THE HIGH ENERGY LIMIT
OF
QCD

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GOAL to analyse the QCD dynamics in the $s \gg |t|$ limit: the high energy limit (HEL)

FACT in HEL the scattering processes are dominated by sub-processes with gluon exchange in the t channel

BFKL theory resums multiple gluon radiation out of the gluon exchanged in the t channel

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PHENOM. Process-dependent questions:

- ☛ does a fixed-order expansion in α_S suffice to describe the data ?
- ☛ can the data be described in terms of other, e.g. Sudakov, resummations ?
- ☛ in phase space, where do sub-processes with gluon exchange in the t channel dominate over the other sub-processes ?

Forward Scattering

Parton-parton scattering In the c.m. frame, $t = -s(1-\cos\theta)/2$, with θ the scattering angle. $s \gg |t|$:

- forward, i.e. small angle, scattering: $d\sigma/dt \sim 1/t^2$
- the scattering process is dominated by sub-processes with gluon exchange in the t channel: $q\,Q \to q\,Q$, $q\,g \to q\,g$, $g\,g \to g\,g$

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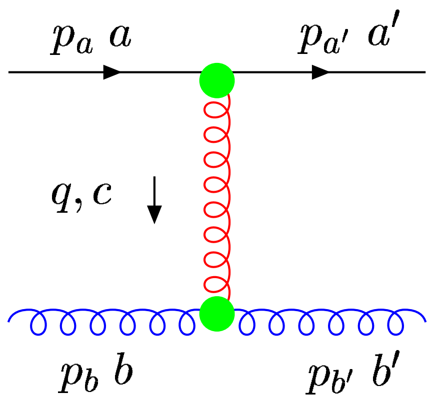
$q\,g \to q\,g$ scattering amplitude in the $s \gg |t|$ limit:



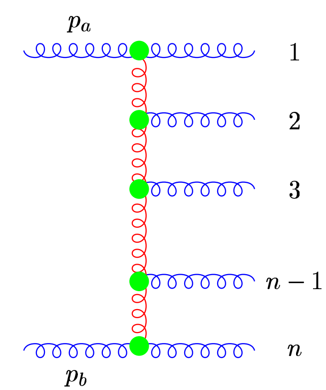
$$\mathcal{A}^{\rm tree}_{q\,g\to q\,g}(p_a, p_{a'}|p_{b'}, p_b) = 2s\left[g\,T^c_{a'\bar{a}}\,C^{q;q}(p_a;p_{a'})\right]\frac{1}{t}\left[ig\,f^{bb'c}\,C^{g;g}(p_b;p_{b'})\right]$$

- $C^{g;g}$ ($C^{q;q}$): gluon (quark) high energy effective vertices
- high energy factorisation: to obtain $q\,Q \to q\,Q$ or $g\,g \to g\,g$ replace

$$ig\,f^{bb'c}\,C^{g;g}(p_b;p_{b'}) \leftrightarrow g\,T^c_{b'\bar{b}}\,C^{q;q}(p_b;p_{b'})$$

BFKL Resummation

- in any scattering process with $s \gg |t|$ gluon exchange in the t channel dominates



- BFKL is a resummation of multiple gluon radiation out of the gluon exchanged in the t channel

- for $s \gg |t|$ BFKL resums the Leading Log (and Next-to-Leading Log) contributions, in $\log(s/t)$, of the radiative corrections to the gluon propagator in the t channel, to all orders in α_S

- the LL terms are obtained in the approximation of strong rapidity ordering $(y_1 \gg y_2 \gg \ldots \gg y_n)$ and no k_t ordering of the emitted gluons

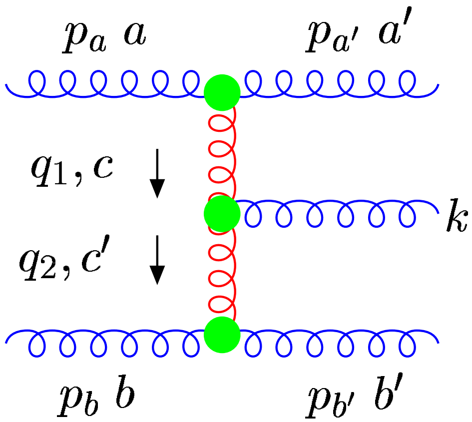
- the NLL terms are universal

- the resummation yields a 2-dim integral equation for the evolution of the gluon propagator in the t channel

LL BFKL RESUMMATION

❋ the universal building blocks of the LL BFKL resummation are:

- the real term: the emission of a gluon along the gluon ladder



$$\mathcal{A}^{\rm tree}_{gg\to 3g}(p_a, p_{a'}|k|p_{b'}, p_b) = s\left[ig\, f^{aa'c}\, C^{g;g}(p_a;p_{a'})\right] \times \frac{1}{t_1}\left[ig\, f^{cdc'}\, C^{g}(q_1,k,q_2)\right] \times \frac{1}{t_2}\left[ig\, f^{bb'c'}\, C^{g;g}(p_b;p_{b'})\right]$$

➥ $C^g(q_1, k, q_2)$ is the gluon emission (Lipatov) vertex

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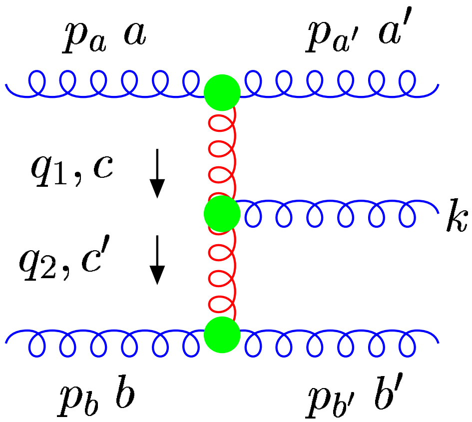

