

Constraints on Light Sterile Neutrinos from CMB and Cosmological Measurements



Based on [SG et al., JHEP 1311 (2013) 211] [SG et al., arxiv:1412.7405]

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Introduction

- Cosmological Observations
- Tensions between local and CMB measurements
- Neutrino Oscillation Anomalies

2 Light Sterile Neutrino in Cosmology

- Cosmological Model
- Planck 2013 constraints
- Large Scale Structures constraints

Inflationary Freedom

- The Inflationary Paradigm
- Primordial Power Spectrum Parametrization
- Results

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Cosmic Microwave Background (CMB)

First predicted in 1948 (Alpher, Herman): blackbody background radiation at $T \simeq 5$ K. Discovery (accidental): Penzias, Wilson 1964 \rightarrow Nobel prize 1978

Observations: perfect black body spectrum at $T_{\rm CMB}=2.72548\pm0.00057$ K $_{\rm [Fixsen,\ 2009]}$ \rightarrow CMB is a remnant of the Big Bang.

Anisotropies at the level of 10^{-5} : very high precision measurements are needed. Improvement of the CMB experiments in 20 years:



Planck DR1 results



Planck DR2 results - I



Planck DR2 results - II

- TE cross-correlation and EE auto-correlation measured with high precision;
- ΛCDM explains very well the data;
- Note: in the plots, the red curve is the prediction based on the TT only best-fit for ∧CDM model → very good consistency between temperature and polarization spectra.

[Planck Collaboration, 2015]



The BICEP2 experiment

[BICEP2, 2014]: claim for detection of primordial tensor modes.

Non-zero value for tensor-to-scalar ratio r.

March 2014: $r = A_t(k_\star)/A_s(k_\star) = 0.2^{+0.07}_{-0.05}$



Conclusion, from the joint analysis: $r_{0.05} < 0.12$ at 95% CL.

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[Planck Intermediate Results XXX, 2014]

Power

South

Tension I: Hubble parameter

Hubble parameter today: $v = H_0 d$, with $H_0 = H(z = 0)$

Local measurements: H(z = 0), local and independent on evolution (model independent, systematics?)

CMB measurements

(probe $z \simeq 1100$):

 H_0 from the cosmological evolution (model dependent, well controlled systematics)

o Efstathiou Planck ∆CDM, Spergel Planck ACDM Planck QACDM Planck wCDN Planck N_{eff}CDM 50 60 70 80 90 100 H_0 (km s⁻¹ Mpc⁻¹) (HST Cepheids) Riess et al., 2011] (SNe la calibrated distance): $H_0 = 73.8 \pm 2.4 \,\mathrm{Km} \,\mathrm{s}^{-1} \,\mathrm{Mpc}^{-1}$ [Efstathiou 2013] (NGC 4258 calibrated distance): $H_0 = 70.6 \pm 3.3 \,\mathrm{Km} \,\mathrm{s}^{-1} \,\mathrm{Mpc}^{-1}$ (ACDM - CMB data only) [Planck 2013]: $H_0 = 67.3 \pm 1.2 \,\mathrm{Km} \,\mathrm{s}^{-1} \,\mathrm{Mpc}^{-1}$

[Planck 2015]: $H_0 = 67.27 \pm 0.66 \,\mathrm{Km} \,\mathrm{s}^{-1} \,\mathrm{Mpc}^{-1}$

[Cuesta et al., 2014] 68% CL error bars

Tension II: Cosmic Shear measurements

Cosmic shear: distortion of distant galaxy images by gravitational lensing of large scale structures \Rightarrow sensitive to non-linear matter density along the line of sight, amplitude of matter power spectrum.

