Appendice A

Vettori

A.1 Identità vettoriali generali

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) \tag{A.1}$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\,\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\,\mathbf{C} \tag{A.2}$$

$$\boldsymbol{\nabla}\,(\phi\psi) = \phi\boldsymbol{\nabla}\psi + \psi\boldsymbol{\nabla}\phi \tag{A.3}$$

$$\boldsymbol{\nabla} \cdot (\phi\mathbf{A}) = \mathbf{A} \cdot \boldsymbol{\nabla}\phi + \phi\boldsymbol{\nabla} \cdot \mathbf{A} \tag{A.4}$$

$$\boldsymbol{\nabla} \times (\phi\mathbf{A}) = \phi\boldsymbol{\nabla} \times \mathbf{A} - \mathbf{A} \times \boldsymbol{\nabla}\phi \tag{A.5}$$

$$\boldsymbol{\nabla}\,(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\boldsymbol{\nabla} \times \mathbf{B}) + \mathbf{B} \times (\boldsymbol{\nabla} \times \mathbf{A}) + (\mathbf{A} \cdot \boldsymbol{\nabla})\,\mathbf{B} + (\mathbf{B} \cdot \boldsymbol{\nabla})\,\mathbf{A} \tag{A.6}$$

$$\boldsymbol{\nabla} \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\boldsymbol{\nabla} \times \mathbf{A}) - \mathbf{A} \cdot (\boldsymbol{\nabla} \times \mathbf{B}) \tag{A.7}$$

$$\boldsymbol{\nabla} \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}\,(\boldsymbol{\nabla} \cdot \mathbf{B}) - \mathbf{B}\,(\boldsymbol{\nabla} \cdot \mathbf{A}) + (\mathbf{B} \cdot \boldsymbol{\nabla})\,\mathbf{A} - (\mathbf{A} \cdot \boldsymbol{\nabla})\,\mathbf{B} \tag{A.8}$$

A.2 Relazioni con operatori differenziali

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla}\phi = 0 \tag{A.9}$$

$$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \mathbf{A}) = 0 \tag{A.10}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla}\phi = \boldsymbol{\nabla}^2\phi \tag{A.11}$$

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{A}) = \boldsymbol{\nabla}\,(\boldsymbol{\nabla} \cdot \mathbf{A}) - \boldsymbol{\nabla}^2\mathbf{A} \tag{A.12}$$

A.3 Teoremi integrali

Teorema di Gauss

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = \int_V (\boldsymbol{\nabla} \cdot \mathbf{A})\, dV \tag{A.13}$$

con analoga relazione per una quantità scalare

$$\oint_S \phi d\mathbf{S} = \int_V (\boldsymbol{\nabla} \phi)\, dV \tag{A.14}$$

Teorema della circolazione di Stokes

$$\oint_C \mathbf{A} \cdot d\mathbf{l} = \int_S (\boldsymbol{\nabla} \times \mathbf{A}) \cdot d\mathbf{S} \tag{A.15}$$

Appendice B

Tensori

Rappresentazione matriciale

$$T_{ij} = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix}$$

Delta di Kronecker:

$$\delta_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Rappresentazione tensoriale di operazioni vettoriali:0

$$\mathbf{A} \cdot \mathbf{B} = A_i B_i = A_1 B_1 + A_2 B_2 + A_3 B_3 \tag{B.1}$$

$$\boldsymbol{\nabla} \cdot \mathbf{A} = \frac{\partial A_i}{\partial x_i} = \frac{\partial A_1}{\partial x_1} + \frac{\partial A_2}{\partial x_2} + \frac{\partial A_3}{\partial x_3} \tag{B.2}$$

$$\boldsymbol{\nabla} \psi = \frac{\partial \psi}{\partial x_i}$$

$$\int_V \frac{\partial T_{ij}}{\partial x_j} dV = \oint_S T_{ij} dS_j \tag{B.3}$$

