Power Laws in Solar Flares: Self-Organized Criticality or Turbulence?

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The statistics of quiescent times τ_L between successive bursts of solar flares activity, performed using 20 years of data, displays a power law distribution with exponent $\alpha \simeq 2.4$. This is an indication of an underlying complex dynamics with long correlation times. The observed scaling behavior is in contradiction with the self-organized criticality models of solar flares which predict Poisson-like statistics. Chaotic models, including the destabilization of the laminar phases and subsequent restabilization due to nonlinear dynamics, are able to reproduce the power law for the quiescent times. A shell model of MHD turbulence correctly reproduces all the observed distributions.

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Solar flares are sudden, transient energy release above active regions of the sun [1]. Energy is released in various form [thermal soft x-ray emission, accelerated particles, hard x-ray (HXR) emission, and so on]. Parker [2] conjectured that flares represent the dissipation at the many tangential discontinuities arising spontaneously in the bipolar fields of the active regions of the sun as a consequence of random continuous motion of the footpoints of the field in the photospheric convection [2]. Probability distributions, calculated for various observed quantities, x, can be well represented by power laws of the form $P(x) = Ax^{-\alpha}$. In particular from HXR emission, the distribution of peak flux yields $\alpha \simeq 1.7$, that of total energy associated with a single event yields $\alpha \simeq 1.5$, and finally the distribution of flare duration yields $\alpha \simeq 2$ [3].

The conjecture by Parker and the power laws found in the distributions of real flares stimulated a new way of looking at impulsive events like flares. In fact, Lu and Hamilton [4] pointed out that self-organized criticality (SOC), introduced earlier [5], could describe the main features of HXR flares [6,7]. What is usually called SOC is a mechanism of charging and discharging, apparently without tuning parameters, which reproduces self-similarity in critical phenomena.

Lu and Hamilton [4] assume that the coronal magnetic field evolves in a self-organized critical state in which an "event" can give rise to other similar "events" through an avalanche process (sandpile model). An active region on the sun is thus modeled through a magnetic field $\boldsymbol{B}$ on a uniform 3D lattice. In order to have a statistically stationary state, energy is injected into the system by adding a small magnetic field increment $\delta\boldsymbol{B}$ at a random site on the grid. When an avalanche takes place, the energy input is suspended until all sites become stable. In this sense the avalanches (flares) are fast phenomena, on a time scale much smaller than the injection mechanism. The large popularity of these models settles on their capability to reproduce the power law behavior in the distribution functions of the total energy, the peak luminosity, and the duration of avalanches.

What we want to stress in this Letter is the fact that a different kind of statistics can be studied on solar flare signals: the distribution of laminar or waiting times, i.e., the time intervals between two successive bursts. This distribution has been recently studied on solar flares HXR events [8,9]. Wheatland *et al.* [9] have also emphasized the fact that this kind of distribution is crucial from the point of view of the avalanche model. SOC models, indeed, are expected to display an exponential waiting time distribution $P(\tau_L) = \langle\tau_L\rangle^{-1}\exp(-\tau_L/\langle\tau_L\rangle)$, where $\langle\tau_L\rangle$, the average laminar time, depends on the parameters of the model. This behavior is related to the fact that the probability to give rise to an avalanche as a consequence of each single throw of the magnetic field in a random position is simply the ratio between the volume occupied by *clusters of minimally stable states* [5] and the total volume of the system. Then one expects no correlation between successive bursts and thus a trivial statistics (binomial or Poisson distribution) for the laminar times. This is clearly observed in a simulation of the SOC automaton that we have done (see the inset of Fig. 1). On the contrary, when looking at solar flares data, authors [8,9] found a more or less well defined power law distribution. In particular, Wheatland *et al.* [9], by performing a careful statistical analysis of waiting time distribution on 8 years of solar flares HXR bursts observed by the ICE/ISEE 3 spacecraft, have shown that the distribution in no way can be attributed to a nonstationary Poisson process.

