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G.A. 754496

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SEZIONE DI TORINO

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(Light) Sterile neutrinos, from A to Z

Seminar at Universidad Adolfo Ibañez, Stgo / online, 11/06/2021

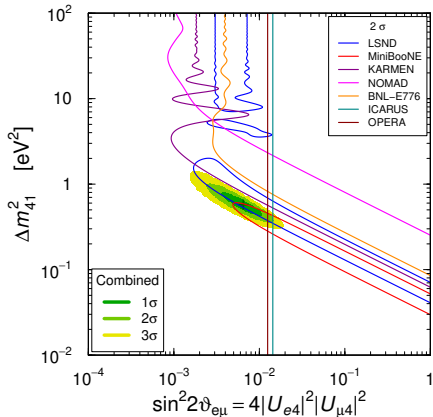
A

Appearance probes

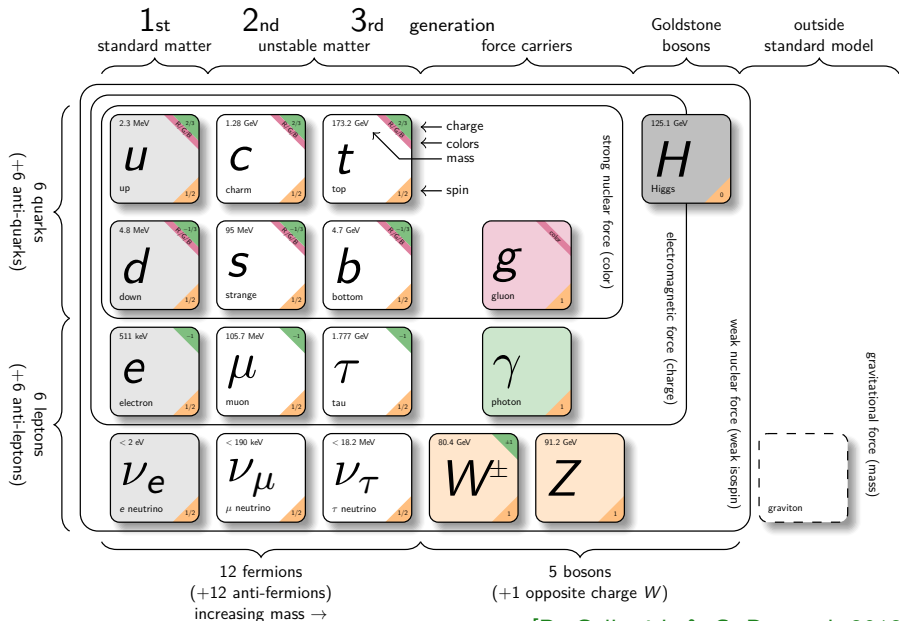
Appearance: the first anomaly

Based on:

- JPG 43 (2016) 033001
- LSND
- MiniBooNE
- in preparation

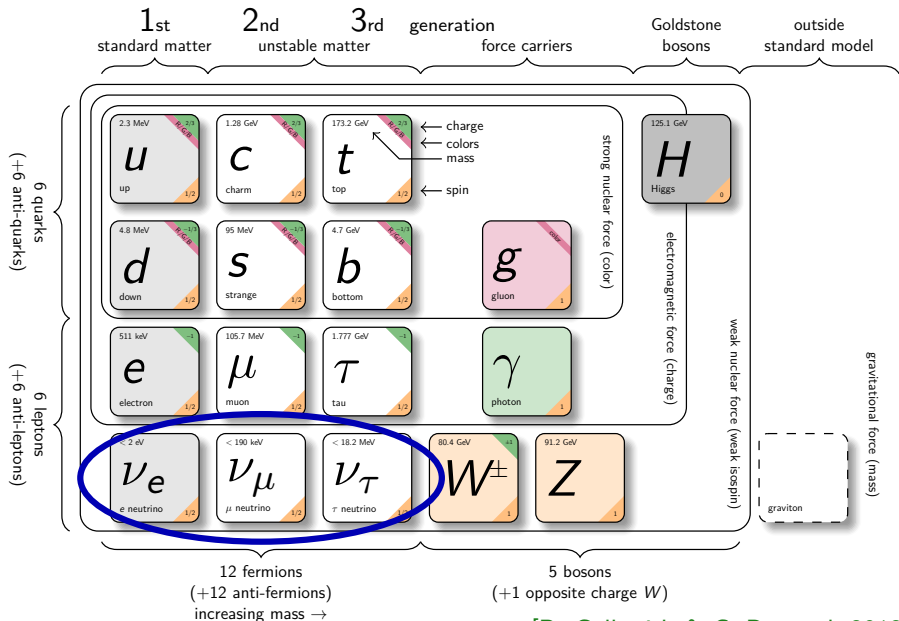


The Standard Model of Particle Physics



[D. Galbraith & C. Burgard, 2012]

The Standard Model of Particle Physics

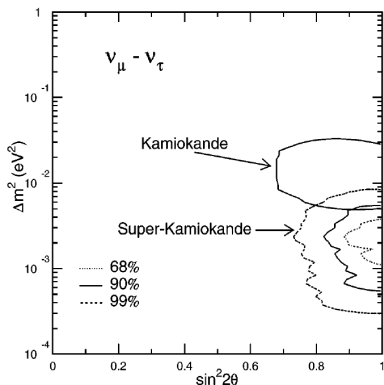


[D. Galbraith & C. Burgard, 2012]

Neutrino oscillations

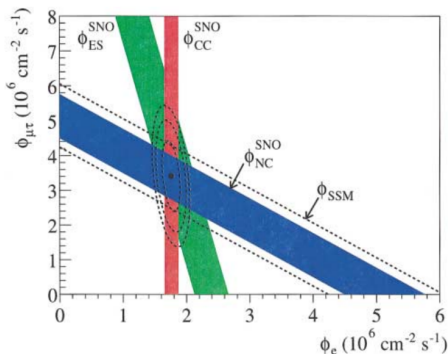
Major discoveries:

[SuperKamiokande, 1998]



first discovery of $\nu_\mu \rightarrow \nu_\tau$
oscillations from atmospheric ν

[SNO, 2001-2002]



first discovery of $\nu_e \rightarrow \nu_\mu, \nu_\tau$
oscillations from solar ν

Nobel prize in 2015

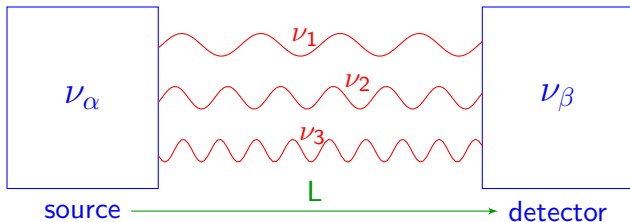
Two neutrino bases

flavor neutrinos ν_α

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k} |\nu_k\rangle$$

massive neutrinos ν_k

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U_{\alpha 1} |\nu_1\rangle + U_{\alpha 2} |\nu_2\rangle + U_{\alpha 3} |\nu_3\rangle$$



$$|\nu(t > 0)\rangle = |\nu_\beta\rangle = U_{\alpha 1} e^{-iE_1 t} |\nu_1\rangle + U_{\alpha 2} e^{-iE_2 t} |\nu_2\rangle + U_{\alpha 3} e^{-iE_3 t} |\nu_3\rangle \neq |\nu_\alpha\rangle$$

$$E_k^2 = p^2 + m_k^2 \longleftarrow \text{define} \longrightarrow t = L$$

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = |\langle \nu_\beta | \nu(L) \rangle|^2 = \sum_{k,j} U_{\beta k} U_{\alpha k}^* U_{\beta j}^* U_{\alpha j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

Three Neutrino Oscillations

$$\nu_\alpha = \sum_{k=1}^3 U_{\alpha k} \nu_k \quad (\alpha = e, \mu, \tau)$$

$U_{\alpha k}$ described by 3 mixing angles θ_{12} , θ_{13} , θ_{23} and one CP phase δ

Current knowledge of the 3 active ν mixing: [JHEP 02 (2021)]

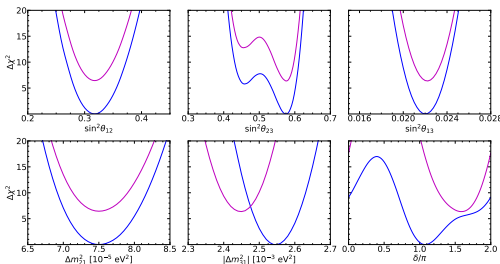
NO/NH: Normal Ordering/Hierarchy, $m_1 < m_2 < m_3$

IO/IH: Inverted O/H, $m_3 < m_1 < m_2$

$$\begin{aligned} \Delta m_{21}^2 &= (7.50^{+0.22}_{-0.20}) \cdot 10^{-5} \text{ eV}^2 \\ |\Delta m_{31}^2| &= (2.55^{+0.02}_{-0.03}) \cdot 10^{-3} \text{ eV}^2 \text{ (NO)} \\ &= (2.45^{+0.02}_{-0.03}) \cdot 10^{-3} \text{ eV}^2 \text{ (IO)} \end{aligned}$$

$$\begin{aligned} 10 \sin^2(\theta_{12}) &= 3.18 \pm 0.16 \\ 10^2 \sin^2(\theta_{13}) &= 2.200^{+0.069}_{-0.062} \text{ (NO)} \\ &= 2.225^{+0.064}_{-0.070} \text{ (IO)} \\ 10 \sin^2(\theta_{23}) &= 5.74 \pm 0.14 \text{ (NO)} \\ &= 5.78^{+0.10}_{-0.17} \text{ (IO)} \end{aligned}$$

$$\begin{aligned} \delta/\pi &= 1.08^{+0.13}_{-0.12} \text{ (NO)} \\ &= 1.58^{+0.15}_{-0.16} \text{ (IO)} \end{aligned}$$



mass ordering
still unknown

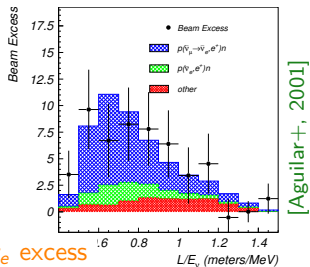
δ still unknown

see also: <http://globalfit.astroparticles.es>

Do three-neutrino oscillations explain all experimental results?

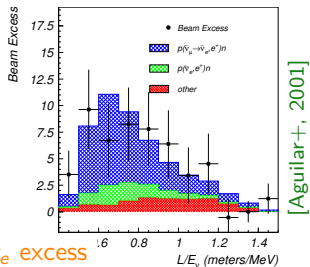
Do three-neutrino oscillations explain all experimental results?

LSND

 3.8σ $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ excess

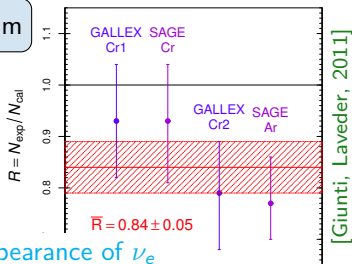
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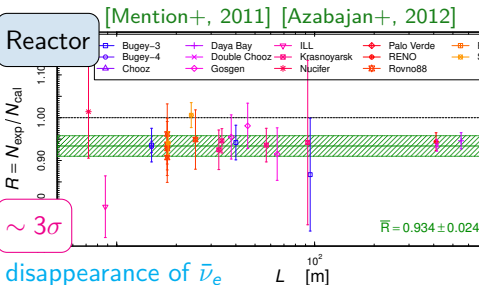
3.8σ

Gallium



2.7σ

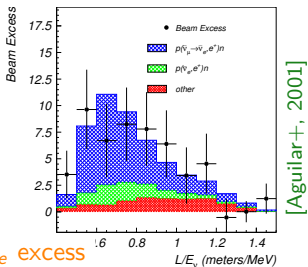
Reactor



$\sim 3\sigma$

Do three-neutrino oscillations explain all experimental results?

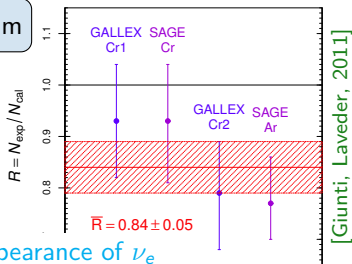
LSND



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$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ excess

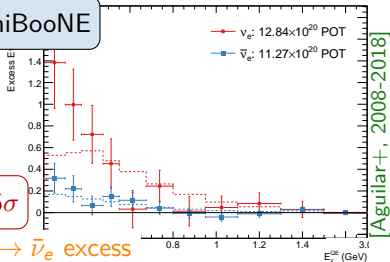
Gallium



2.7σ

disappearance of ν_e

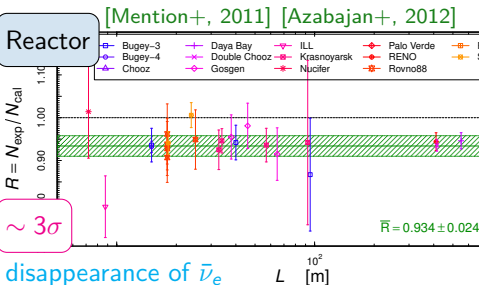
MiniBooNE



$\sim 5\sigma$

$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ excess

Reactor

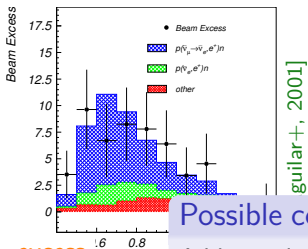


$\sim 3\sigma$

disappearance of $\bar{\nu}_e$

Do three-neutrino oscillations explain all experimental results?

