Turbulence and nonlinear dynamics in solar wind and laboratory plasmas

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PLASMAS IN ASTROPHYSICS AND IN THE LABORATORY:
THE IGNITOR CHALLENGE

UNIVERSITY OF CALABRIA
Department of Physics

A brief review of the interplay between *in situ* observations of turbulence in space plasma and laboratory experiments (competences available in Italy)

To be pointed out: fruitful experience of common projects between University of Calabria in Italy and scientists of the Institute for Cosmic Researches, Russian Academy of Sciences.

1) INTAS project: 1998/2001 “Multiscale dynamical structuring in planetary magnetotails”
2) INTAS project: 2007/2009 “Non Gaussian transport”
3) People - Marie Curie - 7FP - no. 269198 2011/2014 “Dissipative structures and kinetic processes in the near Earth plasmas”

What actually "turbulence" means

$$\partial_t u_i + u_\alpha \partial_\alpha u_i = -\partial_i P + \nu \partial_\alpha^2 u_i$$

$$\tau_\ell \sim \frac{\ell}{u_\ell} \qquad \tau_D \sim \frac{\ell^2}{\nu}$$

From (for example) Navier-Stokes equations we can find two characteristic times for the two basic processes: a convective (transfer) time and a diffusive (dissipative) time

$$R = \frac{\tau_D}{\tau_L} = \frac{UL}{\nu}$$

Their ratio, at the (largest) scale L, is the Reynolds number

$$\epsilon_D \sim \frac{U^2}{\tau_D} \sim \frac{U^2 \nu}{L^2}$$

$$\epsilon_L \sim \frac{U^2}{\tau_L} \sim \frac{U^3}{L} = R\epsilon_D$$

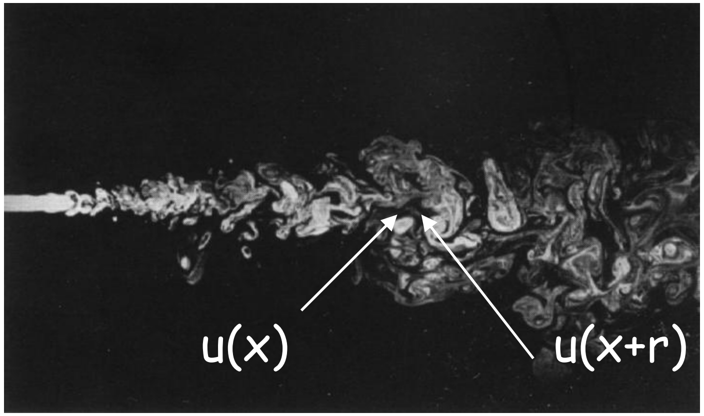
At the largest scale L the energy injection rate (per unit mass) turns out to be R times greater than the energy dissipation rate

Turbulence is nothing but the way chosen by the fluid system to dissipate the excess energy injected at large scales.

Since dissipation is efficient only at very small scales, the system dissipates energy by transferring it to small scales → **nonlinear energy cascade.**

Two-points correlation tensor: statistical predictability

Two-points differences separated by a distance r are the main quantities we investigate.
They represent characteristic fluctuations across eddies at the scale r.



$$\left\langle [u(x+r)-u(x)]^2 \right\rangle = 2\int_0^\infty E(k)\left[1-\frac{\sin kr}{kr}\right]dk$$

Assuming homogeneity the 2-th order moment of two-points differences is related to the energy spectra

Gaussian process: the 2-th order moment suffices to fully determine probability density functions (pdf). High-order moments are uniquely defined from the 2-th order (in this sense energy spectra are interesting!)

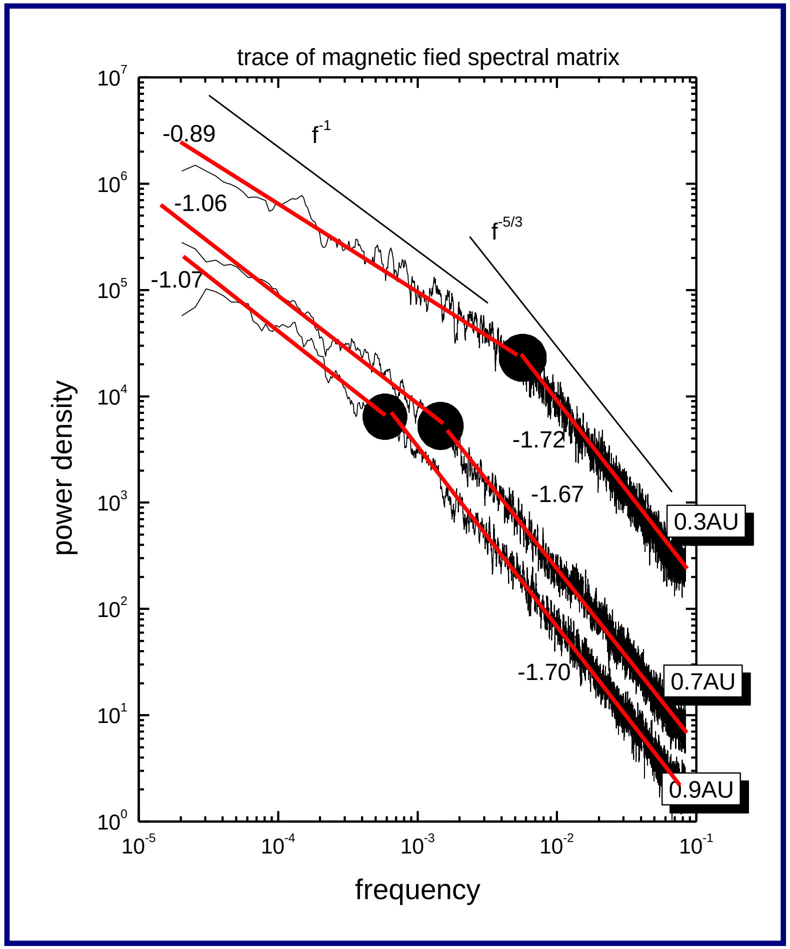
High-order moments of two-points differences represent probes for non-gaussian behaviour of fluctuations

$$S_n(r) = \left\langle [u(x+r)-u(x)]^n \right\rangle \approx r^{\zeta_n}$$

Kolmogorov's standard turbulence:

$$\zeta_n = n/3 \;\Rightarrow\; E(k) \approx k^{-5/3}$$

Evidences of power spectrum for magnetic fluctuations in the solar wind and Tokamak plasmas can be attributed to fully developed turbulence



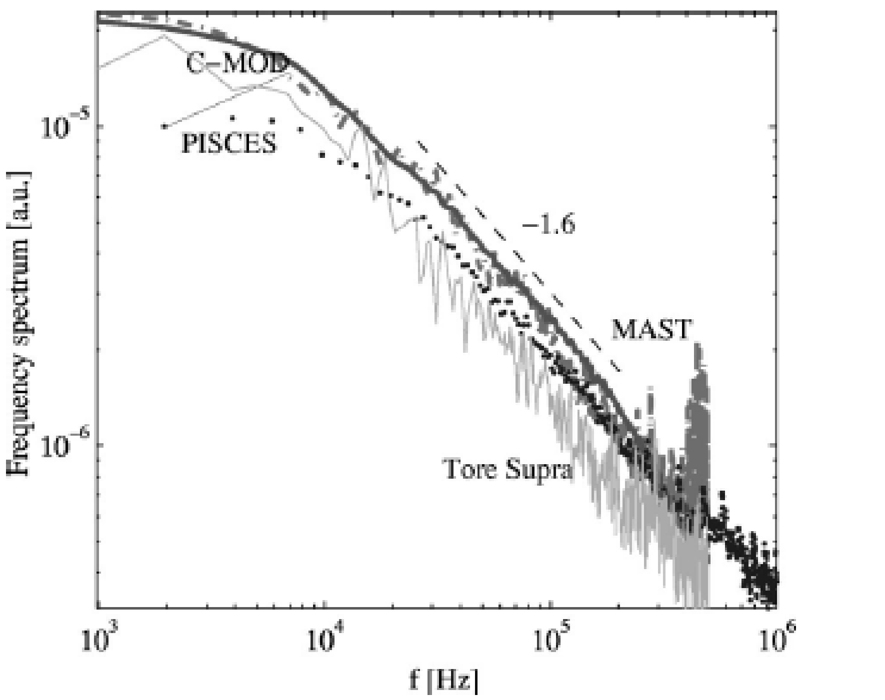


FIG. 4. A log–log plot of the power spectrum of the ion saturation current fluctuations in Tore Supra (solid line), Alcator C-Mod (thick solid line), MAST (dashed–dotted line), and PISCES (dots). The ion saturation currents, taken in the SOL, were normalized to the standard deviations.

How the presence of a turbulent energy cascade can be evidenced? An exact Yaglom's relation for incompressible MHD turbulent cascade

$$\frac{\partial \vec{z}^{\pm}}{\partial t} + \left(\vec{z}^{\mp} \cdot \nabla\right)\vec{z}^{\pm} = -\nabla P + \nu \nabla^2 \vec{z}^{\pm}$$

$$\vec{z}^{\pm} = \vec{u} \pm \vec{b} = \vec{u} \pm \vec{B} / \sqrt{4\pi\rho}$$

The MHD equations in terms of Elsasser variables

$$\Delta Z_i^{\pm} = Z_i^{\pm}(x_i') - Z_i^{\pm}(x_i) \qquad x_i' = x_i + \ell_i$$

Two-points vector differences

From MHD equations, assuming local isotropy and homogeneity, in the limit of vanishing dissipation, it can be derived an exact relation for the third-order mixed moment (assuming turbulence be in the stationary state)

$$\langle \Delta Z_\ell^{\mp} | \Delta Z_i^{\pm} |^2 \rangle = -\frac{4}{3} \epsilon^{\pm} \ell$$

THE YAGLOM'S RELATION FOR TURBULENCE IS THE ONLY RESULT OF TURBULENCE THAT IS BOTH EXACT AND NONTRIVIAL.

$$\epsilon_{ij}^{\pm} = \langle \left(\partial_i Z_j^{\pm}\right) \left(\partial_j Z_i^{\pm}\right) \rangle$$

Energy dissipation rate tensor per mass unit

$$\langle \Delta Z_\ell^\mp |\Delta Z_i^\pm|^2 \rangle = -\frac{4}{3}\epsilon^\pm \ell$$

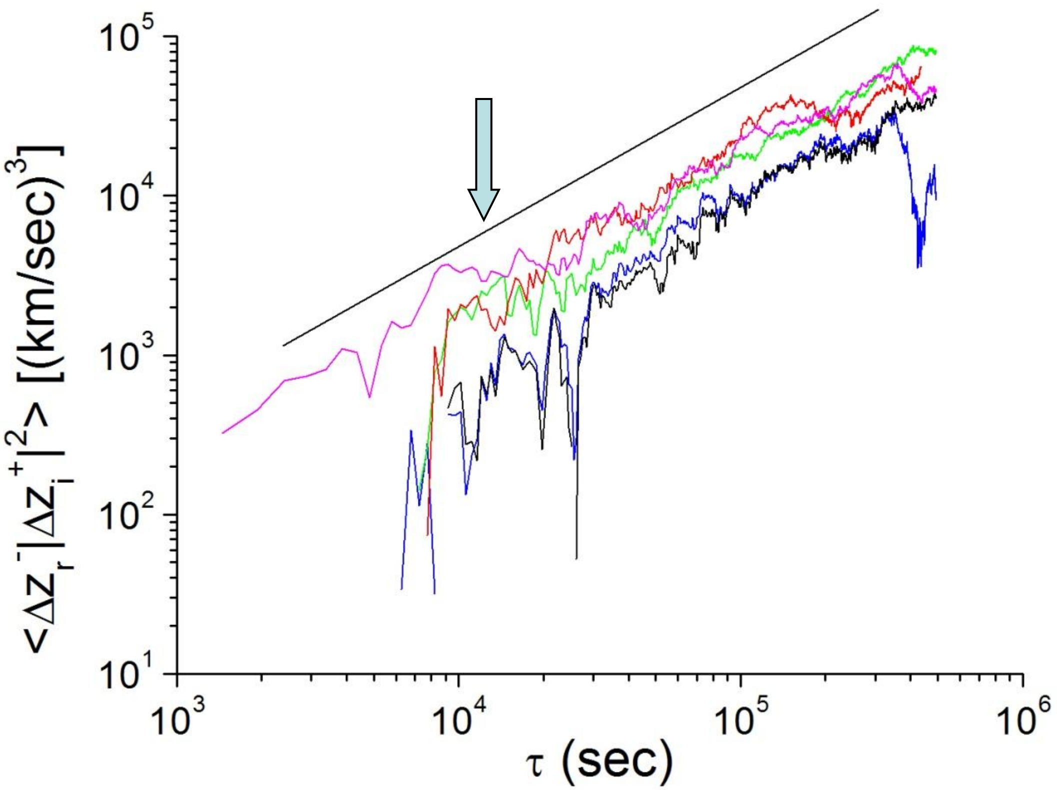
a) The well defined (negative) sign IS CRUCIAL → energy cascade implies fluctuations with asymmetric PDFs (irreversibility);

b) The third-order moment is different from zero → turbulence MUST have some nongaussian features, at least within the inertial range.

c) The third-order moment of fluctuations is related to the energy dissipation rate, thus it can be used to estimate this quantity;

The Yaglom relation is satisfied by most datasets of Ulysses spacecraft (polar wind → high correlations!)

$$\langle \Delta Z_\tau^\mp |\Delta Z_i^\pm|^2 \rangle = \frac{4}{3} U_{rms} \epsilon^\pm \tau$$



L. Sorriso-Valvo et al., PRL (2007)

Although the presence of inhomogeneity and local anisotropy, the observed scale collapse onto the Yaglom law appears very robust

The first REAL evidence that (low frequency) solar wind can be described in the framework of MHD turbulence

We can measure the energy dissipation rate in solar wind turbulence

Is the **measured** turbulent energy flux enough for solar wind heating ?

Solar wind model → Adiabatic expansion, temperature should decreases with helioscentric distance

$$T(r) \approx r^{-4/3}$$

Spacecraft measurements → temperature decay is slower than expected from adiabatic expansion

$$T(r) \approx r^{-\xi}$$

$$\xi \in [0.7;1]$$

Estimate of the heating rate needed to heat the solar wind (say to obtain the observed small radial cooling)

$$\epsilon_{heat}(r) = \frac{3}{2}\left(\frac{4}{3} - \xi\right)\frac{V_{SW}(r) k_B T(r)}{r m_p}$$

Carbone et al., PRL 2009
R. Marino et al., ApJ (2008)

The measured turbulent energy flux is enough for solar wind heating !

