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# Neutrino mass ordering: current status

*What Bayesian model comparison*
*and available data can tell us*

VI Postgraduate Meeting On Theoretical Physics, Valencia - 29/11/2017

1. Basics of Bayesian statistics
   - Bayes' theorem
   - Bayesian model comparison
2. Constraining the neutrino mass ordering
   - Introducing the problem
   - Comparing models and mass orderings
3. Constraining the neutrino masses
4. Conclusions

# Bayes' theorem

Basic rule to deal with Bayesian probability!

given hypothesis $H$, data $d$, some information $I$ (true):

Bayes theorem:

$$p(H|d, I) = \frac{p(d|H, I)\, p(H|I)}{p(d|I)}$$

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Marginal likelihood:

or "Bayesian evidence",

$$p(d|I) \equiv \sum_H p(d|H,I)\,p(H|I)$$

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posterior → parameter inference

evidence → model comparison

# Bayesian evidence

"Bayesian evidence" or "Marginal likelihood"

$$Z = p(d|\mathcal{M}) = \sum_H p(d|H, I)\, p(H|I)$$

sum over different (discrete) hypothesis
(given that $I$ is true)

# Bayesian evidence

"Bayesian evidence" or "Marginal likelihood"

$$Z = p(d|\mathcal{M}) = \int_{\Omega_{\mathcal{M}}} p(d|\theta, \mathcal{M})\, p(\theta|\mathcal{M})\, d\theta$$

integrate over all possible (continuous) parameters of model $\mathcal{M}$
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What if there are several possible models $\mathcal{M}_i$?

use $Z_i$ to perform bayesian model comparison

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Model posterior:

$$p(\mathcal{M}_i|d) \propto p(\mathcal{M}_i)\, Z_i$$

← proportional to constant that depends only on data

given model prior $p(\mathcal{M}_i)$

# Bayes factor

Posterior odds of $\mathcal{M}_1$ versus $\mathcal{M}_2$:

$$\frac{p(\mathcal{M}_1|d)}{p(\mathcal{M}_2|d)} = B_{1,2}\,\frac{p(\mathcal{M}_1)}{p(\mathcal{M}_2)}$$

Bayes factor:

$$B_{1,2} = \frac{Z_1}{Z_2} \quad \Rightarrow \quad \ln B_{1,2} = \ln Z_1 - \ln Z_2$$

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if priors are the same $[p(\mathcal{M}_1) = p(\mathcal{M}_2)]$,
$B_{1,2}$ tells which one is preferred:

$B_{1,2} > 1$ ($\ln B_{1,2} > 0$)

$\mathcal{M}_1$ preferred

$B_{1,2} < 1$ ($\ln B_{1,2} < 0$)

$\mathcal{M}_2$ preferred

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$\mathcal{M}_1$ preferred

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$\mathcal{M}_2$ preferred

$|B_{1,2}|$ tells the odds in favor of preferred model

# Jeffreys' scale

[Trotta, arxiv:0803.4089]

odds in favor of the preferred model:

$$|B_{1,2}| : 1$$

strength of preference according to Jeffreys' scale:

| $\|\ln B_{1,2}\|$ | Odds | probability | strength of evidence |
|---|---|---|---|
| $< 1.0$ | $\lesssim 3 : 1$ | $< 0.750$ | inconclusive |
| $\in [1.0, 2.5]$ | $(3 - 12) : 1$ | $< 0.923$ | weak |
| $\in [2.5, 5.0]$ | $(12 - 150) : 1$ | $< 0.993$ | moderate |
| $> 5.0$ | $> 150 : 1$ | $> 0.993$ | strong |

odds & strength always valid

probability correct given equal priors and that only two models are possible (see e.g. neutrino mass ordering: normal OR inverted)

# Occam's razor

what the Bayesian model comparison tells us?

Best model strikes optimum balance between

Quality of fit → Occam's razor ← Predictivity

the simplest theory that fits data is preferred

model with more parameters ⟶ better fit (usually)

→ are all the parameters needed?

Bayes factor penalizes unnecessarily complex models!

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what if we compare same model and different priors?

Bayesian evidence depends on priors!

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Bayes factor penalizes unnecessarily wide priors!

Bayes factor DOES NOT penalize models with parameters that are unconstrained by the data