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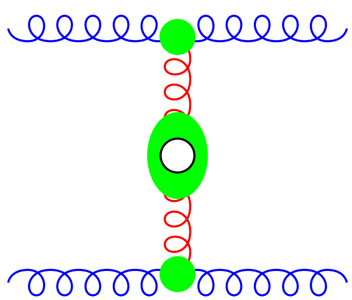
- the virtual term: the reggeisation of the gluon exchanged in the t channel (here in $d = 4 - 2\epsilon$ dimensional regularisation)



$$\mathcal{A}^{1-\text{loop}}_{g\,g\to g\,g} = \tilde{g}^2(t)\alpha^{(1)} \ln\frac{s}{-t}\mathcal{A}^{\text{tree}}_{g\,g\to g\,g}$$

$$\alpha^{(1)} = \frac{2C_A}{\epsilon} \qquad \tilde{g}^2(t) = g^2 c_\Gamma\left(\frac{\mu^2}{-t}\right)^{\epsilon}$$

➥ $\tilde{g}^2(t)\alpha^{(1)}$ is the 1-loop gluon Regge trajectory $\qquad (C_A = N_c)$

NLL BFKL RESUMMATION

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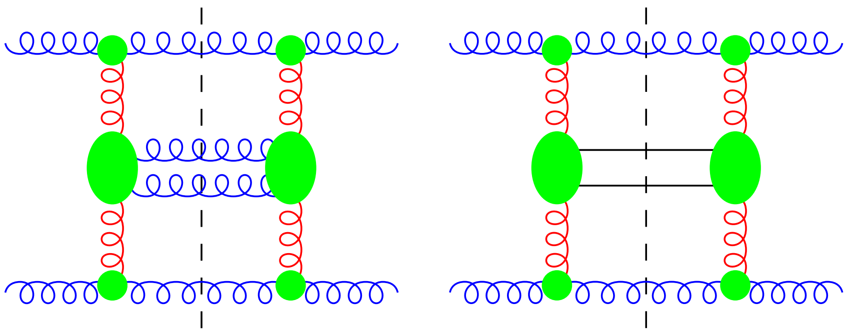
  * corrections to the Lipatov vertex

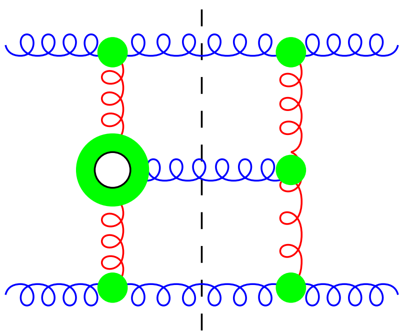
real



Fadin, Lipatov 1989-96

VDD 1996

virtual



Fadin, Lipatov 1993

Fadin, Fiore, Quartarolo 1994

Fadin, Fiore, Kotsky 1996

Bern, Schmidt, VDD 1998

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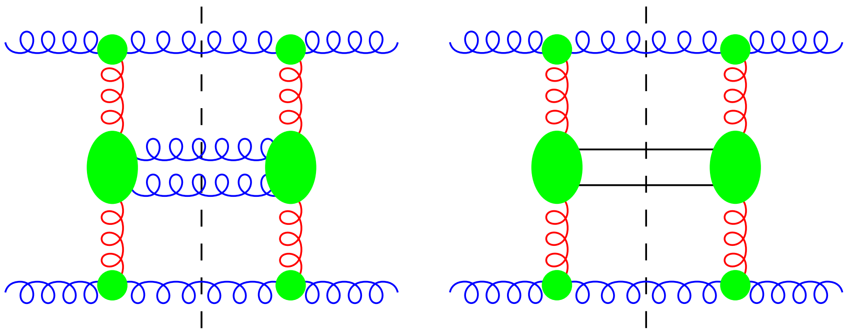

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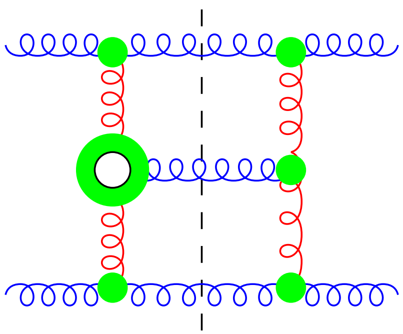

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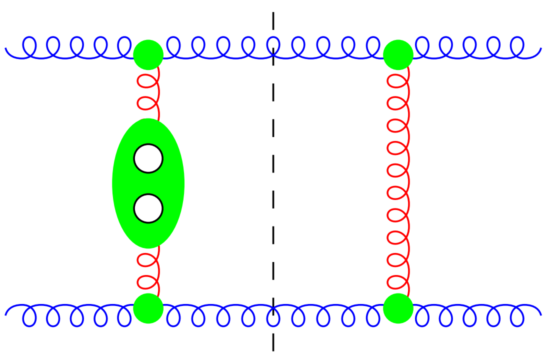
- 2-loop gluon reggeisation



Fadin, Fiore, Kotsky 1995-96

Fadin, Fiore, Quartarolo 1995

Glover, VDD 2001

Gluon Reggeisation

Ansatz in HEL the gluon-gluon scattering amplitude for the exchange of a colour octet of negative signature in the t channel is

$$\mathcal{A}_{g\,g\to g\,g}(p_a,p_{a'}|p_{b'},p_b)$$
$$= s\left[ig\,f^{aa'c}\,C^{g;g}(p_a;p_{a'})\right]\frac{1}{t}\left[\left(\frac{-s}{-t}\right)^{\alpha(t)}+\left(\frac{s}{-t}\right)^{\alpha(t)}\right]\left[ig\,f^{bb'c}\,C^{g;g}(p_b;p_{b'})\right]$$