A comparison of the radial evolution of dissipation energy rate calculated from Yaglom law, with the model of heating (scaling exponent of temperature decay measured independently from Ulysses datasets)

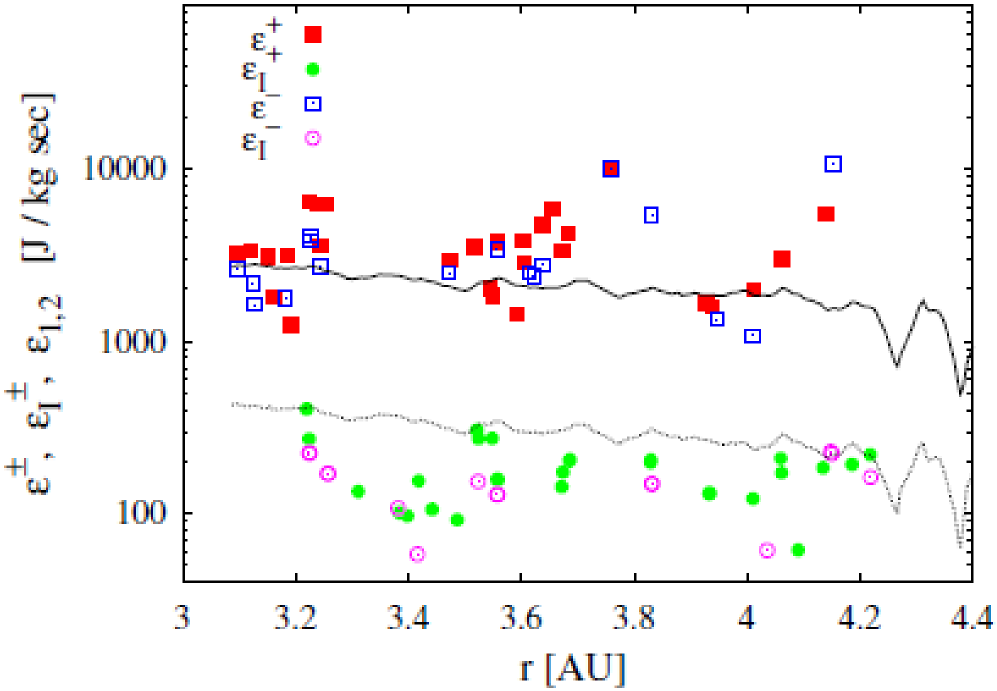


FIG. 3 (color online). Radial profile of the pseudoenergy transfer rates obtained from the turbulent cascade rate through the Yaglom relation, for both the compressive and the incompressive case. The solid lines represent the radial profiles of the heating rate required to obtain the observed temperature profile.

Carbone et al., PRL 2009

"Bursty turbulence" transport in laboratory plasmas: a toy model

High density structures localized both in space and time, observed by probes located at the edge of plasma devices.

Conditional average of many blobs at different radial position → asymmetric dipolar structure

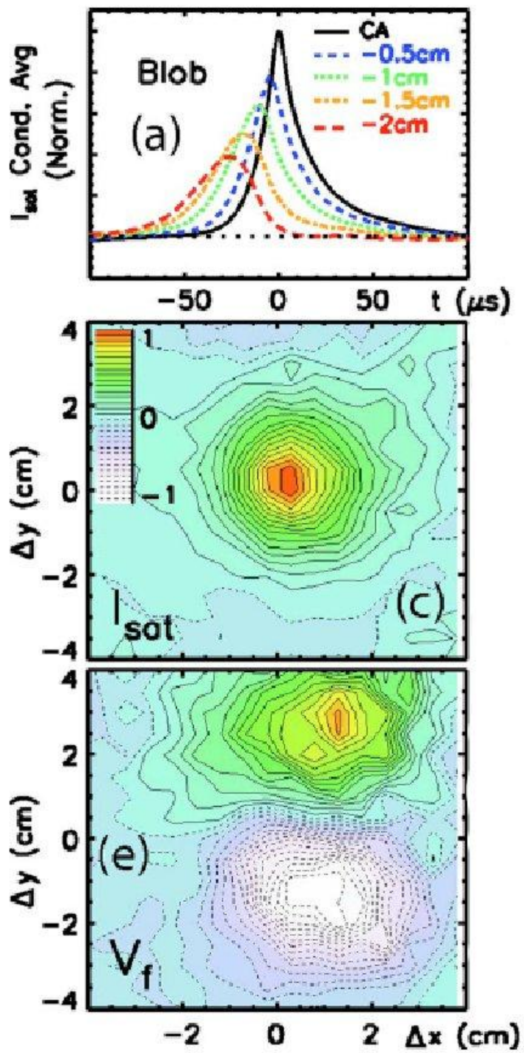
A toy model for the 2D electrostatic E x B convection in the drift approximation can reproduce the main physics

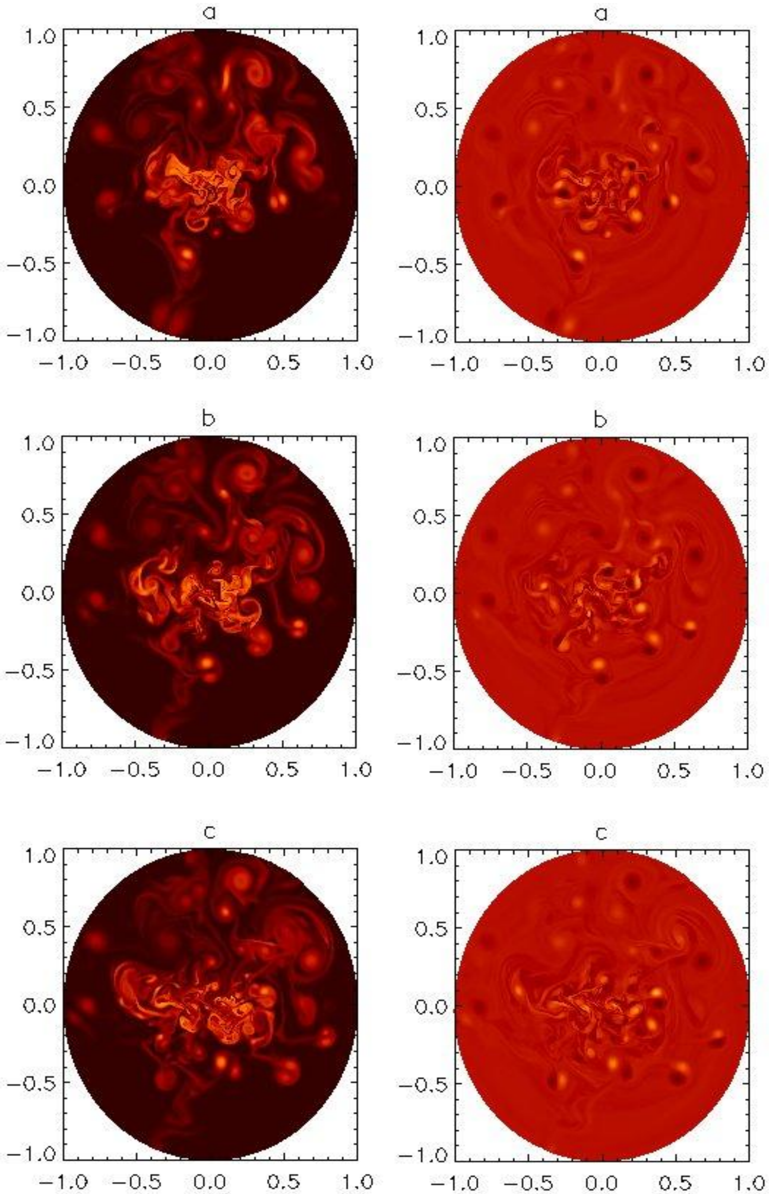
$$\frac{\partial n}{\partial t} + (\mathbf{u}\cdot\nabla)n = f_n, \qquad \nabla^2\psi = \omega$$

$$\frac{\partial \omega}{\partial t} + \hat{\mathbf{z}}\times\nabla\psi\cdot\nabla\omega = f_\omega. \qquad \psi \equiv U/B$$

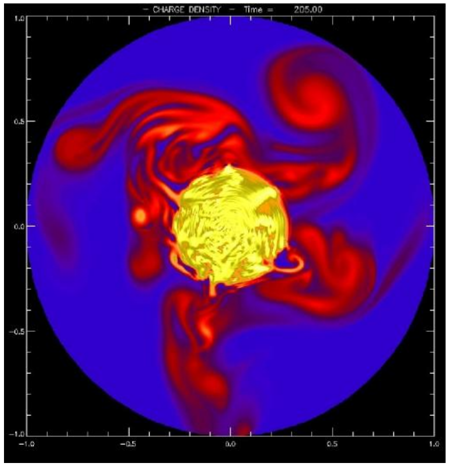
$$\mathbf{u} = \hat{\mathbf{z}}\times\nabla\psi$$

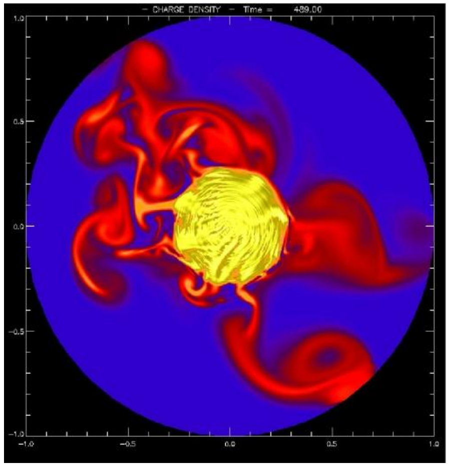
f_n represents plasma sources and losses





density vorticity



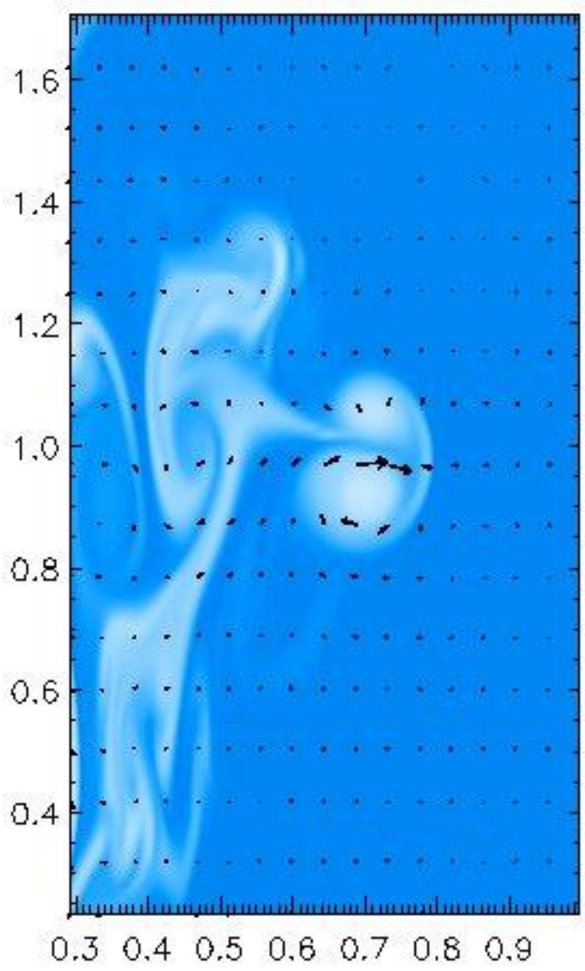


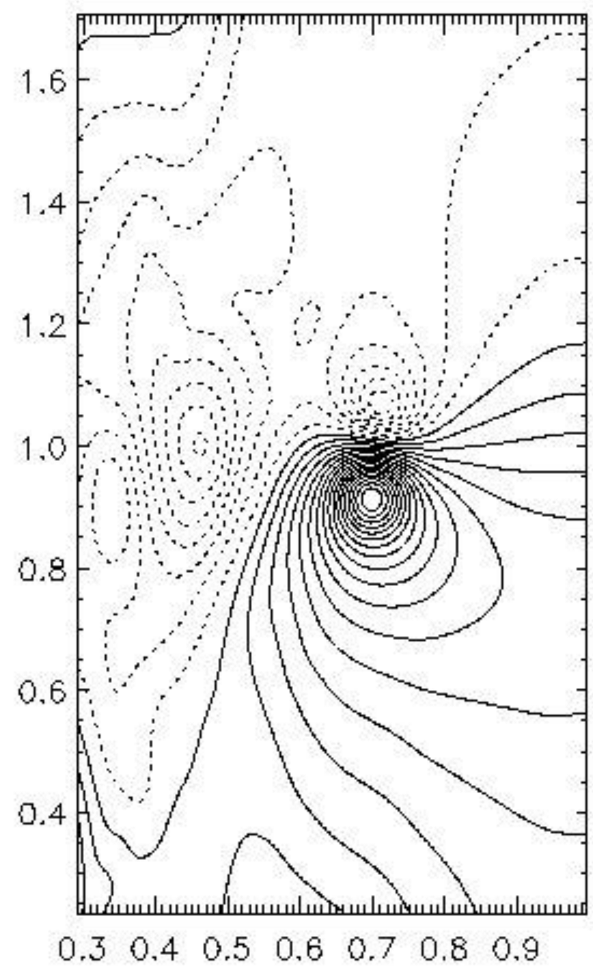
A forcing term at the center of the simulation domain.

Perfectly absorbing boundary conditions.

Blobs on average are convected away towards the boundary by the E x B drift.