Assuming ACDM model:

 $\sigma_8: \mbox{ rms fluctuation in total matter (baryons + CDM + neutrinos) in 8h^{-1} Mpc spheres, today;} $$\Omega_m: total matter density today divided by the critical density $$$

CFHTLenS weak lensing data alone [Heymans et al., 2013] (68% CL):

 $\sigma_8 (\Omega_m/0.27)^{0.46\pm0.02} = 0.774\pm0.04$

Planck + WMAP polarization + ACT/SPT [Planck 2013 Results XVI] (68% CL):

 $\sigma_8 (\Omega_m/0.27)^{0.46} = 0.89 \pm 0.03$

 2σ discrepancy!

Similar results from cluster counts:

Planck SZ Cluster Counts [Planck 2013 Results XX] (68% CL):

 $\sigma_8 (\Omega_m/0.27)^{0.3} = 0.76 \pm 0.03$

Planck + WMAP polarization + ACT/SPT [Planck 2013 Results XVI] (68% CL):

$$\sigma_8 (\Omega_m/0.27)^{0.3} = 0.87 \pm 0.02$$

 3σ discrepancy!

Qualitatively similar results from SPT clusters, Chandra Cluster Cosmology Project.

Unexplained discrepancies! Solutions?

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Solving the Tensions

Possible solution

Non-zero neutrino masses can help reconciling local Universe with CMB measurements.

Reasons:

- neutrino are relativistic in the primordial Universe
 - \Rightarrow free-streaming reduces the perturbations at small scales \Rightarrow lower σ_8 ;
- additional content in the early Universe
 - \Rightarrow shift in the matter-radiation equality \Leftrightarrow perturbation evolution is delayed.

Aim: to study if the neutrinos can help reconciling the different measurements.

Method:

- assume a cosmological model (ΛCDM + neutrinos);
- integrate Boltzmann equations to generate predictions;
- compare predictions with observations;
- put constraints on the theoretical model.

Framework: Bayesian analysis, Markov Chain Monte Carlo approach.

Datasets for the analysis

CAMB for Boltzmann equation integration +

CosmoMC for Markov Chain Monte Carlo (MCMC),

with different cosmological data:

- Planck: Planck 2013 TT spectra.
- WP: WMAP 9-year polarization data.
- *high-ℓ* spectra from Atacama Cosmology Telescope (ACT) and South Pole Telescope (SPT).
- *Barionic Acoustic Oscillations (BAO)*: values obtained from the SDSS-DR7, the SDSS BOSS-DR9 and the 6dFGS.
- LSS: WiggleZ Dark Energy Survey matter power spectrum at 4 different redshifts.
- H_0/HST : $H_0 = 73.8 \pm 2.4$ Km s⁻¹ Mpc⁻¹, using Cepheids and SN Ia calibration.
- LGC: Local Galaxy Cluster data from the Chandra Cluster Cosmology Project.
- *CFHTLens*: the CFHTLens 2D cosmic shear correlation function (from redshifts and shapes of 4.2 million galaxies with 0.2 < z < 1.3).
- *PSZ*: 189 galaxy clusters identified through the Sunayev Zel'Dovich (SZ) effect from Planck SZ (2013) catalogue.

In the following: CMB = Planck 2013 TT + WMAP 9-year polarization + ACT + SPT.

Neutrino Oscillations

Analogous to CKM mixing for quarks:

$$u_{lpha} = \sum_{k=1}^{3} U_{lpha k}
u_k \quad (lpha = e, \mu, au)$$

 ν_{α} flavour eigenstates, $U_{\alpha k}$ PMNS mixing matrix, ν_{k} mass eigenstates. Oscillations sensitive only to mass differences, not to absolute mass scale!

