



Vniver§itat te València





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# Neutrinos and Cosmology

Strenghts and weaknesses of cosmological bounds on effective number and masses of neutrinos

European Neutrino "Town" Meeting, CERN, 22-24/10/2018

#### 1 Introduction

- Neutrinos and early Universe
- Relativistic neutrinos in the early Universe
- Massive neutrinos in the late Universe

#### 2 Current constraints

- Cosmological observables
- Current status
- Extending the cosmological model
- Mass ordering

#### <sup>3</sup> Direct detection of relic neutrinos

#### 4 Conclusions

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# Three Neutrino Oscillations

Analogous to CKM mixing for quarks:

[Pontecorvo, 1968] [Maki, Nakagawa, Sakata, 1962]

$$u_{\alpha} = \sum_{k=1}^{3} U_{\alpha k} \nu_k \quad (\alpha = e, \mu, \tau)$$

 $u_{\alpha}$  flavour eigenstates,  $U_{\alpha k}$  PMNS mixing matrix,  $\nu_k$  mass eigenstates.

Current knowledge of the 3 active  $\nu$  mixing: [de Salas et al. (2018)]

 $\Delta m_{ji}^2 = m_j^2 - m_i^2$ ,  $\theta_{ij}$  mixing angles NO: Normal Ordering,  $m_1 < m_2 < m_3$ IO: Inverted Ordering,  $m_3 < m_1 < m_2$ 



# Three Neutrino Oscillations

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# Relic neutrinos in cosmology: N<sub>eff</sub>

Radiation energy density  $\rho_{\it r}$  in the early Universe:

$$\rho_r = \left[1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right] \rho_\gamma = \left[1 + 0.2271 N_{\text{eff}}\right] \rho_\gamma$$

 $ho_\gamma$  photon energy density, 7/8 is for fermions,  $(4/11)^{4/3}$  due to photon reheating after neutrino decoupling

- $N_{
  m eff} 
  ightarrow$  all the radiation contribution not given by photons
- $N_{\rm eff} \simeq 1$  correspond to a single family of active neutrino, in equilibrium in the early Universe
- Active neutrinos:

 $N_{\rm eff} = 3.046$  [Mangano et al., 2005] (damping factors approximations)  $\sim N_{\rm eff} = 3.045$  [de Salas et al., 2016] (full collision terms) due to not instantaneous decoupling for the neutrinos

= + Non Standard Interactions:  $3.040 < N_{
m eff} < 3.059$  [de Salas et al., 2016]

Observations:  $N_{\rm eff}\simeq 3.0\pm 0.2$  [Planck 2018] Indirect probe of cosmic neutrino background!



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Additional Radiation in the Early Universe



#### Starting configuration:



#### If we increase $N_{\text{eff}}$ , all the other parameters fixed:



⇒ decrease of the angular scale of the acoustic peaks  $\theta_s = r_s/D_A$ ⇒ shift of the peaks at higher  $\ell$ 

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If we increase  $N_{\text{eff}}$ , plus  $\omega_m$  to fix  $z_{\text{eq}}$ :



- Contribution from early ISW effect restored (first peak)
- different slope of the Sachs-Wolfe plateau, peak positions, envelope of high- $\ell$  peaks  $\Rightarrow$  due to later  $z_{\Lambda}$

If we increase  $N_{\text{eff}}$ , plus  $\omega_m$ ,  $\omega_{\Lambda}$  to fix  $z_{\text{eq}}$ ,  $z_{\Lambda}$ :



- peak positions recovered;
- slope of the Sachs-Wolfe plateau recovered;
- peak amplitude not recovered!

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# N<sub>eff</sub> and BBN

BBN: production of light nuclei at  $t \sim 1$ s to  $t \sim \mathcal{O}(10^2)$ s

temperature  $T_{fr} \simeq 1 \text{ MeV}$ from nucleon freeze-out:

$$\Gamma_{n\leftrightarrow p} \sim G_F^2 T^5 = H \sim \sqrt{g_\star G_N} T^2$$

$$\downarrow$$

$$T_{fr} \simeq (g_\star G_N / G_F^4)^{1/6}$$

enters  
$$n/p = \exp(-Q/T_{fr})$$

. .