LSND

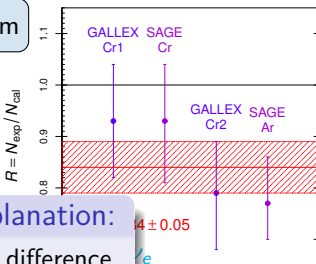


guilard+, 2001]

3.8σ

$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ excess

Gallium

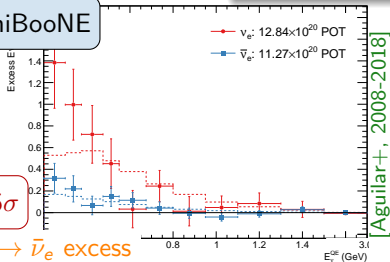


[Giunti, Laveder, 2011]

Possible common explanation:

Additional squared mass difference
 $\Delta m_{\text{SBL}}^2 \approx 1 \text{ eV}^2$

MiniBooNE

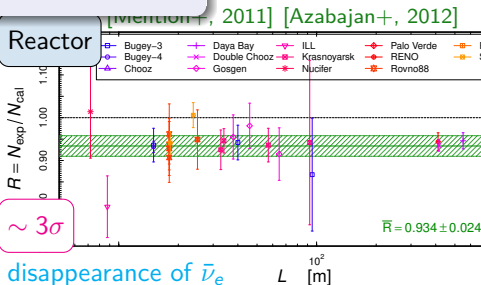


[Aguilar+, 2008-2018]

$\sim 5\sigma$

$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ excess

Reactor



[Aguilar+, 2008-2018] [Azababan+, 2012]

$\sim 3\sigma$

disappearance of $\bar{\nu}_e$

$\bar{R} = 0.934 \pm 0.024$

A large family

In principle, previous discussion is valid for N neutrinos

only constraint: there are exactly three flavor neutrinos in the SM

[LEP, Phys. Rept. 427 (2006) 257,
arXiv:hep-ex/0509008]

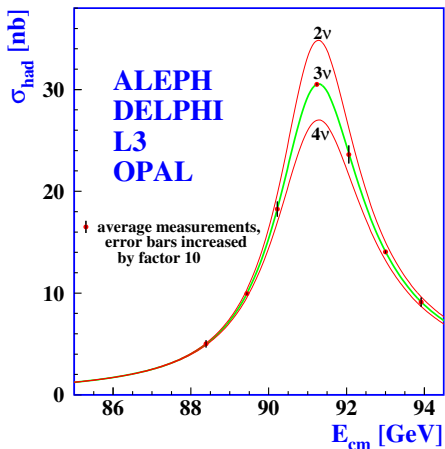
$$N_{\nu}^{(Z)} = 2.9840 \pm 0.0082$$

through the measurement
of the Z resonance

$$e^+ e^- \rightarrow Z \rightarrow \sum_{a=e,\mu,\tau} \nu_a \bar{\nu}_a$$

neutrinos $\alpha > 3$ must be sterile

sterile neutrino = SM singlet: no couplings with other SM particles



A large family

In principle, previous discussion is valid for N neutrinos

$N \times N$ mixing matrix, N flavor neutrinos, N massive neutrinos

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \\ |\nu_{s1}\rangle \\ \dots \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \vdots \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} & \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} & \\ U_{s11} & U_{s12} & U_{s13} & U_{s14} & \\ \dots & & & & \ddots \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \\ |\nu_4\rangle \\ \dots \end{pmatrix}$$

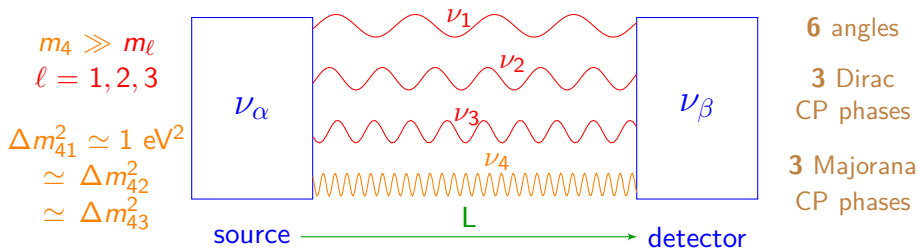
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Our case will be 3 (active)+1 (sterile), a perturbation of 3 neutrinos case



Short BaseLine (SBL)

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = |\langle \nu_\alpha | \nu(L) \rangle|^2 = \sum_{k,j} U_{\beta k} U_{\alpha k}^* U_{\beta j}^* U_{\alpha j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

If $m_4 \gg m_\ell$, faster oscillations

ν_4 oscillations are averaged in most neutrino oscillation experiments

Effect of 4th neutrino only visible as global normalization

Short BaseLine (SBL) oscillations: $\frac{\Delta m_{41}^2 L}{E} \simeq 1$

At SBL, oscillations due to Δm_{21}^2 and $|\Delta m_{31}^2|$ do not develop

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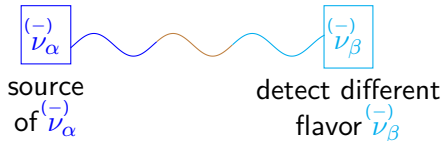
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APPearance ($\alpha \neq \beta$)



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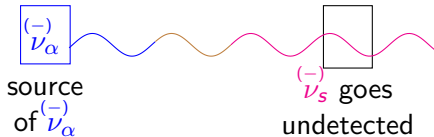
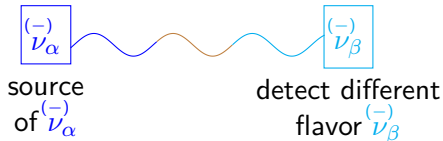
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APPEARance ($\alpha \neq \beta$)

DISappearance



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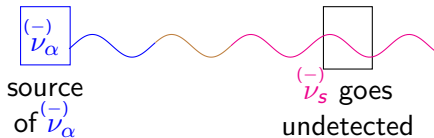
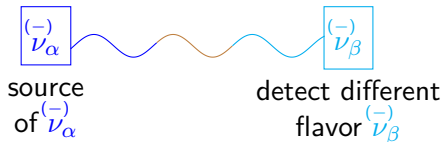
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Short BaseLine (SBL) oscillations: $\frac{\Delta m_{41}^2 L}{E} \simeq 1$

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APPEARANCE ($\alpha \neq \beta$)

DISAPPEARANCE



CP violation cannot be observed in SBL experiments!

New mixings in the 3+1 scenario

4 × 4 mixing matrix:

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s11} & U_{s12} & U_{s13} & U_{s14} \end{pmatrix}$$

New mixings in the 3+1 scenario

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DISappearance

$$P_{\nu_{\alpha}^{(-)} \rightarrow \nu_{\alpha}^{(-)}}^{\text{SBL}} \simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2(1 - |U_{\alpha 4}|^2)$$

$\nu_e^{(-)} \rightarrow \nu_e^{(-)}$

reactor
gallium

$$|U_{e4}|^2 = \sin^2 \vartheta_{14}$$

$\nu_{\mu}^{(-)} \rightarrow \nu_{\mu}^{(-)}$

accelerator
atmospheric

$$|U_{\mu 4}|^2 = \cos^2 \vartheta_{14} \sin^2 \vartheta_{24}$$

New mixings in the 3+1 scenario

4 × 4 mixing matrix:

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atmospheric

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APPEARance

$$P_{\nu_{\alpha} \rightarrow \nu_{\beta}}^{\text{SBL}(-)} \simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

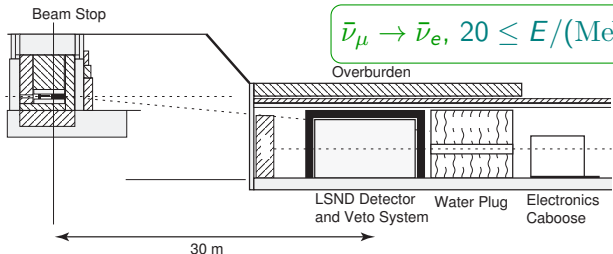
$$\nu_{\mu}^{(-)} \rightarrow \nu_e^{(-)}$$

LSND
MiniBooNE
KARMEN
OPERA
...

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2 |U_{\mu 4}|^2$$

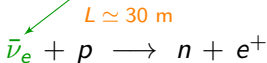
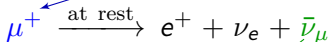
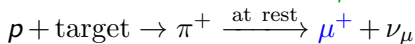
quadratically suppressed!

for small $|U_{e4}|^2$, $|U_{\mu 4}|^2$



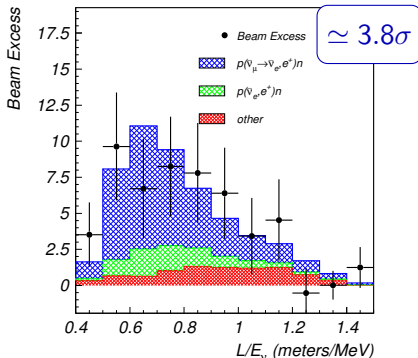
$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e, 20 \leq E/(\text{MeV}) \leq 52.8$$

well known source of $\bar{\nu}_\mu$:



No signal seen in KARMEN ($L \simeq 18 \text{ m}$)

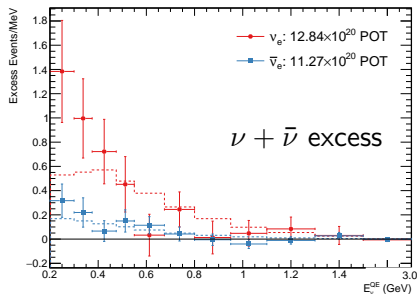
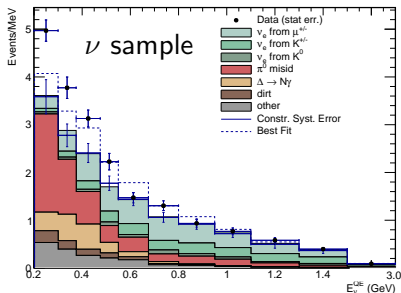
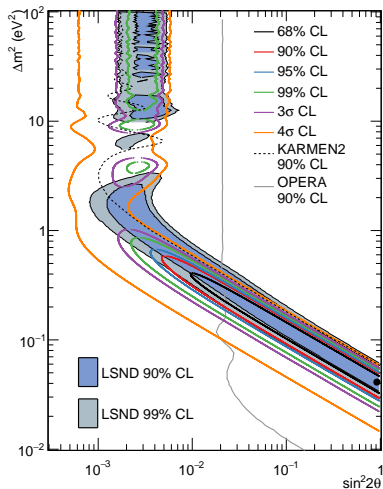
[PRD 65 (2002) 112001]



purpose: check LSND signal

$L \simeq 541$ m, $200 \text{ MeV} \leq E \lesssim 3 \text{ GeV}$

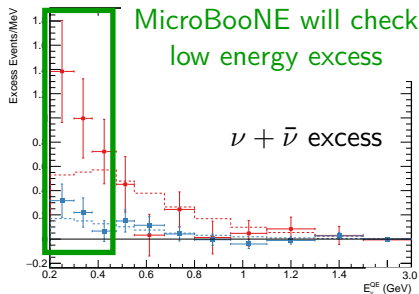
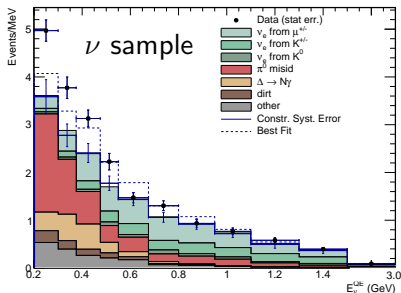
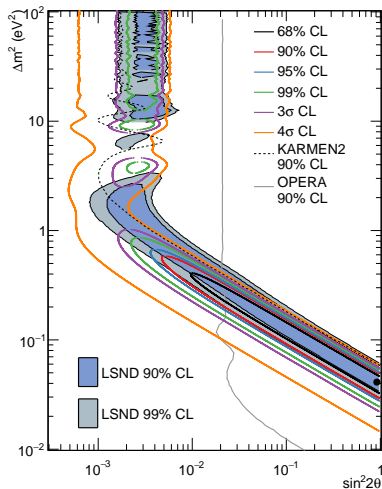
no money, no near detector

