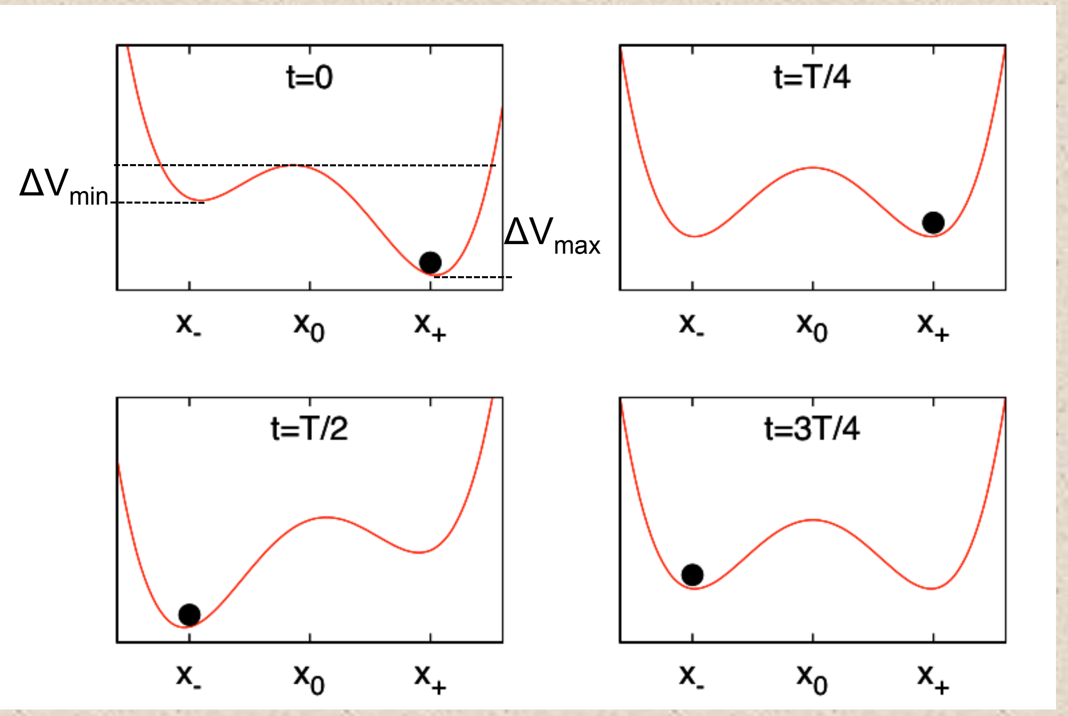
$$\tau \simeq \tau_0 e^{\frac{\Delta V}{D}}$$

The key point is the strong dependence of τ on ΔV

Condition for resonance:

$$\tau_0 e^{\frac{\Delta V_{max}}{D}} \gg \frac{T}{2}$$

$$\tau_0 e^{\frac{\Delta V_{min}}{D}} \ll \frac{T}{2}$$



Stochastic resonance / 2

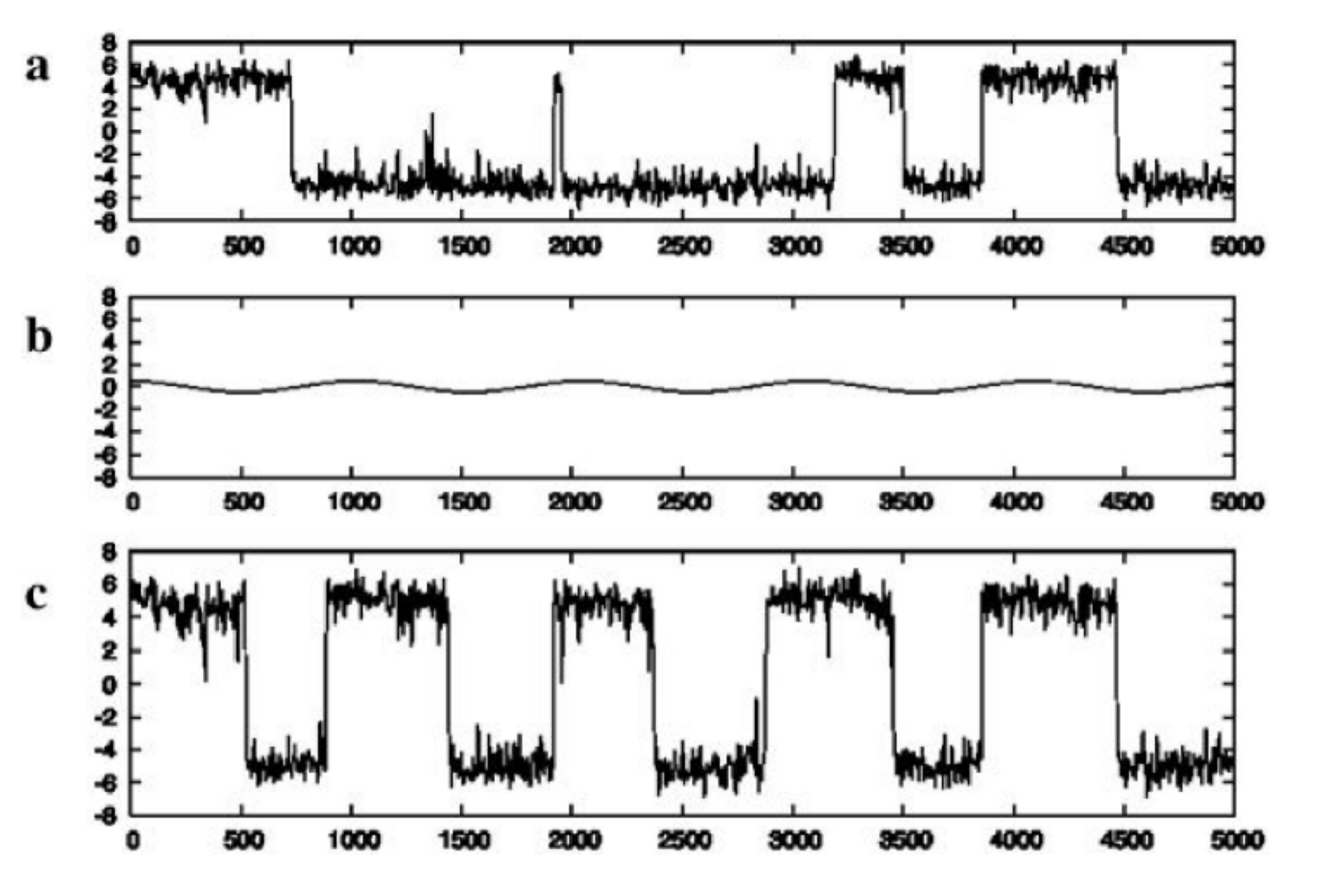
Strong noise
random jumps between
the two states