Two neutrino mixing ($\Delta m_{21}^2 = m_2^2 - m_1^2$, θ_{12} mixing angle):

$$P_{lpha
ightarrow eta, lpha
eq eta} = \sin^2(2 heta_{12})\sin^2\left(rac{\Delta m_{21}^2 L}{4E}
ight)$$

Current knowledge of the 3 active neutrino mixing: [PDG - Olive et al. (2014)]

$$\begin{split} \Delta m^2_{21} &= (7.53 \pm 0.18) \cdot 10^{-5} \, \mathrm{eV}^2 \\ |\Delta m^2_{32}| &= (2.44 \pm 0.06) \cdot 10^{-3} \, \mathrm{eV}^2 \ \rightarrow \ \text{hierarchy unknown} \\ \sin^2(2\theta_{12}) &= 0.846 \pm 0.021 \\ \sin^2(2\theta_{23}) &= 0.999^{+0.001}_{-0.018} \\ \sin^2(2\theta_{13}) &= 0.093 \pm 0.008 \\ \mathrm{CP} \ \text{violating phase} \ \delta_{\mathrm{CP}} \ \text{still unknown} \\ 2 \ \text{Majorana phases? only if } \nu \ \text{is Majorana particle} \end{split} \right\} U_{\alpha k} \end{split}$$

Short Baseline (SBL) anomaly

Neutrino oscillations $\Rightarrow \theta_{ij}$, Δm_{ij}^2 (and $\delta_{\rm CP}$). Problem: anomalies in SBL experiments $\Rightarrow \begin{cases} \text{ error in flux calculations?} \\ \text{deviations from 3-}\nu \text{ description?} \end{cases}$

A short review: [Abazajian et al., 2012]

- LSND: search for $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$, with $L/E = 0.4 \div 1.5$ m/MeV. Observed a 3.8 σ excess of $\bar{\nu}_{e}$ events [Aguilar et al., 2001]
- *MiniBooNE*: search for $\nu_{\mu} \rightarrow \nu_{e}$ and $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$, with $L/E = 0.2 \div 2.6$ m/MeV. No ν_{e} excess detected, but $\bar{\nu}_{e}$ excess observed at 2.8 σ [MiniBooNE Collaboration, 2013]
- Reactor anomaly: re-evaluation of the expected anti-neutrino flux \Rightarrow excess of $\bar{\nu}_e$ events compared to predictions ($\sim 3\sigma$) with L < 100 m [Azabajan et al, 2012]
- Gallium anomaly: GALLEX and SAGE Gallium solar neutrino experiments give a 2.7 σ anomaly (disappearance of ν_e) [Giunti, Laveder, 2011]

Possible explanation: oscillations between active ν and a sterile ν at eV scale, driven by

$$\Delta m^2_{
m SBL} \simeq 1 \,\, {
m eV}^2$$

Possible commonly used models: [Giunti et al., 2013]

- 3 active $(m_i \ll 1 \text{ eV}) + 1$ sterile $(m_s \simeq 1 \text{ eV}) \rightarrow$ minimal extension
- 3 active ($m_i \ll 1$ eV) + 2 sterile ($m_s \simeq 1$ eV) ightarrow CP violation in SBL experiments

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Cosmological Model: Neutrino Sector

Additional neutrinos \Rightarrow effects on Universe evolution!

3 active + 1 sterile ν scenario, we assume: $m_1 \simeq 0 \rightarrow \Delta m_{\text{SBL}}^2 = \Delta m_{41}^2 \simeq m_4^2$. Furthermore, sterile ν_s is weakly mixed with active ν : $m_s \simeq m_4 \simeq \sqrt{\Delta m_{\text{SBL}}^2}$

Sterile ν contribution in cosmology parametrized with: [Acero, Lesgourgues, 2009]

- energy density in the early universe, described by $\Delta N_{\text{eff}} = N_{\text{eff}} 3.046$: ν_s contribution to $\rho_R = \left[1 + \frac{7}{8} \left(\frac{T_{\nu}}{T_{\gamma}}\right)^4 N_{\text{eff}}\right] \rho_{\gamma};$
- energy density today, described by $m_s^{\rm eff} = (94.1 \ {\rm eV}) \ \omega_s = \rho_s / \rho_c^0$. [Non relativistic: $\rho_s = m_s \ n_s$] Constant is given by $\sum m_i = (94.1 \ {\rm eV}) \ \omega_{\nu}$ for SM neutrinos.

Problem: not independent observables (ΔN_{eff} , m_s^{eff} in cosmology, m_s from oscillations)!