We have done the same statistics on laminar phases using 20 years of data from the National Geophysical Data

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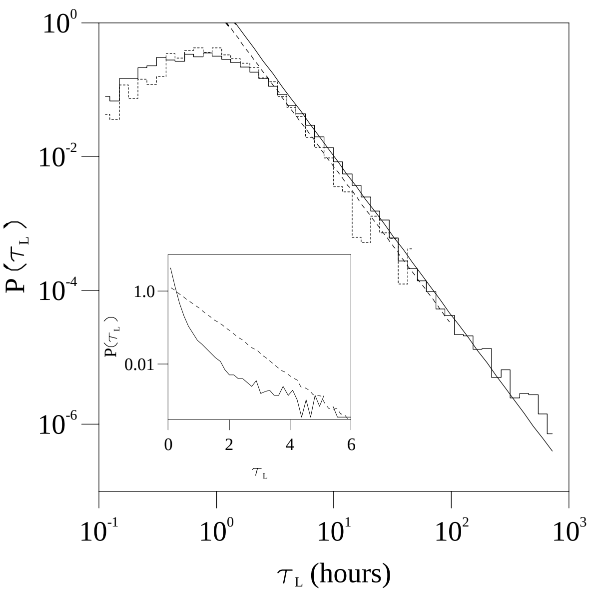

FIG. 1. Probability distribution of the laminar time $P(\tau_L)$ between two x-ray flares for data set A (dashed line) and data set B (full line). The straight lines are the respective power law fits. In the inset we show, in linear-log scale, the distribution for data set B (full line) and the distribution obtained through the SOC model (dashed line) which displays a clear exponential law. The variables shown in the inset have been normalized to the respective root-mean-square values.

Center of USA. In this database starting and ending times as well as peak times of HXR bursts associated with flares from 1976 up to 1996, measured at the Earth by satellites in the 0.1 to 0.8 nm band, are stored. We calculated the laminar times as the time differences between two successive maxima of the flares intensity recorded during periods of activity of the same instrument. Two different kinds of analysis have been performed: in the first one we have built up a data set of about 1100 samples (hence on data set A) by calculating only the differences between the time of occurrence of flares within the same active region, as identified through the Hα flares occurrence. To build up the second data set (data set B), we have considered the sun as a unique physical system, and we have calculated the time differences between two successive maxima of flare intensity regardless of the position of the flare on the sun surface. In this way we get a data set of about 32 000 samples. The analysis of these data shows that, in both cases, laminar times display a clear power law distribution $P(\tau_L) = A\tau_L^{-\alpha}$ with $\alpha = 2.38 \pm 0.03$ in the range $6 \leq \tau_L \leq 67$ (reduced $\chi^2 = 2.2$) for data set B and $\alpha = 2.4 \pm 0.1$ (reduced $\chi^2 = 1.1$) for data set A (Fig. 1). The exact value of the exponent can be affected by the finite length of the observation times, which underestimates the occurrence of long waiting times. However, these results, as well as those obtained by previous authors [8,9], allow us to consider the power law distribution of waiting times as firmly established as the power laws observed for total energy, peak luminosity, and time duration and force us to investigate whether models different from SOC can account for all these distributions.

It is worth noting that the occurrence of a power law in the distribution of the laminar times is analogous for the solar flares of the Omori's law for the earthquakes [10] and represents a clear indication of the existence, in the flare dynamics of strong correlations between successive bursts, at variance with the SOC model. The unique possible origin of the correlations arises from nontrivial evolution equations of the phenomenon. In particular, let us note that Parker's conjectures about solar flares dynamics could be usefully investigated in the framework of MHD turbulence.

Turbulence is a common phenomenon in fluids, where chaotic dynamics and power law statistics coexist. Moreover turbulence displays time intermittency, in that dissipative events are not uniformly but burstly distributed in time, so that flares might be tentatively identified with such events. Before introducing a turbulence model for the Parker conjecture, it is worth noting that the intermittent behavior can be observed also in simple dynamical models. Let us consider the one dimensional random map $x_{t+1} = r_t x_t(1 - x_t)$, where at each step t the random variable r_t is extracted according to a given distribution [11]. The dynamics is such that x_t remains close to zero for a certain time (laminar time) then there is a short interval of strong activity (burst) after which a new quiescent phase takes place. In this simple model one can observe power laws not only for the energy distribution (here defined as the integral of x_t over the burst), but also for the laminar times τ_L. The exponent of power law for τ_L turns out to be $\alpha = 1.5$ [12]. In spite of its simplicity, the random map model contains some basic ingredients of the relevant features of time intermittency in dynamical systems, i.e., the destabilization of the laminar phase by linear instability and the subsequent restabilization due to the nonlinear dynamics.

Some attempts to reproduce the flare statistics from direct numerical simulations of MHD equations have been performed using 2D codes [13]. These attempts suffer for some basic limitations. In fact, because of the limited capability of computers, only low Reynolds numbers are allowed and the statistics obtained are poor. Moreover, to avoid the antidynamo theorem of 2D MHD, the external forcing is supposed to act on the magnetic field, at variance with the hypothesis that the driver of the turbulent dynamics is related to the random motion due to photospheric convection [2].