which controls element abundances  

$$g_{\star}$$
 depends on  $N_{eff}$   
abundances depend on  $N_{eff}$   
 $G_F$  Fermi constant  $Q = 1.293$  MeV neutron-proton density number  
 $G_N$  Newton constant  $Q = 1.293$  MeV neutron-proton mass difference  
 $S_N$  Garazzo "Neutrinos and Cosmology" CERN. 22/10/2018 7/28











$$k_{fs}(z) \equiv \sqrt{rac{3}{2}} rac{H(z)}{(1+z)\sigma_{
u,
u}(z)} \simeq 0.7 \left(rac{m_
u}{1 ext{ eV}}
ight) \sqrt{rac{\Omega_M}{1+z}} h/ ext{Mpc}$$

 $\rho$  energy density of a given fluid  $\delta = \delta \rho / \rho$  perturbation (single fluid)

 $c_s$  sound speed of the fluid

 $\sigma_{v,\nu}(z) \nu$  velocity dispersion H = H(z) Hubble factor at redshift z h reduced Hubble factor today

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# Free-streaming - II

#### Damping occurs for all $k \gtrsim k_{nr}$

 $k_{nr}$ : corresponding to  $\nu$  non-relativistic transition

["Neutrino Cosmology", Lesgourgues et al.] (fixed h,  $\omega_m$ ,  $\omega_b$ ,  $\omega_\Lambda$ )



Expected constraints from future surveys: Planck CMB + DES:  $\sigma(m_{\nu}) \simeq 0.04-0.06$  eV [Font-Ribera et al., 2014] Planck CMB + Euclid:  $\sigma(m_{\nu}) \simeq 0.03$  eV [Audren et al., 2013]

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# CMB spectra as of 2018

#### [Planck Collaboration, 2018]

0.05°

ĒΕ

BB

ΤE

lensing

3000

2000

 $0.1^{\circ}$ 



4000

# (Linear) matter power spectrum



# Tension I: the Hubble parameter $H_0$

#### [Planck Collaboration, 2018]

$$v = H_0 d,$$
  
with  $H_0 = H(z = 0)$ 

Local measurements: H(z = 0), local and independent on evolution (model independent, but systematics?)

#### CMB measurements

(probe  $z \simeq 1100$ ):  $H_0$  from the cosmological evolution (model dependent, well controlled systematics)



68% CL error bars

### Tension I: the Hubble parameter $H_0$



Local measurements: H(z = 0), local and independent on evolution (model independent, but **systematics?**)

#### CMB measurements

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(probe  $z \simeq 1100$ ):  $H_0$  from the cosmological evolution (model dependent, well controlled systematics)



Using HST Cepheids: [Efstathiou 2013]  $H_0 = 72.5 \pm 2.5 \text{ Km s}^{-1} \text{ Mpc}^{-1}$ [Riess et al., 2016]  $H_0 = 73.24 \pm 1.74 \text{ Km s}^{-1} \text{ Mpc}^{-1}$ GW: [Abbott et al., 2017]  $H_0 = 70^{+12}_{-8} \text{ Km s}^{-1} \text{ Mpc}^{-1}$ (ACDM model - CMB data only) [Planck 2013]:  $H_0 = 67.3 \pm 1.2 \text{ Km s}^{-1} \text{ Mpc}^{-1}$ [Planck 2018]:  $H_0 = 67.27 \pm 0.60 \text{ Km s}^{-1} \text{ Mpc}^{-1}$ 

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Riess2011 Efstathiou2013 Riess2016 GW170817+EM (2017) WMAP 9yr + ACT + SPT -- ACDM Planck2013 -- ACDM Planck2015 -- ACDM Planck2018 -- ACDM Planck2018 + lens + BAO -- ACDM+N<sub>eff</sub> Planck2018 + lens + BAO --  $\Lambda CDM + \Omega_k$ Planck2018 + lens + BAO -- wCDM 50 55 45 60 65 70 75 80 85 90  $H_0$  [Km s<sup>-1</sup> Mpc<sup>-1</sup>]

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CERN, 22/10/2018

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#### Tension II (?): the matter distribution at small scales Assuming ACDM model:

 $\sigma_8$ : rms fluctuation in total matter (baryons + CDM + neutrinos) in  $8h^{-1}$  Mpc spheres, today;

 $\Omega_m$ : total matter density today divided by the critical density



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# Planck and CMB lensing



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### $N_{\rm eff}$ and the local tensions



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#### $\Sigma m_{\nu}$ and the local tensions - 1



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# Neutrino masses and CMB lensing



# Neutrino masses and CMB lensing





Bayes theorem:

$$p( heta|d,\mathcal{M}) = \mathcal{L}( heta)rac{\pi( heta|\mathcal{M})}{Z_{\mathcal{M}}}$$

posterior depends on prior!