Two different possibilities:

[Dodelson, Widrow, 1994] (DW) model: $m_{DW}^{\rm eff} = m_s \Delta N_{\rm eff}^{DW}$ Thermal (TH) distribution for ν_s : $m_{TH}^{\text{eff}} = m_s (\Delta N_{\text{off}}^{TH})^{3/4}$

SBL data included as a prior on m_s .

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Cosmological Model: ACDM sector

In the following we will study the Universe evolution considering a

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\Lambda \text{CDM} + \nu_s \text{ model}
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with 8 free parameters:

 $\{\omega_{\text{CDM}}, \omega_b, \theta_s, \tau, \mathsf{ln}(10^{10} A_s), n_s\} + \{N_{\text{eff}}, m_{DW, TH}^{\text{eff}}\}$

 $ω_{\text{CDM}}$ - CDM density today $ω_b$ - baryon density today $θ_s$ - angular sound horizon τ - optical depth to reionization $\ln(10^{10}A_s)$ - amplitude and n_s tilt of the primordial power spectrum

 $N_{\rm eff}$ effective number of ν_s $m_{DW,TH}^{\rm eff}$ physical mass of ν_s (DW or TH scenarios)

Primordial Power Spectrum (PPS) of scalar perturbations:

 $P_s(k) = A_s(k/k_0)^{n_s-1}$ with k_0 pivot scale, n_s and A_s as above.

Assume:

- $\sum m_{
 u, \mathrm{active}} = 0.06$ eV (minimal value for Normal Hierarchy)
- $0 \le m_{DW,TH}^{\mathrm{eff}} \le 5$
- 3.046 $\leq N_{\rm eff} \leq 6$

Neutrino Constraints with Planck DR1



- ν_s as Warm Dark Matter (WDM): $N_{\rm eff} \simeq 3.046$, large $m_s^{\rm eff}$ (large m_s);
- SBL prior: $m_s\simeq 1.2$ eV, but $N_{
 m eff}=$ 4 (u_s thermalized as $u_{
 m SM}$) disfavoured;
- (DW), (TH) models give similar results ($N_{\rm eff}$ slightly higher in (DW));
- only without SBL prior: positive correlation among $N_{\rm eff}$ and $H_0 \rightarrow$ tension with local measurements partially solved at large $N_{\rm eff}$.

Adding BAO and HST



• stronger limits on $m_s^{\rm eff}$, no ν_s WDM tail at small $N_{\rm eff}$;

- no SBL prior: higher $N_{\rm eff}$ admitted \rightarrow higher H_0 (correlation with $N_{\rm eff}$ holds);
- with SBL prior: slightly smaller $N_{\rm eff}$;
- with SBL prior: improvement in solving H_0 tension (driven by H_0 prior), but still low values. Due to direction in m_s^{eff} , N_{eff} plane forced by SBL prior on m_s .

MPK constraints and mass evidence



- LGC results give preference towards non-zero $m_s^{\text{eff}} \rightarrow$ non-zero m_s : smaller σ_8 from LGC can be addressed with massive ν_s (due to free streaming);
- no SBL prior: $N_{\rm eff}$ constraints almost unchanged;
- with SBL prior: preference for $N_{\rm eff} > 3.046$ at more than 2σ ;
- with SBL prior: $N_{\rm eff} = 4$ still hardly disfavoured.

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Why Inflation?

Inflation developed in the 1980s to solve several shortcomings in the Big Bang model:

 Horizon problem: why is the Universe homogeneous and isotropic? widely separated regions cannot equilibrate during gravitational expansion, since there is no causal contact during the Universe evolution.

Solution: parts of the Universe in casual contact before inflation were widely separated during inflation, while today they are re-entering the expanding causal horizon.

• Flatness problem: is the Universe flat? Planck DR2: $\Omega_K = 0.000 \pm 0.005$ today, but this corresponds to exponentially small values in the early Universe $(|\Omega_{tot} - 1| < 10^{-18} \text{ at nucleosynthesis, even smaller at earlier times})$. Fine-tuning?

