Light Sterile Neutrino and Inflationary Freedom

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Abstract

We perform a cosmological analysis in which we allow the primordial power spectra of scalar perturbations to assume a shape that is different with respect to the usual power-law that arise from the simplest models of cosmological inflation. We parametrize the primordial power spectra with a piecewise monotone cubic Hermite function and we use it to investigate how the constraints on the other cosmological parameters change: we find that the limits we obtain are slightly relaxed with respect to the power-law case. Major changes occur in the neutrino sector.

Moreover, the cosmological analysis provides us some indications about the shape of the reconstructed primordial power spectra and the obtained best-fitting functions present a feature around $k = 0.002\,\mathrm{Mpc}^{-1}$, at more than 2σ. If confirmed in future analysis that will use enhanced experimental data, this suggests that the simplest cosmological inflation models must be extended in order to accomodate the feature.

This work was mainly inspired by [de Putter et al., 2014]. This poster is based on [Gariazzo et al., 2014].

Introduction

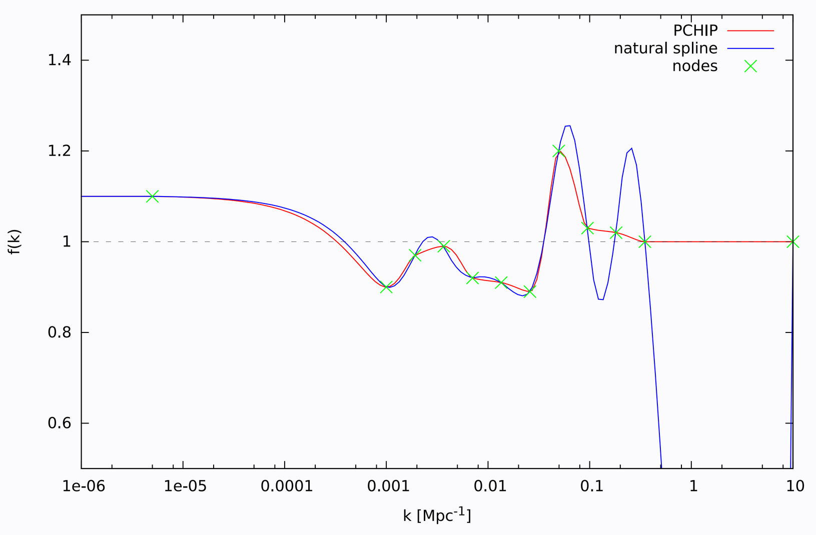
One of the main assumptions about the early Universe in the cosmological analysis is the power-law form of the Primordial Power Spectrum (PPS), that is predicted by the simplest models of inflation. Deviations from the simplest inflationary model can in principle lead to different shapes or deviations in the PPS with respect to the power-law [Baumann and Peiris, 2009]. We have no direct probe about the inflation scales and any cosmological analysis performed assuming a power-law PPS can in principle result in biased constrains.

The cosmological observable we can access is the late time power spectrum $P(k)$: this is a convolution of the PPS, encoding information about the inflationary physics, and the transfer function, that can be obtained numerically from well tested physics and is described by a small number of cosmological parameters.

Our goal is to study how the freedom in the PPS form can affect the limits on the cosmological parameters and the existing bounds on the presence in the early Universe of additional sterile neutrinos, in particular the impact of a light sterile neutrino with mass $m_s \simeq 1$ eV in cosmology.

Piecewise cubic Hermite function and natural cubic spline

In this work we decided to parametrize the scalar PPS not with a natural cubic spline function, but with a piecewise cubic Hermite function [Fritsch and Carlson, 1980] usually named `PCHIP`, in order to avoid some unwanted oscillating behaviour related to the natural cubic spline function.



A comparison between the spline PPS (blue) and the `PCHIP` (red) obtained with the same nodes (green crosses).

The ***natural cubic spline*** is a piecewise cubic function: the function, the first and second derivatives must be continue in the nodes.

It has a problem since if the data series has the same value in all the nodes but one, for example the node k_i, the interpolating spline is constant between all the nodes before k_i, but oscillates in $k > k_i$: since the computation of the first derivatives starts from $f''(k)|_{k_1} = 0$ and it is performed under the assumption that $f'(k)$ is smooth in the nodes, if at some point the function deviate from the constant value, the $f'(k)$ cannot be restored to be 0 in the points where also $f''(k) = 0$.