Dipolar structure of bursts are easily reproduced by the toy model

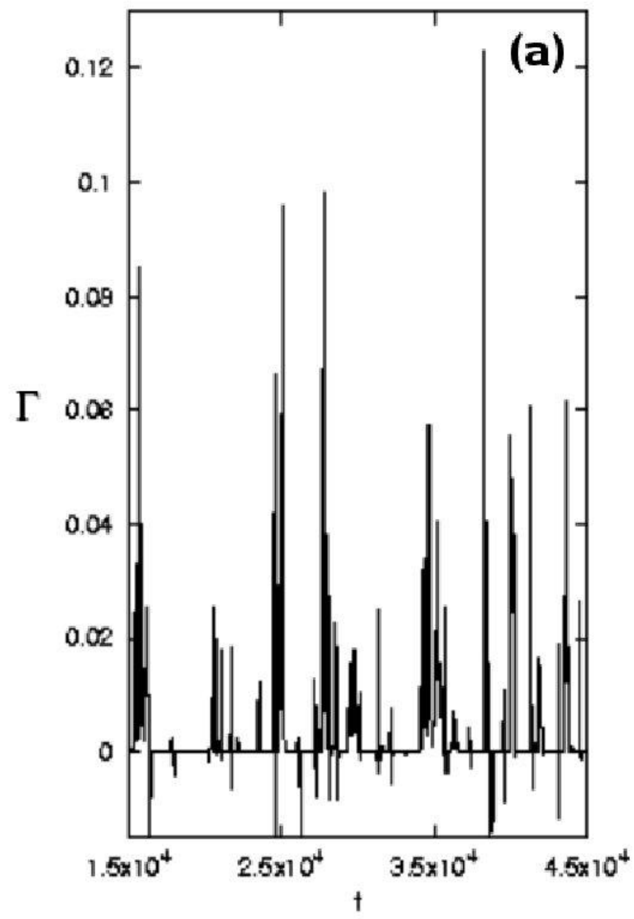




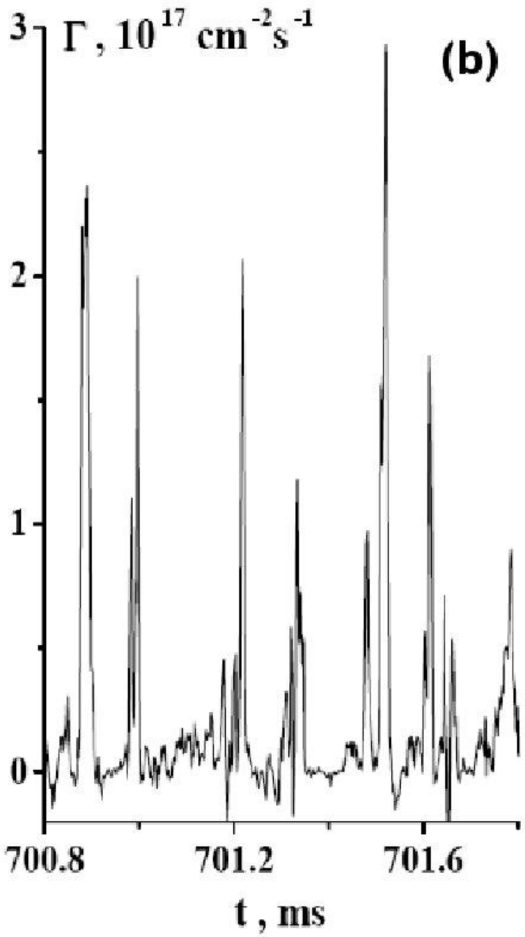
Flux at the edge

$$\Gamma(r,\theta,t) = nu_r = -\frac{n}{r}\frac{\partial\phi}{\partial\theta}.$$

Toy model



T-10 tokamak

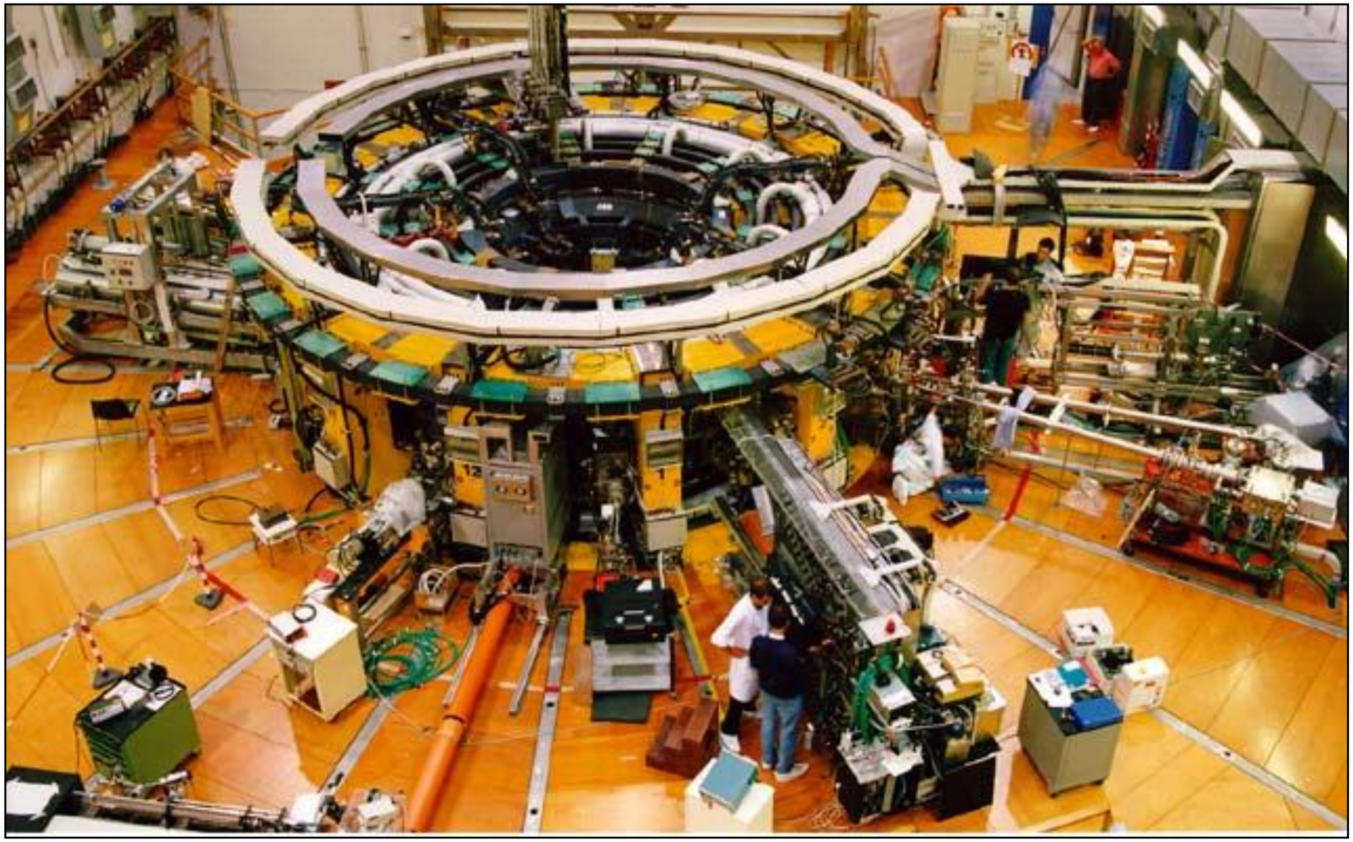


Interesting: A Yaglom's law for E x B bursty turbulence

$$\langle \Delta u_r (\Delta n)^2 \rangle = -\frac{1}{\ell}\int_0^{\ell} dy \left\langle u_r \left[\frac{\partial}{\partial r} + \frac{\partial}{\partial r'}\right](\Delta n)^2 \right\rangle y + \frac{2}{\ell}\int_0^{\ell} dy \langle \Delta n \Delta f \rangle y .$$

The occurrence of this law (say the occurrence of a nonlinear energy cascade) can be verified through laboratory measurements of velocity and density fluctuations at two points inside the device

Results from Reversed Field Pinch



Plasma generated for nuclear fusion, confined in a Reversed Field Pinch configuration (RFX, Padova - Italy).

High amplitude fluctuations of magnetic field and floating potential measured at the edge of the device.

Reversed field pinch (RFX) measurements

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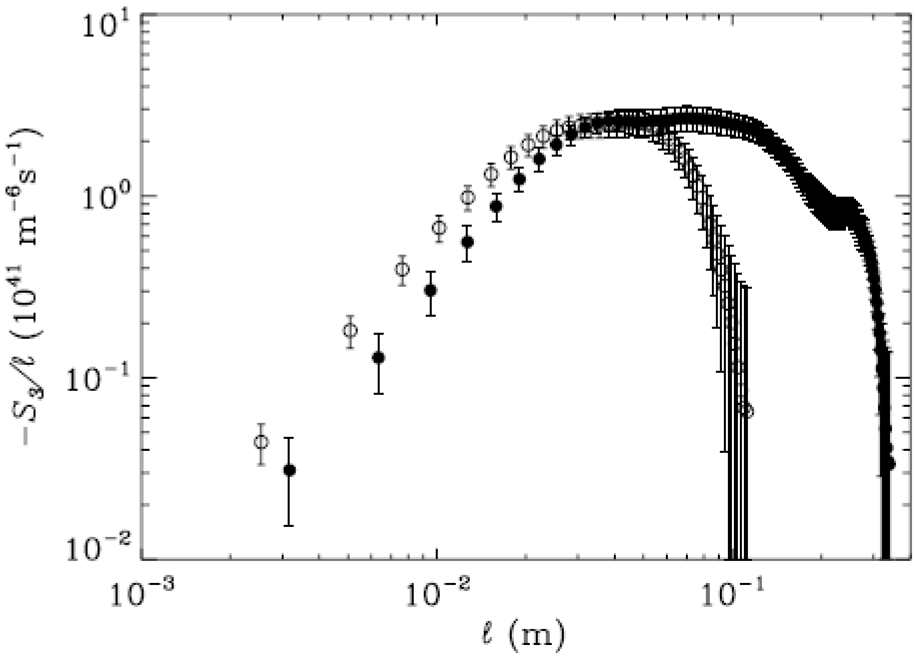
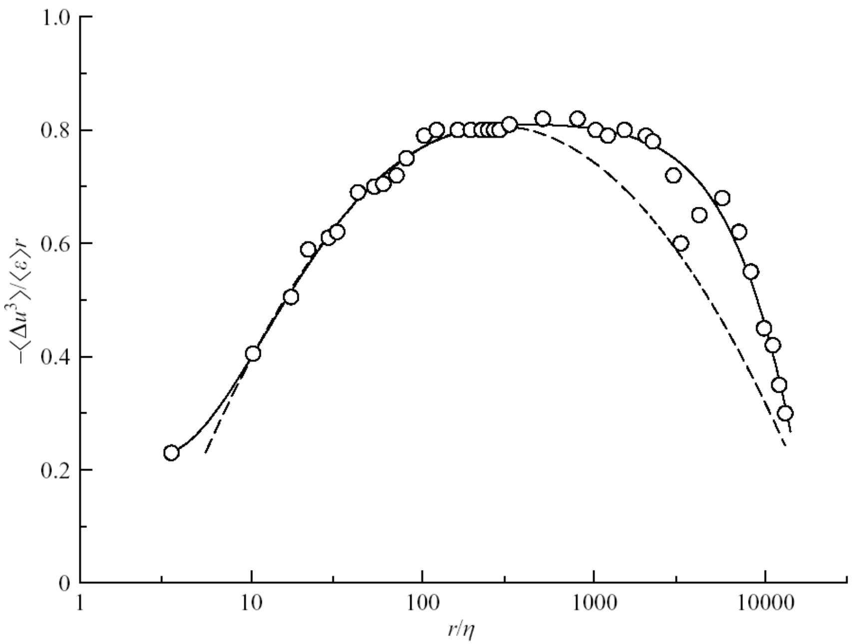
Yaglom law for electrostatic turbulence in laboratory magnetized plasmas

F. Lepreti[1(a)], V. Carbone[1,2], M. Spolaore[3], V. Antoni[3], R. Cavazzana[3], E. Martines[3], G. Serianni[3], P. Veltri[1], N. Vianello[3] and M. Zuin[3]

$$3\,\mathrm{cm} \lesssim \ell \lesssim 10\,\mathrm{cm}.$$

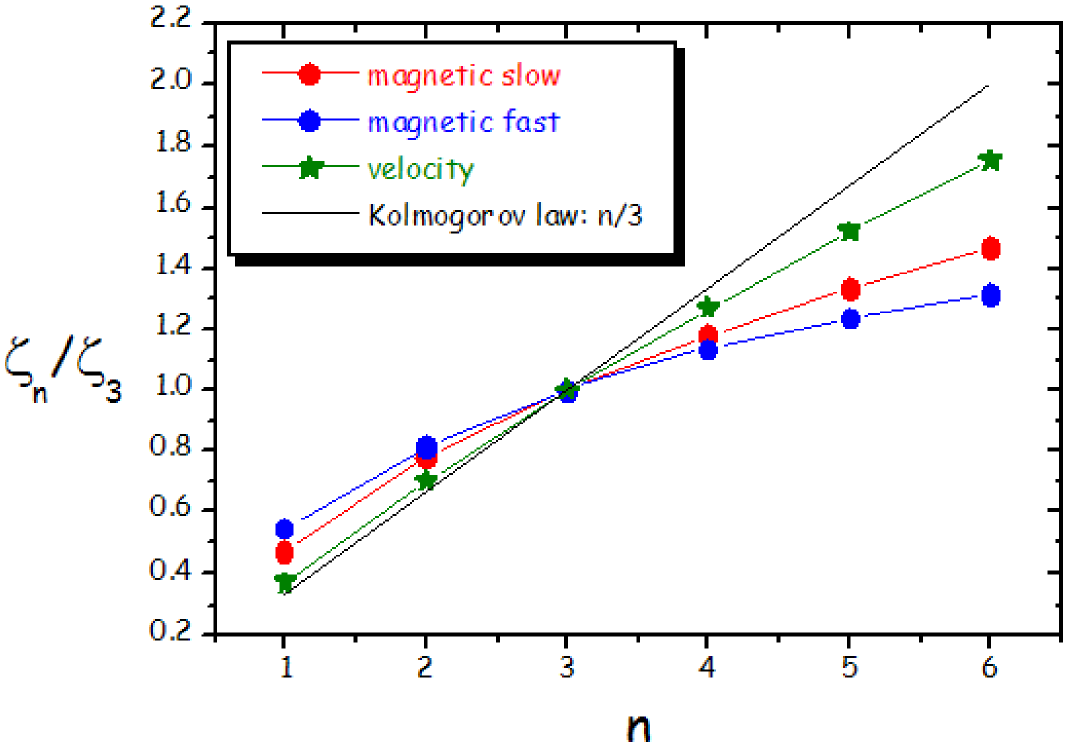
Sreenivasan & Dhruva (1998), atmospheric turbulence





Despite both the Yaglom-law and the 5/3-spectrum are observed, measurements show a strong departure from the Kolmogorov's conjecture for higher-order moments

$$S_n(\tau) = \langle [u(t+\tau) - u(t)]^n \rangle \sim \tau^{\zeta_n}$$

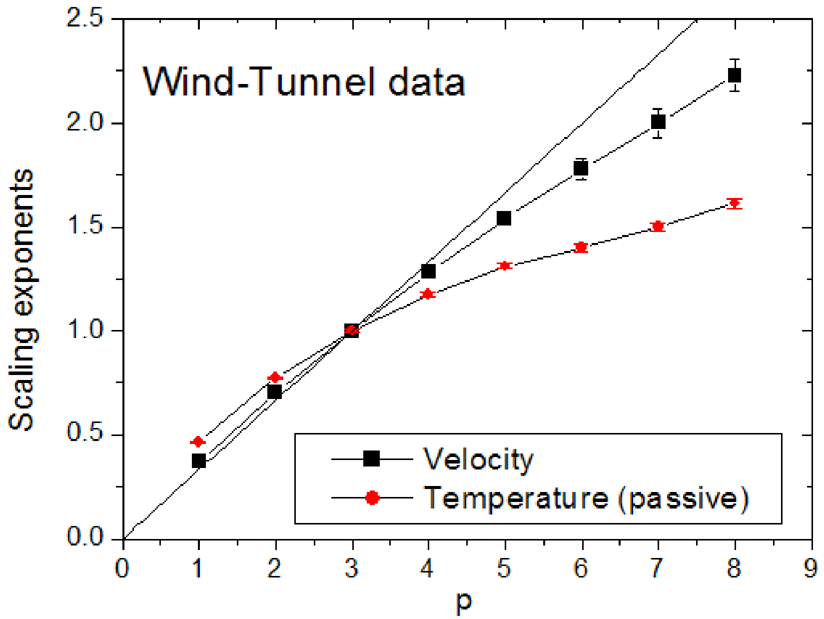


1) u along the sun-earth (longitudinal) direction;
2) Taylor hypothesis to transform length scales in time scales

The departure has been attributed to **INTERMITTENCY** in fully developed turbulence

Solar wind: Intermittency (measured as the distance of the scaling exponents from n/3) is stronger for magnetic field than for velocity field. Scaling laws for velocity field in the solar wind coincide with that observed in fluid flows