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posterior depends on prior!

 $\begin{array}{ll} \mbox{strongest upper limit (95\%):} \\ \Sigma m_{\nu} &< 113 \mbox{ meV} \\ \mbox{(CMB+lens+BAO+SN)} \end{array}$ 

corresponding to  $\Sigma m_{\nu} < 53.6 \text{ meV} (68\%)$ 

below minimum for NO! does it make sense?

parameters  $\theta$ , model M, data  $d = \pi(\theta|M)$  prior  $p(\theta|d, M)$  posterior  $\mathcal{L}(\theta)$  likelihood  $Z_M$  Bayesian evidence S. Gariazzo "Neutrinos and Cosmology" CERN, 22/10/2018 20/28

Bayes theorem:

$$p( heta|d,\mathcal{M}) = \mathcal{L}( heta) rac{\pi( heta|\mathcal{M})}{Z_{\mathcal{M}}}$$

posterior depends on prior!

Different limits if you consider simply  $\Sigma m_{\nu} > 0$  or you take into account oscillation results...

 $\pi(\theta | \mathcal{M})$  prior

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[Wang+, 2017] degenerate (DH) vs normal (NH) vs inverted (IH) hierarchy

(i.e. change the prior lower bound)



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parameters  $\theta$ , model  $\mathcal{M}$ , data d

Bayes theorem:

$$p( heta|d,\mathcal{M}) = \mathcal{L}( heta) rac{\pi( heta|\mathcal{M})}{Z_{\mathcal{M}}}$$

posterior depends on prior!

You can artificially tighten the bounds on  $\Sigma m_{\nu}$ with different priors... [SG+, 2018] logarithmic vs linear prior

on  $m_{\rm lightest}$ 



parameters  $\theta$ , model  $\mathcal{M}$ , data  $d = \pi(\theta|\mathcal{M})$  prior  $p(\theta|d, \mathcal{M})$  posterior  $\mathcal{L}(\theta)$  likelihood  $Z_{\mathcal{M}}$  Bayesian evidence S. Gariazzo "Neutrinos and Cosmology" CERN, 22/10/2018 20/28

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Bayes theorem:

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posterior depends on prior!



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[Hannestad+, 2017] Jeffreys prior  $(\pi_I)$  for  $\Sigma m_{\nu}$ 

> $\pi_{I}$  makes the posterior maximally sensitive to data for constrained parameter, compensate border effect



what if we release the assumption of the  $\Lambda CDM$  model?

CMB TT + lens CMB TT,TE,EE

 $\Sigma m_{
u} < 0.68 \text{ eV}$  $\Sigma m_{
u} < 0.49 \text{ eV}$  [Planck 2015]

 $\Sigma m_{
u} < 0.25 \text{ eV}$  $\Sigma m_{
u} < 0.17 \text{ eV}$ 

CMB TT + lens + BAO

CMB TT.TE.EE + BAO

what if we release the assumption of the  $\Lambda CDM$  model?

CMB TT + lens CMB TT,TE,EE CMB TT + lens + BAO CMB TT,TE,EE + BAO



wCDM

 $\Sigma m_{\nu}$  < 0.37 eV [Planck 2015]  $\Sigma m_{\nu}$  < 0.27 eV [Wang+, 2016]

free dark energy equation of state  $w \neq -1$ 

what if we release the assumption of the  $\Lambda CDM$  model?



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[de Salas et al., arxiv:1806.11051]

# Constraining the mass ordering



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# Bayesian) results

Bayes theorem for models:

 $p(\mathcal{M}|d) \propto Z_{\mathcal{M}} \pi(\mathcal{M})$ 

Bayesian evidence:

$$\left( Z_{\mathcal{M}} = \int_{\Omega_{\mathcal{M}}} \mathcal{L}( heta) \, \pi( heta) \, d heta 
ight)$$

Bayes factor NO vs IO:

 $B_{\rm NO,IO} = Z_{\rm NO}/Z_{\rm IO}$ 

Posterior probability:

 $\begin{array}{ll} P_{\mathrm{NO}} &= B_{\mathrm{NO,IO}}/(B_{\mathrm{NO,IO}}+1) \\ P_{\mathrm{IO}} &= 1/(B_{\mathrm{NO,IO}}+1) \end{array}$ 