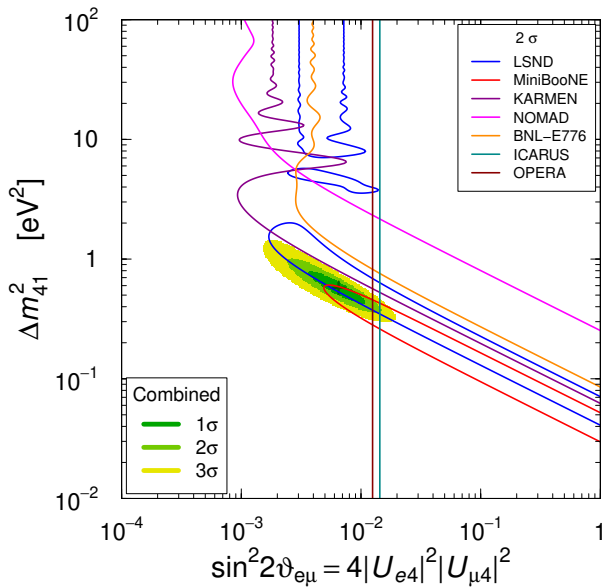


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no money, no near detector





with full MiniBooNE data

ICARUS and OPERA

exclude

MiniBooNE best fit

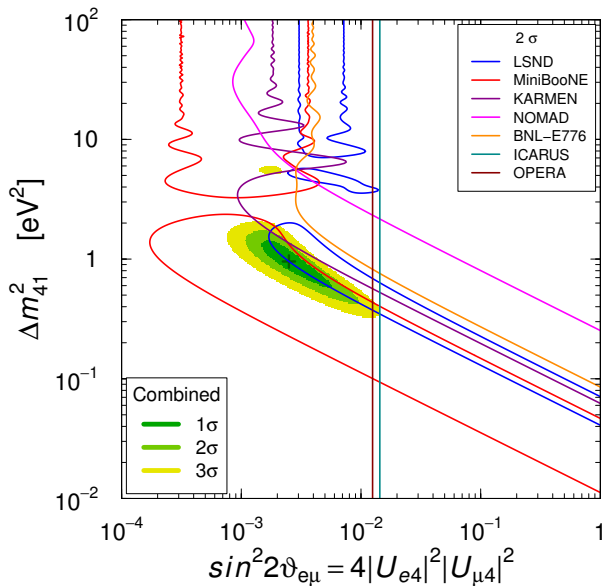
LSND and MiniBooNE

only partially

in agreement

KARMEN cuts part

of LSND region



ICARUS and OPERA

exclude

MiniBooNE best fit

LSND and MiniBooNE

only partially
in agreement

KARMEN cuts part
of LSND region

without MiniBooNE low energy bins

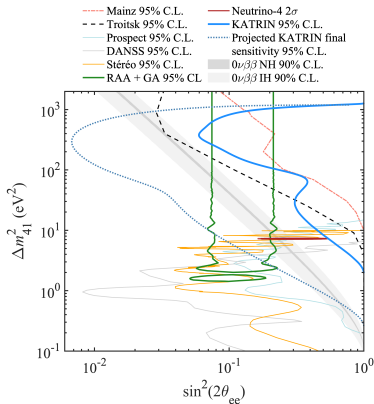
B

Beta/double beta constraints

i.e. non-oscillation probes, first part

Based on:

- KATRIN
- Giunti+ JHEP 2015



β decay



$$Q_\beta = M_i - M_f - m_e$$

total available energy

$$E_\nu = Q_\beta - T = Q_\beta - (E_e - m_e)$$

neutrino energy

notice that max electron energy is:

$$T_{\max} = Q_\beta - m_{\bar{\nu}_e}$$

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Kurie function: (degenerate ν masses)

$$K(T) = \left[(Q_\beta - T) \sqrt{(Q_\beta - T)^2 - m_{\bar{\nu}_e}^2} \right]^{1/2}$$

Useful to describe
the e^- spectrum
near the endpoint

notice: flavor neutrinos have no definite mass!

$$m_{\bar{\nu}_e}^2 = \sum |U_{ei}|^2 m_i^2$$

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Full expression:

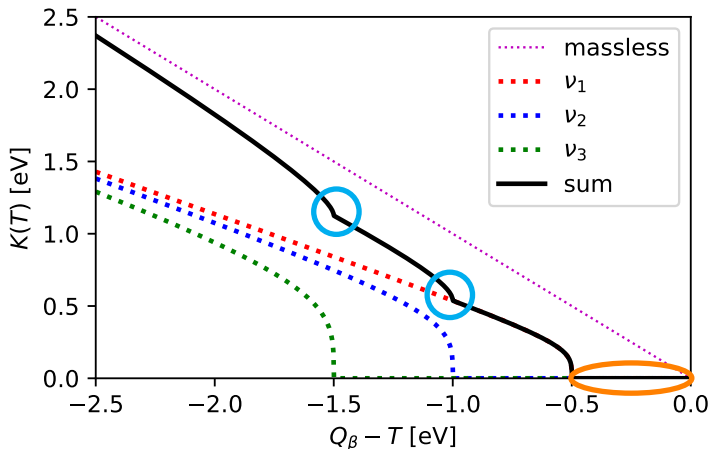
$$K(T) = \left[(Q_\beta - T) \sum_{i=1}^{N_\nu} |U_{ei}|^2 \sqrt{(Q_\beta - T)^2 - m_i^2} \right]^{1/2}$$

N_ν neutrinos
with different
masses m_i

mixing angles
enter ($|U_{ei}|^2$)

β decay

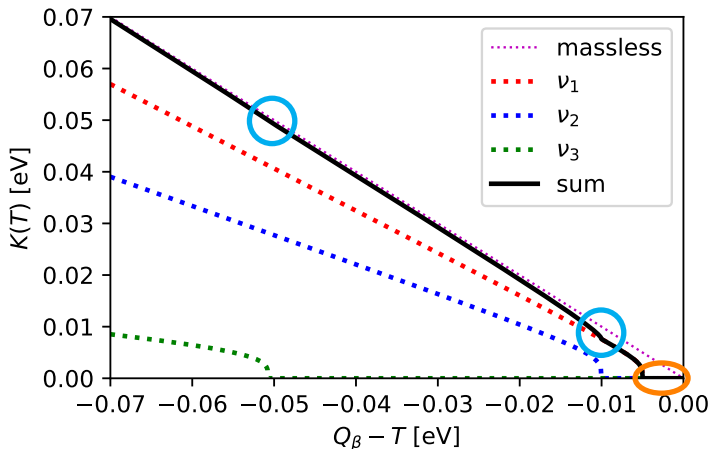
$$K(T) = \left[(Q_\beta - T) \sum_{i=1}^{N_\nu} |U_{ei}|^2 \sqrt{(Q_\beta - T)^2 - m_i^2} \right]^{1/2}$$



Fake case:
3 neutrinos
masses:
 $m_i = i \cdot 0.5$ eV,
mixings:
 $|U_{ei}|^2 = 1/3$

endpoint shifted + one kink for each mass eigenstate

$$K(T) = \left[(Q_\beta - T) \sum_{i=1}^{N_\nu} |U_{ei}|^2 \sqrt{(Q_\beta - T)^2 - m_i^2} \right]^{1/2}$$



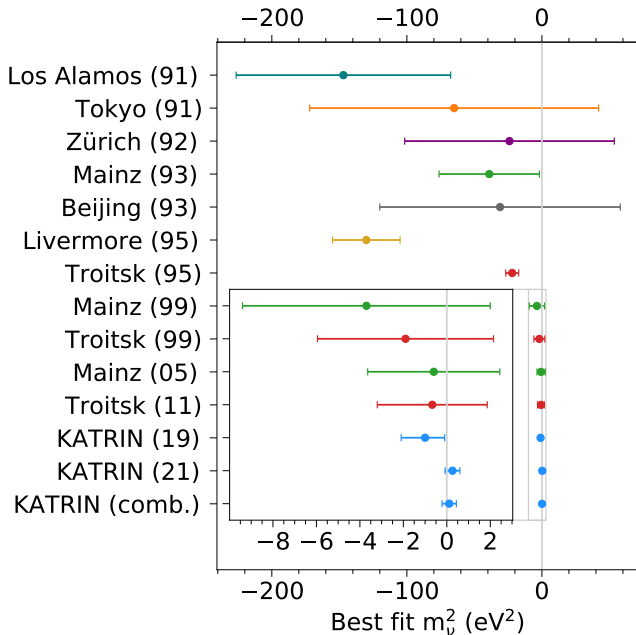
Realistic case:

3 neutrinos,
normal
ordering

masses: $m_i =$
[5, 10, 51] meV,

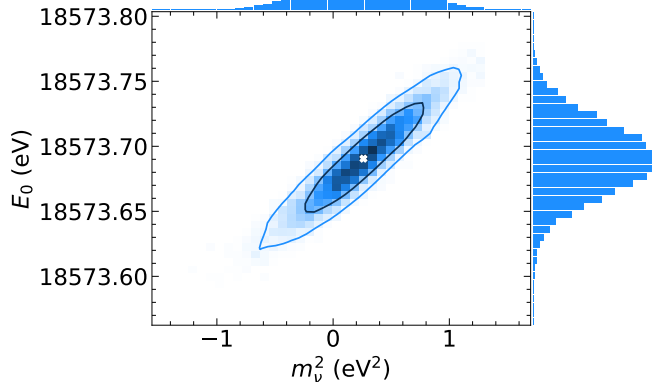
mixings:
 $|U_{ei}|^2 =$
[0.67, 0.31, 0.02]

Much harder to see the endpoint shift and kinks!



strongest bound on $m_\nu (\equiv m_{\bar{\nu}_e})$ are from KATRIN

[Katrin Neutrino Mass 2]



KNM1+KNM2:
 $m_\nu^2 = (0.1 \pm 0.3) \text{ eV}^2$

Upper limit 90%:

$$m_\nu < 0.8 \text{ eV}$$

Bayesian 90%:

$$m_\nu < 0.7 \text{ eV}$$

statistics dominated!

expected final
 sensitivity (90%):

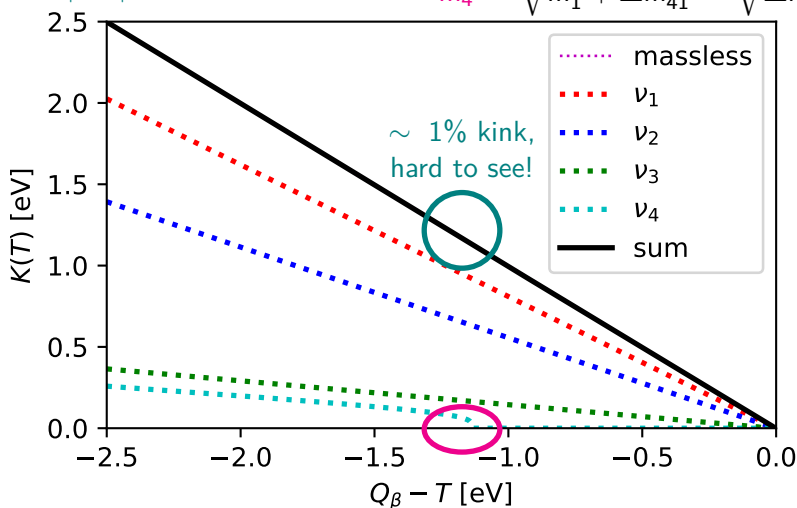
$$m_\nu \lesssim 0.2 \text{ eV}$$

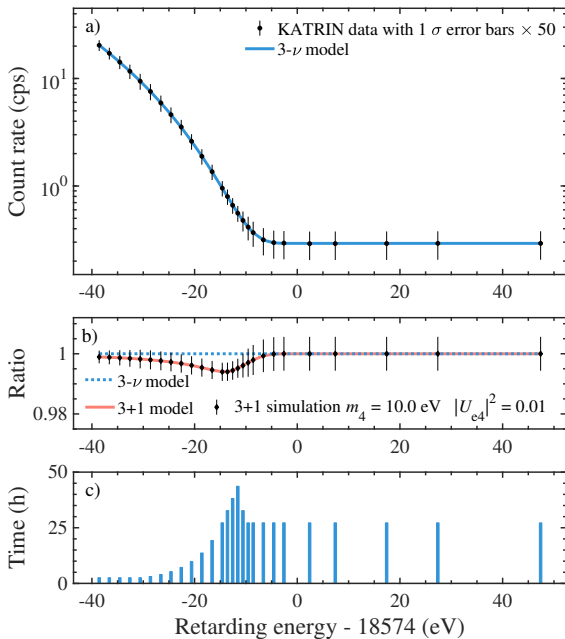
Sterile neutrino in β decay

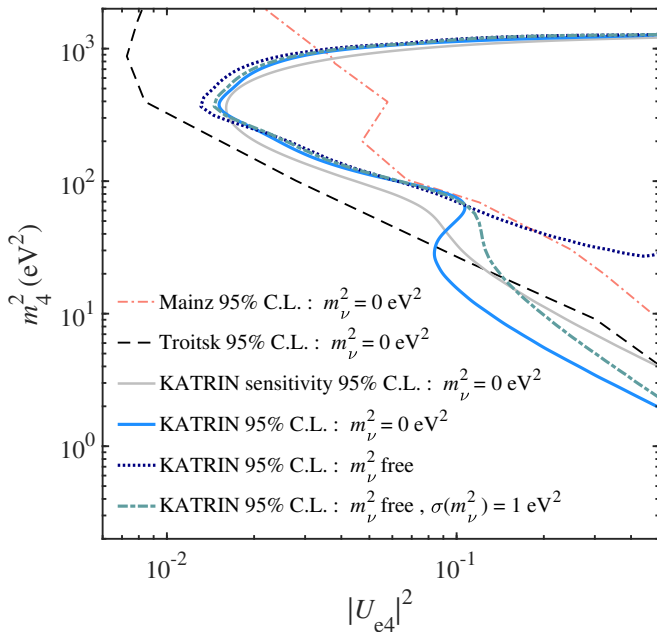
$$K(T) = \left[(Q_\beta - T) \sum_{i=1}^{N_\nu} |U_{ei}|^2 \sqrt{(Q_\beta - T)^2 - m_i^2} \right]^{1/2}$$

