Supplementary Materials

(Dated: August 15, 2019)

First-passage problem

This behavior can be rationalized by considering the vertical motion of sinking particles as a stochastic process with drift.

Stochastic processes are commonly used to describe systems showing random variations in the dynamics. In our work sinking particles are subjected to turbulent fluctuation (assumed Gaussian), which are responsible of the random variation in the strain that, in its turn, has an impact on v_s. This leads us to represent our system as as a stochastic process with drift. In particular, we refer to a specific family of stochastic processes, the so called first passage processes.

We consider, within a domain $x \in [0; a]$, a diffusing particle that starts in $x = x_0$ and fall with a velocity V_d; this conditions is described by the one-dimensional convection-diffusion equation:

$$\frac{\partial p}{\partial t} + V_d \frac{\partial p}{\partial x} = D \frac{\partial^2 p}{\partial x^2} \tag{1}$$

or alternatively

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial x} \mathcal{J} \tag{2}$$

where $p(x, t) = p(x, t \mid x_0, 0)$ is the probability distribution of the particle starting in $x = x_0$ at $t = 0$ and $\mathcal{J} = vp - D\partial_x p$ is the flux.

The next step consists in defining the probability density of the first-passage time as the probability to be within the domain after a time t:

$$\int_0^a p(x, t \mid x_0, 0) dx = \int_t^\infty q(t', x_0) dt' \tag{3}$$

where $q(t, x_0)$ is the first-passage time (FPT) probability density. It can be expressed using the backward convection-diffusion equation:

$$-q(t, x_0) = \int_0^a \frac{\partial}{\partial t} p(x, t \mid x_0, 0) dx = -\int_0^a \frac{\partial}{\partial x} \mathcal{J} dx \tag{4}$$

At this point the aim is to determine the probability density of the first-passage time in $x = 0$ for the sinking diffusive particles started in $x = x_0$, when the absorbing boundary condition $p(x = 0, t) = 0$ is considered. This problem can be solved both using Laplace transform and with the Image method. The latter consists in considering a sort of diffusive antiparticle, that starts in $x = -x_0$ and satisfies the absorbing condition. In this scenario the solution of (1) can be written as:

$$p(x, t) = \frac{1}{\sqrt{4\pi D t}} \left[e^{-(x - x_0 - V_d t)^2 / 4Dt} - e^{V_d x / D} e^{-(x + x_0 + V_d t)^2 / 4Dt} \right] \tag{5}$$

were the prefactor $e^{V_d x/D}$ quickly decays for large negative x and it ensures that $p(x = 0, t) = 0$.

It is now possible to compute the FPT probability distribution by combining 4 and 5. We obtain

$$q(t, x_0) = \mathcal{J} \mid_{x=0} = \frac{x_0}{\sqrt{4\pi D t^3}} e^{-\dfrac{(x_0 + V_d t)^2}{4Dt}} \tag{6}$$

that is the generic expression for a sinking diffusive particles that is started in $x = x_0$ and absorbed in $x = 0$.