# Neutrino masses

**Normal ordering (NO)**

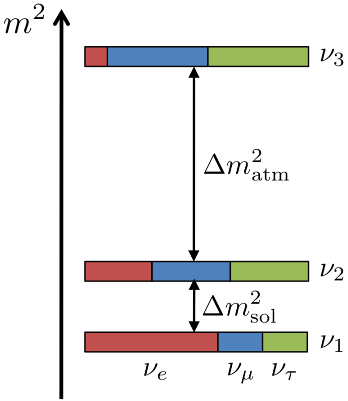
$m_1 < m_2 < m_3$

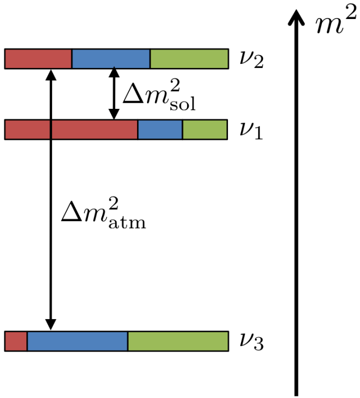
$\sum m_k \gtrsim 0.06$ eV

**Inverted ordering (IO)**

$m_3 < m_1 < m_2$

$\sum m_k \gtrsim 0.1$ eV



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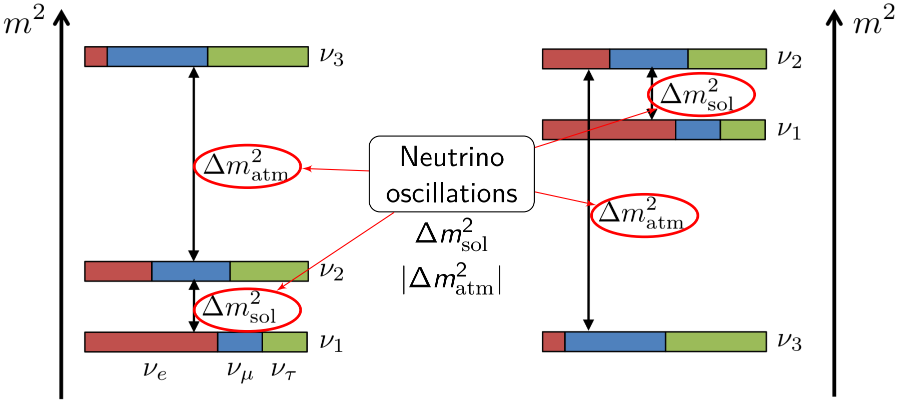

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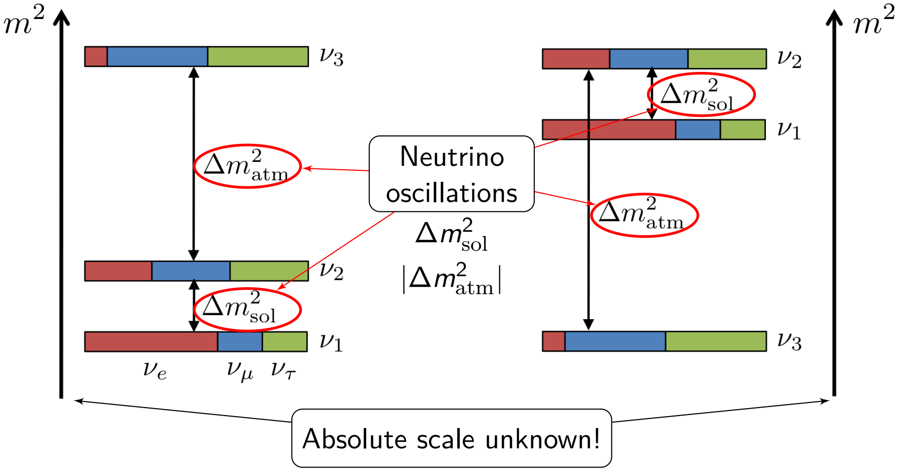


Constrain mass ordering by constraining $\sum m_k$

# Constraining the absolute scale of neutrino masses

Neutrino effects on $\beta$ decay endpoint

Mainz/Troitsk limits, $m_{\nu_e} \lesssim 2$ eV

Katrin, (expected) $m_{\nu_e} \lesssim 0.2$ eV

$$m^2_{\nu_e} = \sum_k |U_{ek}|^2 \, m^2_k$$

$U_{ek}$ mixing matrix

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$U_{ek}$ mixing matrix

(if neutrino is Majorana)

Constraints from neutrinoless double beta decay

Measure $T^{0\nu}_{1/2}$, convert into $m_{\beta\beta}$ using $m_e/m_{\beta\beta} = \mathcal{M}'^{\nu} \sqrt{G_{0\nu}\, T^{0\nu}_{1/2}}$

and then use $m_{\beta\beta} = \left| \sum_k e^{i\alpha_k} \, U^2_{ek} \, m_k \right|$

$\alpha_k$ Majorana phases

$m_e$ electron mass,
$G_{0\nu}$ phase space,
$\mathcal{M}'^{\nu}$ matrix element

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**Cosmological constraints from neutrino mass effects**

Mainly through CMB effects and free streaming

Currently can constrain $\sum_k m_k \lesssim 0.1X$ eV, single $m_k$ in future?

# Can current data tell us the neutrino mass ordering?

- [Hannestad, Schwetz, 2016]: extremely weak (2:1, 3:2) preference for NO (cosmology + [Bergstrom et al., 2015] neutrino oscillation fit) Bayesian approach;
- [Gerbino et al, 2016]: extremely weak (up to 3:2) preference for NO (cosmology only), Bayesian approach;
- [Simpson et al., 2017]: strong preference for NO (cosmological limits on $\sum m_\nu$ + constraints on $\Delta m^2_{21}$ and $|\Delta m^2_{31}|$) Bayesian approach;
- [Capozzi et al., 2017]: $2\sigma$ preference for NO (cosmology + [Capozzi et al., 2016, updated 2017] neutrino oscillation fit) frequentist approach;
- [Caldwell et al., 2017] very mild indication for NO (cosmology + neutrinoless double-beta decay + [Esteban et al., 2016] readapted oscillation results) Bayesian approach;
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# Parameterizations, priors and data

[Gariazzo et al., in preparation]

Neutrino oscillations

full $\chi^2 = -2 \log \mathcal{L}_{\rm osc}$
from global fit

[de Salas et al, 2017]

Neutrino mixing

| Parameter | Prior |
|---|---|
| $\sin^2\theta_{12}$ | 0.1 – 0.6 |
| $\sin^2\theta_{13}$ | 0.00 – 0.06 |
| $\sin^2\theta_{23}$ | 0.25 – 0.75 |

Masses: see later!