An alternative way to look at a turbulent MHD system is represented by the so-called shell models [14]. In these models one tries to reproduce at best the nonlinear dynamics, but ignores from the beginning the details of the spatial structure and its associated topology. A single (complex) scalar variable u_n (and b_n) is considered to be representative of the velocity (magnetic) fluctuation associated with a wave number $k_n = k_0 2^n$ $(n = 1, \ldots, N)$. The fact that the wave numbers k_n are exponentially spaced allows one to reach very large Reynolds numbers with a moderate number of degrees of freedom and then to investigate regimes of 3D MHD turbulence which are not accessible by direct numerical simulation.

The evolution equations for the dynamical variables u_n and b_n are built up by retaining only the interactions between nearest and next nearest neighbor shells in the form of quadratic nonlinearities. The coupling coefficients of nonlinear terms are determined by imposing the inviscid conservation of the 3D MHD quadratic invariants [14,15]. The particular shell model we used in our simulation reads [16]

$$\frac{du_n}{dt} = -\nu k_n^2 u_n + f_n + ik_n\left\{(u_{n+1}u_{n+2} - b_{n+1}b_{n+2}) - \frac{1}{4}(u_{n-1}u_{n+1} - b_{n-1}b_{n+1}) - \frac{1}{8}(u_{n-2}u_{n-1} - b_{n-2}b_{n-1})\right\}^*,$$

$$\frac{db_n}{dt} = -\eta k_n^2 b_n + ik_n(1/6)\{(u_{n+1}b_{n+2} - b_{n+1}u_{n+2}) + (u_{n-1}b_{n+1} - b_{n-1}u_{n+1}) + (u_{n-2}b_{n-1} - b_{n-2}u_{n-1})\}^*,$$

where ν and η are, respectively, the viscosity and the resistivity and f_n is an external forcing term acting only on velocity fluctuations.

The external forcing term has been set up in order to model the driving mechanism due to the large scale random motion of magnetic field lines footpoints. f_n is a stochastic variable acting only on the first two shells of the velocity fluctuations. It is calculated according to the Langevin equation $df_n/dt = -f_n/\tau_0 + \mu$, where τ_0 is the characteristic time of the largest shells ($\tau_0 \simeq 10$ in our units) and μ is a Gaussian white noise with $\sigma = 0.1$.

This model has been shown to be able to reproduce a dynamolike effect (inverse cascade of magnetic helicity), which is typical of 3D MHD [16] as well as the time intermittency through the analysis of the scaling laws of the structure functions (paper in preparation, 1999). In particular, time intermittency can also be observed by looking at the energy dissipation defined as $\epsilon(t) = \nu \sum_{n=1}^{N} k_n^2 |u_n|^2 + \eta \sum_{n=1}^{N} k_n^2 |b_n|^2$ (see Fig. 2).

As stated above we identify the intermittent spikes of dissipation with the intermittent spikes on data [HXR, extreme ultraviolet (EUV), etc.] associated with flares. Then we have performed on $\epsilon(t)$ the same statistical analysis done for the solar flare signals. We have defined a burst of dissipation by the condition $\epsilon(t) \geq \epsilon_c$. This definition allows us to calculate the distribution functions for the peak values of the bursts, their total energy (defined as the integral of the signal above ϵ_c) and the duration of the bursts (defined as the time during which the dissipation is above ϵ_c). We have chosen the threshold as $\epsilon_c = \langle \epsilon(t) \rangle + 2\sigma$, where the average and the standard deviation have been calculated on the time intervals in between the bursts, through an iterative process in order to take into account only the background contribution. The results of the analysis are shown in Fig. 3. Also in this case we observe clear power law distribution functions with exponents $\alpha \simeq 2.05$ for the peak distribution, $\alpha \simeq 1.8$ for the total energy distribution, and $\alpha \simeq 2.2$ for the burst durations. The exponents are close to those obtained in analyzing solar

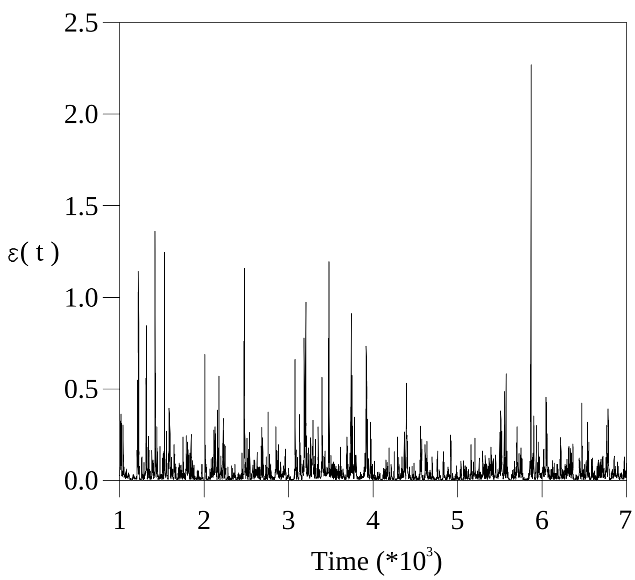

FIG. 2. Time series of energy dissipation $\epsilon(t)$ for the shell model. The parameters used in the simulation are $N = 19$, $\nu = \eta = 10^{-7}$, and $k_0 = 1$.

