Sudden freeze-out vs continuous emission: duality in hydro-kinetic approach to A+A collisions

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Abstract. The problem of spectra formation in hydrodynamic approach to A+A collisions is discussed. It is analyzed in terms of the two different objects: distribution and emission functions. We show that though the process of particle liberation, described by the emission function, is, usually, continuous in time, the observable spectra can be also expressed by means of the Landau/Cooper-Frye prescription. We argue that such an approximate duality results from some symmetry properties that systems in A+A collisions reach to the end of hydrodynamic evolution and reduction of the collision rate at post hydrodynamic stage.

Keywords: hydro-kinetic approach, distribution functions, emission functions.

1. Introduction

The two main features of high energy nucleon and nuclear collisions, namely, strong interactions and multiparticle production, suggest an idea to Heisenberg, Watagin and Fermi [1] that the systems created in those collisions could be considered as thermal. The Landau hydrodynamic model for multi-hadron production [2] appears as a method that is alternative to S-matrix one, where the asymptotic states at $t = ±∞$ are not considered but instead the detail space-time evolution of thermal hadronic matter is described basing on the local energy-momentum conservation laws. Instead of the initially well defined asymptotic state of colliding particles, this approach uses initial conditions of hydrodynamic expansion that, in fact, strongly depend on pre-thermal stage of hadronic or nuclear collisions. At the RHIC energies the pre-thermal stage is formed in collisions of the two groups of specifically distributed partons associated with colliding nuclei that results, apparently, in very
dense state of partonic matter: Color Glass Condensate (see, e.g., [3]). Current phenomenological and pQCD estimates give the (proper) time $\tau_0$ of thermalization of that matter between 0.6 and 3 fm/c. Different phenomenological assumptions are used to fix the details of thermalization; well known examples are Landau initial condition: compressed static disk at $t = \tau_0$ [2] and Bjorken one: quasi-inertial flow at proper time $\tau = (t^2 - z^2)^{1/2} = \tau_0$ [4].

The another important aspect of the hydrodynamic approach is the final state of colliding system. If one uses hydrodynamic equations till infinitely large times, the result will be the infinitesimally small final densities that gives no chance to restore microscopic (or particle) picture from continuous medium approach. According to the Landau criterion of freeze-out, the hydro-evolution stops and all particles become free when fluid elements reach the temperature that is equal to the mass of the lightest hadron (pion): $T_{f.o.} \equiv 1/\beta_{f.o.} = m_\pi$. At that temperature the mean free path in pion gas becomes to be equal to transverse radius of hydrodynamic tube in $p + p(\bar{p})$ collisions and system decays. This qualitative criterion is in mysterious qualitative agreement with hydrodynamic descriptions of very wide class of high energy collisions: from $p + p(\bar{p})$ to heavy ion A+A (see, e.g., [5]) where the correspondent fitting freeze-out temperature turns out to be $120\div 150$ MeV. This universality seems like a puzzle since it does not take into account the real dynamics and system sizes. Moreover, the results of many studies of A+A collisions based on cascade (transport) models contradict to an idea of sudden freeze-out at some fixed temperature. The particles escape from the system during the whole period of its evolution and do not demonstrate the local equilibration at the late stages. Though the pure hadronic cascade models as well as hybrid "hydro + cascade" models [6] fail to describe properly experimental data, especially the HBT radii in A+A collisions, the problem of spectra formation in the hydrodynamic approach in itself is very serious and only stress the puzzle of the successful application of the hydrodynamic models to multiparticle production processes.

2. Distribution and emission functions and observables

To clarify the problems we will follow to the basic ideas of Ref. [7] and express the observables through both distribution and emission functions of expanding systems. We start from the Boltzmann equations (BE) as more general than the hydrodynamic ones. The BE for the distribution function $f(x, p)$ in the case of no external forces has the form:

$$\frac{p^\mu}{p^0} \frac{\partial f(x, p)}{\partial x^\mu} = F^{gain}(x, p) - F^{loss}(x, p).$$

The term $F^{gain}$ and $F^{loss}$ are associated with the number of particles which respectively came to the point $(x, p)$ and leave this point because of collisions. The term $F^{loss}(x, p) = R(x, p)f(x, p)$ can easily be expressed in terms of the rate of collisions of the particle with momentum $p$, $R(x, p) = <\sigma v_{rel}> n(x)$. The term
$F^{gain}$ has more complicated integral structure and depends on the differential cross-section. Let us split the distribution function at each space-time point into two parts: $f(x, p) = f_{int}(x, p) + f_{esc}(x, p)$, $x = (t, x)$. The first one, $f_{int}(x, p)$, describes the fraction of the system which will continue to interact after the time $t$. The second one, $f_{esc}(x, p)$, describes the particles that will never interact after the time $t$. According to the probability definition

$$f_{esc}(x, p) = P(x, p) f(x, p), \quad (2)$$

where escape probability $P(x, p)$, or probability for any given particle at $x$ with momentum $p$ not to interact any more, propagating freely, can be expressed explicitly in terms of the rate of collisions along the world line of the free particle with momentum $p$ through the opacity integral

$$P(x, p) = \exp \left( - \int_{t}^{\infty} dt' R(x', p) \right) \quad (3)$$

Since $f_{esc}(x, p)$ is formed from the particles suffering last collisions at space-time point $x$ it is associated with term $PF^{gain}$ and form the emission function $S[7]$

$$F^{gain} = \frac{\partial}{\partial p^\mu} f_{esc}(x, p) = p^0 P(x, p) F^{gain}(x, p) \equiv S(x, p). \quad (4)$$