* the effective vertex $C^{g;g}$ and the gluon Regge trajectory have the perturbative expansion

$$C^{g;g} = C^{g;g(0)}(1+\tilde{g}^2(t)C^{g;g(1)}+\tilde{g}^4(t)C^{g;g(2)})+\mathcal{O}(\tilde{g}^6)$$
$$\alpha(t) = \tilde{g}^2(t)\alpha^{(1)}+\tilde{g}^4(t)\alpha^{(2)}+\mathcal{O}(\tilde{g}^6)$$

* the 2-loop gluon Regge trajectory is

$$\alpha^{(2)}= C_A\left[\beta_0\frac{1}{\epsilon^2}+K\frac{2}{\epsilon}+C_A\left(\frac{404}{27}-2\zeta_3\right)+N_F\left(-\frac{56}{27}\right)\right]$$

where $\beta_0=\frac{(11C_A-2N_F)}{3}$ $\quad K=\left(\frac{67}{18}-\frac{\pi^2}{6}\right)C_A-\frac{5}{9}N_F$

BACKWARD SCATTERING

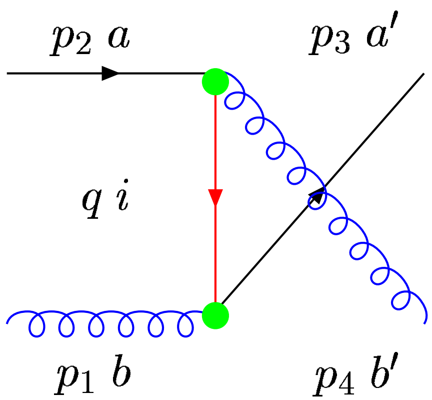
QUARK–GLUON SCATTERING in the c.m. frame $u = -s(1+\cos\theta)/2$.

$s \gg |u|$: backward, i.e. large angle, scattering: $d\sigma/du \sim 1/(su)$

$q\ g \to q\ g$ scattering amplitude in the $s \gg |u|$ limit:

$$\mathcal{A}^{\rm tree}_{qg\to gq} = 2\left[g\,(T^b)_{a'i} C^{(0)}_{gq}(p_1,p_3)\right]\sqrt{\frac{s}{-u}}\left[g\,(T^{b'})_{ia}\,C^{(0)}_{qg}(p_2,p_4)\right]$$

$C^{(0)}_{gq}$: effective vertex



- the quark reggeises

  Fadin, Sherman 1977

- in QED the electron reggeises but the photon does not

Quark Reggeisation

Ansatz in the $s \gg |u|$ limit, the quark–gluon scattering amplitude for the exchange of a colour triplet of positive signature in the u channel is

$$\mathcal{A}^{\rm tree}_{qg\to gq}$$

$$= \quad \left[g\,(T^b)_{a'i} C_{gq}(p_1,p_3)\right] \sqrt{\frac{s}{-u}} \left[\left(\frac{s}{-u}\right)^{\delta(u)} + \left(\frac{-s}{-u}\right)^{\delta(u)}\right] \left[g\,(T^{b'})_{ia}\, C_{qg}(p_2,p_4\right.$$

$\delta(u)$ is the quark Regge trajectory, with expansion

$$\delta(u) = \tilde{g}^2(u)\delta^{(1)} + \tilde{g}^4(u)\delta^{(2)} + \mathcal{O}(\tilde{g}^6) \qquad\qquad \tilde{g}^2(u) = g^2 c_\Gamma \left(\frac{\mu^2}{-u}\right)^{\epsilon}$$

the 1-loop trajectory is $\delta^{(1)} = \dfrac{2C_F}{\epsilon}$

the 2-loop trajectory is — Bogdan, Fadin, Glover, VDD 2002

$$\delta^{(2)} = C_F \left[\beta_0 \frac{1}{\epsilon^2} + K\frac{2}{\epsilon} + C_A\left(\frac{404}{27} - 2\zeta_3\right) + N_F\left(-\frac{56}{27}\right) + (C_F - C_A)\,(16\zeta_3)\right]$$

❋ by mapping $C_F \to C_A$ we obtain the gluon trajectory

BFKL PHENOMENOLOGY

- in principle, the BFKL resummation can be applied to any scattering process with $s \gg |t|$, where t is a typical (squared) transverse energy scale

  - in $p\,p$ collisions $\left\{ \begin{array}{c} \text{dijet} \\ V,\, H + 2 \text{ jet} \\ \text{heavy diquark} \end{array} \right\}$ production at large rapidities

  - in DIS $\left\{ \begin{array}{c} F_2 \text{ scaling violations} \\ \text{forward jet production} \end{array} \right.$

  - in e^+e^-, $\gamma^*\gamma^* \to$ hadrons at large Y

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- in HEL, the partonic cross section is $\hat{\sigma}(AB \to j_1 j_2) \sim \mathcal{I}(j_1)\mathcal{F}_{BFKL}\mathcal{I}(j_2)$

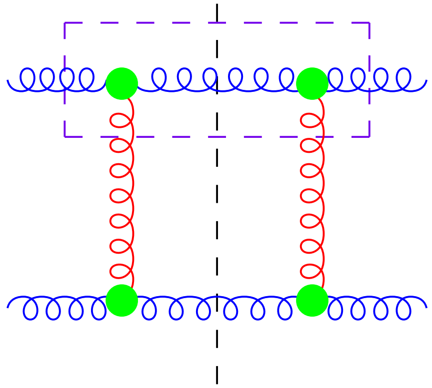
- the BFKL ladder $\mathcal{F}_{BFKL}$ is universal