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**$\beta\beta0\nu$ data**

Likelihood approximations as in [Caldwell et al, 2017], from [Gerda, 2017] (Ge), [KamLAND-Zen, 2016], [EXO-200, 2014] (Xe)

**Neutrino oscillations**

full $\chi^2 = -2\log\mathcal{L}_{\rm osc}$ from global fit [de Salas et al, 2017]

| $\beta\beta0\nu$ parameters | | Neutrino mixing | |
|---|---|---|---|
| Parameter | Prior | Parameter | Prior |
| $\alpha_2$ | $0 - 2\pi$ | $\sin^2\theta_{12}$ | 0.1 – 0.6 |
| $\alpha_3$ | $0 - 2\pi$ | $\sin^2\theta_{13}$ | 0.00 – 0.06 |
| $\mathcal{M}^{0\nu}_{^{76}Ge}$ | 4.07 – 4.87 | $\sin^2\theta_{23}$ | 0.25 – 0.75 |
| $\mathcal{M}^{0\nu}_{^{136}Xe}$ | 2.74 – 3.45 | | |

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# Parameterizations, priors and data

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**Cosmological data**

Full CMB temperature and polarization spectra from [Planck, 2015], working with ΛCDM model as basis

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| Cosmological | | $\beta\beta0\nu$ parameters | | Neutrino mixing | |
|---|---|---|---|---|---|
| Parameter | Prior | Parameter | Prior | Parameter | Prior |
| $\omega_b$ | 0.019 – 0.025 | $\alpha_2$ | $0 - 2\pi$ | $\sin^2\theta_{12}$ | 0.1 – 0.6 |
| $\omega_c$ | 0.095 – 0.145 | $\alpha_3$ | $0 - 2\pi$ | $\sin^2\theta_{13}$ | 0.00 – 0.06 |
| $\Theta_s$ | 1.03 – 1.05 | $\mathcal{M}^{0\nu}_{76Ge}$ | 4.07 – 4.87 | $\sin^2\theta_{23}$ | 0.25 – 0.75 |
| $\tau$ | 0.01 – 0.4 | $\mathcal{M}^{0\nu}_{136Xe}$ | 2.74 – 3.45 | | |
| $n_s$ | 0.885 – 1.04 | | | | |
| $\log(10^{10}A_s)$ | 2.5 – 3.7 | | | | |

Masses: see later!

# Modeling neutrino masses

[Gariazzo et al., in preparation]

[Simpson et al, 2017]

using $m_1$, $m_2$, $m_3$ (A)

[Caldwell et al, 2017]

using $m_{\rm lightest}$, $\Delta m^2_{21}$, $|\Delta m^2_{31}|$ (B)

intuition says: (B) is closer to observable quantities! Better than (A)?

Should we use linear or logarithmic priors on $m_k$ ($m_{\rm lightest}$)?

Can data help to select (A) or (B), linear or log?

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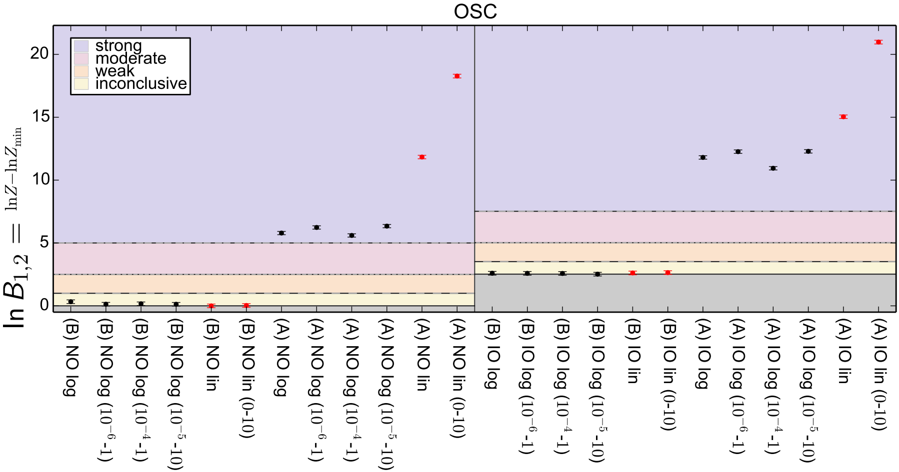
| Model A | | | Model B | | |
|---|---|---|---|---|---|
| Parameter | Prior | Range | Parameter | Prior | Range |
| $m_1/eV$ | linear<br>log | $0-1$<br>$10^{-5}-1$ | $m_{\rm lightest}/{\rm eV}$ | linear<br>log | $0-1$<br>$10^{-5}-1$ |
| $m_2/eV$ | linear<br>log | $0-1$<br>$10^{-5}-1$ | $\Delta m^2_{21}/{\rm eV}^2$ | linear | $5\times10^{-5}-10^{-4}$ |
| $m_3/eV$ | linear<br>log | $0-1$<br>$10^{-5}-1$ | $|\Delta m^2_{31}|/{\rm eV}^2$ | linear | $1.5\times10^{-3}-3.5\times10^{-3}$ |

# Comparing models

[Gariazzo et al., in preparation]



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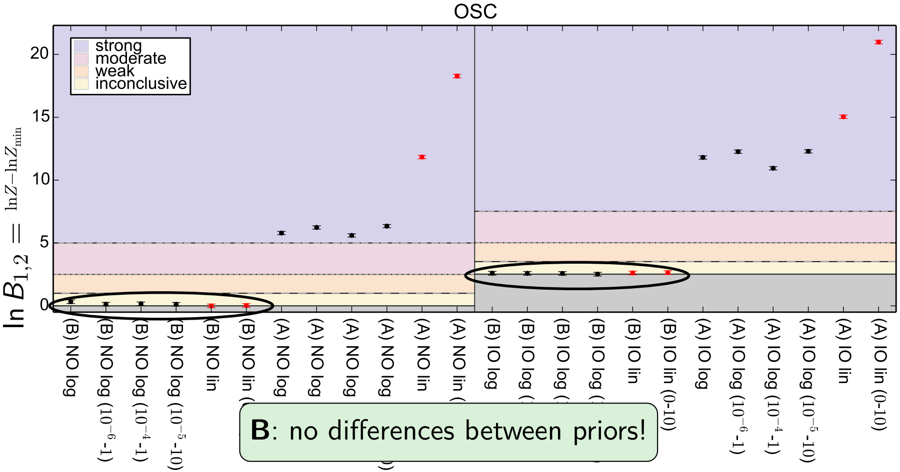


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(no penalty for unconstrained parameter $m_{\text{lightest}}$!)