Wind tunnel experiments



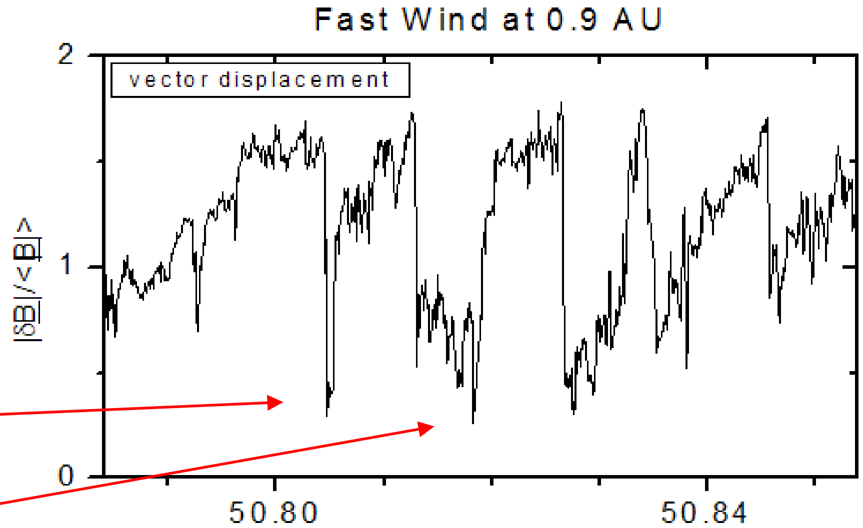
Fluid flows: Intermittency is stronger for passive scalar

Structure functions for the passive scalar

$$\langle \Delta T_\tau^p \rangle = \langle [T(t+\tau) - T(t)]^p \rangle$$

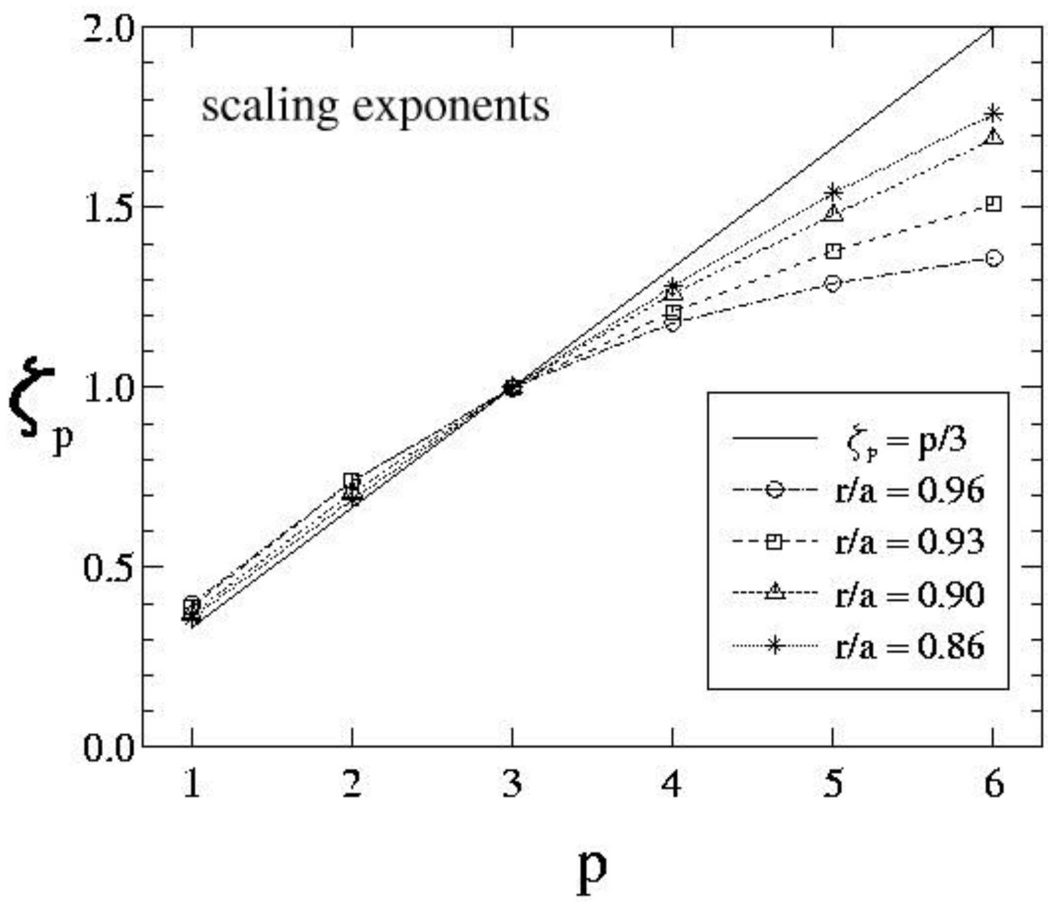
A comparison with solar wind: Same scaling laws for velocity, similar scaling laws for passive scalar and magnetic field

THIS DOES NOT IMPLY THAT THE MAGNETIC FIELD IS A "PASSIVE VECTOR": statistics cannot prove just disprove



Strong jumps of magnetic orientation are responsible for the strong intermittency

Magnetic turbulence in Reversed Field Pinch (RFX, Padua- Italy)



r/a → normalized distance

The departure from the linear scale increases going towards the wall

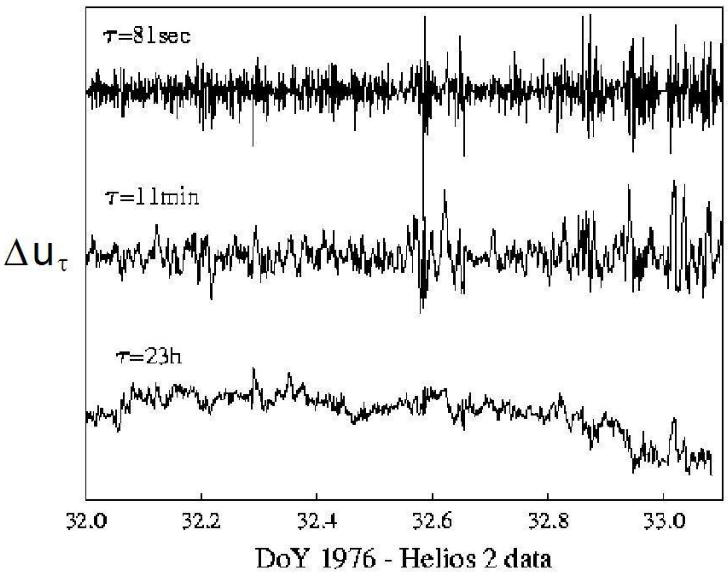
↓

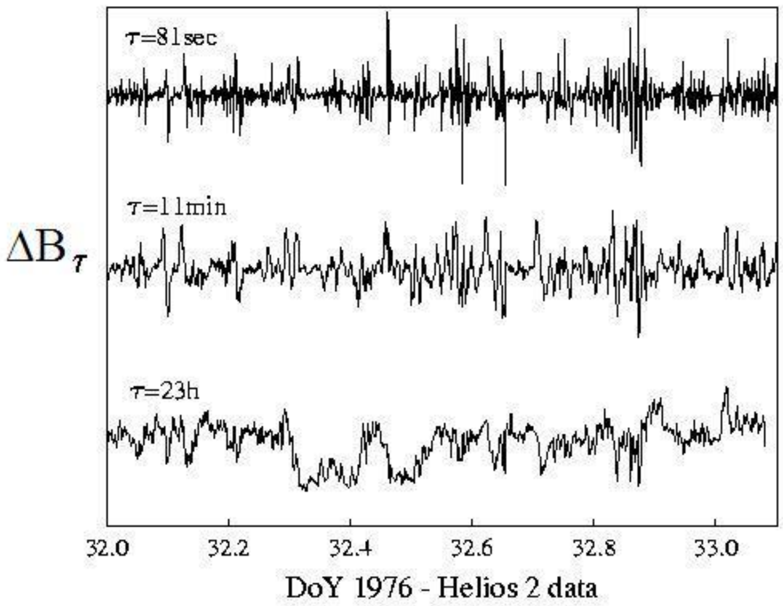
Turbulence more intermittent near the external wall

Similar to edge turbulence in laboratory fluid flows

What is “intermittent” in turbulence

Velocity and magnetic differences at three different separation scales



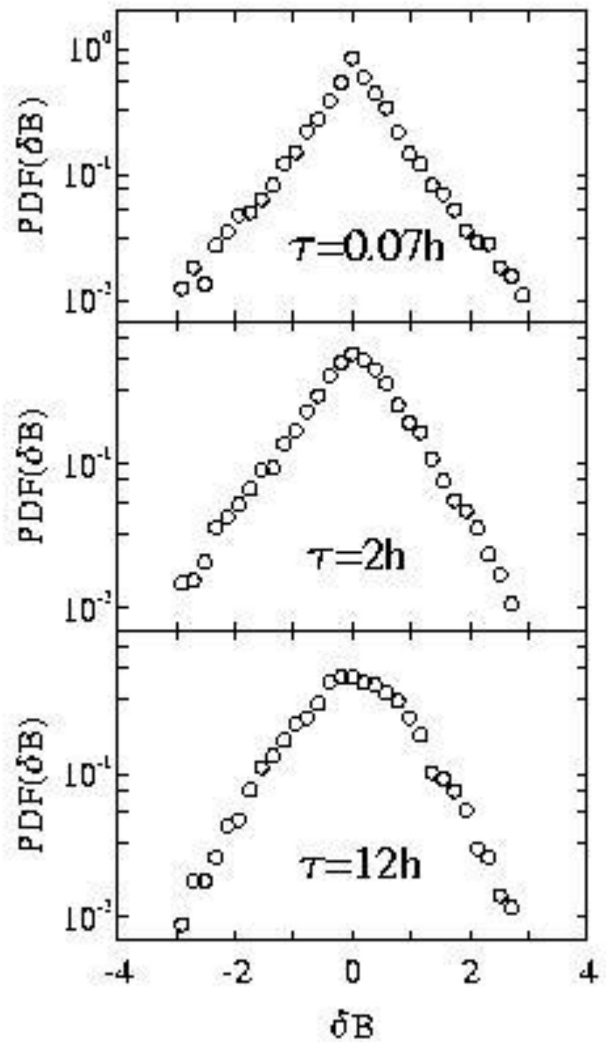


1) A random signal at large separations;
2) Bursts of activity at smaller separations

Intermittency implies a departure from global self-similarity

Stretched exponential PDF at small scales

Gaussian PDF at large scales



PDFs of normalized variables changes with scale

$$\delta w_r = \frac{\delta u_r}{\left\langle \left(\delta u_r\right)^2 \right\rangle^{1/2}}$$

Probability of occurrence of strongest events are higher than a Gaussian
→ Random events, with phases highly correlated, are present, they are an unavoidable characteristic of real turbulence.

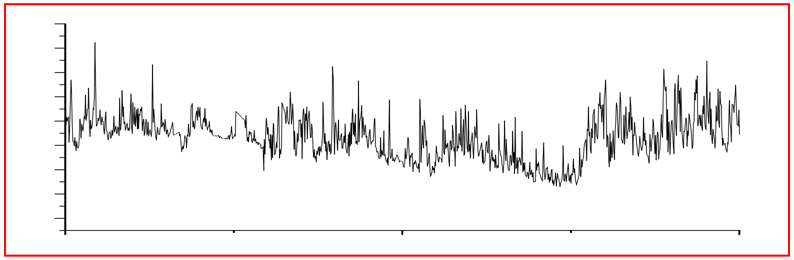
⇩

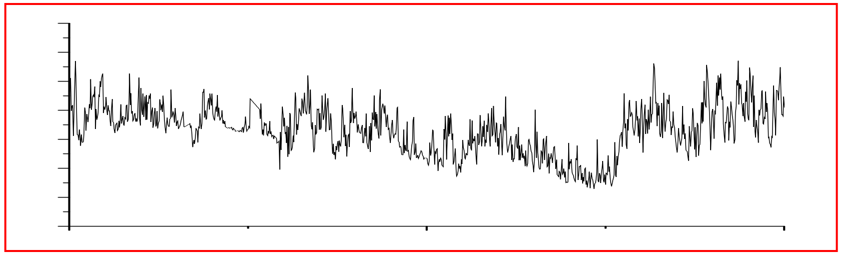
Turbulence CANNOT be described by a random phase random process

Turbulence: intermittent “structures” and background fluctuations

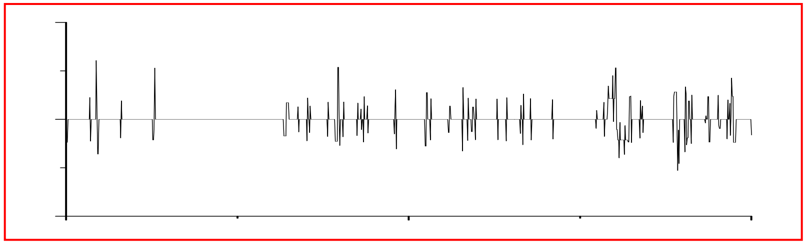
Intermittent “coherent” events within turbulence, on all scales, can be isolated (for example) through wavelets.