- the impact factors $\mathcal{I}(j) \sim |C^{g;g}|^2$ are process dependent

IMPACT FACTORS

LO IMPACT FACTOR

$g\, g^* \to g$:



☛ at LO the impact factors are known for all the processes of interest (see next Table)

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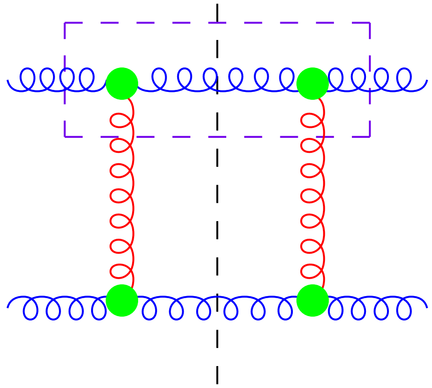

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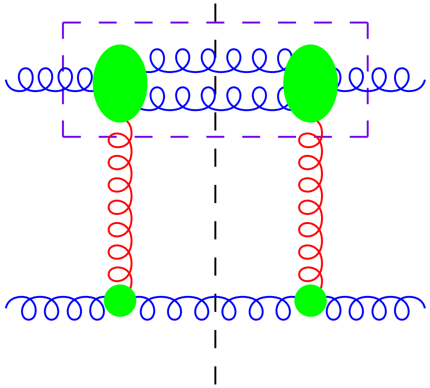
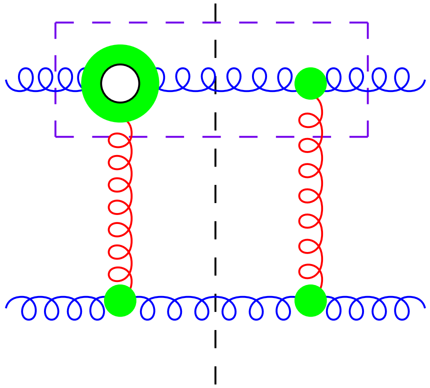
NLO IMPACT FACTOR





☛ at NLO the impact factors are known for $q\, g^* \to q$, $g\, g^* \to g$ and $\gamma^* g^* \to q\bar{q}$

Bartels, Colferai, Gieseke, Vacca 2001-02

Table of Impact Factors

$p\,g^* \to p \quad p = q, g$

$q\,g^* \to q\,V$

$g\,g^* \to H \quad p\,g^* \to H\,p$

$g\,g^* \to Q\,\bar{Q}$

$\gamma^* g^* \to q\,\bar{q}$

DIJET PROD.

$V + 2$ JET PROD.

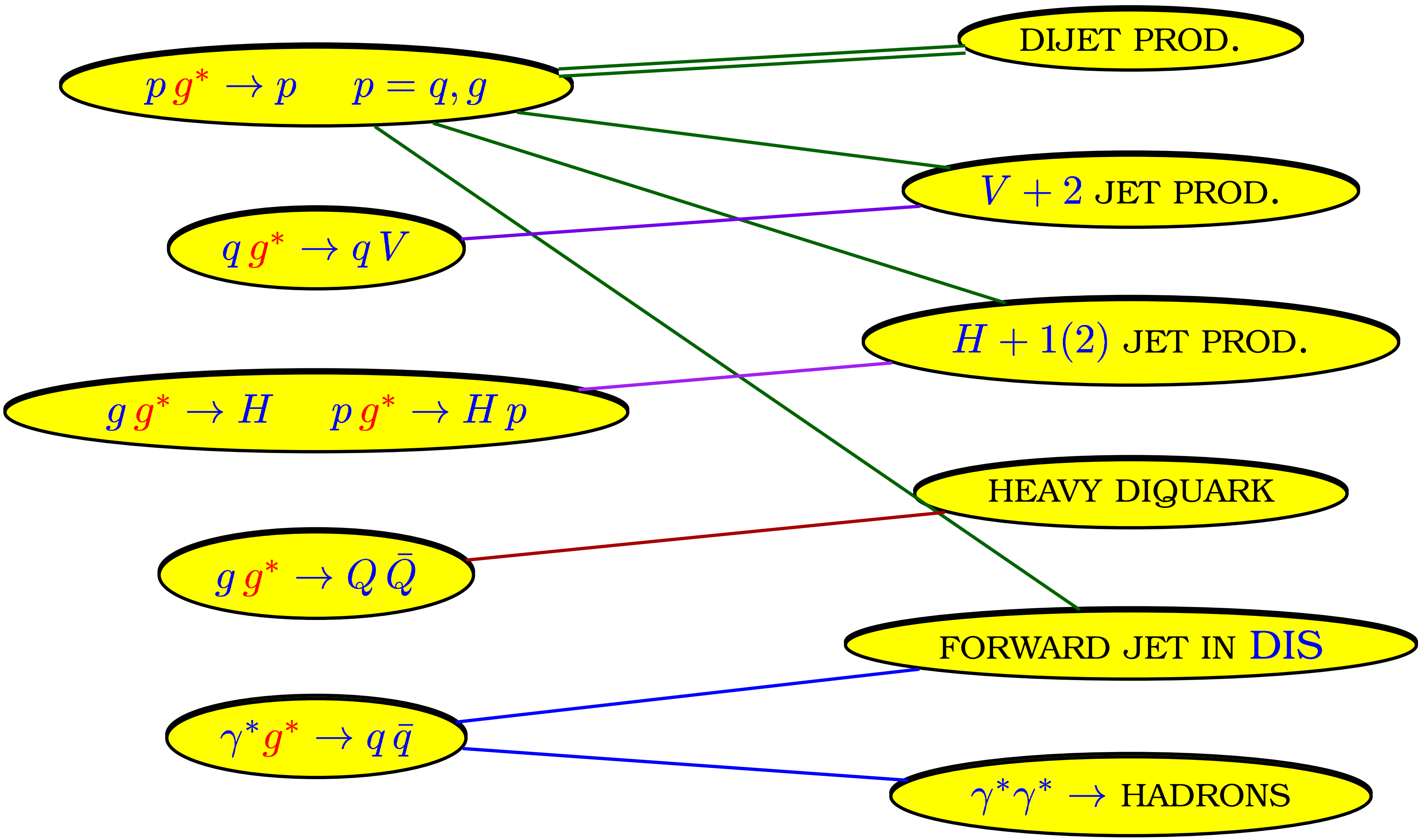
$H + 1(2)$ JET PROD.