Fot this reason, the interpolation gives oscillations even where the nodes have a constant behaviour.

The ***`PCHIP`*** function is proposed to preserve the monotonicity of the original points: the first derivatives in the nodes are calculated taking into account the values of the secants between successive nodes, adding the requirement that $f'(k_x) = 0$ if in k_x there is a change in the monotonicity in the point series. Moreover, the function is forced to be constant in $k_i \le k \le k_{i+1}$ if the consecutive nodes i and $i + 1$ have the same value.

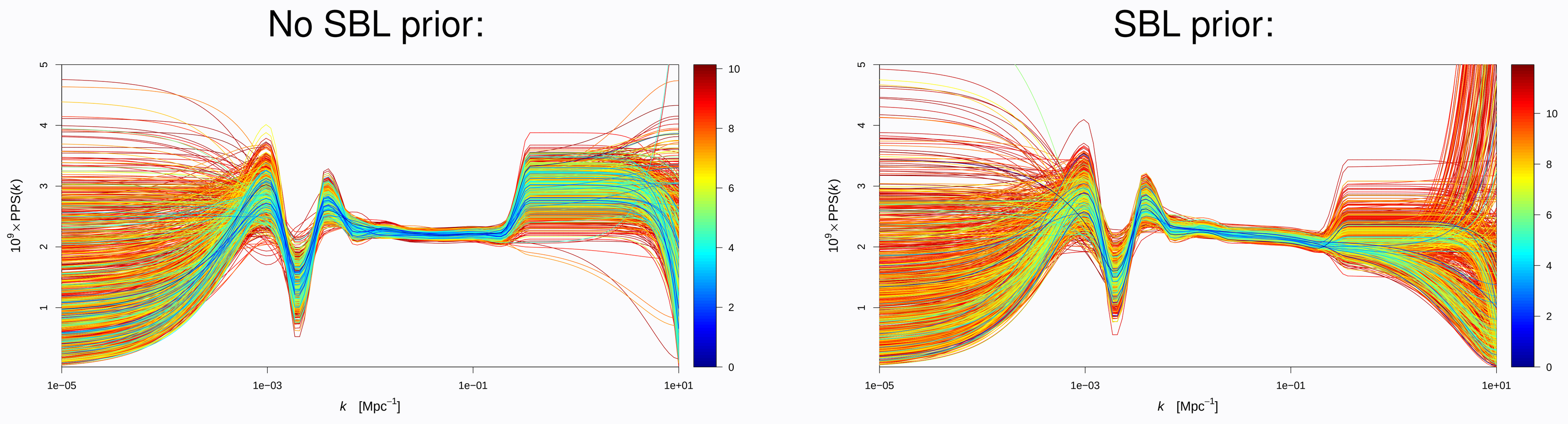
With these prescriptions, the `PCHIP` function is continue only up to the first derivative, while the second derivative can present discontinuities. On the other hand, the function is forced to follow the behaviour of the values in the nodes, without the addition of unwanted oscillations.

In the figure you can see a comparison of the unwanted behaviour for the spline function in the region $k > 0.07\,\mathrm{Mpc}^{-1}$, compared to the expected one given by the `PCHIP` function. The nodes to be interpolated are the same and the spacing of the nodes is the one we used for the scalar PPS parametrization.

Results on PPS

An helpful way to visualize how the `PCHIP` parametrization of the PPS is constrained by data in our model is the superposition of a large number of PPS that correspond to models giving a good fit to the data. The PPS are color-coded depending on the $\Delta\chi^2$ with respect to the best-fit.

No SBL prior: SBL prior:



Superposition of the best-fitting PPS in different combinations of datasets.

The reconstructed PPS can be described in this way:

- the least constrained nodes are in $k = 5 \cdot 10^{-6}\,\mathrm{Mpc}^{-1}$ and $k = 10\,\mathrm{Mpc}^{-1}$;
- nodes from $k \simeq 0.007\,\mathrm{Mpc}^{-1}$ and $k \simeq 0.2\,\mathrm{Mpc}^{-1}$ are the best constrained, with a few percent sensitivity at 1σ;
- there is a significant dip ($\gtrsim 2\sigma$) at $k \simeq 0.002\,\mathrm{Mpc}^{-1}$
- there is a small bump ($\simeq 1\sigma$) at $k \simeq 0.0035\,\mathrm{Mpc}^{-1}$

In all the plots there is a region where the power-law is a good approximation of the free PPS: this is the well collimated band in $0.007\,\mathrm{Mpc}^{-1} \le k \le 0.2\,\mathrm{Mpc}^{-1}$.