Zero noise
no jumps, small deterministic
modulation of the minimum

Resonance condition

$$\Delta V_{min} < D \log \frac{T}{2\tau_0} < \Delta V_{max}$$

jumps driven by noise
syncronized with potential
modulations



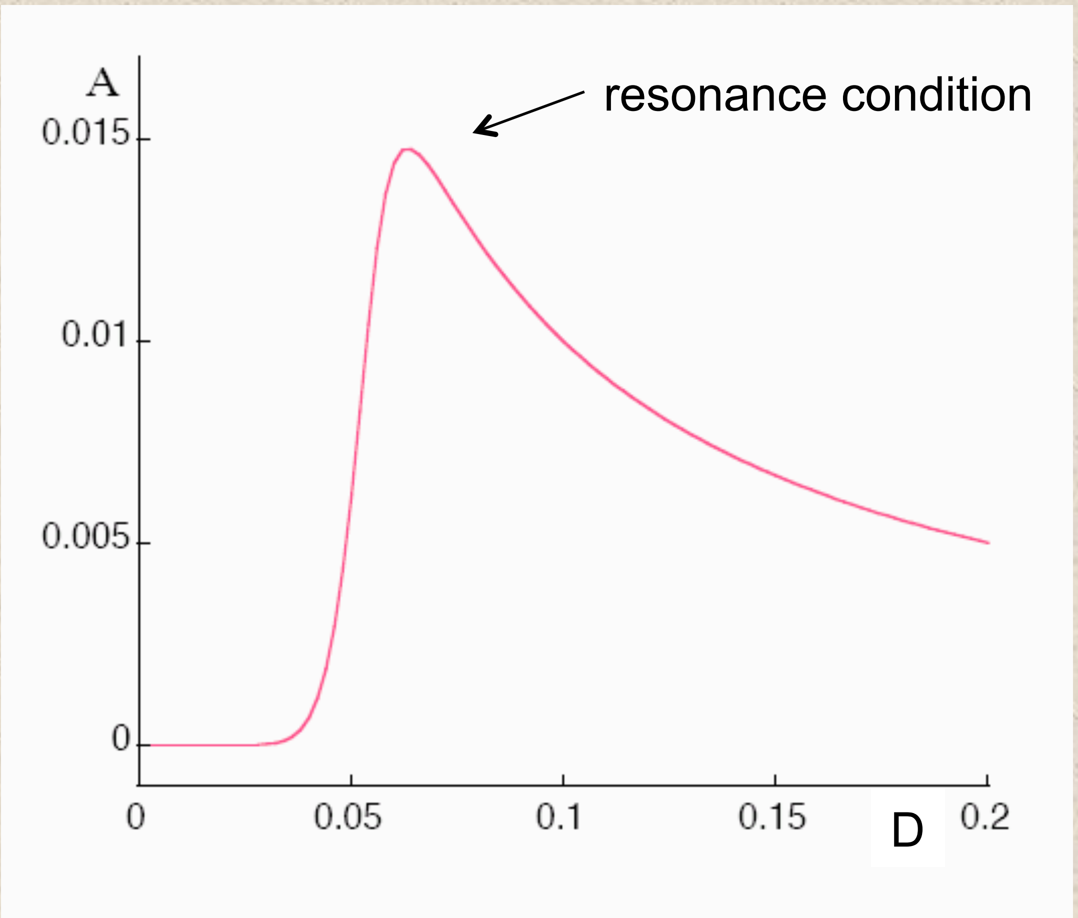
R.Benzi, *Nonlin. Proc. Geophys.* **17** (2010)