$$\delta_{ij} A_j = A_i \tag{B.4}$$

$$\delta_{ii} = 3 \tag{B.5}$$

$$\begin{aligned} \varepsilon_{123} &= \varepsilon_{231} = \varepsilon_{312} = 1 \\ \varepsilon_{132} &= \varepsilon_{213} = \varepsilon_{321} = -1 \\ \text{altri} \quad \varepsilon_{ijk} &= 0 \qquad \text{(simbolo di Levi-Civita)} \end{aligned} \tag{B.6}$$

$$C_i = (\mathbf{A} \times \mathbf{B})_i = \varepsilon_{ijk} A_j B_k \tag{B.7}$$

$$(\boldsymbol{\nabla} \times \mathbf{A})_i = \varepsilon_{ijk} \frac{\partial A_k}{\partial x_j} \tag{B.8}$$

$$\boldsymbol{\nabla}^2\psi = \frac{\partial^2\psi}{\partial x_i \partial x_i}$$

$$\varepsilon_{ijk}\varepsilon_{pqk} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp} \tag{B.9}$$

Appendice C

Diadi

Tensori del second'ordine possono venire rappresentati in forma diadica:

$$\mathbb{P} = \mathbf{e}_i P_{ij} \mathbf{e}_j \tag{C.1}$$

$$\mathbb{I} = \mathbf{e}_i \delta_{ij} \mathbf{e}_j \tag{C.2}$$

$$\mathbf{A} \cdot \mathbb{B} = A_k \mathbf{e}_k \cdot \mathbf{e}_i B_{ij} \mathbf{e}_j = A_k \delta_{ki} B_{ij} \mathbf{e}_j = A_i B_{ij} \mathbf{e}_j \tag{C.3}$$

$$\mathbb{B} \cdot \mathbf{A} = \mathbf{e}_i B_{ij} \mathbf{e}_j \cdot \mathbf{e}_k A_k \mathbf{e}_k = A_k \delta_{ki} B_{ij} \mathbf{e}_j A_j = B_{ij} A_i \mathbf{e}_j \tag{C.4}$$

$$\mathbb{A} \colon \mathbb{B} = \mathbf{e}_i A_{ij} \mathbf{e}_j \colon \mathbf{e}_k B_{kl} \mathbf{e}_l = A_{ij} B_{kl} \left(\mathbf{e}_j \cdot \mathbf{e}_k\right) \left(\mathbf{e}_i \cdot \mathbf{e}_l\right) = A_{ij} B_{ji} \tag{C.5}$$

Relazione con rappresentazioni tensoriali:

$$\begin{aligned}
\mathbb{A} &\rightarrow A_{ij} \\
\mathbf{AB} &\rightarrow A_i B_j \\
\mathbf{A} \cdot \mathbb{B} &\rightarrow A_i B_{ij} \\
\mathbb{B} \cdot \mathbf{A} &\rightarrow B_{ij} A_i \\
\mathbb{A} \colon \mathbb{B} &\rightarrow A_{ij} B_{ji} \\
\boldsymbol{\nabla} \mathbf{A} &\rightarrow \frac{\partial A_j}{\partial x_i} \\
\boldsymbol{\nabla} \boldsymbol{\nabla} \psi &\rightarrow \frac{\partial^2 \psi}{\partial x_i \partial x_j} \\
\boldsymbol{\nabla} \cdot \mathbb{A} &\rightarrow \frac{\partial A_{ij}}{\partial x_i} \\
\boldsymbol{\nabla} \cdot \mathbf{AB} &\rightarrow \frac{\partial \left(A_i B_j\right)}{\partial x_i} \\
\boldsymbol{\nabla} \boldsymbol{\nabla} \colon \mathbb{A} &\rightarrow \frac{\partial^2 A_{ij}}{\partial x_i \partial x_j}
\end{aligned}$$

Appendice D

Integrali della teoria cinetica

$$
\begin{aligned}
I_n &= \int_0^\infty e^{-\beta x^2} x^n dx \\
I_0 &= \frac{1}{2}\sqrt{\frac{\pi}{\beta}} \\
I_1 &= \frac{1}{2\beta} \\
I_2 &= \frac{1}{4}\sqrt{\frac{\pi}{\beta^3}} \\
I_{n+2} &= -\frac{dI_n}{d\beta}
\end{aligned}
$$

Appendice E

Funzioni di Bessel