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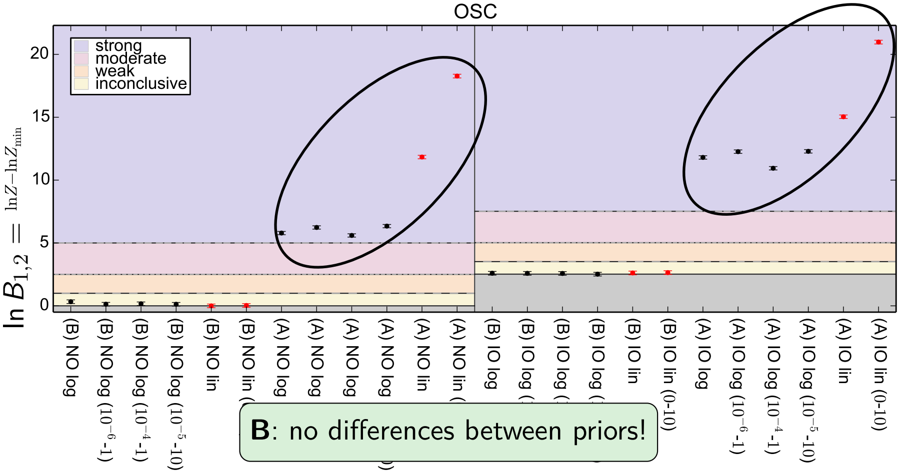


**B**: no differences between priors!

(no penalty for unconstrained parameter $m_{\text{lightest}}$!)

**A**: always strongly disfavored!

(waste of parameter space, no unconstrained parameter due to $\Delta m^2_{i1}$!)

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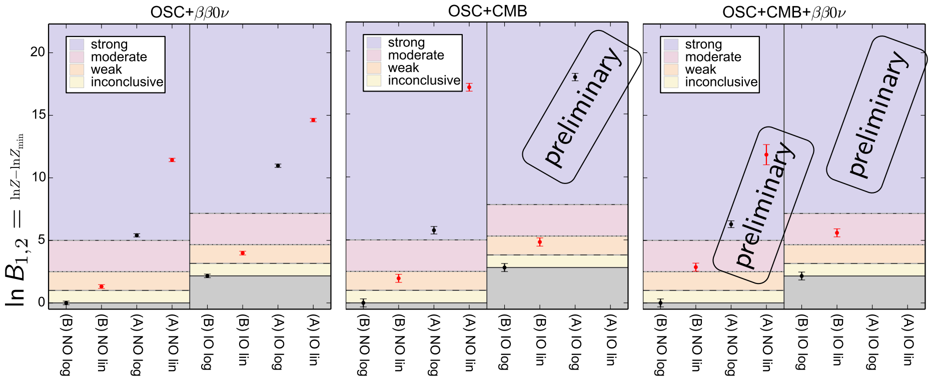


compare linear versus **logarithmic**

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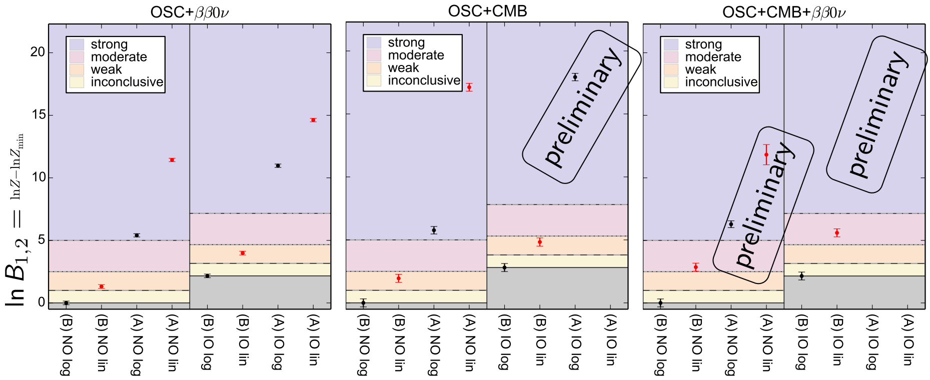


compare linear versus **logarithmic**

$\ln B_{\log,\,\mathrm{lin}}$ prefers **log** priors:
weakly-to-moderately for model B, moderately-to-strongly for model A

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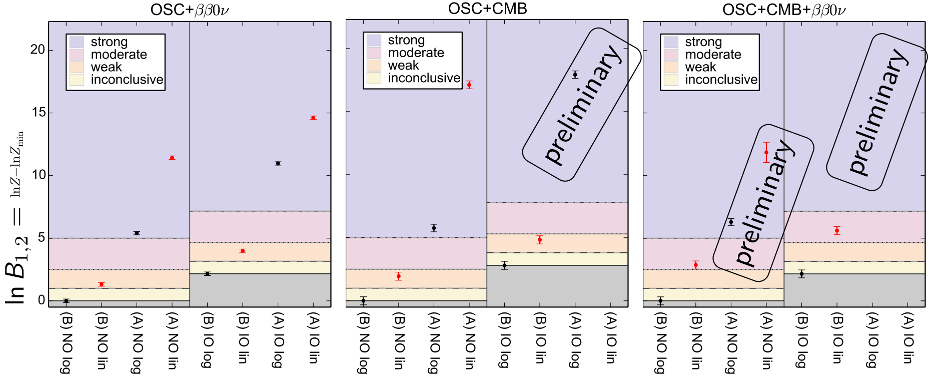


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summary: model B, log priors is better!