In stochastic resonance noise is used to amplify small perturbations

Stochastic resonance / 3

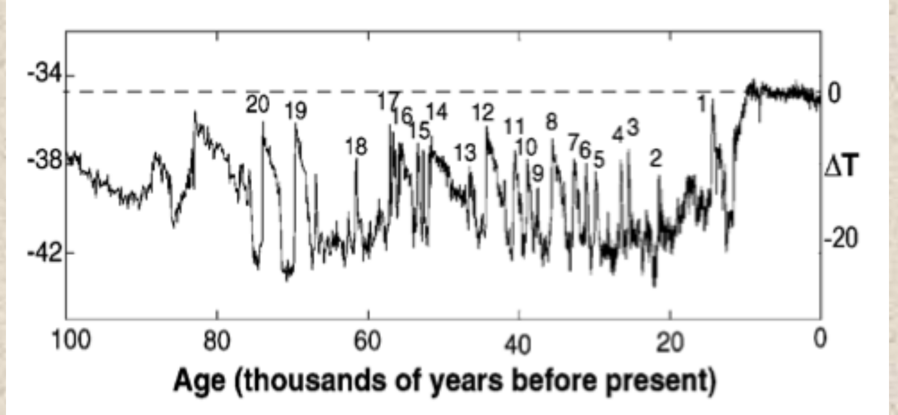
A= amplitude of periodic component in the output signal

$$\Delta V_{min} < D \log \frac{T}{2\tau_0} < \Delta V_{max}$$



Back to climate: DO events

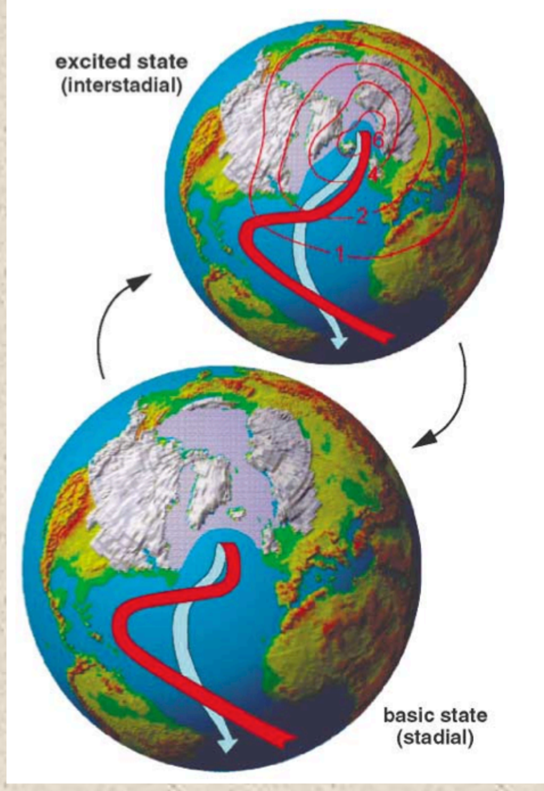
R.B.Alley et al, *Paleocean.* **16** (2001)

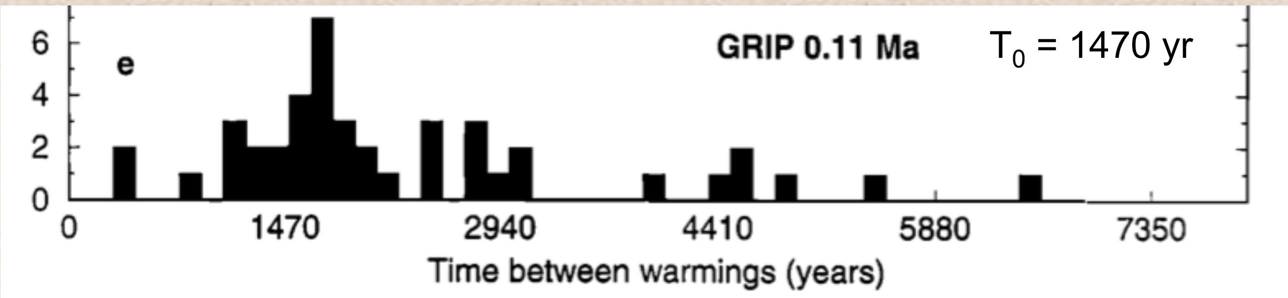


Dansgaard-Oeschger events

Rapid (50 years) warming of about 8°C with
a recurrence time given by multipes of 1470 years.
First observed in Greenland ice core
(later also in Antartica, but weaker).

Physical origin unclear but correlated with
oscillations in North Atlantic circulation





Stochastic resonance signature in the distribution of waiting times:

$\Delta T = T_0$ or $2T_0$ or $3T_0$ ...

but statistical significance is low:
almost compatible with Poisson process
[Ditlevsen et al, *Clim. Past.* **3** (2007)]