For initially finite system with a short-range interaction among particles, the system becomes free, in fact, at large enough times $t_{out}$, so $P(x, p) \to 1$ and $f_{esc}(x, p) \to f(x, p)$ in this limit. Therefore, to describe the inclusive spectra of particles

$$p^0 \frac{dN}{dp} = \langle a_{p_1}^+ a_{p_2} \rangle, \quad p^0 \frac{dN}{dp_1 dp_2} = \langle a_{p_1}^+ a_{p_2}^+ a_{p_1} a_{p_2} \rangle, \quad (5)$$

the asymptotic equality $f_{esc}(x, p) = f(x, p)$ can be used, replacing the total distribution function $f$ in all irreducible averages in (5),

$$\langle a_{p_1}^+ a_{p_2} \rangle = \int_{\sigma_{out}} d\sigma_{\mu} p^\mu \exp(iqx) f(x, p), \quad (6)$$

by $f_{esc}$. Here, $p = (p_1 + p_2)/2$, $q = p_1 - p_2$ and the hypersurface $\sigma_{out}$ just generalizes $t_{out}$. Applying the Gauss theorem and recalling that $\partial_{\mu}[p^\mu \exp(iqx)] = 0$ for particles on mass shell, one obtains, using respectively general equations (4) and (1) and supposing their analytical continuation off mass shell,

$$\langle a_{p_1}^+ a_{p_2} \rangle = p^\mu \int_{\sigma_0} d\sigma_{\mu} f_{esc}(x_0, p_0) e^{iqx} + \int_{\sigma_0}^{\sigma_{out}} d^4 x S(x, p) e^{iqx}, \quad (7)$$

$$\langle a_{p_1}^+ a_{p_2} \rangle = p^\mu \int_{\sigma_0} d\sigma_{\mu} f(x_0, p_0) e^{iqx} + p_0 \int_{\sigma_0}^{\sigma_{out}} d^4 x (F^{gain}(x, p) - F^{loss}(x, p)) e^{iqx} \quad (8)$$
where $S(x, p)$ is defined through the product $P F^{gain}$ by Eq. (4), $f_{esc}(x_0, p)$ corresponds to the portion of the particles, which is already free at initial time $t_0$, or, more generally, at the initial hypersurface $\sigma_0$, and $f(x_0, p)$ is the distribution function at $\sigma_0$.

Thus, the use of escaping function as the asymptotic interpolation to the solution of BE is equivalent to taking, as the source function for the spectra and correlations, the 4-volume emission function $S = p^0 P F^{gain}$ together with direct emission $f_{esc}(x_0, p)$ from an initial 3D hypersurface $\sigma_0$. The Landau criterion of freeze-out of locally equilibrium (l.eq.) hydrodynamic momentum spectra and correspondent Cooper-Frye prescription, defined by Eq. (6) with substitutions $\sigma_{out} \rightarrow \sigma_{f.o.}$ and $f \rightarrow f_{l.eq.}$, treats particle spectra as results of rapid conversion of a l.eq. hadron system into a gas of free particles at some hypersurface $\sigma_{f.o.}$. Formally, it corresponds to taking the cross-section tending to infinity at $t < t_{\sigma_{f.o.}}$ (to keep system in l.eq. state) and zero beyond $t_{\sigma_{f.o.}}$. Then $P(t, x, p) = \theta(t - t_{\sigma_{f.o.}}(x))$ (and so $f_{esc} = 0$ at $t < t_{\sigma_{f.o.}}$), and $S = p^0 P F^{gain} = p^0 \delta(t - t_{\sigma_{f.o.}}(x)) f_{l.eq.}$ in Eq. (7). The proportionality between the $S$ and $f_{l.eq.}$, like $S(x, p) = \rho(\tau) f_{l.eq.}(x, p)$ is used in many papers devoting to a description of the data in a hydrodynamically motivated way (see review [8]). One should understand, however, that in the realistic case of no sudden freeze-out the emission function $S(x, p)$ loses completely its proportionality to the distribution function $f(x, p)$, it is just two different objects! This was shown by direct calculations in Ref. [7] and also is clear from Eqs. (2), (4) since the escape probability $P(x, p)$ in finite systems is very sensitive to an asymmetry of the positions $x$ as for the effective "boundary” and, thereby, the $S(x, p)$ becomes anisotropic in momentum $p$ in the rest frame of a fluid element unlike to the hydrodynamic distribution function $f_{l.eq.}(x, p)$.