Complete signal





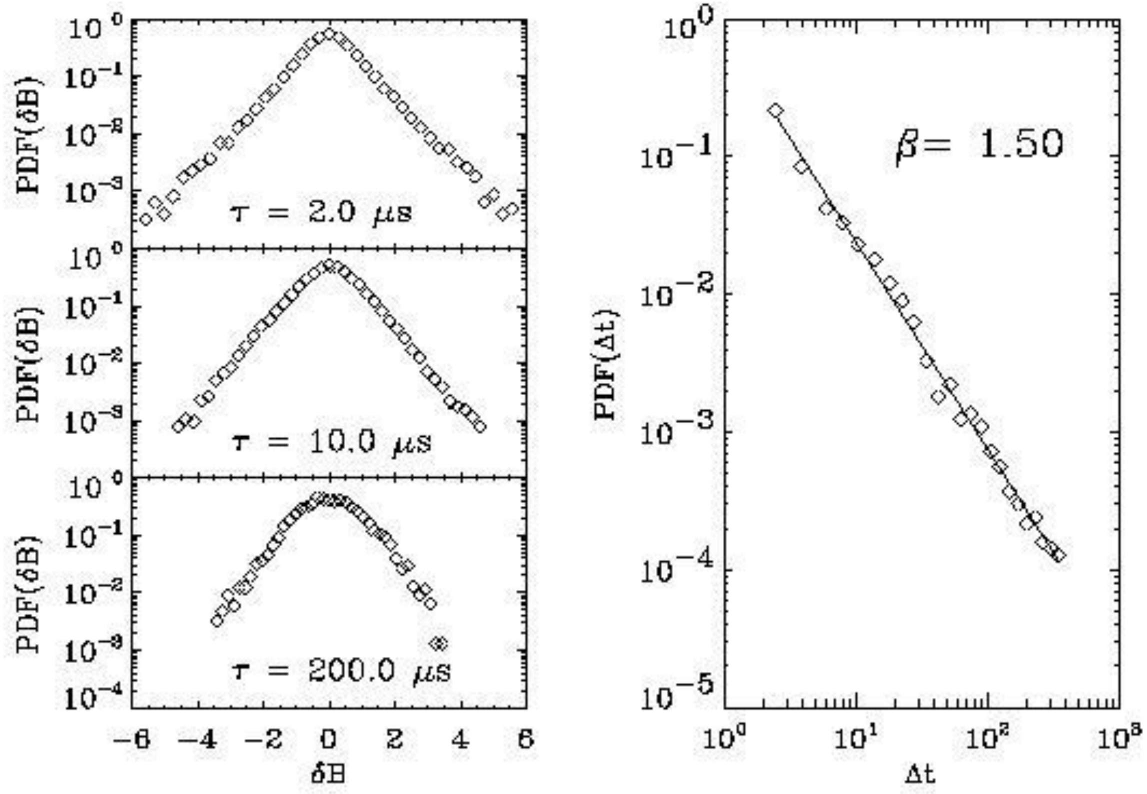
Gaussian background



Isolated structures

Waiting times between structures

Laboratory plasma (RFX)



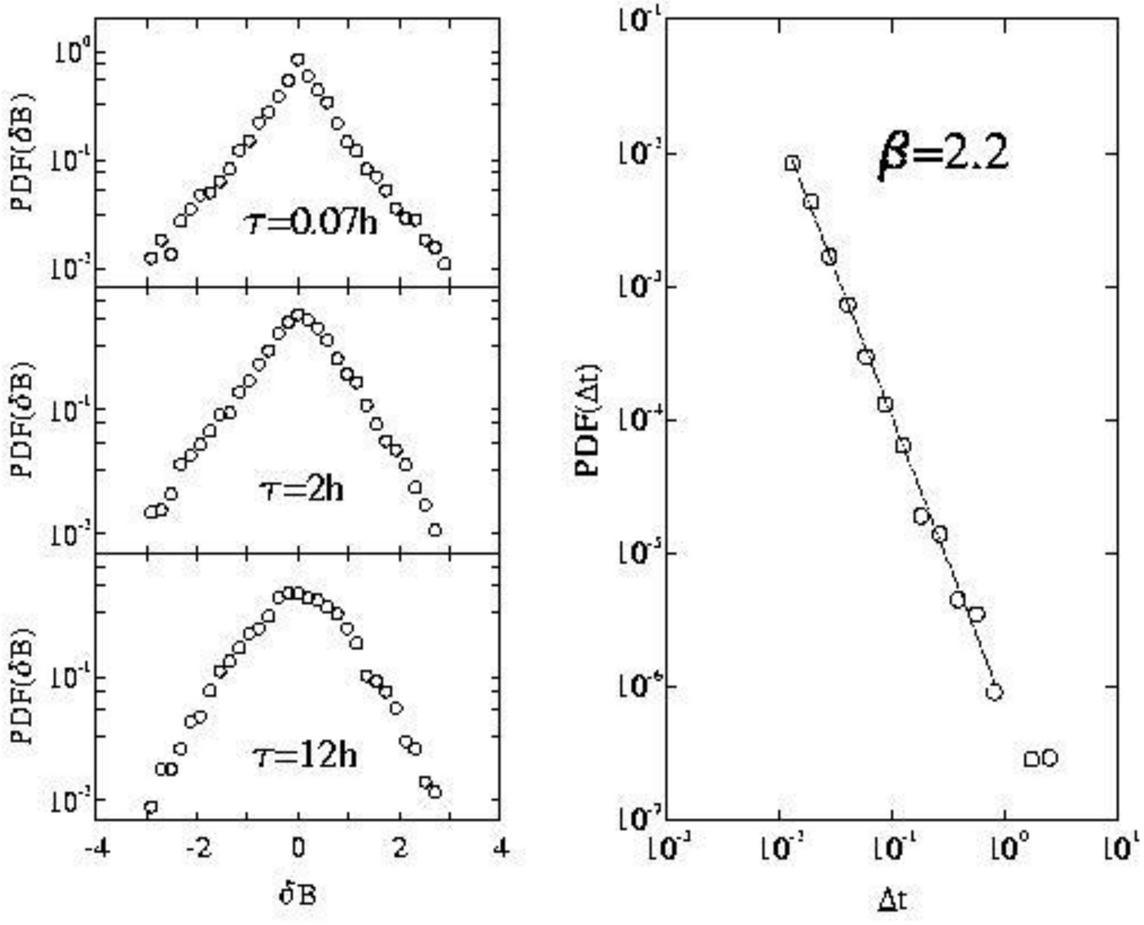
The times between events are distributed according to a power law

$Pdf(\Delta t) \sim \Delta t^{-\beta}$

The turbulent energy cascade generates intermittent "coherent" events.

Interesting! the underlying cascade process is NON POISSONIAN, that is the intermittent (more energetic) bursts are NOT INDEPENDENT (memory) → Self-Organized-Criticality CANNOT properly describe turbulence.

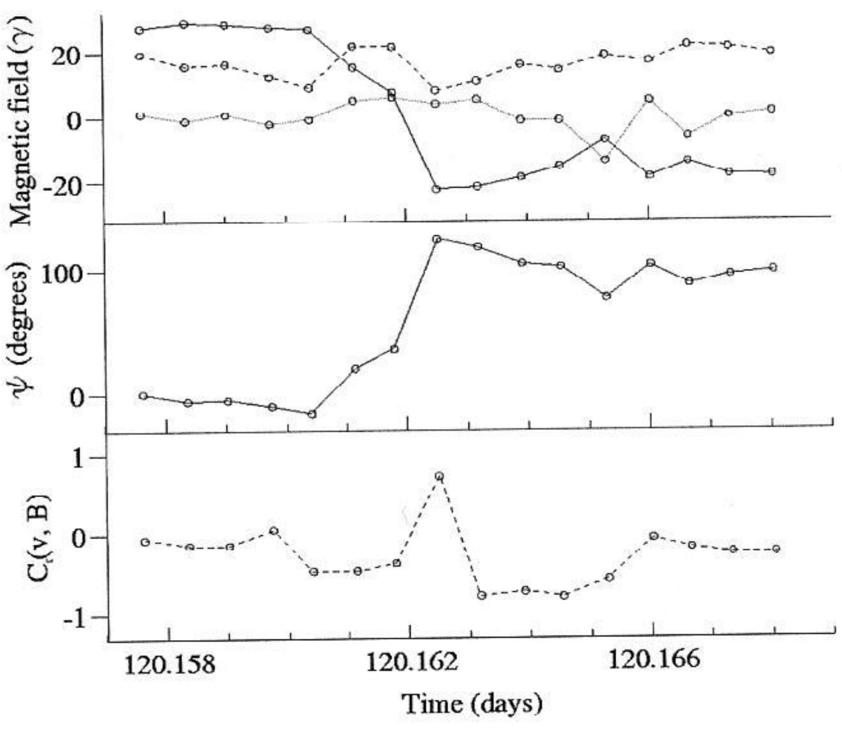
Solar Wind data share the same characteristics

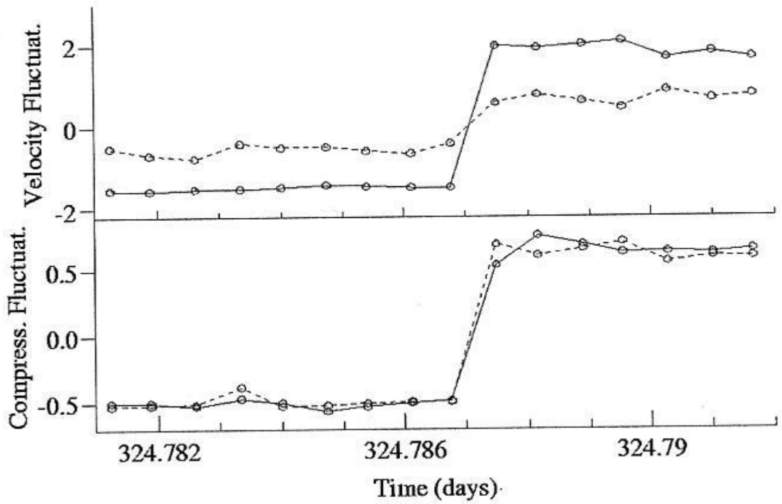


What kind of intermittent structures in solar wind

(identified through minimum variance)

Compressive structures



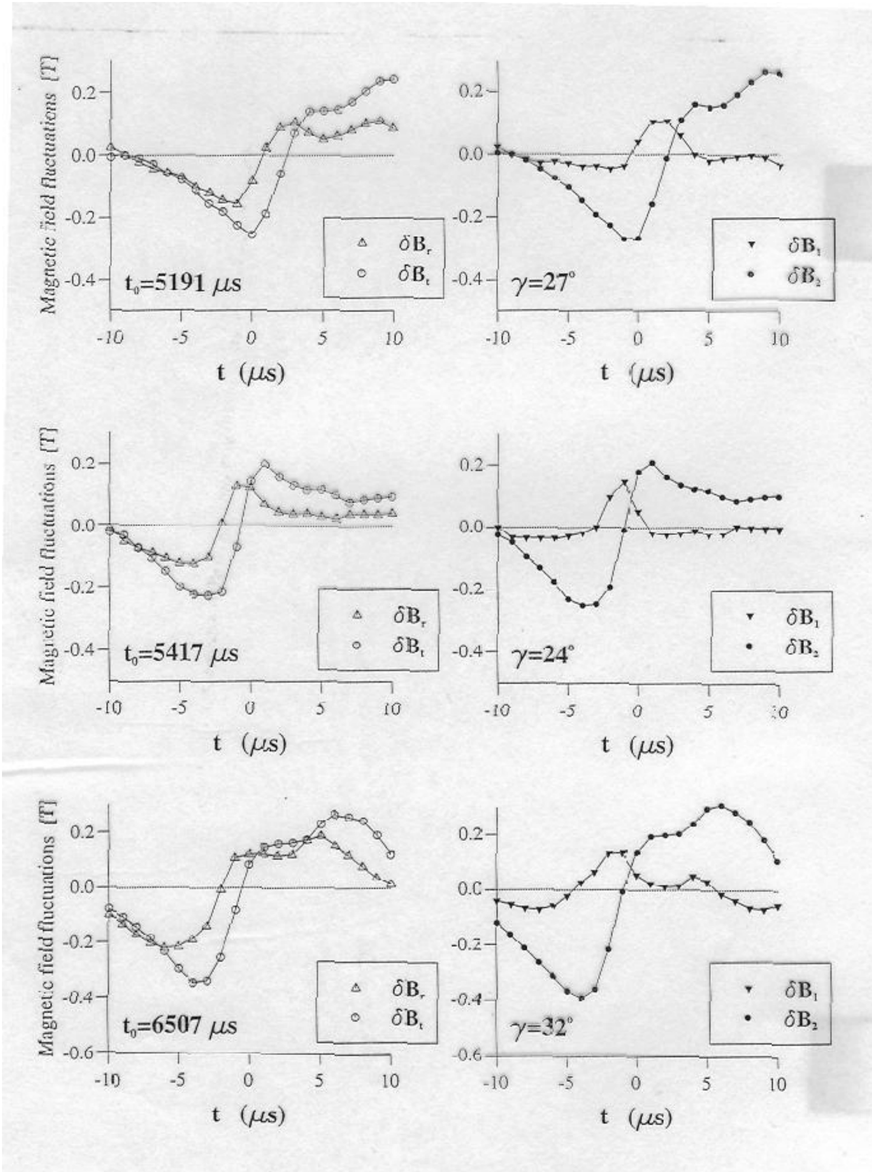


Tangential discontinuity (current sheet)

Magnetic structures in laboratory plasmas

RFX edge magnetic turbulence:
current sheets

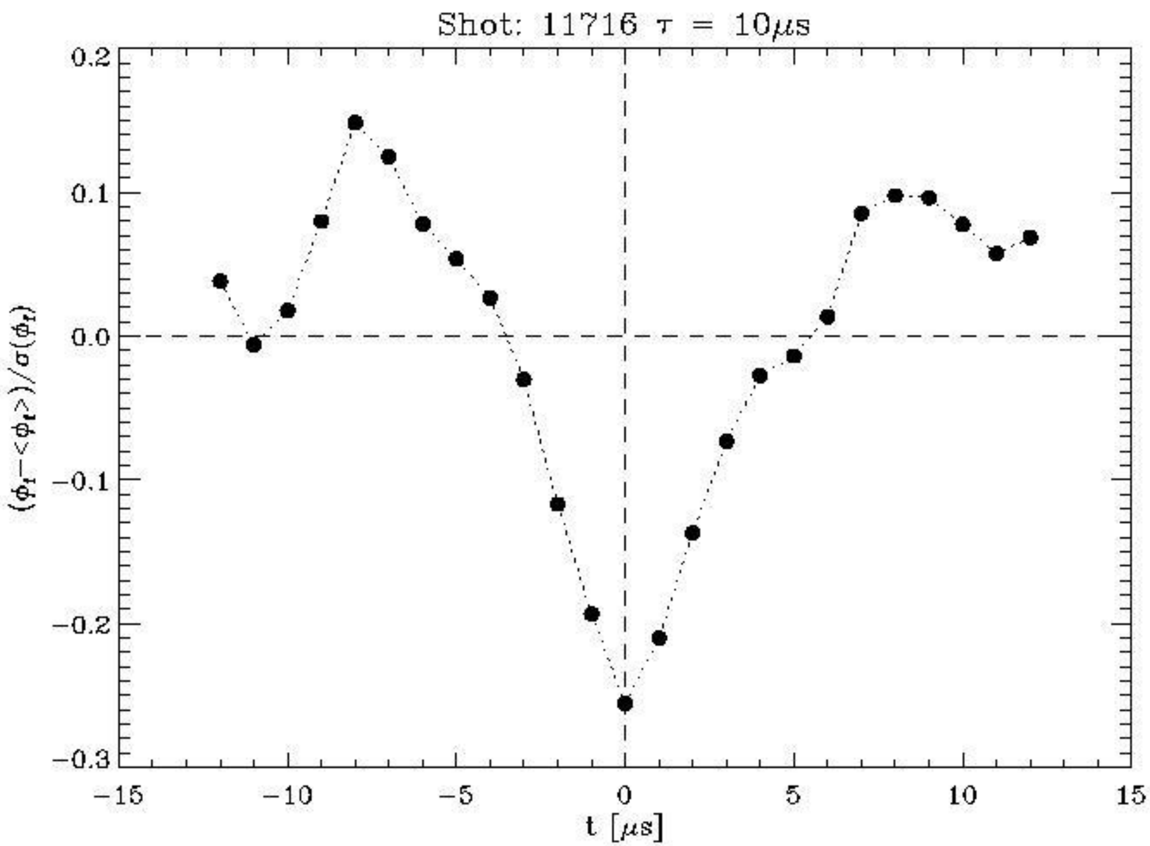
Current sheets are naturally produced as coherent, intermittent structures by the nonlinear turbulent dynamics ON ALL SCALES in a laboratory plasma.
→ Reconnection