Solution: $|\Omega_{tot}(t) - 1| \propto \exp\left(-\sqrt{\frac{4\Lambda}{3}}t\right)$. If inflation lasts enough (at least 60 *e*-folds, namely $a_{end}/a_{begin} \simeq e^{60}$), Ω_{tot} is very small still today.

Inflation:
$$H^2 \simeq \frac{\Lambda}{3} \implies \dot{a} = \sqrt{\frac{\Lambda}{3}} a \implies a(t) \propto \exp\left(\sqrt{\frac{\Lambda}{3}}t\right) = \exp\left(Ht\right)$$

H Hubble parameter and Λ cosmological constant during inflation, a scale factor

Primordial Power Spectrum from Slow Roll Inflation

Slow roll inflation [Linde, 1982]:

inflation occurred by a scalar field (Inflaton) rolling down a potential energy hill.

End of inflation depends on

- the shape of the inflaton potential $V(\phi)$;
- the spatially variating perturbation of the inflaton field $\delta \phi(t, \vec{x})$.

Fluctuations in the inflaton modulate the end of inflation: in different regions, inflation ends at different times. $\delta\phi(t, \vec{x})$ converted into energy density fluctuations $\delta\rho$ after inflation.

⇒ small scale dependence of the PPS: we define $(n_s - 1) \equiv \frac{d \ln P_s(k)}{d \ln k} = 2 \frac{V''}{V} - 3 \left(\frac{V'}{V}\right)^2$,

Is n_s constant? Can the PPS deviate from a power-law?

more general than $P_s(k) = A_s(k/k_*)^{n_s-1}$.

Beyond Power-Law PPS Theory



Constraints on Light Sterile Neutrinos from CMB and Cosmological Measurements

PCHIP Parametrization Fix the PPS form leads to possible bias:

 \Rightarrow analysis with free, non-parametric form for the PPS.

Proposal: fix a series of nodes and use an interpolating function among them,

$$P_s(k) = P_0 \times f(k; P_{s,1}, \dots, P_{s,12})$$

 $P_0 = 2.36 \times 10^{-9}$

PCHIP

In our case:

"piecewise cubic Hermite interpolating polynomial" $f(k; P_{s,1}, ..., P_{s,12}) = \text{PCHIP}(k; P_{s,1}, ..., P_{s,12})$

Interpolate piecewise a series of nodes $P_{s,j} = P_s(k_j)$ with $j \in [1, 12]$:

- continue and derivable;
- preserve monotonicity of the nodes:
 - 1st derivative in the node fixed using the secants between consequent nodes;
 - if the monotonicity changes, the node is a local extremum;
- 2nd derivative not continue in the nodes.

Advantage over *natural cubic splines*: no spurious oscillations.



Light Sterile Neutrino Results - I

Change in the parametrization: $ACDM(PL PPS) + \nu_s$ model with

 $\{\omega_{ ext{CDM}}, \omega_b, heta_s, au, \ln(10^{10}A_s), n_s\} + \{N_{ ext{eff}}, m_s^{ ext{eff}}\}.$

Light Sterile Neutrino Results - I

Change in the parametrization: $\Lambda CDM(PCHIP PPS) + \nu_s$ model with

```
\{\omega_{\text{CDM}}, \omega_b, \theta_s, \tau, P_{s,1}, \ldots, P_{s,12}\} + \{\Delta N_{\text{eff}}, m_s\}.
```

We consider only thermal sterile neutrinos, physical mass m_s .

Results in Λ CDM sector almost unchanged (variations well inside 1σ range).