HEAVY DIQUARK

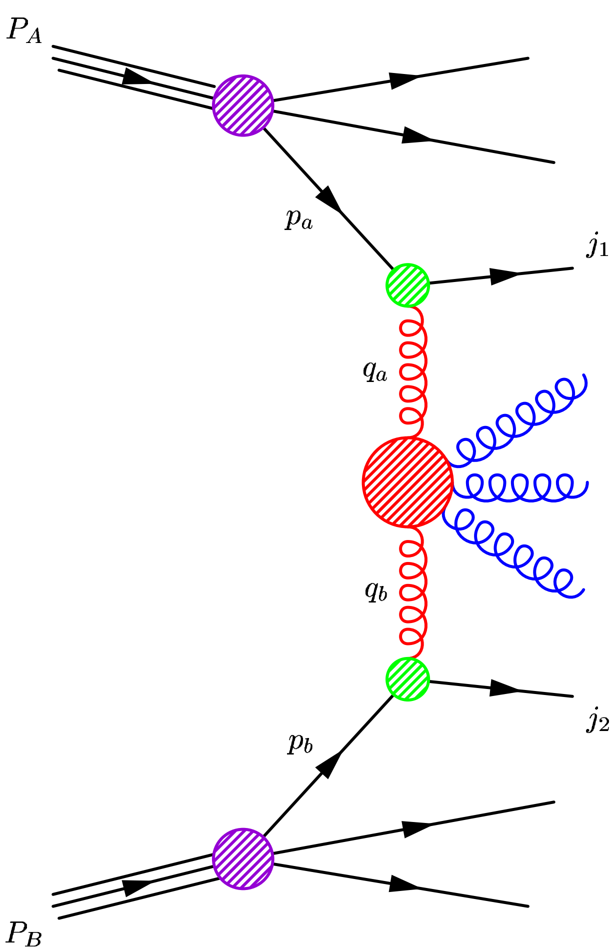
FORWARD JET IN DIS

$\gamma^* \gamma^* \to$ HADRONS

TABLE OF IMPACT FACTORS



Dijet Production in pp Collisions



Kinematics

$p_a = x_a P_A \quad p_b = x_b P_B$:

incoming parton momenta

S : hadron c.m. energy

$s = x_a x_b S$: parton c.m. energy

$E_{j_{1,2}\perp}$: jet transverse energy

$Q^2 = -t$: typical momentum transfer

➥ $Q^2 \sim E^2_{j\perp}$

$\Delta y = |y_{j_1} - y_{j_2}|$:

rapidity difference between the jets

❋ $\ln \frac{S}{Q^2} = \ln \frac{1}{x_a} + \ln \frac{s}{Q^2} + \ln \frac{1}{x_b}$

❋ $x_{a,b} = \mathcal{O}(1) \qquad \ln \frac{s}{Q^2} \simeq \Delta y \gg 1$

➥ physics of large rapidity intervals, and not small-x physics

Dijet Production in HEL

the cross section for dijet production in HEL:

$$\frac{d\sigma}{d^2p_{j_1\perp}\, d^2p_{j_2\perp}\, dy_{j_1}\, dy_{j_2}} = x_a^0 f_{\rm eff}(x_a^0, \mu_F^2)\, x_b^0 f_{\rm eff}(x_b^0, \mu_F^2)\, \frac{d\hat{\sigma}_{gg}}{d^2p_{j_1\perp}\, d^2p_{j_2\perp}}$$

the parton momentum fractions in HEL:

$$x_a^0 = \frac{|p_{j_1\perp}|}{\sqrt{S}} e^{y_{j_1\perp}}\,, \qquad x_b^0 = \frac{|p_{j_2\perp}|}{\sqrt{S}} e^{-y_{j_2\perp}}$$

the effective p.d.f.

$$f_{\rm eff}(x, \mu_F^2) = G(x, \mu_F^2) + \frac{4}{9} \sum_f \left[Q_f(x, \mu_F^2) + \bar{Q}_f(x, \mu_F^2)\right]$$

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parton cross section in terms of BFKL ladder f and impact factors:

$$\frac{d\hat\sigma_{gg}}{d^2p_{j_1\perp}\, d^2p_{j_2\perp}} = \left[\frac{C_A\alpha_s}{|p_{j_1\perp}|^2}\right] f(q_{a_\perp}, q_{b_\perp}, \Delta y) \left[\frac{C_A\alpha_s}{|p_{j_2\perp}|^2}\right]$$

asymptotically, the LL BFKL ladder is:

$$\lim_{\Delta y \gg 1} f(q_{a_\perp}, q_{b_\perp}, \Delta y) \sim \frac{e^{4C_A \ln 2\alpha_S \Delta y/\pi}}{\sqrt{7\zeta_3 C_A \alpha_S \Delta y/2}}$$