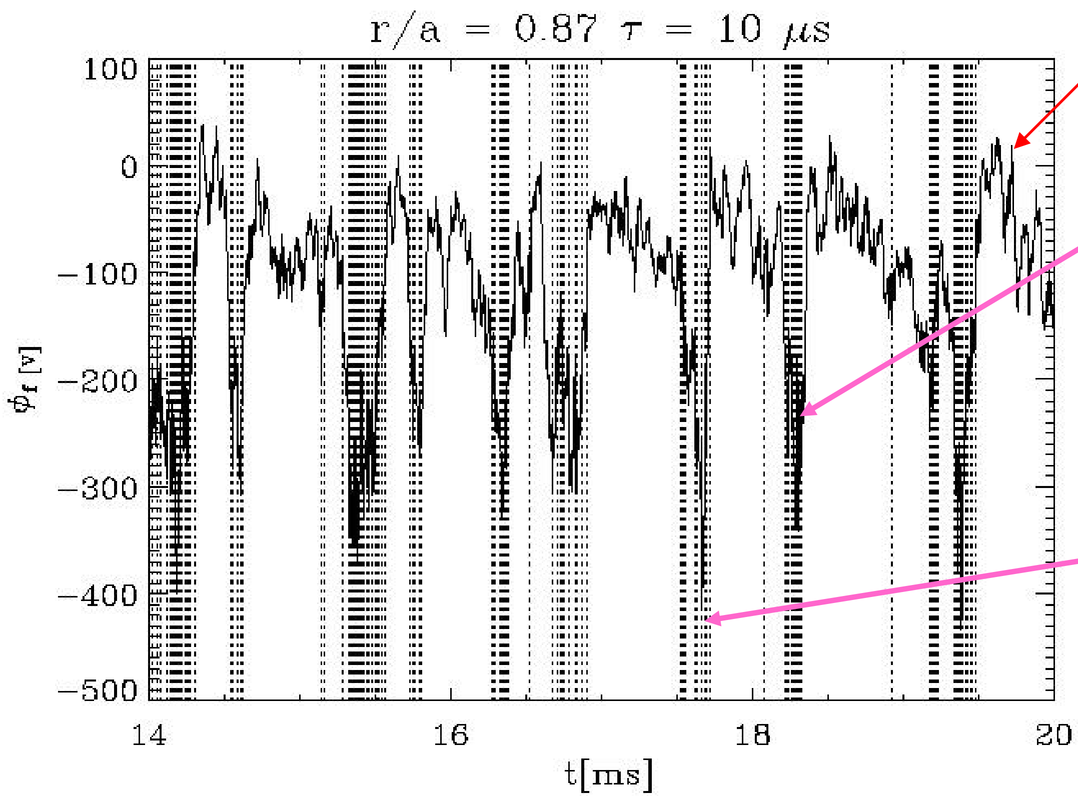
Intermittent structures in laboratory plasmas: floating potential

Structures are identified as potential holes

Should these structures be related to disruptions?



Relationship between intermittent structures of edge turbulence and disruptions of the plasma columns at the center of RFX



Time evolution of floating potential at edge

Minima are related to disruptions of the magnetic structure (at the center)

Appearance of intermittent structures in the electrostatic turbulence at the edge of the plasma column (vertical lines)

Interesting for control: Apparently coherent structures seems to be responsible for disruptions (rather than the Gaussian background)

Anomalous transport due to non-Gaussian features

$$\frac{dx_i}{dt} = u_i$$

$$\left\langle \left| x_i(t) - x_i(0) \right|^2 \right\rangle = 2t \int_0^t dt' \left\langle u_i(x(t')) u_i(x(0)) \right\rangle$$

Typical problem:
Lagrangian evolution of test-particles (or magnetic field lines) in a “complex” medium

Anomalous diffusion is far from a trivial problem!
Diffusion is anomalous (non-Gaussian) when the central limit theorem is broken. This leads to very restrictive conditions:

$$1)\ \left\langle u^2 \right\rangle = \infty \quad \Rightarrow \quad \text{Lèvy flights (physically unrealistic infinite variance)}$$

$$2)\ \left\langle u(x(\tau)) u(x(0)) \right\rangle \approx \tau^{-\beta} \quad \text{with } \beta < 1 \text{ (very strong Lagrangian correlations)}$$

$$\left\langle \left| x(t) - x(0) \right|^2 \right\rangle \approx t^{2\nu} \qquad \text{(in the limit } t \to \infty)$$

$$1)\ \nu < 1/2 \quad \Rightarrow \quad \text{subdiffusion (trapping in compressible fields)}$$

$$2)\ \nu > 1/2 \quad \Rightarrow \quad \text{superdiffusion (particles make "long" jumps)}$$

This kind of anomalous transport evidenced ALSO in very simple “laminar” flows due to coherent structures!

Magnetic field lines diffusion with turbulence modeling

Low level of fluctuations favours the birth of "large-scale structures"

Diffusion is ENHANCED when turbulent fluctuations are WEAK

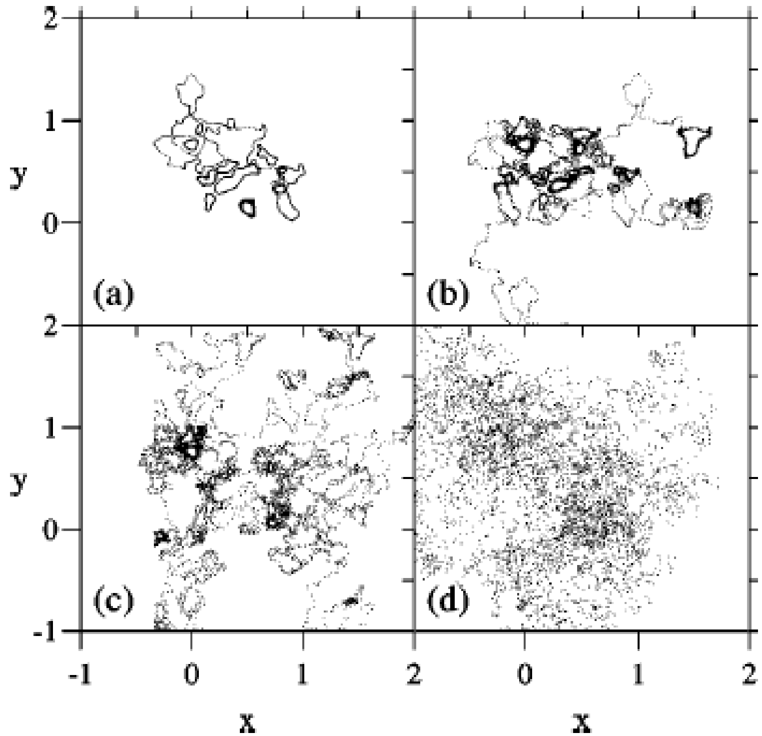


FIG. 4. Poincaré surfaces of section for Run 5 for different fluctuation levels. (a) $\delta B/B_0 = 0.075$; (b) $\delta B/B_0 = 0.15$; (c) $\delta B/B_0 = 0.2$; (d) $\delta B/B_0 = 0.4$.

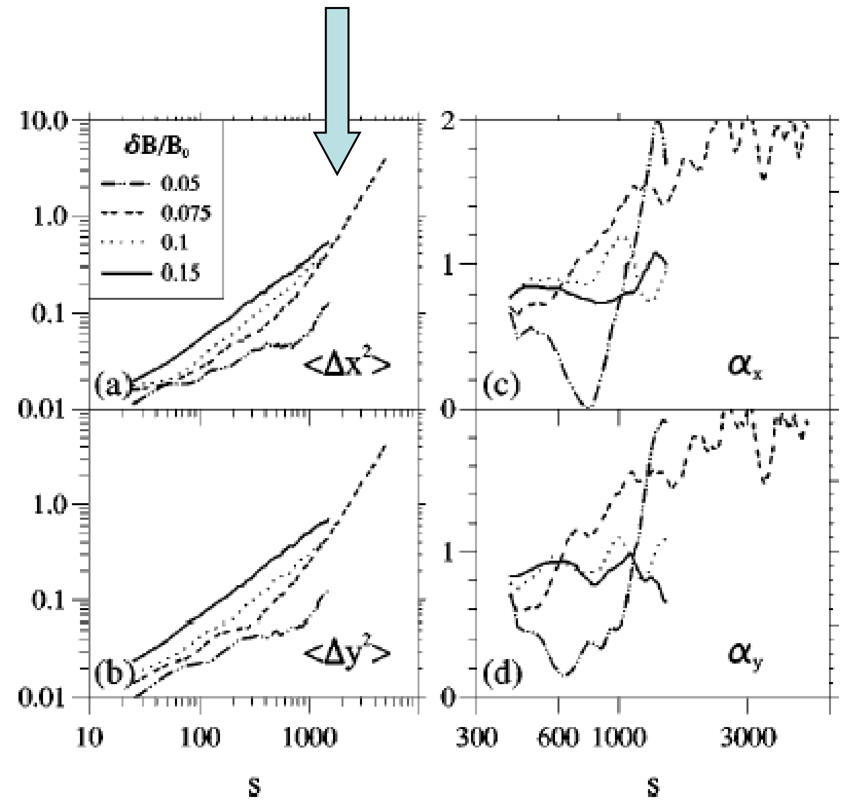
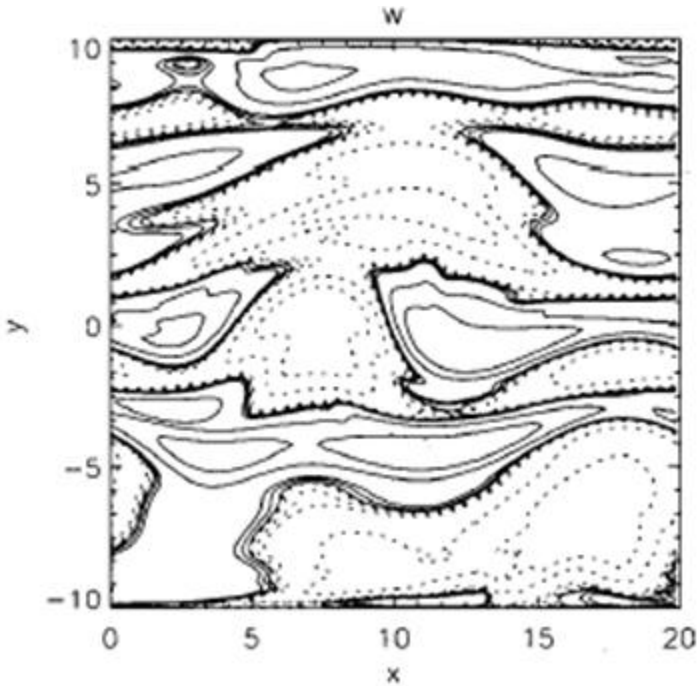


FIG. 11. Values of the mean square displacements $\langle \Delta x_i^2 \rangle$ and of the coefficients α_i ($i = x,y$) for Run 5 vs the field line length s, for small values of the fluctuation levels.

E x B transport in numerical simulations of Hasegawa-Mima equations and Electron-Temperature-Gradient model



Contour plot of the vorticity w

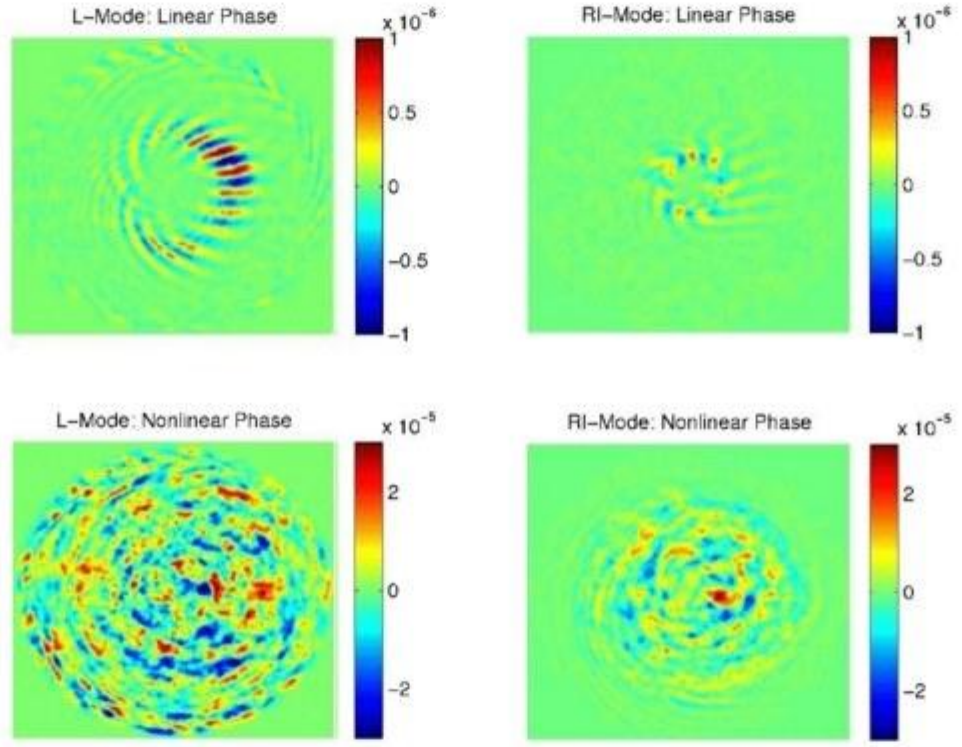
FIG. 2. Characteristic ϕ contours in the outboard x-y plane. This snapshot was taken at the end of the ETG run shown in Fig. 1. The figure is $256\rho_e \times 64\rho_e$.

Turbulence generates coherent structures (streamers, zonal flows,..) which enhance transport properties leading to anomalous diffusion

Barriers for transport

Shear flows are able to decorrelate turbulent eddies.