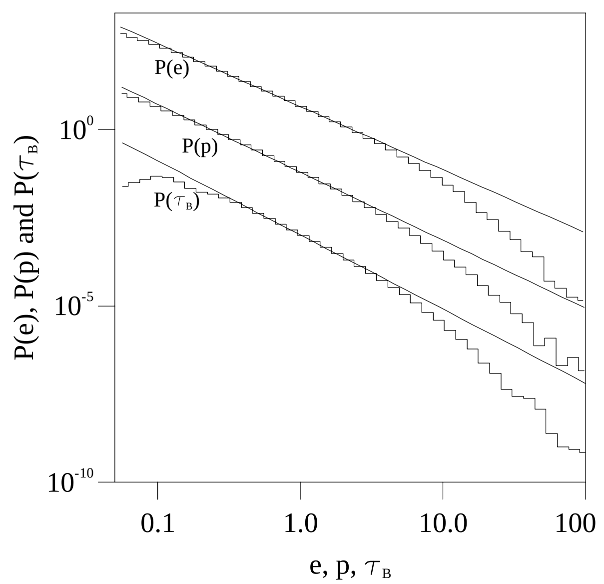

FIG. 3. Total energy distribution $P(e)$, energy peak distribution $P(p)$, and bursts duration distribution $P(\tau_B)$ for the shell model. The variables have been normalized to the respective root-mean-square values. The straight lines are the fits with power laws. The values of $P(e)$ and $P(\tau_B)$ are offset by a factor of 100 and 10^{-2}, respectively.

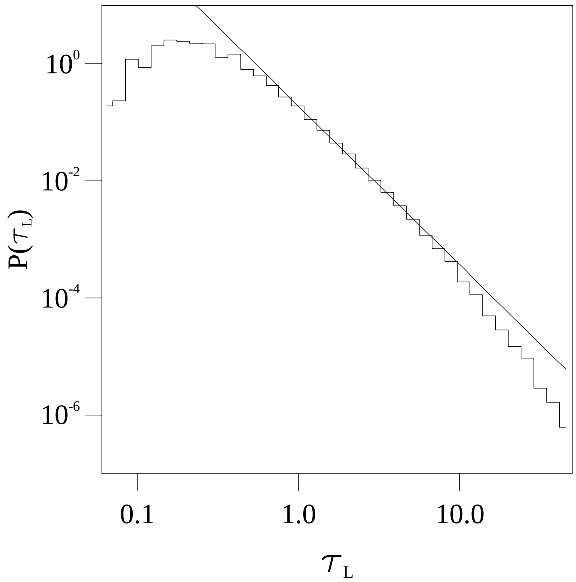

FIG. 4. The distribution of laminar times $P(\tau_L)$ for the shell model, normalized to the root-mean-square. The straight line is the fit with a power law.

flares data, but we do not think that the agreement is particularly significant as the model exponents depend on the value chosen for the threshold. In particular, scaling exponents for the total energy, time duration, and peak value distributions increase in absolute value when increasing the threshold, the contrary occurring for the waiting time distribution.

The relevant point is that, at variance with SOC models, MHD shell models display a power law statistics also for the laminar times, as shown in Fig. 4. The scaling exponent turns out to be $\alpha \simeq 2.70$, close to the one obtained from the experimental data. The different behavior of SOC models and turbulent MHD shell models is related to the conceptually different mechanisms underlying the SOC phenomenon and the phenomenon of intermittency in fully developed turbulence. SOC models represent self-similar phenomena, while the intermittent behavior of turbulence is related to its chaotic nature [15]. The statistics of the laminar times between two bursts is due to global properties of the system (its memory) and thus it is more relevant to characterize the dynamical behavior of the system than the statistics of the properties of the single burst. Moreover, our analysis shows that, since shell models are able to reproduce the power law for the quiescent time, the mechanism for destabilization of the laminar phases and subsequent nonlinear restabilization is more directly related to the nonlinear dynamics than to the particular instability associated with magnetic field topology [17].

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