BFKL Monte Carlo

Drawbacks of the BFKL ladder

- the (N)LL BFKL resummation is performed at fixed α_S
  - any variation in the scale of α_S occurs in the (N)NLL terms

- energy and longitudinal momentum are not conserved:

  in dijet production, the exact x's are

$$x_a = \frac{e^{y_{j_1}}}{\sqrt{S}}\left(|p_{j_1\perp}| + |p_{j_2\perp}|e^{-\Delta y} + \sum_{i=1}^{n} p_{i\perp}e^{y_i - y_{j_1}}\right)$$

$$x_b = \frac{e^{-y_{j_2}}}{\sqrt{S}}\left(|p_{j_2\perp}| + |p_{j_1\perp}|e^{-\Delta y} + \sum_{i=1}^{n} p_{i\perp}e^{-y_i + y_{j_2}}\right)$$

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- a Monte Carlo solution of the BFKL equation can account for
  - running of α_S
  - energy and longitudinal momentum conservation

Schmidt 1996; Orr, Stirling 1997

Dijet Production in HEL

- in an event with two or more jets, tag the most forward and the most backward jets

$$\Delta y = |y_{j_1} - y_{j_2}| \simeq \ln \frac{x_a x_b S}{E_{j_{1\perp}} E_{j_{2\perp}}}$$

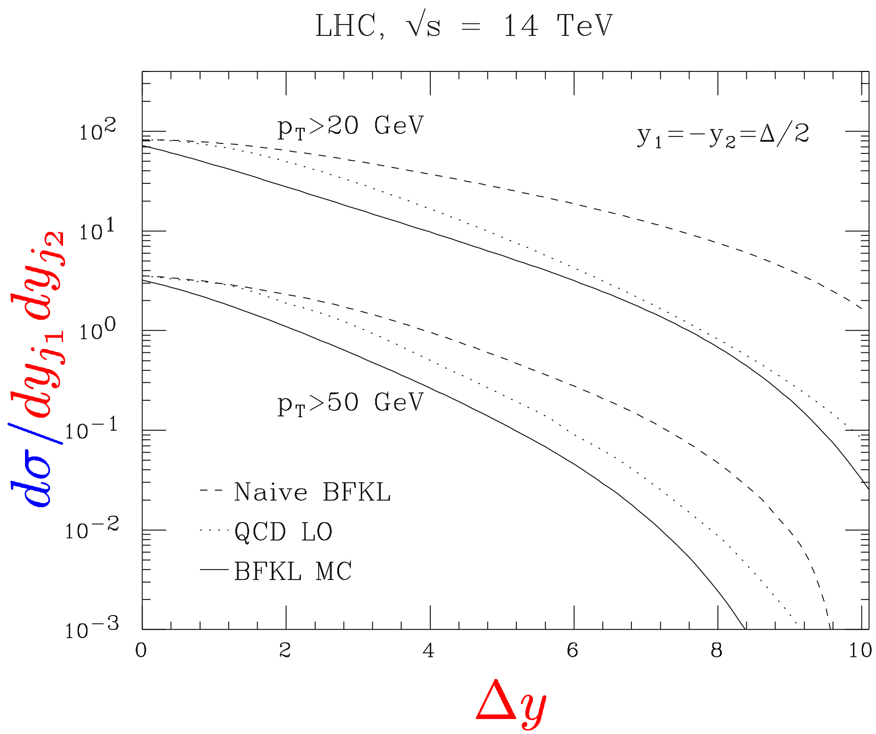
- minimise the jet transverse energy
- maximise $s = x_a x_b S$
  - in a collider with ramping-up energy S, fix $= x_{a,b}$

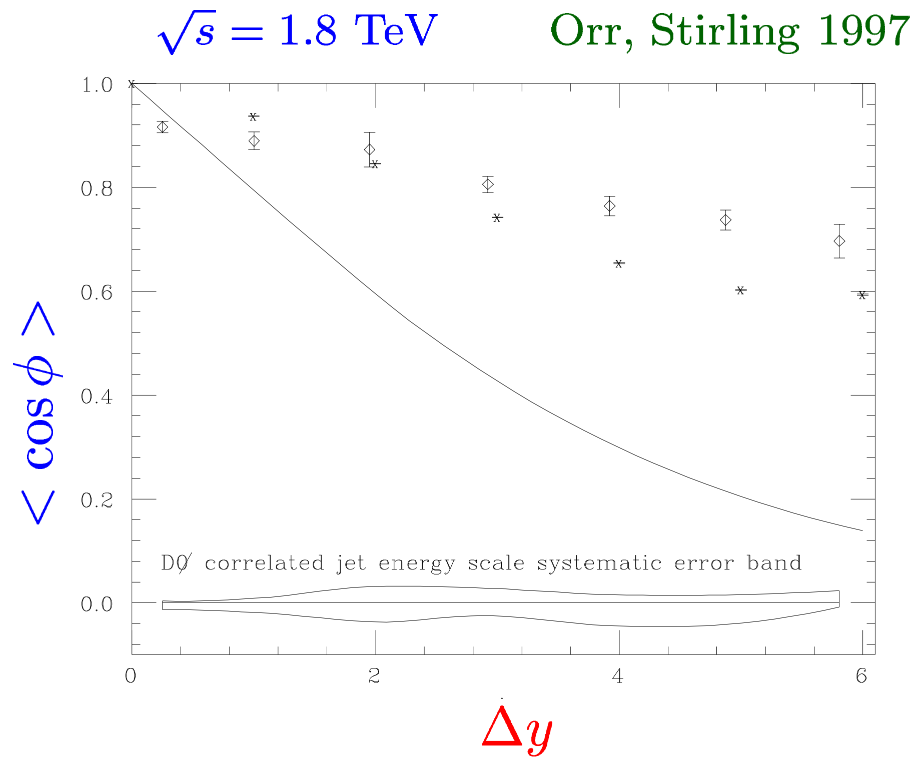
    analyse $\dfrac{d\sigma}{dx_a dx_b}$ for different values of S — Mueller, Navelet 1987
  - in a fixed energy S collider, increase $= x_{a,b}$

    analyse $\begin{cases} \dfrac{d\sigma}{d\Delta y} & \text{for different values of } \Delta y \\ \dfrac{d\sigma}{d\Delta y\, d\phi} & \phi: \text{ azimuthal angle between tagged jets} \end{cases}$