Believed mechanism: stretching and distortion of eddies lead to a decreasing of coherence → turbulent fluctuations are reduced



A simple equation in mind:

absence of turbulent fluctuations → reduction of anomalous transport

“Confining” turbulence ?

$$\nabla\psi(\mathbf{r},t) = (\psi_0/L)\sum_n a_n \mathbf{e}_\perp^{(a_n)} \cos(\mathbf{k}_n^{(a)}\cdot\mathbf{r} - \omega_n t) + b_n \mathbf{e}_\perp^{(b_n)} \sin(\mathbf{k}_n^{(b)}\cdot\mathbf{r} - \omega_n t) \quad (2)$$

$$\frac{d\vec{r}}{dt} = \vec{B}_0 \times \frac{\nabla\psi}{B_0^2}$$

Test-particle simulations using a simple model for electrostatic turbulence with coherent structures at all dynamical scales:

1) Amplitudes a_n and b_n are related to the imposed energy spectra.
2) Wave vectors have random directions and amplitudes $k_n = 2^n k_0$
3) Time evolution is related to the eddy-turnover time of turbulence.

$$a_n^2 = b_n^2 = \frac{k_{n+1} - k_{n-1}}{2} E(k_n)$$

$$\omega_n = \frac{1}{2}\sqrt{k_n^3 E(k_n)}$$

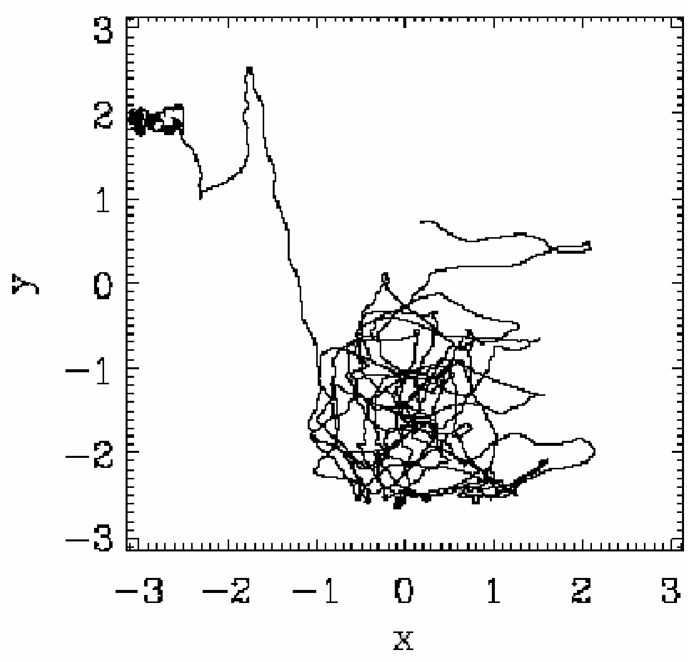
Should phase-correlations be the main ingredient for enhanced diffusion?

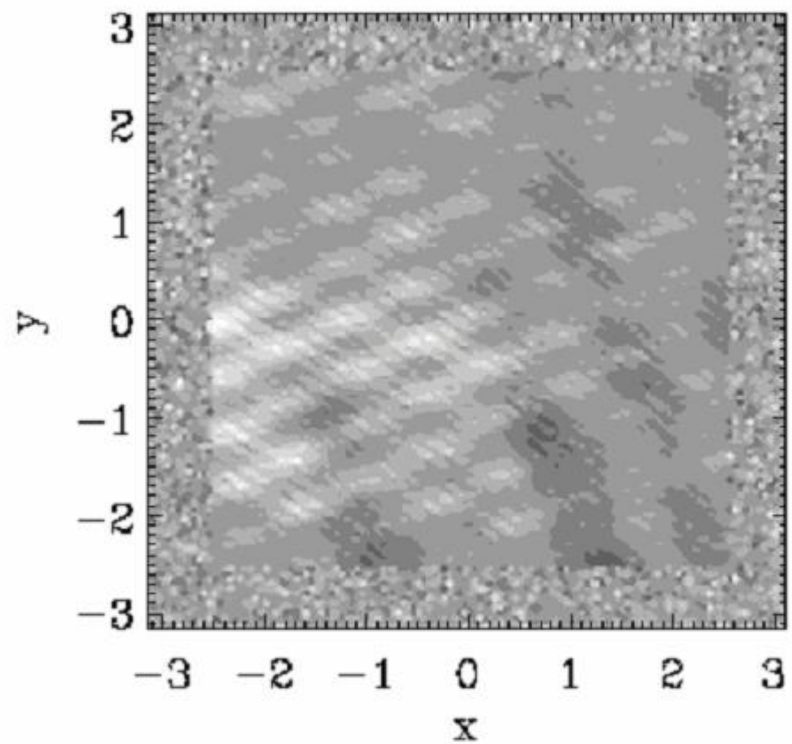
Particle flux

$$\Gamma = \langle \delta u_{drift} \delta \rho \rangle = \langle (\delta u_{drift})^2 \rangle^{1/2} \langle \delta \rho^2 \rangle^{1/2} \cos \Delta_{u,\rho}$$

Cross-correlation term

A barrier has been generated by externally randomizing the phases of the field ONLY within a narrow strip at the border of the integration domain → turbulence still exist but coherent structures disappear.





Q(x,y) = strain^2 – vorticity^2

Diffusive properties

$$\left\langle \left| x(t) - x(0) \right|^2 \right\rangle = D_e t^{2\nu} \quad \text{(in the limit } t \to \infty\text{)}$$

1) $\nu < 1/2 \quad \Rightarrow \quad$ subdiffusion

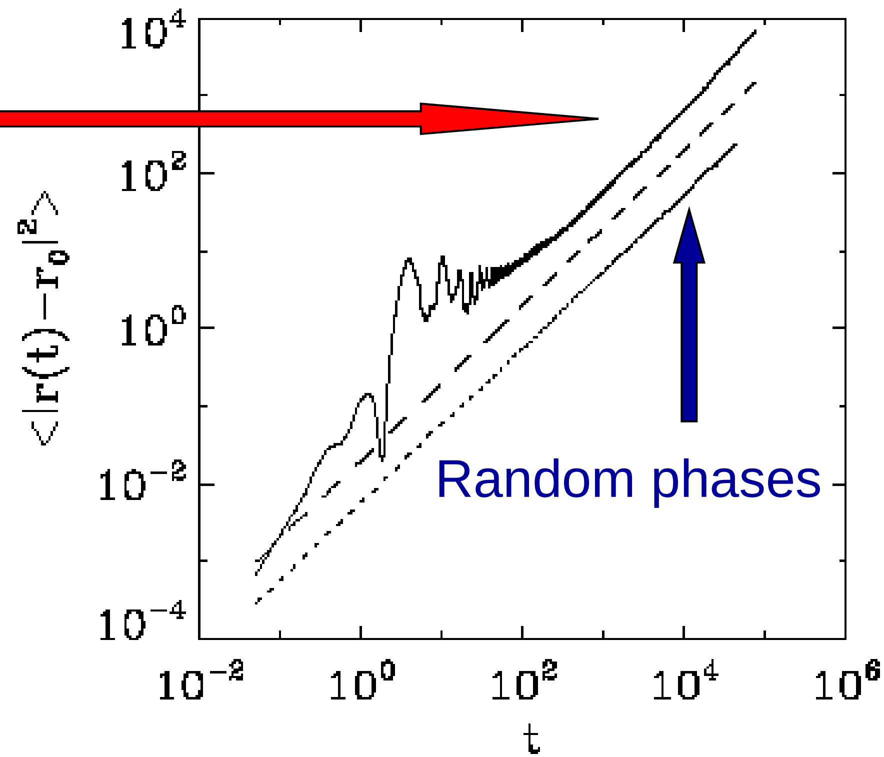
2) $\nu > 1/2 \quad \Rightarrow \quad$ superdiffusion (particles can make "long" jumps)

Correlated phases
(weak superdiffusion)

D_e ~ 1 □ 0.1

□ ~ 0.68

Weak Superdiffusion with a high diffusion coefficient

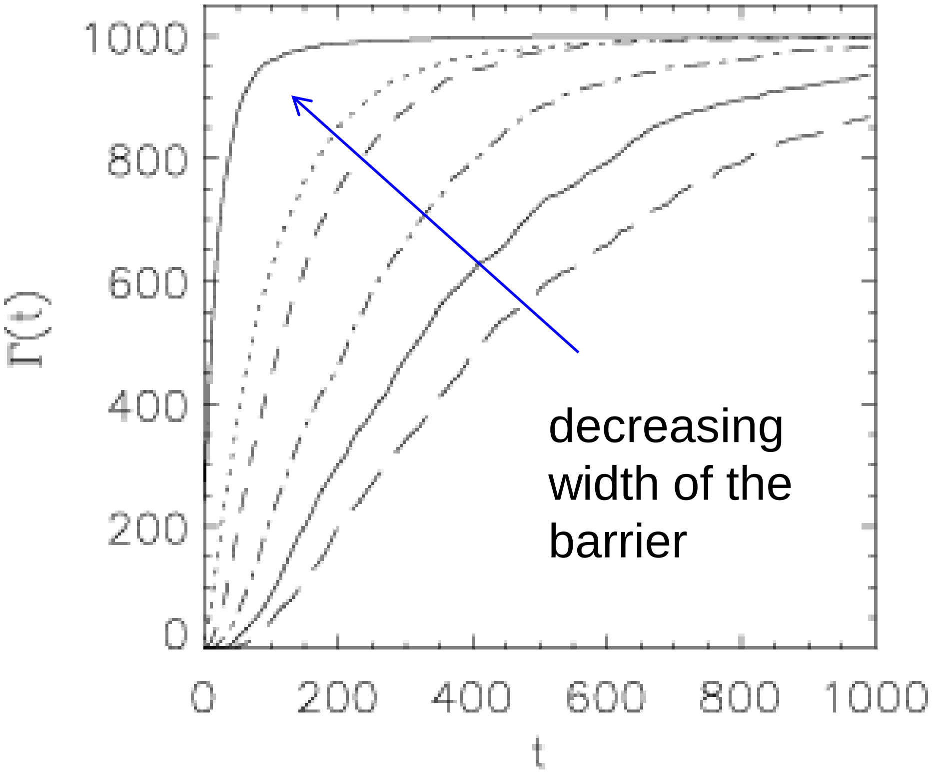


$D_e \sim 10^{-3}$

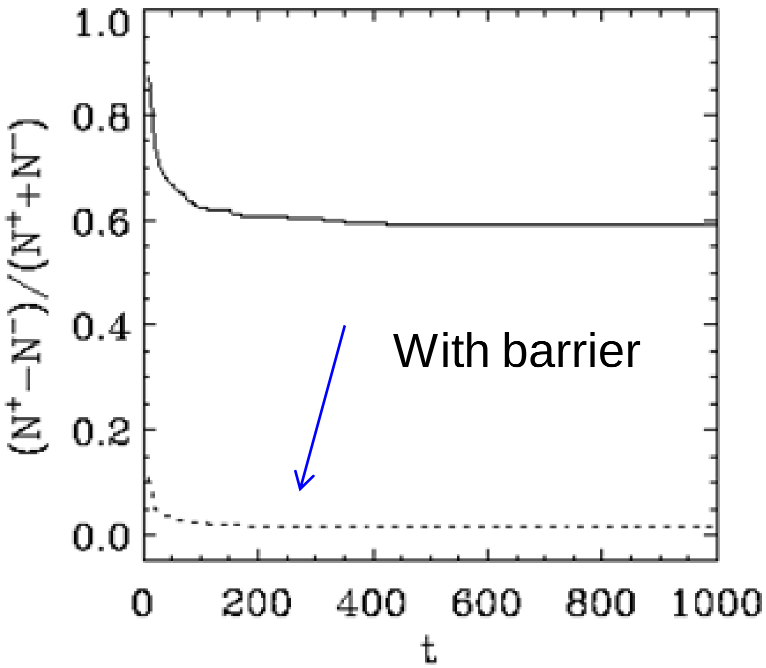
□ = 0.5

Brownian diffusion with a low diffusion coefficient

Reduced flux and symmetry



Cumulative number of particles as a function of time which escape from the integration region.

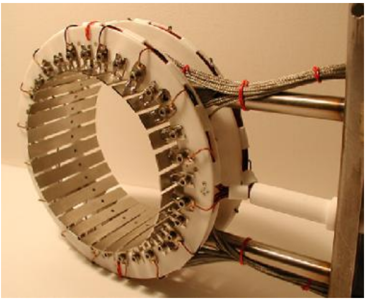


When the barrier is active we observe a symmetrization of the particle flux.

F. Lepreti et al., PRL

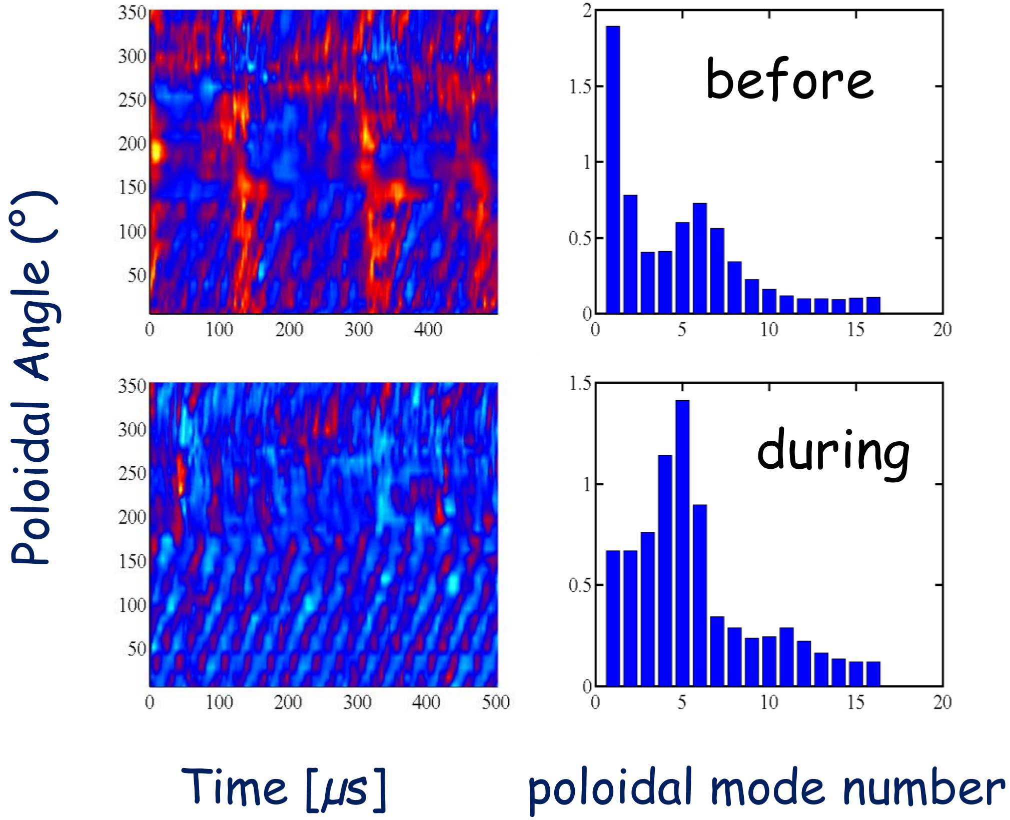
Experiments on Castor Tokamak

A barrier have been generated by biasing the electric field
with a weak perturbation on a region near the external wall (Castor Tokamak, Prague)



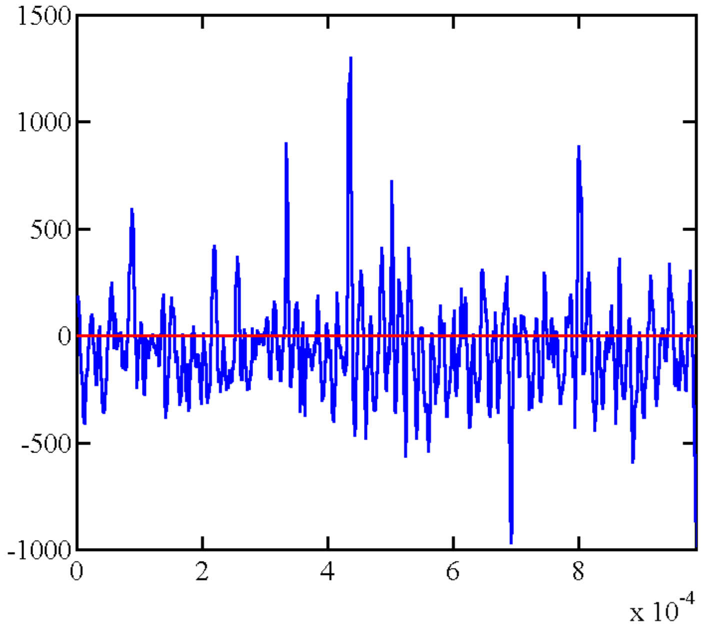
Control ring

Control obtained by biasing PHASES of fluctuations rather than amplitudes. Turbulence still exists but transport is reduced to a brownian-like diffusive process.