Changes in the Sterile neutrino sector:

 $\textit{COSMO} = \mathsf{CMB}(\mathsf{Planck13} + \mathsf{WMAP}\ \mathsf{Polarization} + \mathsf{ACT}/\mathsf{SPT}) + \mathsf{LSS}(\mathsf{WiggleZ}) + \mathsf{HST}(\mathsf{Riess2011}) + \mathsf{CFHTLenS} + \mathsf{PlanckSZ}$



Light Sterile Neutrino Results - II

Change in the parametrization: $\Lambda CDM(PCHIP PPS) + \nu_s$ model with

 $\{\omega_{\text{CDM}}, \omega_b, \theta_s, \tau, P_{s,1}, \ldots, P_{s,12}\} + \{\Delta N_{\text{eff}}, m_s\}.$

We consider only thermal sterile neutrinos, physical mass m_s .

Results in Λ CDM sector almost unchanged (variations well inside 1σ range).

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PPS Results



- CMB constraints for $1 \times 10^{-4} \text{ Mpc}^{-1} (\ell = 2) \le k \le 0.3 \text{ Mpc}^{-1} (\ell \simeq 2500);$
- outer k are not constrained by data;
- power-law is a good approximation in the range $7 \times 10^{-3} \text{ Mpc}^{-1} \le k \le 0.2 \text{ Mpc}^{-1}$;
- feature at $k = 2 \times 10^{-3} \text{ Mpc}^{-1}$ correspond to dip $\ell \simeq 22$ in CMB spectrum;
- feature at $k = 3.5 \times 10^{-3} \,\mathrm{Mpc}^{-1}$ correspond to small bump $\ell \simeq 40$ in CMB spectrum.

Conclusions

- ACDM explains very well CMB measurements;
- tension between CMB observations and local observations;
 - unaccounted systematics?
 - wrong models for the Universe evolution?
- sterile neutrinos suggested by SBL oscillation anomalies can help solving the tensions,
 - but problems in producing them with small $N_{\rm eff}$ (preferred by cosmology);
- non-standard inflation can help reconciling tensions through sterile neutrino presence in the early Universe.

Thank you for the attention!

Talks, Posters and Conferences

Talks

- ISAPP 2013, International Doctoral School, Canfranc (ES), July 20, 2013. "Testing 3+1 Neutrino Mass Models with Cosmology and Short-Baseline Experiments".
- New Frontiers in Theoretical Physics, Cortona (IT), May 29, 2014. "Reconciling cosmology and short-baseline experiments with invisible decay of light sterile neutrinos".

Posters

- **ISAPP 2013**, *International Doctoral School*, Canfranc (ES), July 14–23, 2013. "Testing 3+1 Neutrino Mass Models with Cosmology and Short-Baseline Experiments".
- Planck 2014, Ferrara (IT), December 1–5. "Light Sterile Neutrinos and Inflationary Freedom".
- The Primordial Universe after Planck, Paris (FR), December 15–19. "Light Sterile Neutrinos and Inflationary Freedom".

Other Conferences and Schools

- ISAPP 2014, International Doctoral School, Belgirate (IT), July 21–30. "Multi-Wavelength and Multi-Messenger Investigation of the Visible and Dark Universe".
- Neutrino Oscillation Workshop (NOW) 2014, Conca Specchiulla, Otranto (IT), September 8–14.

Papers

S. Gariazzo, C. Giunti, M. Laveder.

"Light Sterile Neutrinos in Cosmology and Short-Baseline Oscillation Experiments". *JHEP* 1311 (2013), p. 211. arXiv: 1309.3192 [hep-ph].

M. Archidiacono, N. Fornengo, S. Gariazzo, C. Giunti, S. Hannestad et al. "Light sterile neutrinos after BICEP-2". JCAP 1406 (2014), p. 031. arXiv: 1404.1794 [astro-ph.CO].

S. Gariazzo, C. Giunti, M. Laveder. "Cosmological Invisible Decay of Light Sterile Neutrinos". Submitted for publication (2014). arXiv: 1404.6160 [astro-ph.CO].

S. Gariazzo, C. Giunti, M. Laveder. "Light Sterile Neutrinos and Inflationary Freedom". Submitted for publication (2014). arXiv: 1412.7405 [astro-ph.CO].