$$|U_{e4}|^2 \sim 0.01$$

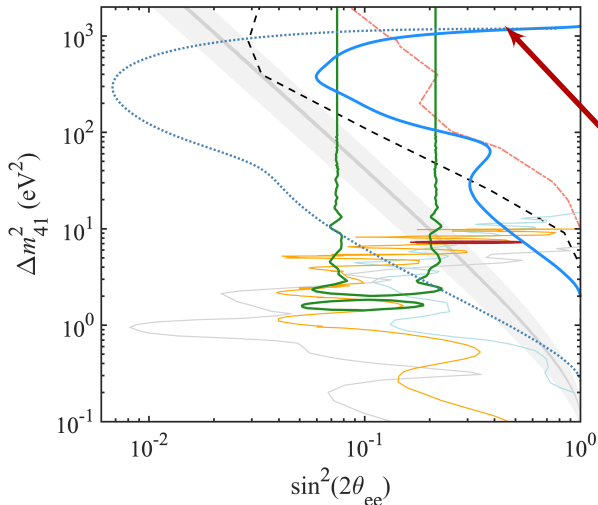
$$m_4 = \sqrt{m_1^2 + \Delta m_{41}^2} \sim \sqrt{\Delta m_{41}^2}$$







- Mainz 95% C.L.
- - - Troitsk 95% C.L.
- Prospect 95% C.L.
- DANSS 95% C.L.
- Stéréo 95% C.L.
- RAA + GA 95% CL
- Neutrino-4 2σ
- KATRIN 95% C.L.
- ⋯ Projected KATRIN final sensitivity 95% C.L.
- $0\nu\beta\beta$ NH 90% C.L.
- $0\nu\beta\beta$ IH 90% C.L.



final sensitivity will test several oscillation results!

search for keV states needs to measure the spectrum much further from the endpoint...

Neutrino masses from neutrinoless double β decay

(if neutrino is Majorana)

[Schechter&Valle, 1982]

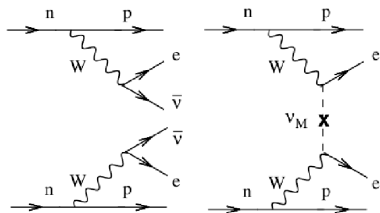
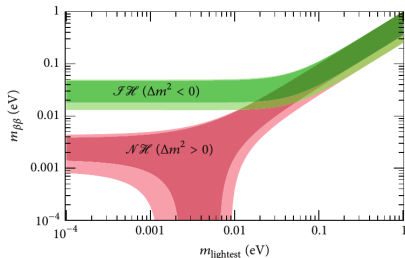
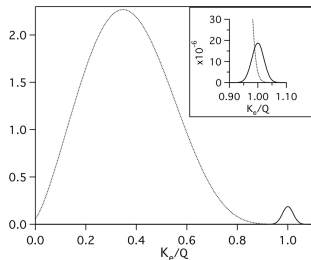


figure from [NEXT] webpage



[Dell'Oro et al., 2016]

Measure $T_{1/2}^{0\nu}$

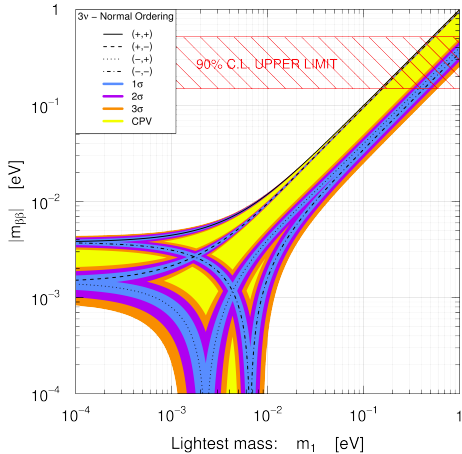
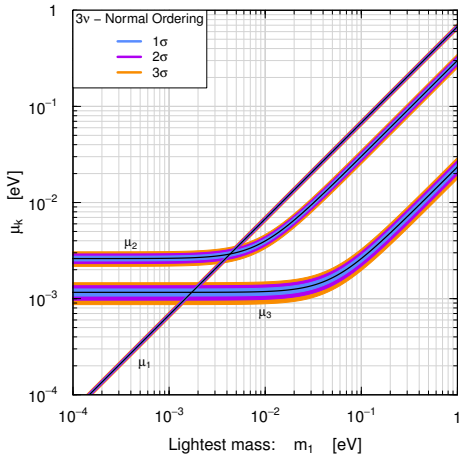
m_e electron mass,
 $G_{0\nu}$ phase space,
 \mathcal{M}'^{ν} matrix element

convert into
$$m_{\beta\beta} = \frac{m_e}{\mathcal{M}'^{\nu} \sqrt{G_{0\nu} T_{1/2}^{0\nu}}}$$

and then use
$$m_{\beta\beta} = \left| \sum_k e^{i\alpha_k} U_{ek}^2 m_k \right|$$

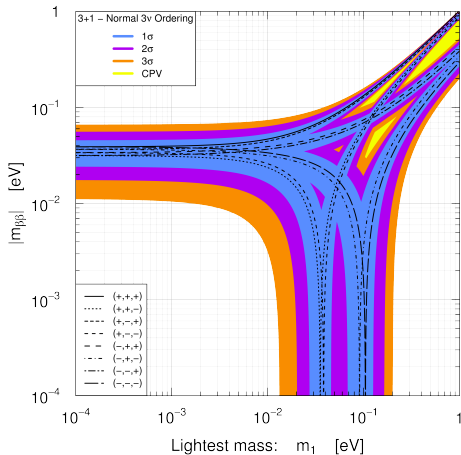
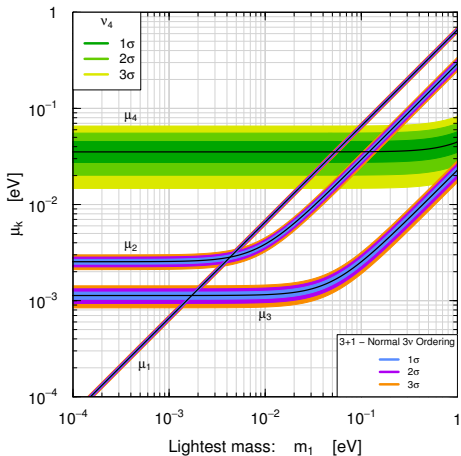
α_k Majorana phases

effective Majorana mass: $m_{\beta\beta} = \left| \sum_k e^{i\alpha_k} \mu_k \right|$, with $\mu_k \equiv U_{ek}^2 m_k$



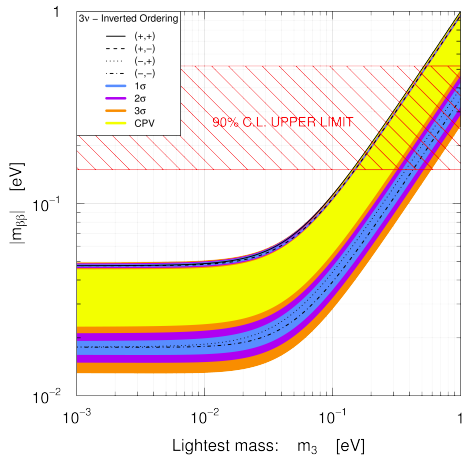
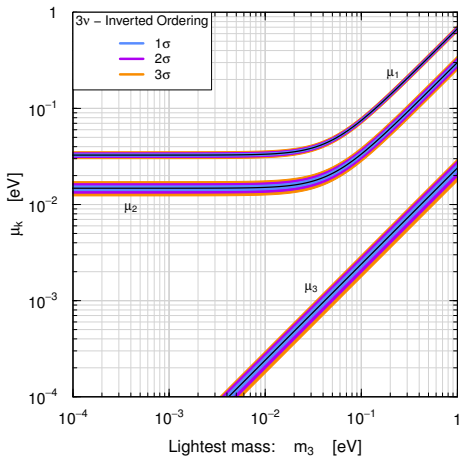
3 neutrinos, normal ordering (NO): $m_1 < m_2 < m_3$, $|U_{e1}| > |U_{e2}| > |U_{e3}|$

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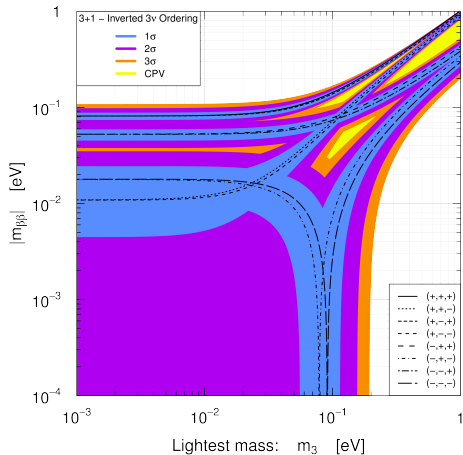
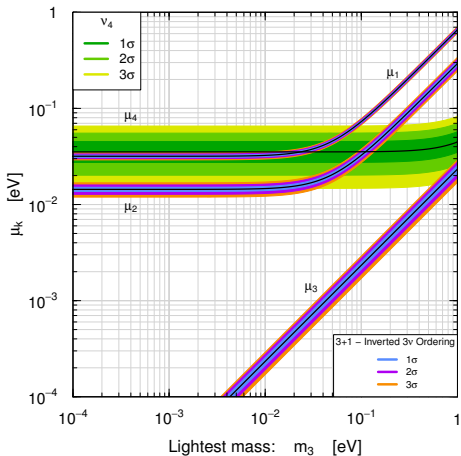
3+1 neutrinos, NO: $m_1 < m_2 < m_3 < m_4$, $|U_{e1}| > |U_{e2}| > |U_{e3}| \gtrsim |U_{e4}|$

effective Majorana mass: $m_{\beta\beta} = \left| \sum_k e^{i\alpha_k} \mu_k \right|$, with $\mu_k \equiv U_{ek}^2 m_k$



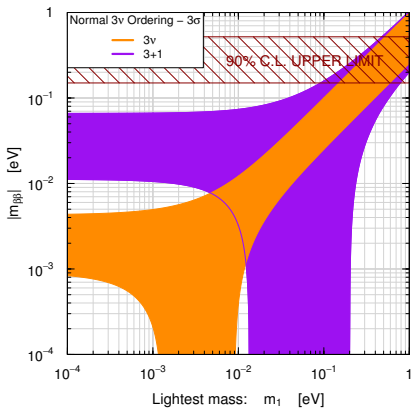
3 neutrinos, inverted ordering (IO): $m_3 < m_1 \lesssim m_2$, $|U_{e1}| > |U_{e2}| > |U_{e3}|$

effective Majorana mass: $m_{\beta\beta} = \left| \sum_k e^{i\alpha_k} \mu_k \right|$, with $\mu_k \equiv U_{ek}^2 m_k$

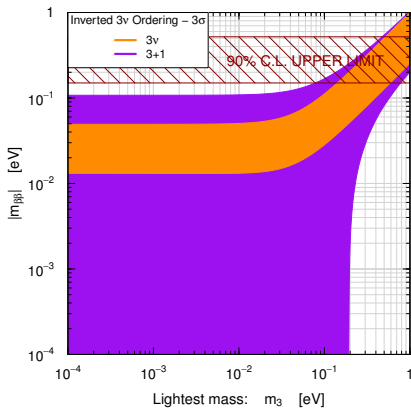


3+1 neutrinos, IO: $m_3 < m_1 \lesssim m_2 < m_4$, $|U_{e1}| > |U_{e2}| > |U_{e3}| \gtrsim |U_{e4}|$

NO for active neutrinos



IO for active neutrinos



one more neutrino completely changes the picture!