Schmidt, VDD; Stirling 1993-95

Dijet Production – Phenomenology





- in $\dfrac{d\sigma}{dy_{j_1}dy_{j_2}}$ the BFKL Monte Carlo yields a depletion rather than an enhancement, both for Tevatron & LHC, due to the falling parton luminosities

- $< \cos\phi >$ shows too much azimuthal decorrelation wrt Tevatron D0 data, while it is well described by a parton-shower Monte Carlo (HERWIG)

Caveat $< \cos\phi >$ is dominated by soft gluon (Sudakov) effects

Mueller-Navelet Jets

Mueller-Navelet proposal for colliders with ramping-up energy S:

❋ take the cross section for dijet production in HEL at fixed x's:

$$\frac{d\sigma}{dx_a^0 dx_b^0} = \int d^2p_{j_{1\perp}} d^2p_{j_{2\perp}} f_{\rm eff}(x_a^0, \mu_F^2)\, f_{\rm eff}(x_b^0, \mu_F^2)\, \frac{d\hat\sigma_{gg}}{d^2p_{j_{1\perp}} d^2p_{j_{2\perp}}}$$

❋ use approximate x's: $x_a^{MN} = \frac{E_\perp}{\sqrt{S}} e^{y_{j_{1\perp}}}$ $\quad x_b^{MN} = \frac{E_\perp}{\sqrt{S}} e^{-y_{j_{2\perp}}}$

➥ the x's are in a one-to-one correspondence with the rapidities

➥ Δy is fixed at its max: $\Delta y = \ln \frac{x_a^{MN} x_b^{MN} S}{E_\perp^2}$

❋ integrate out transverse energies above a threshold $E_\perp$: $\quad |p_{j_{1,2\perp}}| \geq E_\perp$

➥ the Mueller-Navelet gluon-gluon cross section is

$$\hat\sigma_{gg} = \frac{9\pi\alpha_S^2}{2E_\perp^2} \frac{e^{4C_A \ln 2 \alpha_S \Delta y/\pi}}{\sqrt{7\zeta_3 C_A \alpha_S \Delta y/2}}$$

❋ compute the cross section at different c.m. energies S

Mueller-Navelet Jets – D0 Analysis

D0 implementation of Mueller-Navelet: D0 Collaboration 1999

❋ exact LO x's:

$$\begin{cases} x_1 = \dfrac{2|p_{j_1 \perp}|}{\sqrt{S}} e^{\bar{y}} \cosh \dfrac{\Delta y}{2} \\ x_2 = \dfrac{2|p_{j_2 \perp}|}{\sqrt{S}} e^{-\bar{y}} \cosh \dfrac{\Delta y}{2} \end{cases} \qquad \bar{y} = \frac{y_{j_1 \perp} + y_{j_2 \perp}}{2}$$

❋ acceptance cuts:

$$\begin{cases} |p_{j_{1,2} \perp}| \geq 20 \text{ GeV} \qquad Q^2 = |p_{j_1 \perp} p_{j_2 \perp}| \leq Q^2_{MAX} = 1000 \text{ GeV}^2 \\ |y_{j_{1,2} \perp}| \leq 3 \qquad \Delta y \geq 2 \end{cases}$$

❋ measure the cross section at $\sqrt{S_A} = 1800$ GeV and $\sqrt{S_B} = 630$ GeV in 6 (x_1, x_2) bins, with $0.06 \leq x_1, x_2 \leq 0.22$

❋ using the Mueller-Navelet cross section, compute the ratio $R = \dfrac{\sigma(S_A)}{\sigma(S_B)}$

➥ get the BFKL intercept

$$\alpha_{BFKL} = 1.65 \pm 0.07$$

Our Mueller-Navelet/D0 Analysis

❋ D0 uses an upper bound on Q^2, such that $E_\perp^2/Q^2_{MAX} = 0.4$

❋ in HEL $\begin{cases} x_1 \to x_a^0 \neq x_a^{MN} \\ x_2 \to x_b^0 \neq x_b^{MN} \end{cases}$

$x_{a,b}^{MN}$ are not good approximations to $x_{1,2}$

❋ the rapidity interval can be written as

$$\Delta y = Y + \ln \frac{E_\perp^2}{p_{j_{1\perp}} p_{j_{2\perp}}} \quad \text{with} \quad Y = \ln \frac{x_a^0 x_b^0 S}{E_\perp^2}$$

$\Delta y \geq 2$ ☛ $Q^2_{MAX} = E_\perp^2 e^{(Y-2)}$

➥ an effective maximum momentum transfer

$Q^2_{MAX} = \min(1000\,\text{GeV}^2,\, E_\perp^2 e^{(Y-2)})$

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we analysed Mueller-Navelet/D0 with