3. The duality of the hydro and kinetic approaches

As it was advocated in the recent papers [9] the perfect hydrodynamics is a good approximation for the earlier stage of the matter evolution because of the big cross-section of the interaction among colour and white quasi-hadronic states in QGP. As for the matter evolution at the post hadronization stage in A+A collisions it was found the approximate equivalence between chemically frozen hydrodynamics of hadron gas and its evolution within the cascade approach in the temperature region above 0.12 GeV [10]. Below this region the formation of spectra is continuous in time with fairly long "tails" of the emission. That process is characterized by the emission function $S(t, x, p)$ that is far from thermal. The similar situation was analyzed in Ref. [7] based on the exact analytic solution of the BE for expanding fireball.

Unlike to very complicated structure of the emission function $S(t, x, p)$, that was found in [7], the spectra, interferometry radii, averaged phase-space densities have simple and clear analytical forms and correspond to the Cooper-Frye prescription for the thermal distribution function $f_{l.eq.}(x, p)$ in the fireball before it starts to decay. The explanation is based on the duality of Eqs. (7) and (8) that express the
spectra in terms of either the emission or distribution functions. While the emission function, that is proportional to $F_{\text{gain}}$, can have a significant non-zero value in wide space-time region, the integral of the difference $F_{\text{gain}} - F_{\text{loss}}$ over this region could be zero, as in the example discussed in Ref. [7], or small. The latter is typical when the system expands in spherically symmetric way. The analysis of different hydrodynamic solutions demonstrates that the velocity field of expanding systems tends typically to a spherically symmetric one at the late stage of evolution, at least, in the central region where the low $p_T$ spectrum forms. As for the high $p_T$ spectrum formation, one can expect that correspondent particles are radiated mainly from the periphery of the system at the earlier times, because of large hydrodynamic velocities and fast transition to free streaming there: it was argued in Refs. [7] and [11]. Thereby, in a rough approximation, one can apply the generalized Cooper-Frye prescriptions (8) putting there $\sigma_0 = \sigma_{f.o.}$, taking into account the possible $p$-dependence of hypersurface $\sigma_{f.o.}(p)$ at high $p_T \gtrsim 0.8 - 1$ GeV and neglecting the integral over the 4D region situated beyond of the $\sigma_{f.o.}(p)$. That is some kind of duality in the description of the spectra and the interferometry data basing either on (thermal) distribution functions $f_{\text{eq}}(x, p)$ which characterize the system just before decay begins or on the emission function $S(t, x, p)$ that describes the process of continuous particle liberation during the decay of system.

In fact, any kind of hydrodynamic or Boltzmann kinetic approaches loses its applicability when the particle mean free paths become compatible with lengths of homogeneity in the system. The universality of the Landau freeze-out temperature, $T \simeq m_\pi$, an independence on the concrete dynamics and correspondent homogeneity lengths is due to the pions are dominated in hadron system and their mean free paths, $1/\sigma n(T(x))$, start to increase exponentially $\sim \exp(\beta m_\pi)$ when the temperature falls down below $m_\pi$ as it follows from the analytic representation of the thermal density $n(T)$. Thus the temperature $T \simeq m_\pi$ is just lower boundary of the region of applicability of hydrodynamics in wide class of nucleon and nuclear collisions. It does not mean that the hadrons stop to interact then at post hydrodynamic stage but the momentum spectra do not change significantly especially if the above discussed conditions (small value of $F_{\text{gain}}$, symmetry of expansion at the late stage, etc.) are satisfied and so the integral of $F_{\text{gain}} - F_{\text{loss}}$ in Eq. (8) is small at that stage.

4. Conclusions and outlook

We show that universal Landau freeze-out temperature corresponds to lower boundary of the applicability of hydrodynamics that is similar in different collisions. The duality in spectra description between the (generalized) sudden freeze-out prescription, that utilizes the distribution functions $f$, and the detail picture of the particle liberation process, based on the emission function $S$, is argued. The replace of the complicated emission process by the simple Landau [2] criterion of sudden freeze-out at $T \approx 0.12 \div 0.15$ GeV is, of course, a rather rough approximation. Nevertheless,
as it was discussed in Ref. [12], the momentum-energy conservation laws, peculiarities of almost isoentropic and chemically frozen evolution as well as some symmetry features of the late stage of hydro evolution minimize the correspondent uncertainties. Note, that the duality does not mean and even excludes the parametrization of emission function in the form $S \sim f_{\text{eq}}$, excepting for the case of real sudden freeze-out which is, probably, very non-realistic.

The approximation discussed is, unfortunately, not well controlled in practical utilization and we hope to develop an approximate method for spectra calculations within the hydro-kinetic approach [7] based on the escape probabilities and generalized relaxation time approximation. The method will combine the advantages of hydrodynamic approximation and microscopic (kinetic) approach. The former allows one to incorporate the complicated evolution of the system at the possible phase transitions encoded in corresponding equation of state; the latter makes possible to evaluate the observable spectra taking into account the non-equilibrated character of their formation. As a result we hope coherently explain all totality of hadronic observables of various experiments where existing dynamic models fail to accommodate the majority of experimental data.

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References