# Comparing the mass orderings

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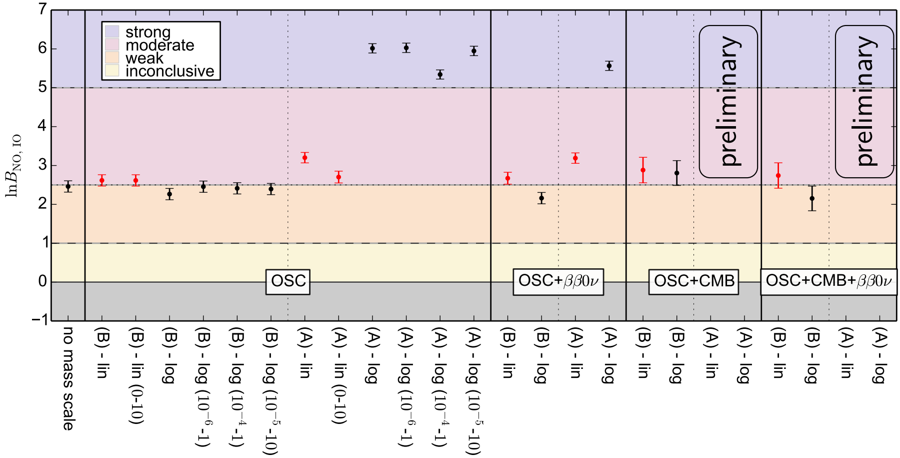

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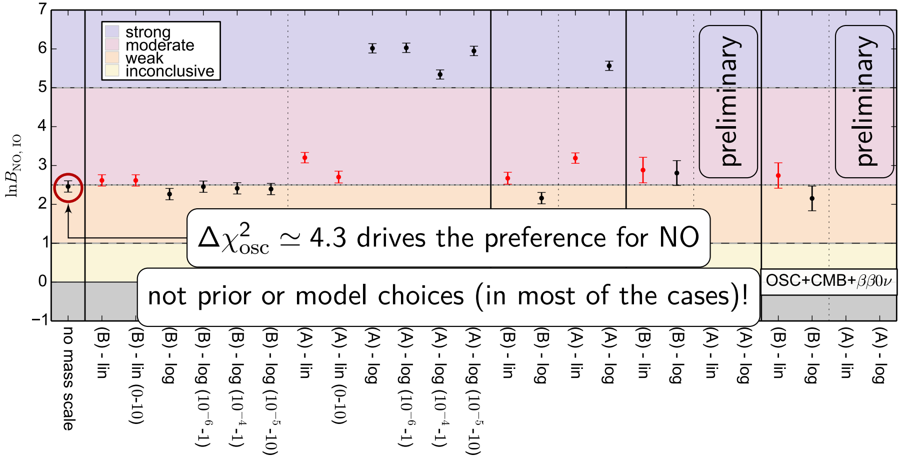

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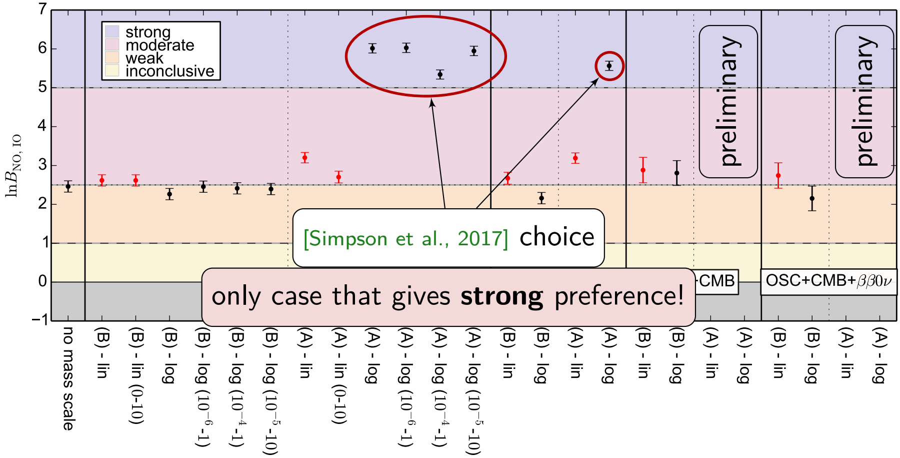


[Simpson et al., 2017] choice

only case that gives **strong** preference!