We underline that the major features we have noticed in the reconstructed PPS were mentioned in [Hazra et al., 2014], where the scalar PPS is reconstructed with a totally different technique, the Richardson-Lucy iteration algorithm.

Experimental data

We used the following experimental data:

- CMB: temperature data by Planck (2013 release), Atacama Cosmlogy Telescope (ACT), South Pole Telescope (SPT) and polarization data by Wilkinson Microwave Anisotropy Probe (WMAP).
- Large Scale Structure (LSS): the matter power spectrum from the WiggleZ Dark Energy Survey.
- H_0 from the Hubble Space Telescope (HST), $H_0 = 73.8 \pm 2.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.
- PSZ: The Planck Sunayev Zel'Dovich catalogue.
- CFHTLenS: the 2D cosmic shear correlation function as determined by the Canada-France Hawaii Telescope Lensing Survey (CFHTLenS).
- SBL: information on the additional sterile neutrino mass by the oscillation anomalies in the Short BaseLine experiments [Giunti et al., 2013].

We considered to be our *base* model the combination CMB+LSS+H_0+PSZ+CFHTLenS with a power-law parametrization for the PPS.

We varyed our *base* model with the addiction of the SBL data and the free PPS.

Parametrization

We used an extended flat ΛCDM model to accomodate the presence of an additional sterile neutrino. The model is described by:

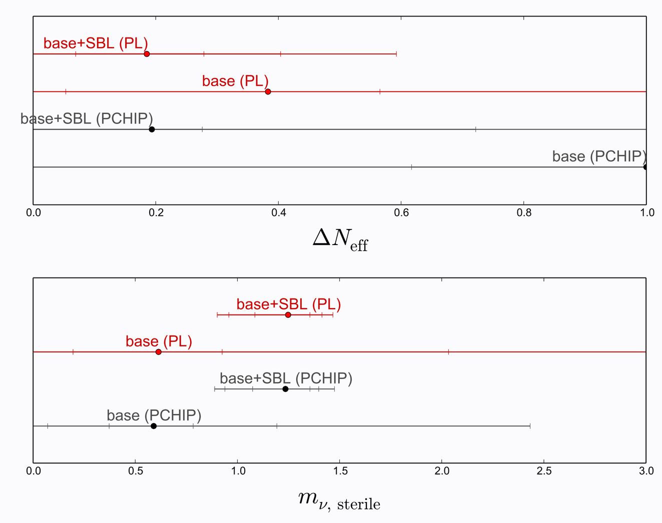
- $\omega_{\mathrm{cdm}} \equiv \Omega_{\mathrm{cdm}} h^2$ and $\omega_{\mathrm{b}} \equiv \Omega_{\mathrm{b}} h^2$, the present-day physical CDM and baryon densities,
- θ_s, the angular the sound horizon,
- τ, the optical depth to reionisation,
- $0 \le m_s/(\mathrm{eV}) \le 3$, the additional sterile neutrino mass,
- $0 \le \Delta N_{\mathrm{eff}} \le 1$, the additional sterile neutrino effective number.

In addition, we have the parameters to describe the scalar PPS.

- When we parametrize the PPS with a ***power-law***, it is described by the usual n_s and $\ln(10^{10} A_s)$, the spectral index and the amplitude respectively.
- To parametrize a ***free*** PPS, we used $N = 12$ nodes to describe a the `PCHIP` function (see dedicated section on the left): ten equally spaced nodes in the range ($k_2 = 0.001\,\mathrm{Mpc}^{-1}$, $k_{11} = 0.35\,\mathrm{Mpc}^{-1}$) that is better constrained from the data, and two nodes $k_1 = 5 \cdot 10^{-6}\,\mathrm{Mpc}^{-1}$ and $k_{12} = 10\,\mathrm{Mpc}^{-1}$ to parametrize a non-constant behaviour in the outermost region of the PPS.
  The spectrum is described by $P_s(k) = P_0 \times \mathtt{PCHIP}(k, P_{s,j})$
  where $P_0 = 2.36 \cdot 10^{-9}$ and $0.01 \le P_{s,j} \le 10$ is the scalar PPS value at the node k_j.