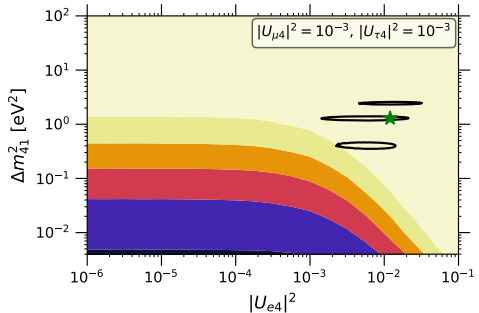
with 3+1 ν s, incoming experiments could see $m_{\beta\beta}$ even if $m_{\text{lightest}} = 0$

C Cosmology

i.e. non-oscillation probes, second part

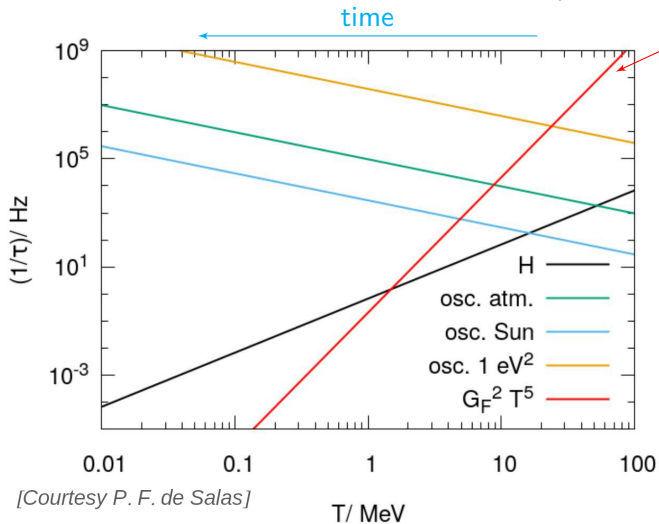
Based on:

- JCAP 04 (2021) 073
- JCAP 07 (2019) 014
- arxiv:2003.02289



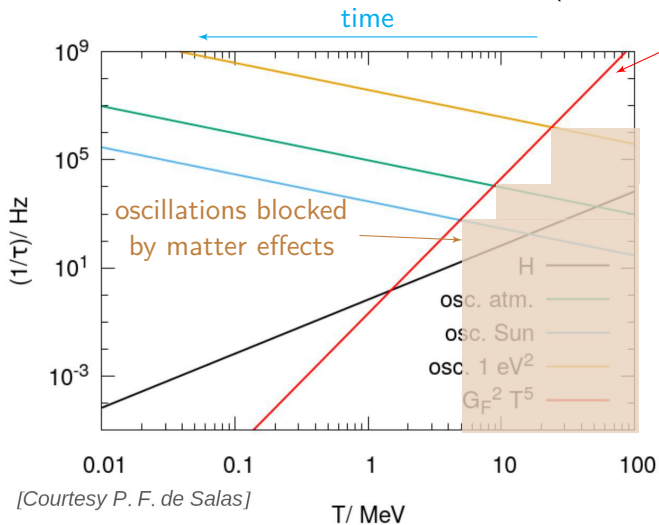
Neutrinos in the early Universe

before BBN: neutrinos coupled to plasma ($\nu_\alpha \bar{\nu}_\alpha \leftrightarrow e^+ e^-$, $\nu e \leftrightarrow \nu e$)



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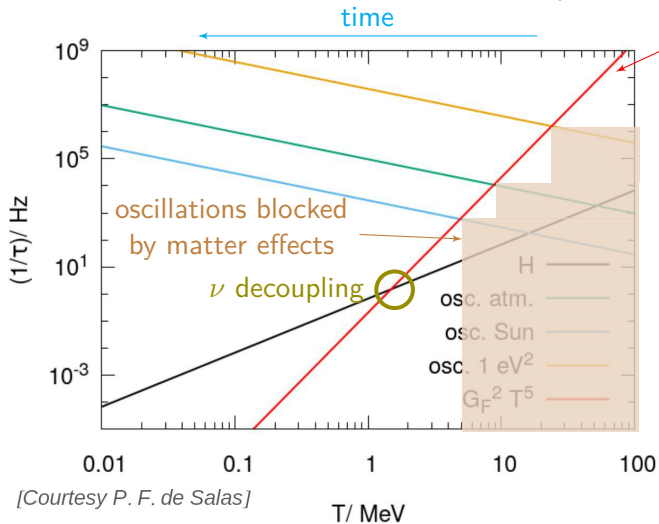
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[Courtesy P. F. de Salas]

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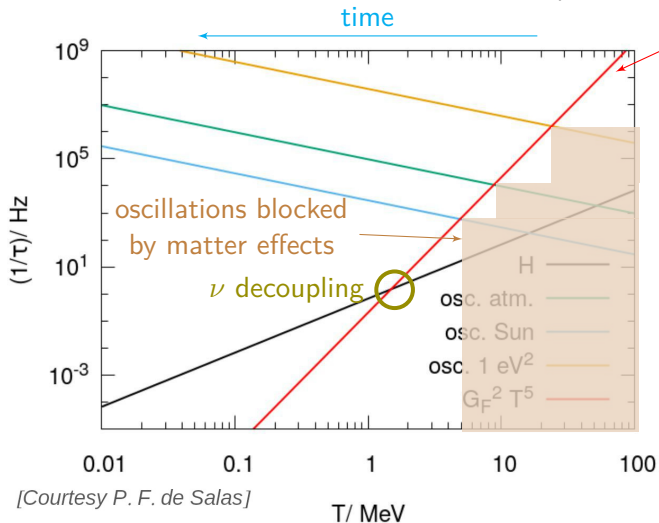


[Courtesy P. F. de Salas]

ν decouple mostly before $e^+ e^- \rightarrow \gamma\gamma$ annihilation!

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$$T_\nu \simeq (4/11)^{1/3} T_\gamma$$

after $e^+ e^- \rightarrow \gamma\gamma$

f_ν : frozen Fermi-Dirac distribution

Today:

$$T_{\nu,0} = 1.945 \text{ K} \simeq 1.676 \times 10^{-4} \text{ eV}$$

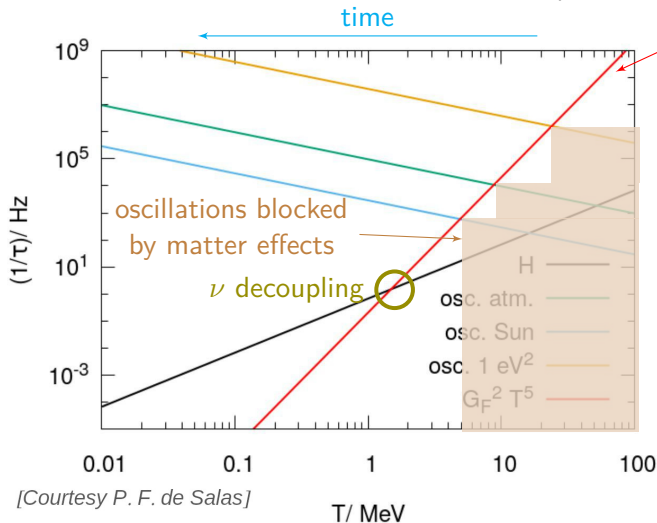
$$\langle E_\nu \rangle \simeq 3.1 T_{\nu,0} \simeq 5 \times 10^{-4} \text{ eV}$$

$$n_0 = n_{\nu,0} = n_{\bar{\nu},0} \simeq 56 \text{ cm}^{-3} \text{ per family}$$

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ν decouple mostly before $e^+ e^- \rightarrow \gamma\gamma$ annihilation!
 actually, the decoupling T is momentum dependent!

distortions to equilibrium f_ν !

comoving coordinates: $a = 1/T$ $x \equiv m_e a$ $y \equiv p a$ $z \equiv T_\gamma a$ $w \equiv T_\nu a$

$$\text{density matrix: } \varrho(x, y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} & \varrho_{es} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} & \varrho_{\mu s} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} & \varrho_{\tau s} \\ \varrho_{se} & \varrho_{s\mu} & \varrho_{s\tau} & \varrho_{ss} \equiv f_{\nu_s} \end{pmatrix}$$

$$\frac{d\varrho(y, x)}{dx} = \sqrt{\frac{3m_{\text{Pl}}^2}{8\pi\rho_T}} \left\{ -i \frac{x^2}{m_e^3} \left[\frac{M_F}{2y} - \frac{2\sqrt{2}G_F y}{x^6/m_e^6} \left(\frac{E_\ell + P_\ell}{m_W^2} + \frac{4E_\nu}{3m_Z^2} \right), \varrho \right] + \frac{m_e^3 G_F^2}{(2\pi)^3 x^4 y^2} \mathcal{I}(\varrho) \right\}$$

m_{Pl} Planck mass – ρ_T total energy density – $m_{W,Z}$ mass of the W, Z bosons – G_F Fermi constant – $[\cdot, \cdot]$ commutator

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$$\mathbb{M}_F = \mathbf{U} \mathbf{M} \mathbf{U}^\dagger$$

$$\mathbf{M} = \text{diag}(m_1^2, \dots, m_N^2)$$

$$\mathbf{U} = R^{34} R^{24} R^{14} R^{23} R^{13} R^{12} \quad \text{e.g. } R^{13} = \begin{pmatrix} \cos \theta_{13} & 0 & \sin \theta_{13} & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta_{13} & 0 & \cos \theta_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$|\mathbf{U}|^2 = \begin{pmatrix} \dots & \dots & \dots & \sin^2 \theta_{14} \\ \dots & \dots & \dots & \cos^2 \theta_{14} \sin^2 \theta_{24} \\ \dots & \dots & \dots & \cos^2 \theta_{14} \cos^2 \theta_{24} \sin^2 \theta_{34} \\ \dots & \dots & \dots & \cos^2 \theta_{14} \cos^2 \theta_{24} \cos^2 \theta_{34} \end{pmatrix}$$

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$$\mathbb{E}_\ell = \text{diag}(\rho_e, \rho_\mu, 0, 0) \quad \mathbb{E}_\nu = S_a \left(\int dy y^3 \varrho \right) S_a \quad \text{with } S_a = \text{diag}(1, 1, 1, 0)$$

lepton densities

neutrino densities

(only for active neutrinos)

take into account matter effects in oscillations

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$\mathcal{I}(\varrho)$ collision integrals

take into account neutrino-electron scattering and pair annihilation,
plus neutrino-neutrino interactions

2D integrals over momentum, take most of the computation time

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$\mathcal{I}(\varrho)$ collision integrals

from continuity
equation

$$\dot{\rho} = -3H(\rho + P)$$

$$\frac{dz}{dx} = \frac{\sum_{\ell=e,\mu} \left[\frac{r_\ell^2}{r} J(r_\ell) \right] + G_1(r) - \frac{1}{2\pi^2 z^3} \int_0^\infty dy y^3 \sum_{\alpha=e}^s \frac{d\varrho_{\alpha\alpha}}{dx}}{\sum_{\ell=e,\mu} \left[r_\ell^2 J(r_\ell) + Y(r_\ell) \right] + G_2(r) + \frac{2\pi^2}{15}}$$

$r = x/z$, $r_\ell = m_\ell/m_e r$ $J(r)$, $Y(r)$ from non-relativistic transition of e^\pm , μ^\pm
 $G_1(r)$ and $G_2(r)$ from electromagnetic corrections

comoving coordinates: $a = 1/T$ $x \equiv m_e a$ $y \equiv p a$ $z \equiv T_\gamma a$ $w \equiv T_\nu a$

$$\text{density matrix: } \varrho(x, y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} & \varrho_{es} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} & \varrho_{\mu s} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} & \varrho_{\tau s} \\ \varrho_{se} & \varrho_{s\mu} & \varrho_{s\tau} & \varrho_{ss} \equiv f_{\nu_s} \end{pmatrix}$$

$$\frac{d\varrho(y, x)}{dx} = \sqrt{\frac{3m_{\text{Pl}}^2}{8\pi\rho_T}} \left\{ -i \frac{x^2}{m_e^3} \left[\frac{\mathbb{M}_F}{2y} - \frac{2\sqrt{2}G_F y}{x^6/m_e^6} \left(\frac{\mathbb{E}_e + \mathbb{P}_e}{m_W^2} + \frac{4\mathbb{E}_\nu}{3m_Z^2} \right), \varrho \right] + \frac{m_e^3 G_F^2}{(2\pi)^3 x^4 y^2} \mathcal{I}(\varrho) \right\}$$

m_{Pl} Planck mass – ρ_T total energy density – $m_{W,Z}$ mass of the W, Z bosons – G_F Fermi constant – $[\cdot, \cdot]$ commutator

$$\mathbb{M}_F = U \mathbb{M} U^\dagger \quad \mathbb{E}_e = \text{diag}(\rho_e, \rho_\mu, 0, 0) \quad \mathbb{E}_\nu = S_a \left(\int dy y^3 \varrho \right) S_a$$