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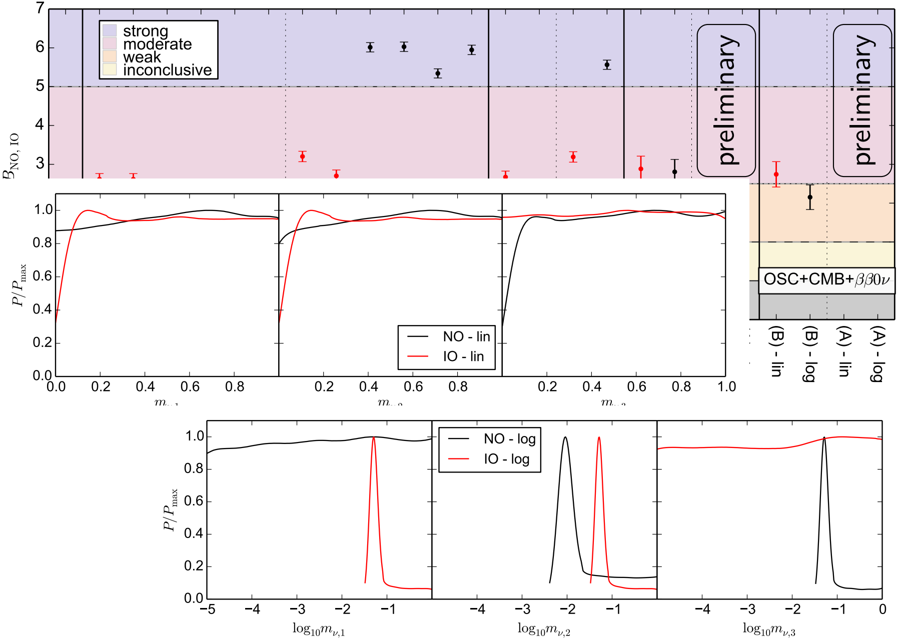

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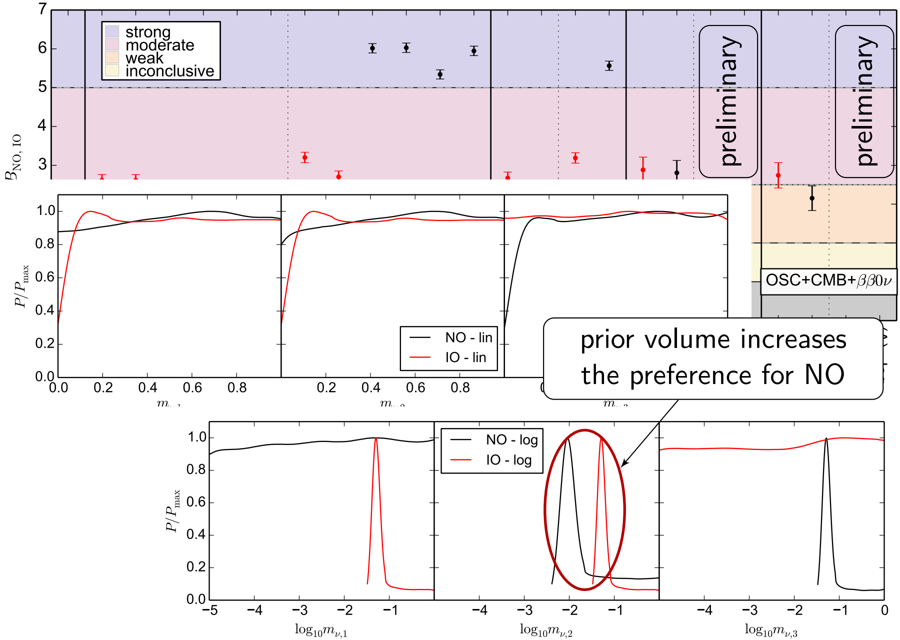


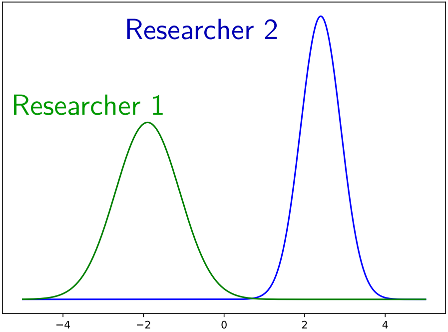

prior volume increases the preference for NO

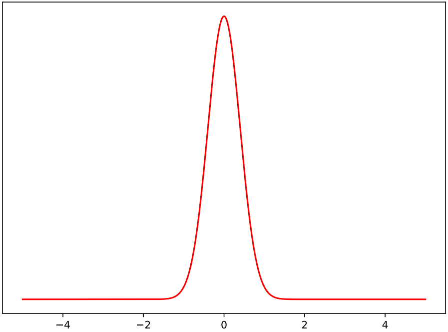
# Bayes theorem in action

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

Prior

Likelihood



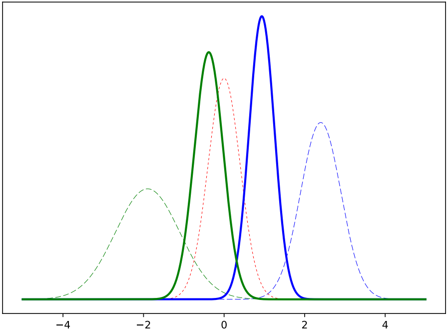

What each researcher knew before the experiment

The result of the experiment

# Bayes theorem in action

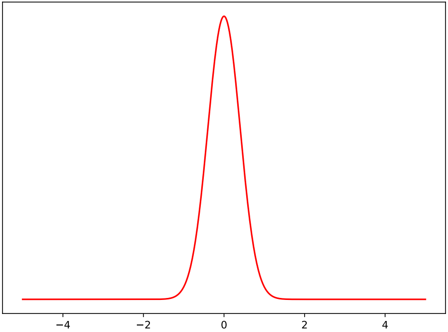
$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

Posterior




What each researcher knows after the experiment

Likelihood




The result of the experiment

Posterior depends on prior!