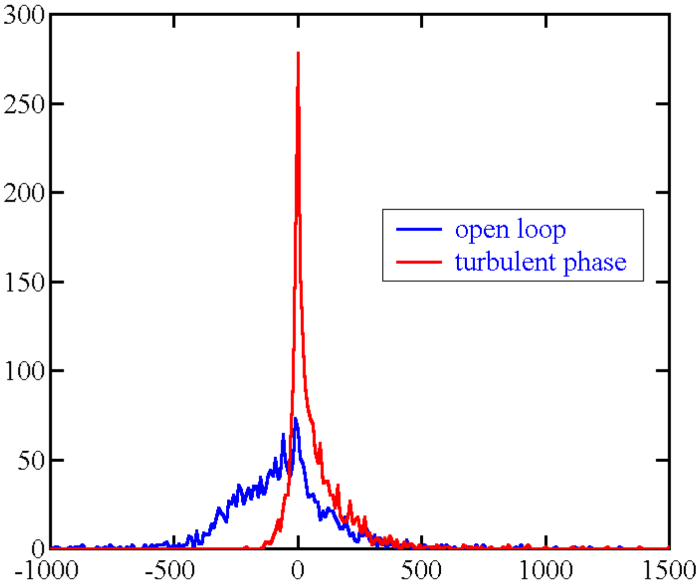


Reduction of the flux by symmetrization similar to our simple model

Particle Flux during « open loop »



PDF of the Particle Flux



The positive bursts (towards the wall) still exist but a backward flux (towards the plasma) is created.

Could a dissipative range be observed in solar wind turbulence?

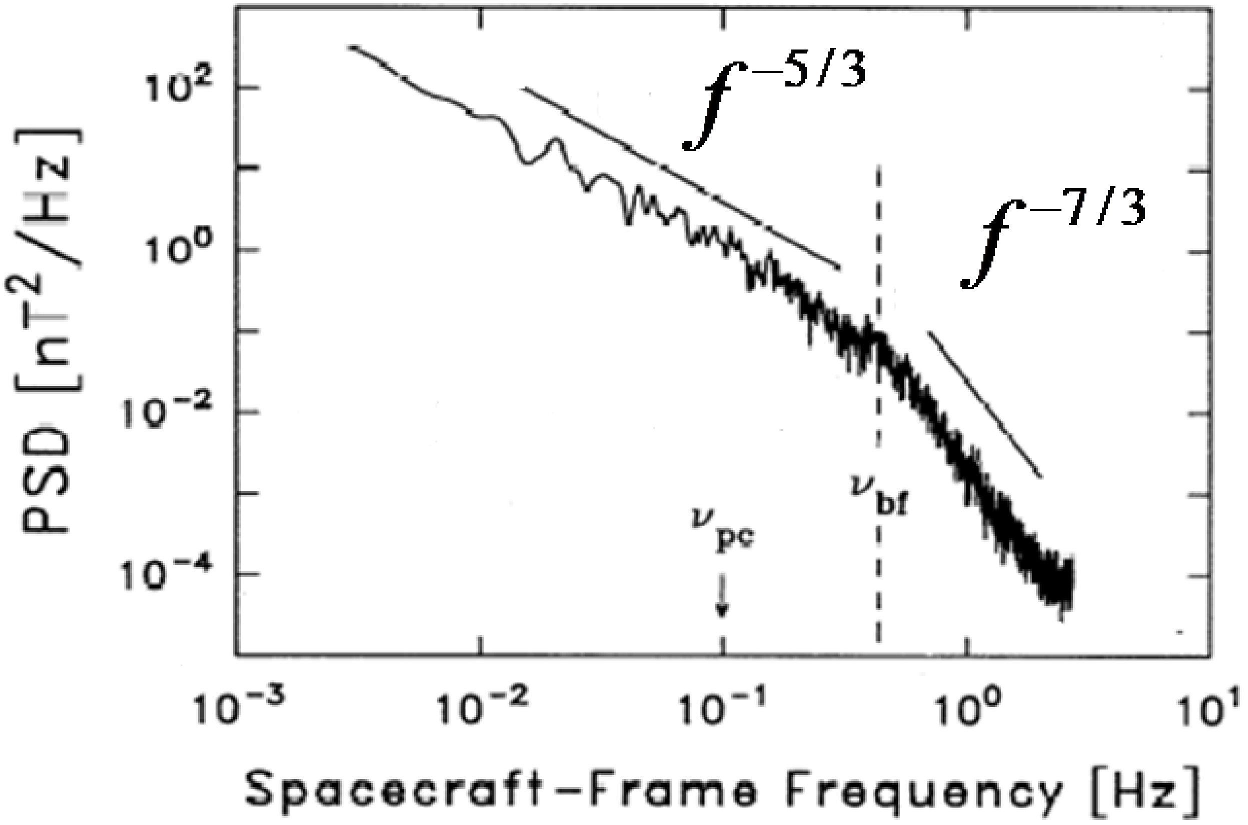
Mean-free-path → $\lambda = 10^{13}$ cm

⇒ Spacecrafts probe a collisionless plasma !
Dispersive properties become important

While large-scales (inertial range) in solar wind can be described (more or less) within a fluid approach, dissipation is much more (perhaps completely) different.

NOTE: The presence of a Yaglom's law, based on MHD where there is a ∇^2 dissipative term, means that in solar wind large-scale turbulence is not affected by the actual form assumed by the dissipation mechanism.

Cross-scale effects: Two ranges with different spectral properties of magnetic fluctuations

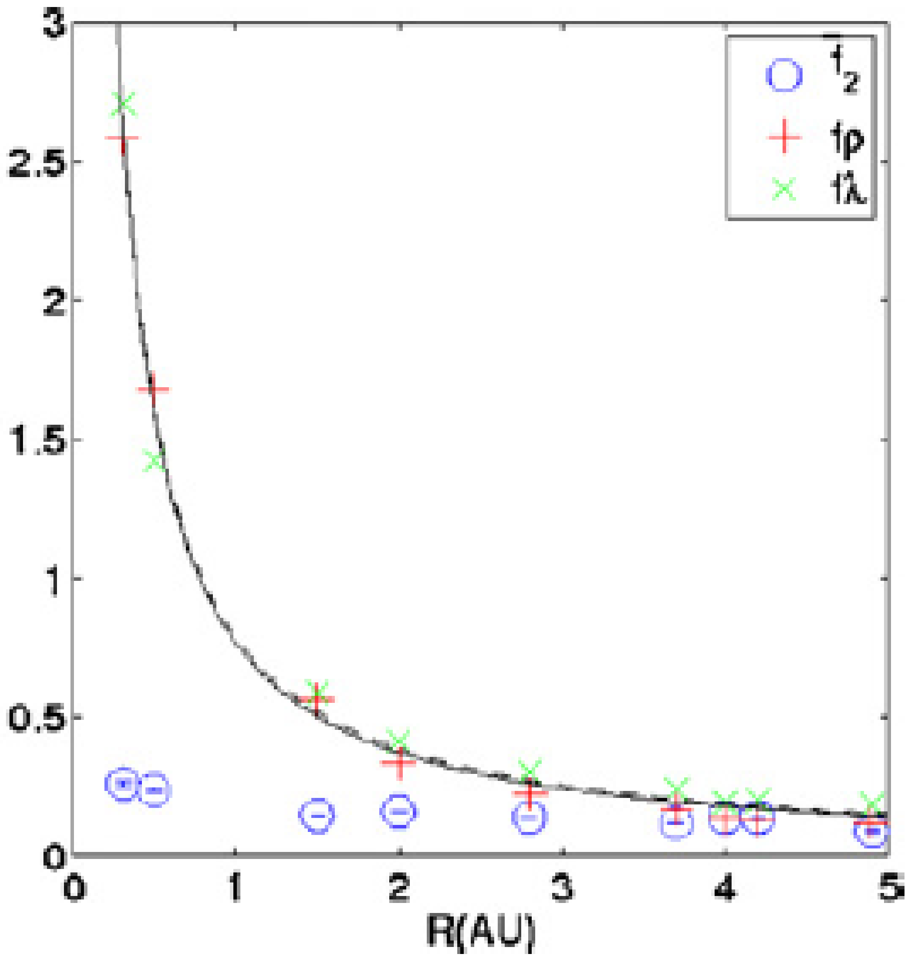


Two distinct power laws: beyond the Alfvènic range a dispersive range is observed.

Two competing scenarios introduced for their generation:

1) Whistler-mode turbulence → spectral break at proton inertial length
2) Kinetic alfven waves turbulence -> spectral break at proton gyroradius

Where does Alfvènic turbulence break down in solar wind turbulence?



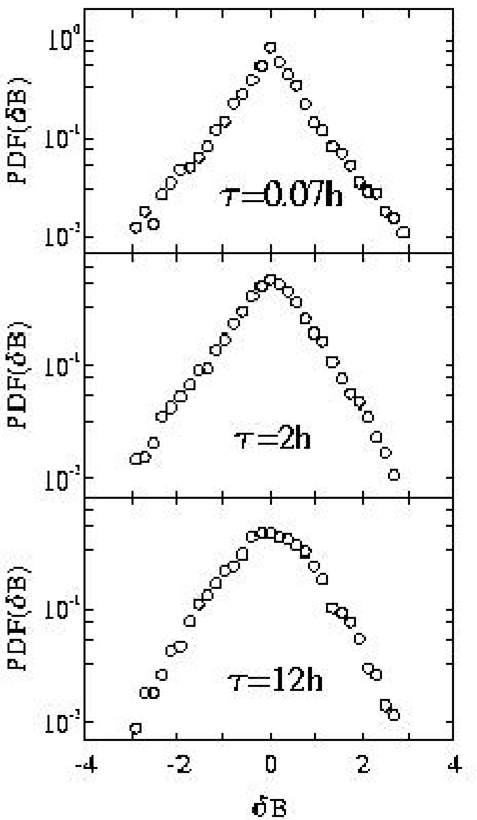
While the characteristic plasma frequencies evolve with distance from the Sun, the spectral break looks to be constant.

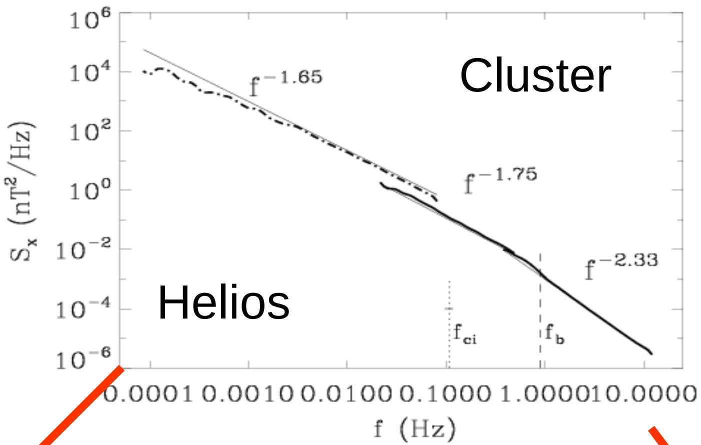
→ Break neither related to proton inertial length nor to proton gyroradius

S. Perri et al., ApJL 2010

Observations of intermittency: strong dependence of PDFs of normalized differences with scale, analogous to usual turbulence

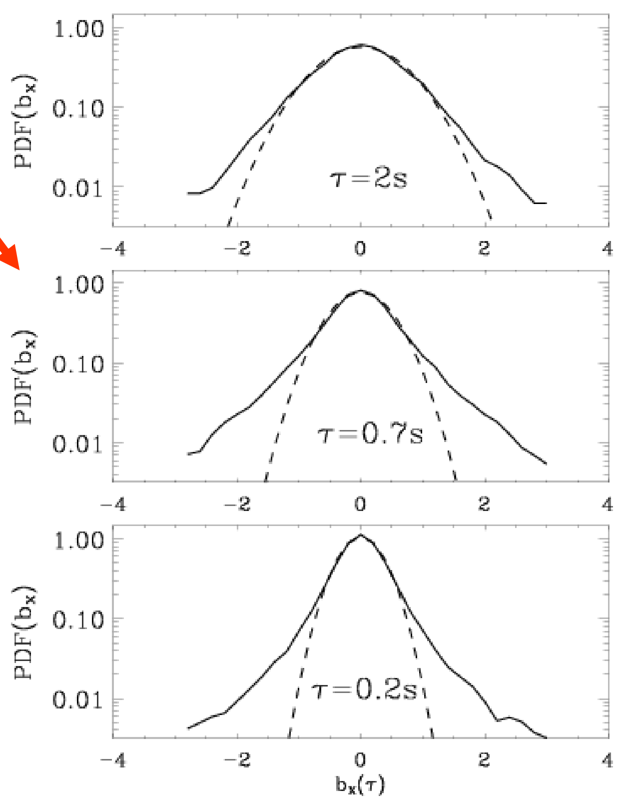
O. Alexandrova et al., ApJ (2008)





A different energy cascade process after the spectral break:

- Hall effect and compressive effect into account
- Enhancement of electrostatic part



Further topics:

1) What physical mechanism replaces "dissipation" in a collisionless plasma? Vlasov simulations show the occurrence of strong bursts of high-requency (10-100 Hz) ion-acoustic turbulent activity (not yet confirmed by observations).

2) Turbulence in the solar atmosphere: Observations show the presence of power-law tails for velocity fluctuations in the solar chromosphere, indicating that shock turbulence is at work. Estimates of the energy dissipation rate (through a Yaglom's law) indicate that turbulence could perhaps be enough for solar chromosphere heating.

3) High Reynolds number turbulence modeling: Turbulent shell models are able to reproduce observations of turbulence in solar wind, coronal loops and statistical features of nanoflares identified as dissipative intermittent bursts within turbulence.

Some few perspectives from an experiment achieving ignition

Important and novel feature of a “burning plasma” is the presence of a highly non thermal population (alpha-particles). Diffusive properties of highly energetic particles in turbulence is a physical process not yet adequately investigated.

1) The strong magnetic field can drives 2D turbulence thus generating some inverse cascade. A unique possibility to investigate this physical process in plasma turbulence (conjectures: different scaling laws, absence of intermittency, …)
2) Interplay between microturbulence and large-scale turbulence. Enhancement of transport, generation of spectral breaks,…?
3) Highly energetic alpha-particles can interact with large-scale turbulence (of the order of the alpha gyroradius). How turbulence can affect alpha-transport ? Interesting also for turbulence in space plasmas
4) Could transport be suppressed through phase-randomization? Should this be the dominant mechanism for the generation of a barrier in “burning plasma”?