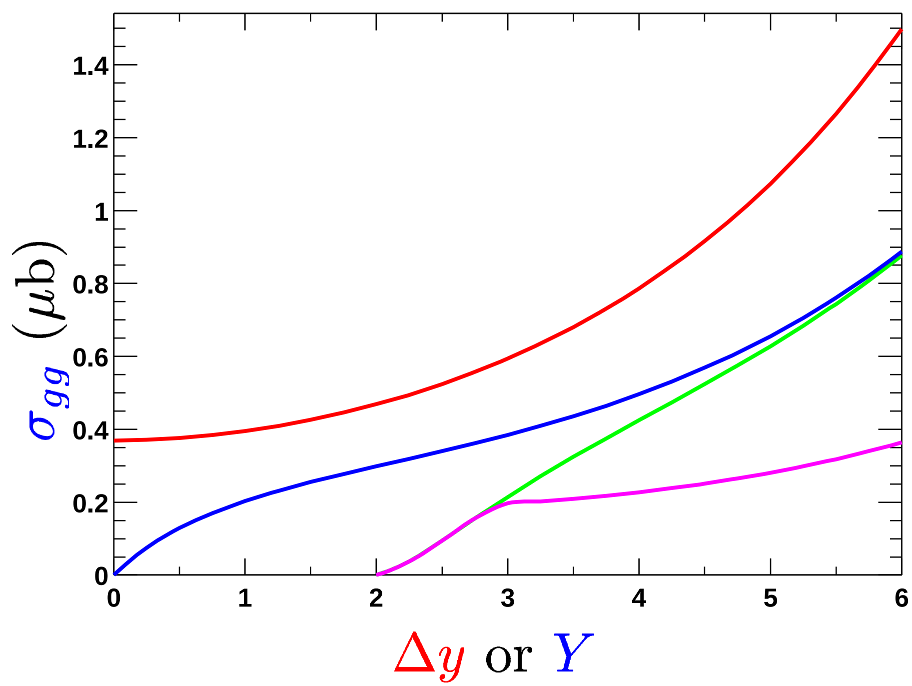
☛ analytic BFKL — Andersen, Frixione, Schmidt, Stirling, VDD 2001

☛ BFKL Monte Carlo

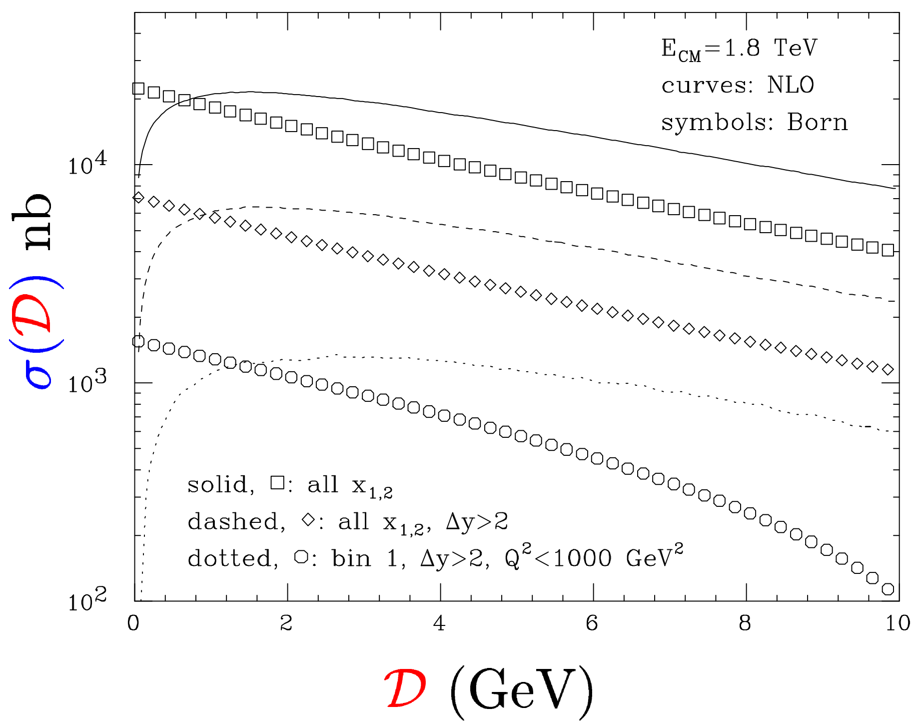
☛ general-purpose NLO partonic Monte Carlo

OUR MUELLER-NAVELET/D0 ANALYSIS

analytic BFKL



NLO partonic Monte Carlo



red: Mueller-Navelet (at fixed $x_{a,b}^{MN}$)

blue: at fixed $x_{a,b}^{0}$

green: at fixed $x_{a,b}^{0}$ with $\Delta y \geq 2$

magenta: green $+ p_{j_{1\perp}} p_{j_{2\perp}} \leq 1000\,\mathrm{GeV}^2$

☛ in σ_{gg} all curves have asymptotically the same shape: sub-leading terms are important

$\mathcal{D} = |\min(p_{j_{1\perp}}) - \min(p_{j_{2\perp}})|$

symbols: LO

curves: NLO

CAVEAT

soft gluon effects at $\mathcal{D} = 0$

CONCLUSIONS

THEORY

- in the high energy limit, scattering amplitudes factorise into a gluon ladder and impact factors
- the BFKL resummation is known at LL and NLL accuracy
- gluon (and quark) reggeisations are known at two-loop order
- impact factors are known at LO accuracy for all processes of interest
- a BFKL Monte Carlo at LL accuracy is available

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DIJET PRODUCTION in $p\,p$ collisions (at Tevatron)

- **AZIMUTHAL DECORRELATION** No evidence of BFKL radiation has been found. Data are described well by parton shower generators (HERWIG)
- **MUELLER-NAVELET JETS** Sub-leading effects forbid the extraction of the BFKL intercept
- **CAVEAT** Both analyses above are contaminated by soft gluon (Sudakov) effects