 $\begin{array}{ll} \pi(\mathcal{M}) \text{ model prior} & \mathcal{L}(\theta) \text{ likelihood} \\ p(\mathcal{M}|d) \text{ model posterior} & \Omega_{\mathcal{M}} \text{ parameter space, for parameters } \theta \\ \text{S. Gariazzo} & "Neutrinos and Cosmology"} \end{array}$ 

[de Salas et al., arxiv:1806.11051]

# (Bayesian) results

Bayes theorem for models:

 $p(\mathcal{M}|d) \propto Z_{\mathcal{M}}\pi(\mathcal{M})$ 

#### Bayesian evidence:

$$\int Z_{\mathcal{M}} = \int_{\Omega_{\mathcal{M}}} \mathcal{L}(\theta) \, \pi(\theta) \, d\theta$$

Bayes factor NO vs IO:

 $B_{\rm NO,IO} = Z_{\rm NO}/Z_{\rm IO}$ 

#### Posterior probability:

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 $\pi(\mathcal{M})$  model prior  $\mathcal{L}(\theta)$  likelihood  $p(\mathcal{M}|d)$  model posterior S. Gariazzo

[de Salas et al., arxiv:1806.11051]



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# A viable detection method

How to directly detect non-relativistic neutrinos?

a process without energy threshold is necessary

[Weinberg, 1962]: neutrino capture in eta-decaying nuclei  $u+n
ightarrow p+e^-$ 

Main background:  $\beta$  decay  $n \rightarrow p + e^- + \bar{\nu}!$ 





$$\Gamma_{\text{CNB}} = \sum_{i=1}^{3} |U_{ei}|^2 [n_i(\nu_{h_R}) + n_i(\nu_{h_L})] N_T \bar{\sigma} \sim \mathcal{O}(10) \text{ yr}^{-1}$$

$$N_T \text{ number of }^{3}\text{H nuclei in a sample of mass } M_T \quad \bar{\sigma} \simeq 3.834 \times 10^{-45} \text{ cm}^2 \quad n_i \text{ number density of neutrino } i$$
(without clustering)



### Detection of the relic neutrinos

[PTOLEMY Lol, arxiv:1808.01892]

using the definition:



if  $A_{CNB} > 0$  at  $N\sigma$ , direct detection of CNB accomplished at  $N\sigma$ 



# PTOLEMY collaboration



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# Conclusions

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Cosmology is an excellent tool for studying neutrino properties! In particular, **masses** and **effective number** 

But beware of systematics/model dependency! Situation less clear than what usually stated? In particular: **priors**, **model extensions** 

> We need more data in order to break degeneracies between different parameters!

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For a not-so-near future: direct detection of relic neutrinos??? A long way to go...

# Conclusions

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Cosmology is an excellent tool for studying neutrino properties! In particular, **masses** and **effective number** 

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> We need more data in order to break degeneracies between different parameters!

Bonus

For a not-so-near future: direct detection of relic neutrinos??? A long way to go...

# Thank you for the attention!

[PTOLEMY Lol, arxiv:1808.01892]

$$\frac{d\widetilde{\Gamma}_{\text{CNB}}}{dE_e}(E_e) = \frac{1}{\sqrt{2\pi\sigma}} \sum_{i=1}^{N_{\nu}} \bar{\sigma} N_T |U_{ei}|^2 f_{c,i} n_0 \times e^{-\frac{[E_e - (E_{\text{end}} + m_i + m_{\text{lightest}})]^2}{2\sigma^2}}$$

$$\frac{d\Gamma_{\beta}}{dE_{e}} = \frac{\bar{\sigma}}{\pi^{2}} N_{T} \sum_{i=1}^{N_{\nu}} |U_{ei}|^{2} H(E_{e}, m_{i})$$

$$\left[\frac{d\widetilde{\Gamma}_{\beta}}{dE_{e}}(E_{e}) = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{+\infty} dx \, \frac{d\Gamma_{\beta}}{dE_{e}}(x) \, \exp\left[-\frac{(E_{e}-x)^{2}}{2\sigma^{2}}\right]\right]$$

 $\bar{\sigma}$  cross section,  $N_T$  number of tritium atoms in  $M_T = 100$  g,  $E_{end}$  endpoint,  $\sigma = \Delta/\sqrt{8 \ln 2}$  standard deviation

[PTOLEMY Lol, arxiv:1808.01892]

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