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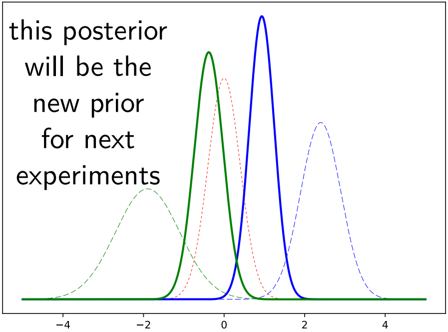


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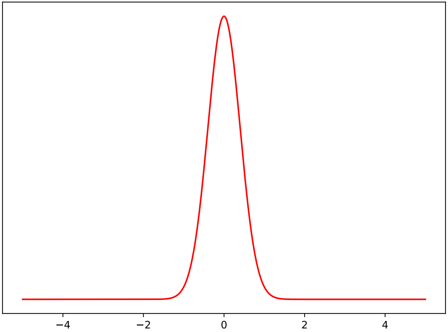


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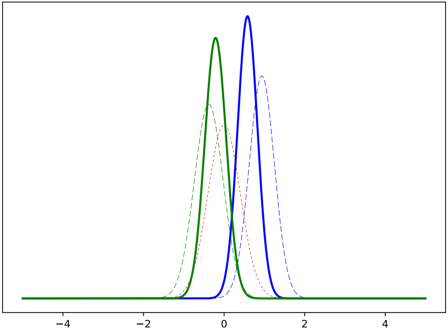
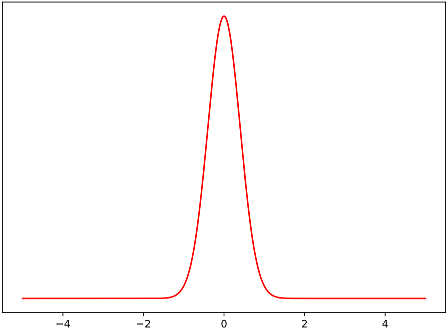



What each researcher knows after the second experiment

The result of the experiment

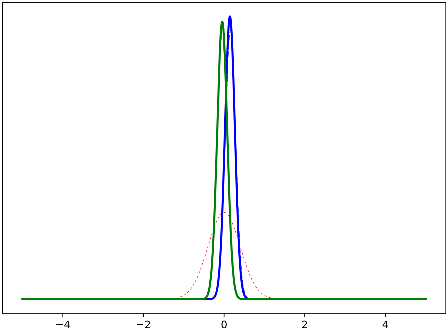
Remember:
$\sigma_N^2 = \sigma^2/N$

Posterior depends on prior!

# Bayes theorem in action

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

Posterior

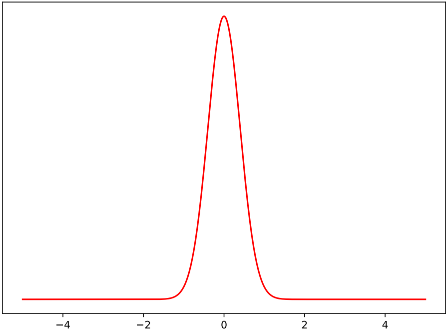



What each researcher knows after 10 experiments

Remember:

$\sigma_N^2 = \sigma^2/N$

Likelihood

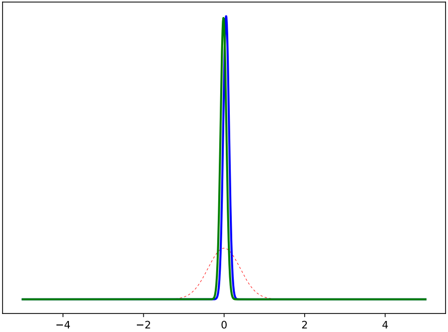



The result of the experiment

# Bayes theorem in action

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

Posterior

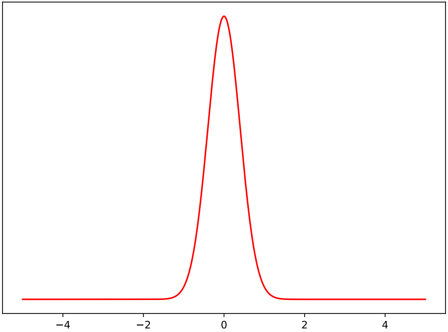



What each researcher knows after 30 experiments

Remember:

$\sigma_N^2 = \sigma^2/N$

Likelihood

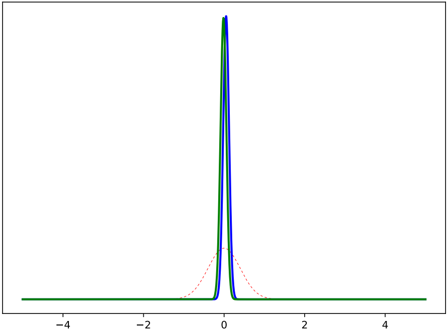



The result of the experiment

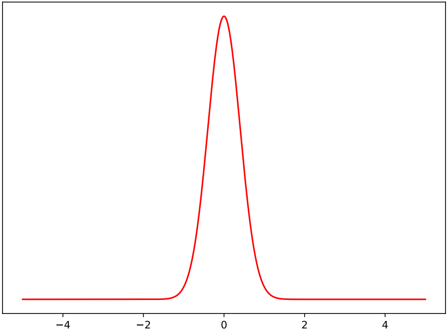
# Bayes theorem in action

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

Posterior




Likelihood




Knowledge converges using information from experiments

# Bayes theorem in action

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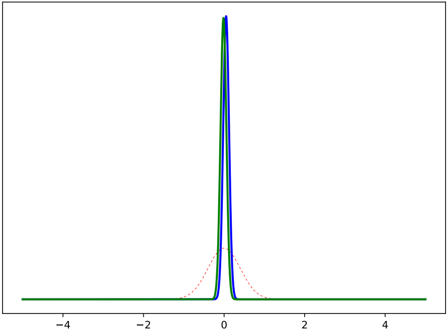
Posterior