In total, we have 8 parameters for describing the model with the power-law PPS, and 18 for model with the free PPS, plus the 31 nuisance parameters used in Planck likelihoods.

Results on Cosmological Parameters



1, 2, 3σ limits for ΔN_{eff}, m_s.

Concerning the cosmological parameters, we do not obtain significant deviations from free PPS model with respect to the the power-law model.

The inflationary freedom affects less the results on the cosmological parameters in the case ***without the SBL prior*** on m_s with respect to the case in which the prior is applied: in the former case $\Omega_{\mathrm{cdm}} h^2$ and θ_s best values changed of about 1σ, while a smaller shift is obtained for $100\,\Omega_{\mathrm{b}} h^2$.

On the contrary, ***with the SBL prior*** inclusion the only shift is the one for τ, well inside the 1σ region.

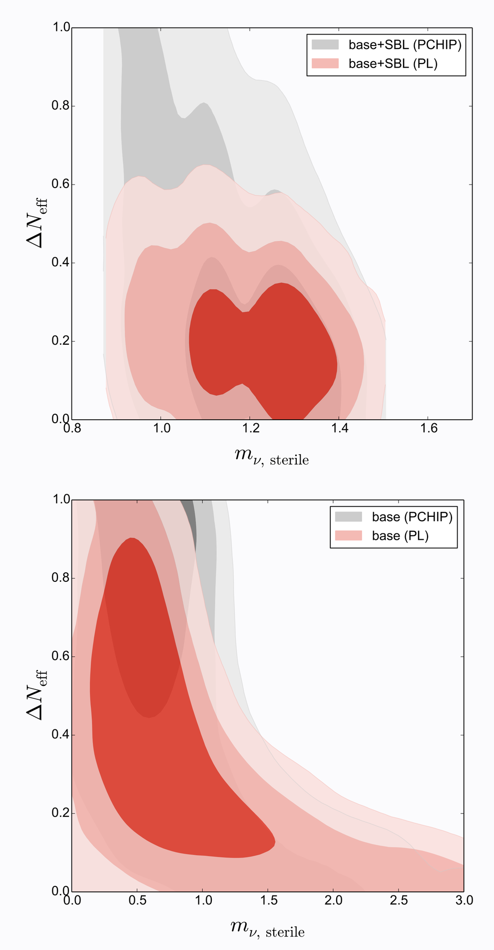
In all the cases, the marginalized limits are slightly weakened for most of the parameters.

In the sterile neutrino sector, the changes are much more evident.

With the SBL prior included,
m_s limits are not affected by the freedom in the PPS.
The free PPS model, however, can compensate the presence of additional relativistic degrees of freedom and the marginalized posterior of ΔN_{eff} is increased in the region towards $\Delta N_{\mathrm{eff}} = 1$.
An additional sterile neutrino with mass just below 1 eV and with $\Delta N_{\mathrm{eff}} = 1$ is inside the 2σ region: this means that a fully thermalized sterile neutrino can be better accomodated in the cosmological model if the PPS is not forced to be described by a power-law.

Without the SBL prior inclusion,
the preferred values for ΔN_{eff} are much higher in the free PPS case with respect to those in the power-law PPS case.
In the former case we have a preference for $\Delta N_{\mathrm{eff}} = 1$, while in the latter the preferred ΔN_{eff} is around 0.4.
Since m_s is not fixed by the SBL prior, in this case the different preference for higher ΔN_{eff} corresponds to a different limit for m_s: if ΔN_{eff} is increased there is less space for a warm dark matter neutrino with high mass and small ΔN_{eff}; the limits on m_s, even if still centered around $m_s = 0.6$ eV, are much tighter.

1, 2, 3σ marginalized contours for ΔN_{eff}, m_s.



Forthcoming Research

The natural prosecution of this work will be the analysis of the effects of a free parametrization on the tensor PPS. We expect that with the current available data the tensor PPS is weakly constrained, being the B-mode polarization measured only by the BICEP2 experiment.

The Planck polarization data will probably give stronger constraints for the analysis concerning the tensor PPS.

The determination of the tensor PPS is more important that the determination of the scalar PPS since it is directly related to the shape of the inflation potential (see [Baumann and Peiris, 2009] and references therein): its determination could help in understanding physics at scales we cannot access directly.

Short Bibliography

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