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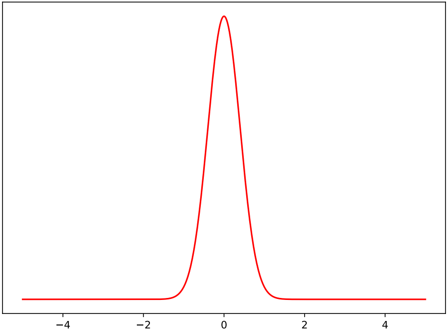
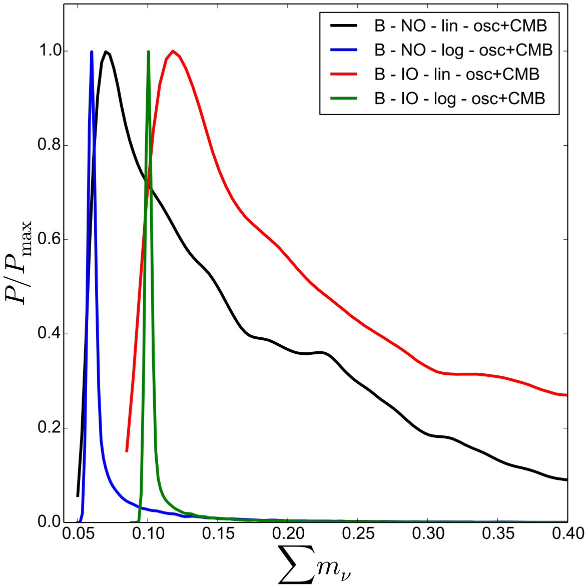



Knowledge converges using information from experiments

Prior dependence (subjectivity) only if not enough information in data!

# The role of priors: $\sum m_\nu$

[Gariazzo et al., in preparation]




showing model B only (only 1 parameter changes)

same for model A, but amplified (3 parameters change!)

logarithmic prior corresponds to $1/m_k$ probability!

↓

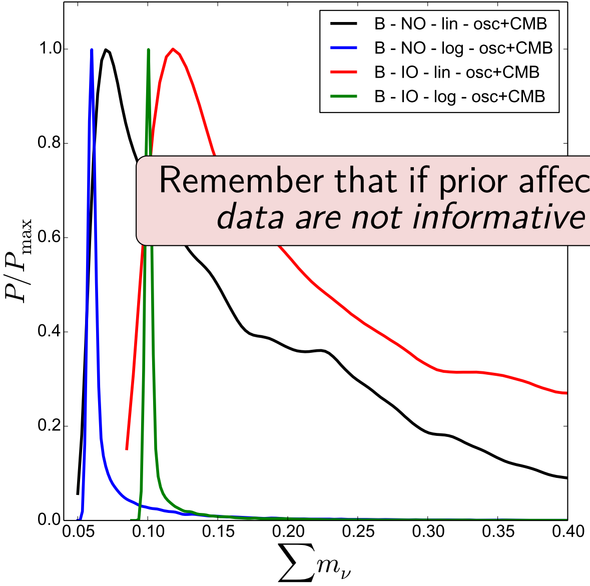
more importance to smaller masses

↓

limits closer to minimum allowed value of $\sum m_\nu$

# The role of priors: $\sum m_\nu$

[Gariazzo et al., in preparation]




Remember that if prior affects posterior,
*data are not informative enough!*

showing model B only (only 1 parameter changes)

same for model A, ...ied (3 ...change!)

logarithmic prior corresponds to $1/m_k$ probability!

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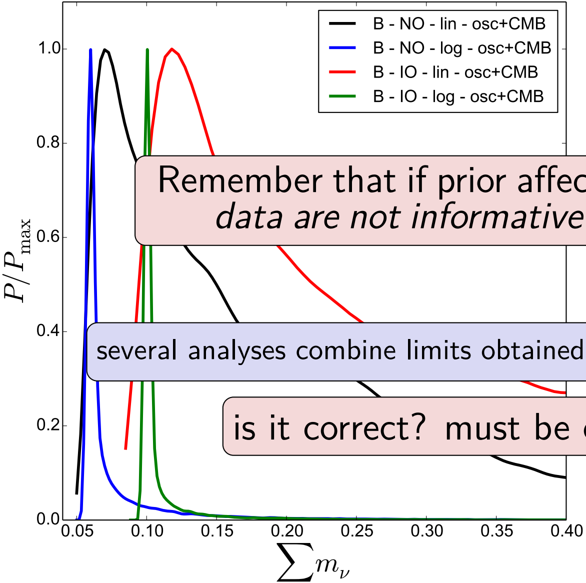


showing model B only (only 1 parameter changes)

same for model A, ...ied (3 ...hange!)

logarithmic prior corresponds to ...ty!

...re importance ...maller masses

↓

limits closer to minimum allowed value of $\sum m_\nu$

Remember that if prior affects posterior, *data are not informative enough!*

several analyses combine limits obtained with different priors

is it correct? must be careful!

# Conclusions

1 Bayesian model comparison
through Bayesian evidence/Bayes factor
to robustly test models/priors against data

2 Be careful with the effects of prior
(or of other subjective choices)
on the results of your calculations

3 data only weakly/moderately prefer normal
versus inverted neutrino mass ordering

# Conclusions

1

Bayesian model comparison
through Bayesian evidence/Bayes factor
to robustly test models/priors against data

2

Be careful with the effects of prior
(or of other subjective choices)
on the results of your calculations

3

data only weakly/moderately prefer normal
versus inverted neutrino mass ordering

**Thank you for the attention!**