$\mathcal{I}(\varrho)$ collision integrals

from continuity
equation
 $\dot{\rho} = -3H(\rho + P)$

$$\frac{dz}{dx} = \frac{\sum_{\ell=e,\mu} \left[\frac{r_\ell^2}{r} J(r_\ell) \right] + G_1(r) - \frac{1}{2\pi^2 z^3} \int_0^\infty dy y^3 \sum_{\alpha=e}^s \frac{d\varrho_{\alpha\alpha}}{dx}}{\sum_{\ell=e,\mu} \left[r_\ell^2 J(r_\ell) + Y(r_\ell) \right] + G_2(r) + \frac{2\pi^2}{15}}$$

neutrino temperature w : same equation as z , but electrons always relativistic

comoving coordinates: $a = 1/T$ $x \equiv m_e a$ $y \equiv p a$ $z \equiv T_\gamma a$ $w \equiv T_\nu a$

$$\text{density matrix: } \varrho(x, y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} & \varrho_{es} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} & \varrho_{\mu s} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} & \varrho_{\tau s} \\ \varrho_{se} & \varrho_{s\mu} & \varrho_{s\tau} & \varrho_{ss} \equiv f_{\nu_s} \end{pmatrix}$$

$$\frac{d\varrho(y, x)}{dx} = \sqrt{\frac{3m_{\text{Pl}}^2}{8\pi\rho_T}} \left\{ -i \frac{x^2}{m_e^3} \left[\frac{\mathbb{M}_F}{2y} - \frac{2\sqrt{2}G_F y}{x^6/m_e^6} \left(\frac{\mathbb{E}_e + \mathbb{P}_e}{m_W^2} + \frac{4\mathbb{E}_\nu}{3m_Z^2} \right), \varrho \right] + \frac{m_e^3 G_F^2}{(2\pi)^3 x^4 y^2} \mathcal{I}(\varrho) \right\}$$

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neutrino temperature w : same equation as z , but electrons always relativistic
initial conditions: $\varrho_{\alpha\alpha} = \text{Fermi-Dirac}$ at $x_{\text{in}} \simeq 0.001$, with $w = z \simeq 1$

comoving coordinates: $a = 1/T$ $x \equiv m_e a$ $y \equiv p a$ $z \equiv T_\gamma a$ $w \equiv T_\nu a$

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$$\frac{d\varrho(y, x)}{dx} = \sqrt{\frac{3m_{\text{Pl}}^2}{8\pi\rho_T}} \left\{ -i \frac{x^2}{m_e^3} \left[\frac{M_{\text{F}}}{2y} - \frac{2\sqrt{2}G_{\text{F}}y}{x^6/m_e^6} \left(\frac{E_\ell + P_\ell}{m_W^2} + \frac{4E_\nu}{3m_Z^2} \right), \varrho \right] + \frac{m_e^3 G_{\text{F}}^2}{(2\pi)^3 x^4 y^2} \mathcal{I}(\varrho) \right\}$$

FORTran-Evolved Primordial Neutrino Oscillations (FortEPiano)

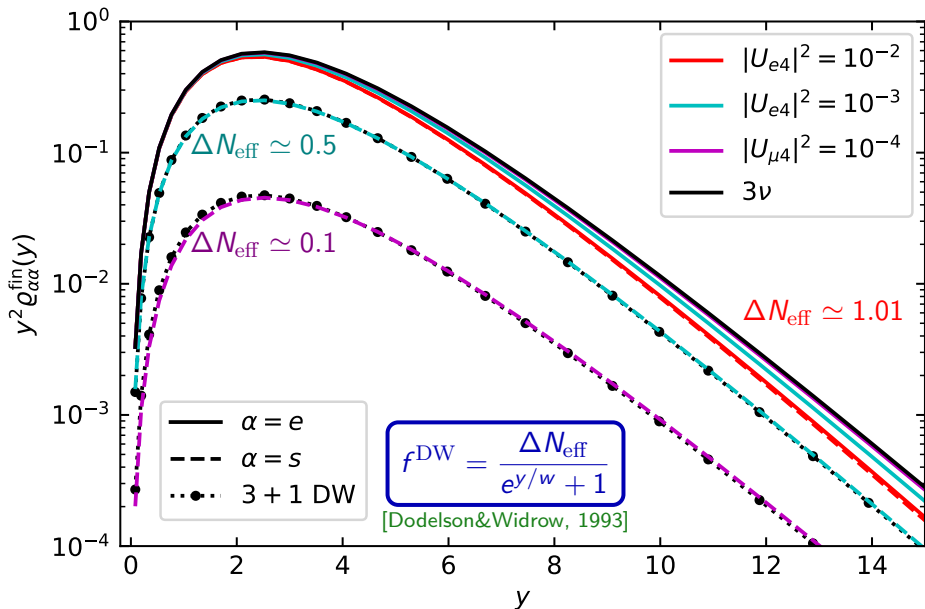
https://bitbucket.org/ahep_cosmo/fortepiano_public

from continuity
equation
 $\dot{\rho} = -3H(\rho + P)$

$$\frac{dz}{dx} = \frac{\sum_{\ell=e,\mu} \left[\frac{r_\ell^2}{r} J(r_\ell) \right] + G_1(r) - \frac{1}{2\pi^2 z^3} \int_0^\infty dy y^3 \sum_{\alpha=e}^s \frac{d\varrho_{\alpha\alpha}}{dx}}{\sum_{\ell=e,\mu} \left[r_\ell^2 J(r_\ell) + Y(r_\ell) \right] + G_2(r) + \frac{2\pi^2}{15}}$$

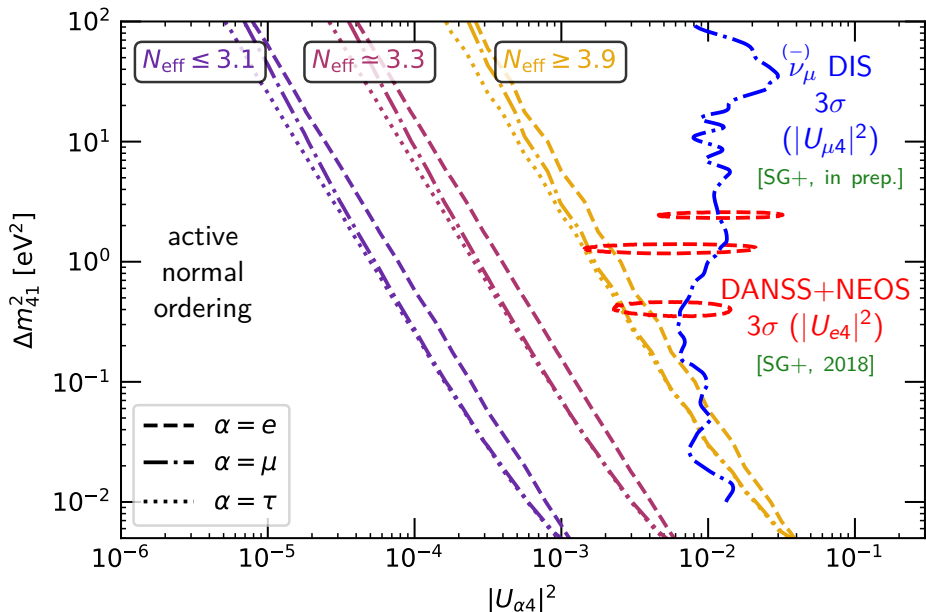
neutrino temperature w : same equation as z , but electrons always relativistic
initial conditions: $\varrho_{\alpha\alpha} = \text{Fermi-Dirac}$ at $x_{\text{in}} \simeq 0.001$, with $w = z \simeq 1$

$$\Delta m_{41}^2 = 1.29 \text{ eV}^2, \text{ other } |U_{\beta 4}|^2 = 0, \Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{active}}$$



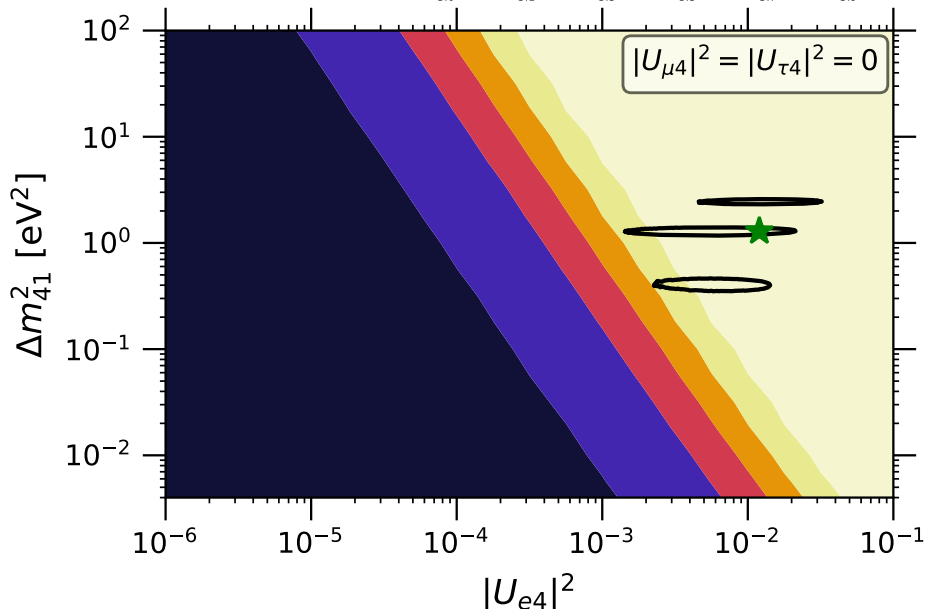
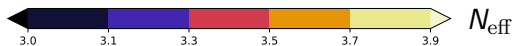
N_{eff} and the new mixing parameters

Only vary one angle and fix two to zero: do they have the same effect?



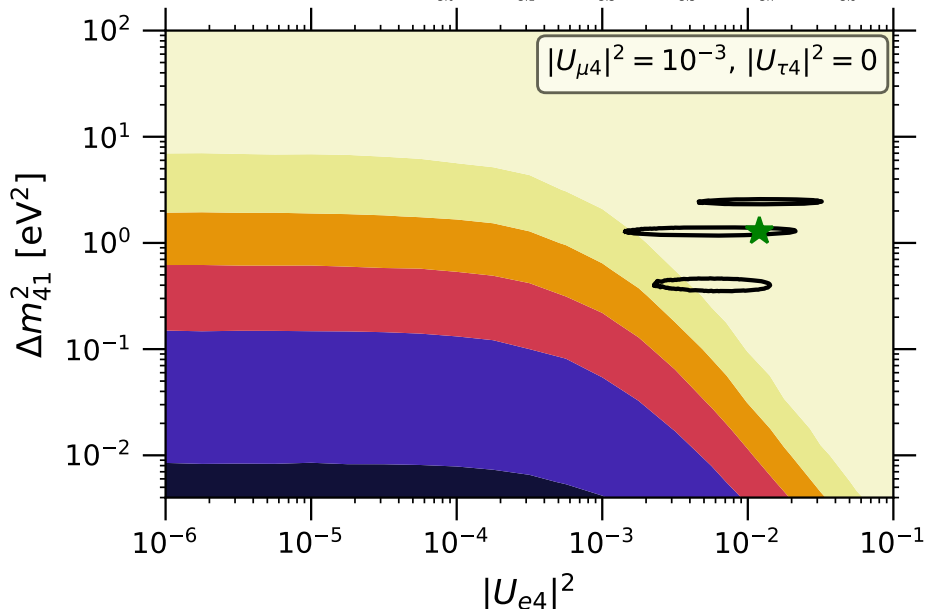
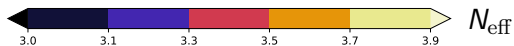
N_{eff} and the new mixing parameters

We can vary more than one angle:



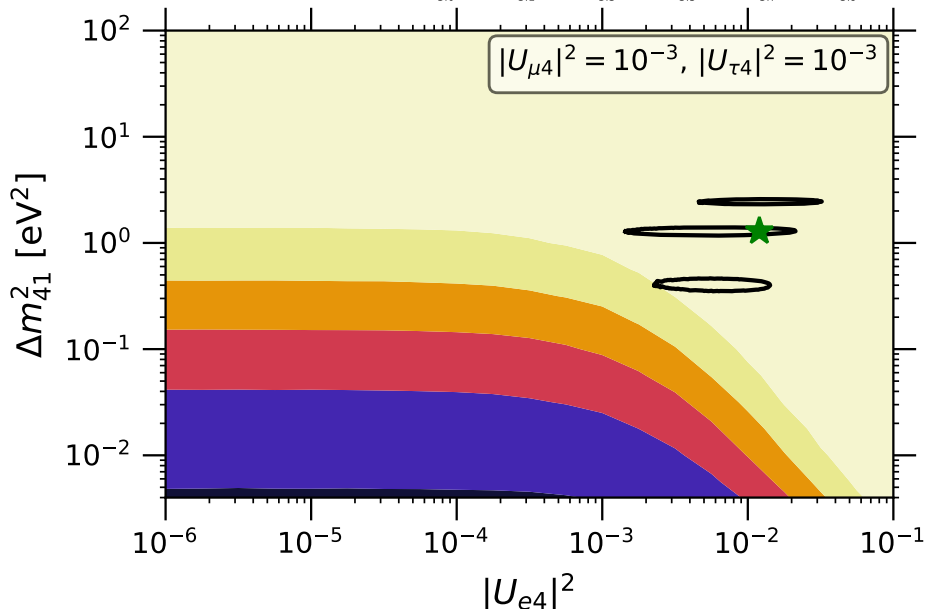
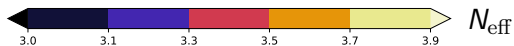
N_{eff} and the new mixing parameters

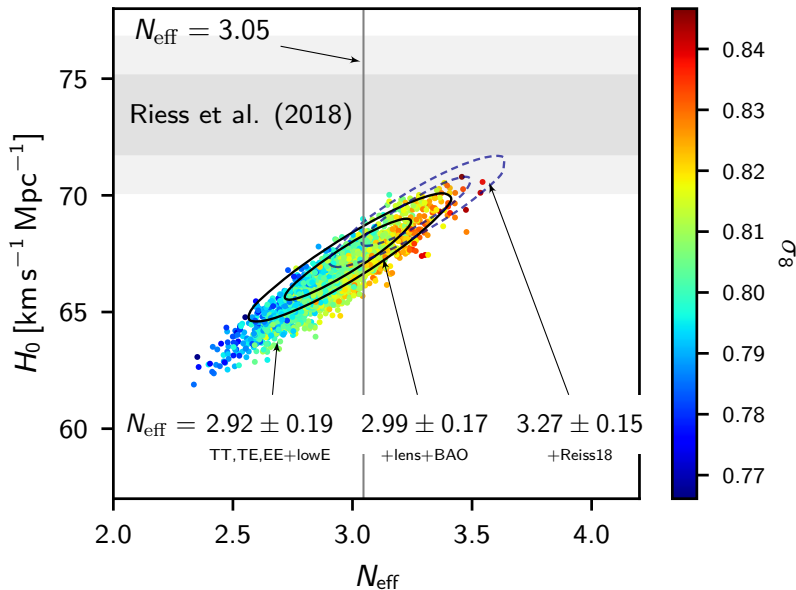
We can vary more than one angle:



N_{eff} and the new mixing parameters

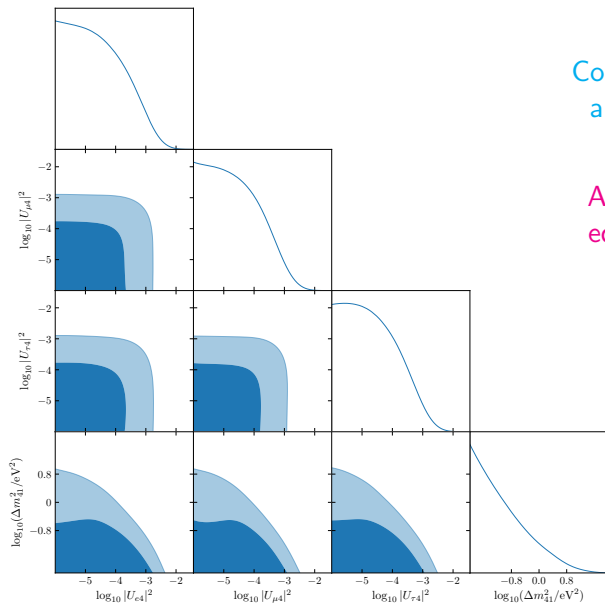
We can vary more than one angle:





Cosmological constraints on $|U_{\alpha 4}|^2$

Use multi-angle results from FortEPiANO to derive constraints on $|U_{\alpha 4}|^2$:



Constraints come from N_{eff}
and late-time density Ω_s

Angles $|U_{\alpha 4}|^2$ are almost
equivalent for cosmology

Prevent ν_s thermalization?

oscillation parameters suggest $\Delta N_{\text{eff}} \simeq 1$ [SG+, 2019]

is there a way to suppress ν_s contribution to N_{eff} ?

suppress oscillations/reduce ΔN_{eff}

large lepton asymmetry

[Foot+1995, Mirizzi+2012, many more]

new neutrino interactions [Bento+2001,

Dasgupta+2014, Hannestad+2014, Sa-

viano+2014, Dentler+2019, de Gouvea+2019,

Moulai+2019, Fischer+2019, Diaz+2019,

Liao+2019, Archidiacono+2020, many more]

very low reheating temperature

[Gelmini+2004, Smirnov+2006, deSalas+2015,

in preparation]

compensate effects of $\Delta N_{\text{eff}} \simeq 1$

time varying dark energy

components [Giusarma+2012]

larger expansion rate at the time

of ν_s production [Rehagen+2014]

freedom in the Primordial Power Spectrum (PPS) of scalar perturbations from inflation compensate damping due to ΔN_{eff} [SG+2015]

These are just some ideas (incomplete list!)

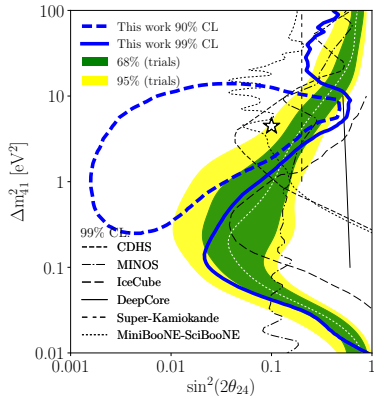
D

Disappearance (Muon channel)

strong constraints, and a recent first hint

Based on:

- IceCube 2016
- DeepCore
- Minos/Minos+
- in preparation
- IceCube 2020



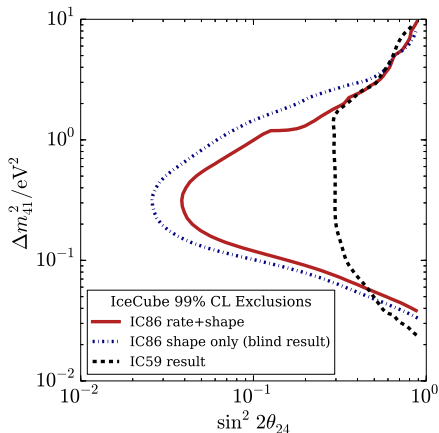
IceCube and DeepCore

IceCube

$\mathcal{O}(10 \text{ km}) \lesssim L \lesssim \mathcal{O}(10^4 \text{ km})$

$\sim 2 \times 10^4$ High energy μ events

$320 \text{ GeV} < E < 20 \text{ TeV}$



[PRL 117 (2016) 071801]

IceCube and DeepCore

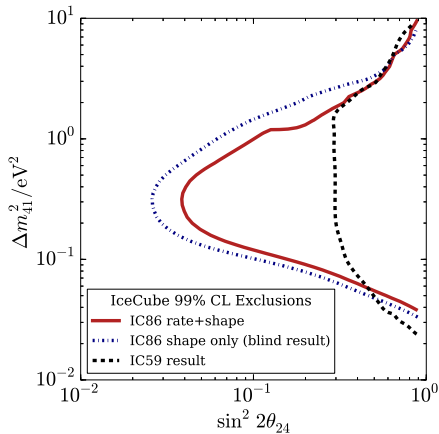
IceCube

$\mathcal{O}(10 \text{ km}) \lesssim L \lesssim \mathcal{O}(10^4 \text{ km})$

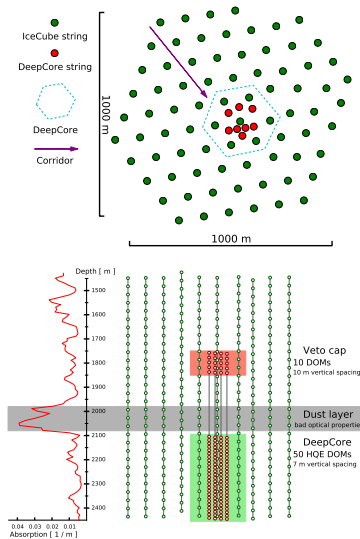
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IceCube and DeepCore

IceCube

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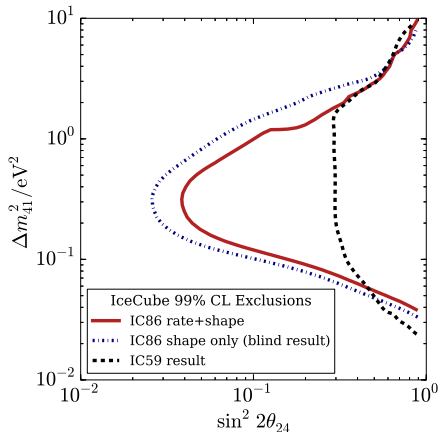
DeepCore

$\sim 2 \times 10^4$ High energy μ events

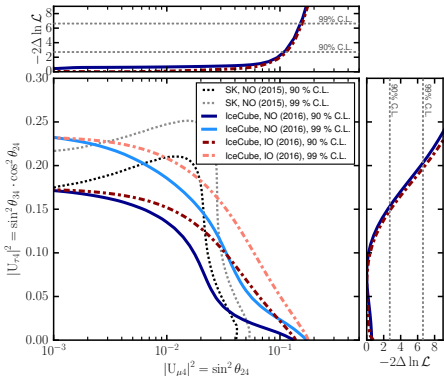
$320 \text{ GeV} < E < 20 \text{ TeV}$

$\sim 5 \times 10^3$ tracklike events

$6 \text{ GeV} \lesssim E \lesssim 60 \text{ GeV}$



[PRL 117 (2016) 071801]

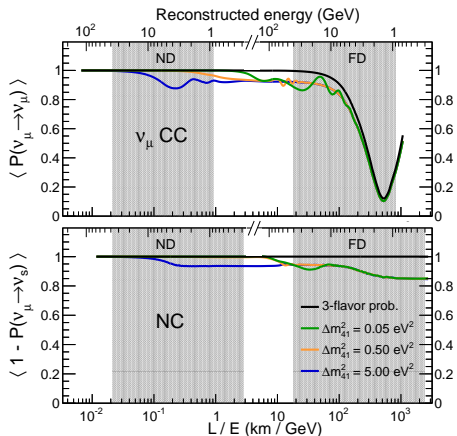


[PRD 95 (2017) 112002]

Both also constrain $|U_{\tau 4}|^2$

Near (ND, $\simeq 500$ m) and
far (FD, $\simeq 800$ km) detector

$1 \text{ GeV} \lesssim E \lesssim 40 \text{ GeV}$,
peak at 3 GeV



[PRL 117 (2016) 151803]:

far-to-near ratio

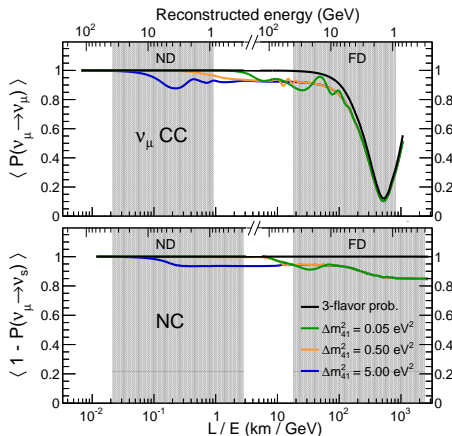
[PRL 122 (2019) 091803]:

full two-detectors fit

MINOS & MINOS+

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peak at 3 GeV

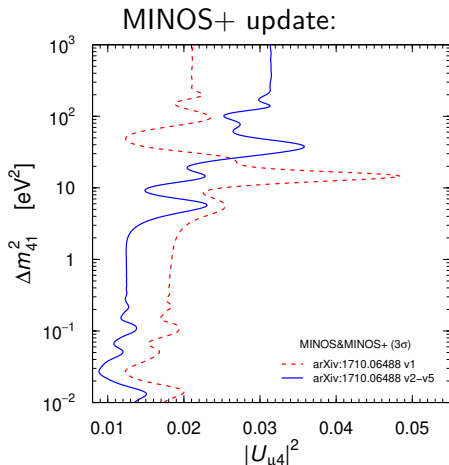


[PRL 117 (2016) 151803]:

far-to-near ratio

[PRL 122 (2019) 091803]:

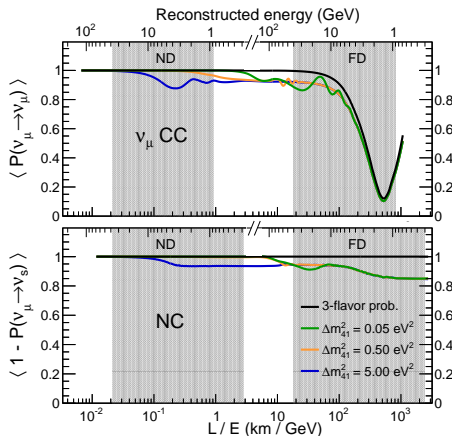
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[SG+, in preparation]

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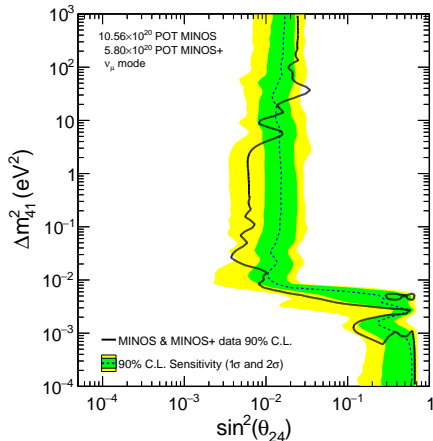
[PRL 117 (2016) 151803]:

far-to-near ratio

[PRL 122 (2019) 091803]:

full two-detectors fit

Sensitivity and exclusion limit:

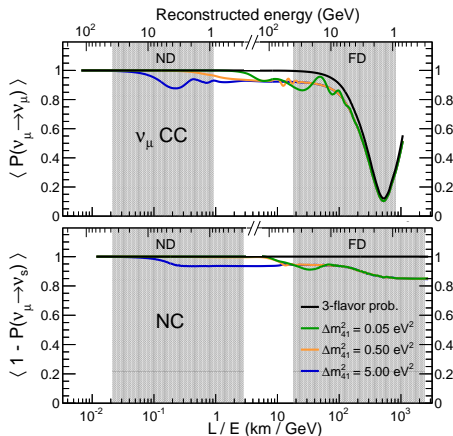


[PRL 122 (2019) 091803]

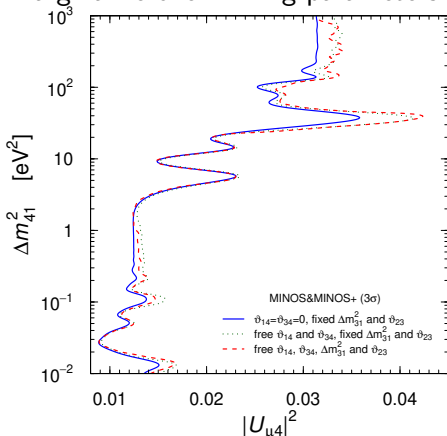
MINOS & MINOS+

Near (ND, $\simeq 500$ m) and
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peak at 3 GeV



Marginalize over mixing parameters:



[PRL 117 (2016) 151803]:

far-to-near ratio

[PRL 122 (2019) 091803]:

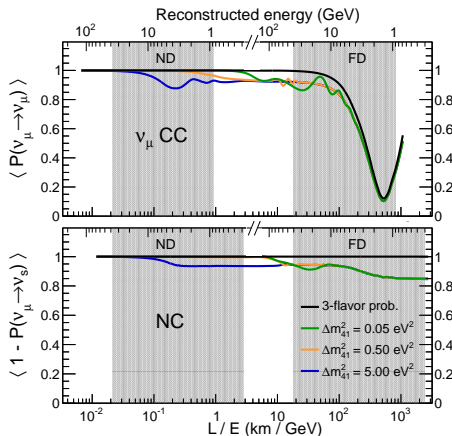
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[SG+, in preparation]

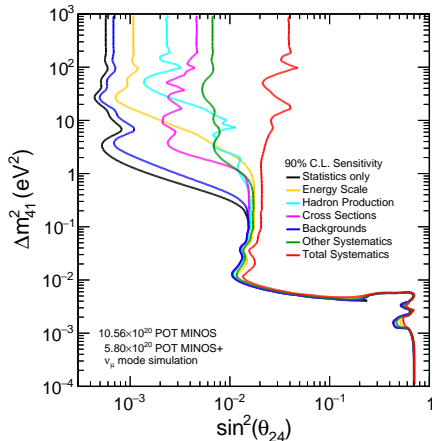
MINOS & MINOS+

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far (FD, $\simeq 800$ km) detector

$1 \text{ GeV} \lesssim E \lesssim 40 \text{ GeV}$,
peak at 3 GeV



Systematics:



[PRL 117 (2016) 151803]:

far-to-near ratio

[PRL 122 (2019) 091803]:

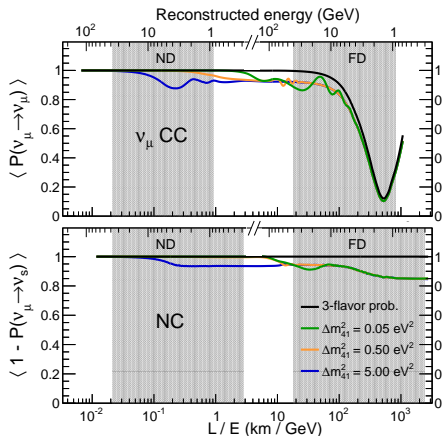
full two-detectors fit

[PRL 122 (2019) 091803]

MINOS & MINOS+

Near (ND, $\simeq 500$ m) and
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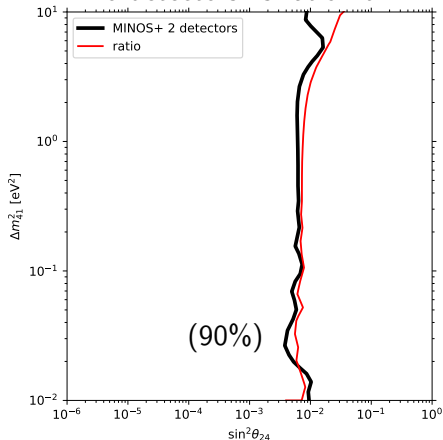
[PRL 117 (2016) 151803]:

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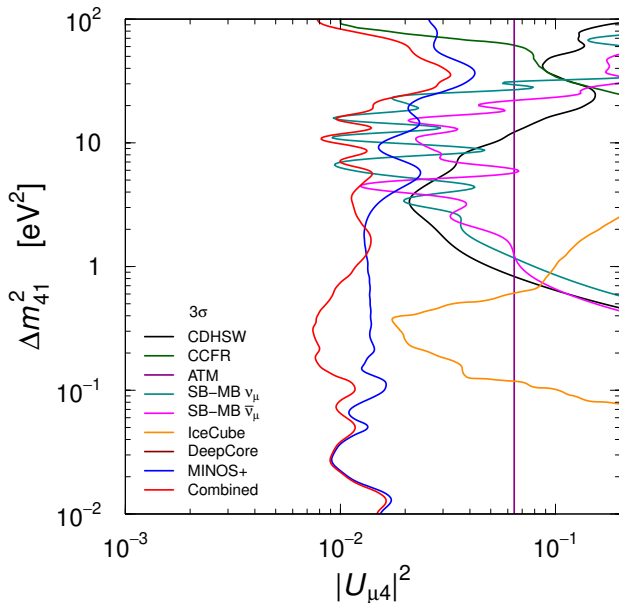
[PRL 122 (2019) 091803]:

full two-detectors fit

Two detectors vs ratio fit:



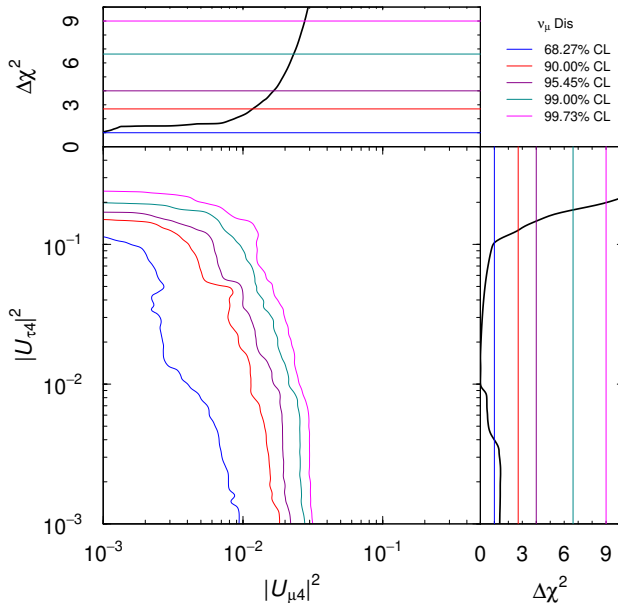
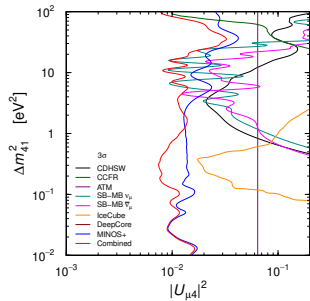
[SG+, in preparation]

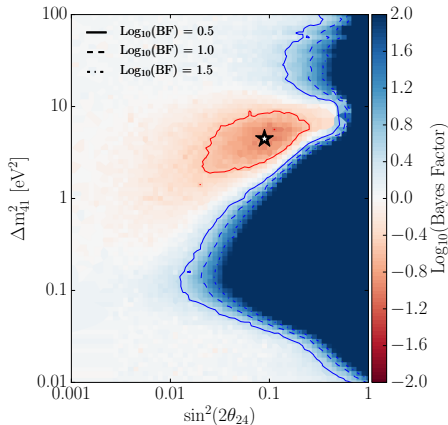
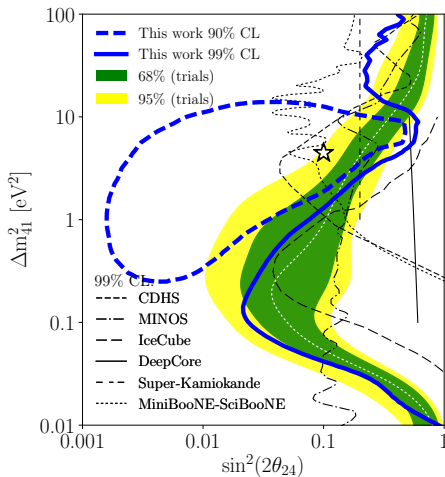


MINOS+
dominates
at small Δm_{41}^2

IceCube (1 yr)
important at
 $\Delta m_{41}^2 \simeq 0.2$ eV²

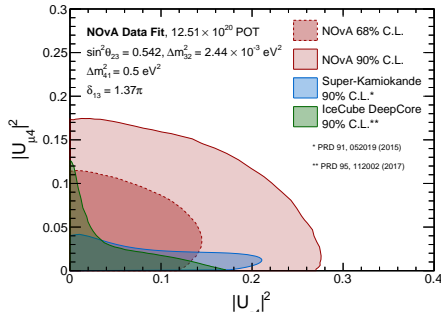
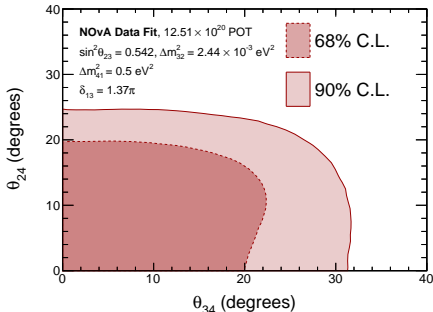
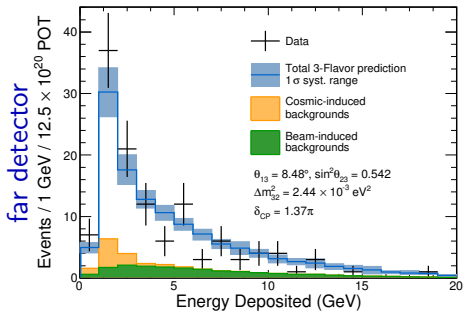
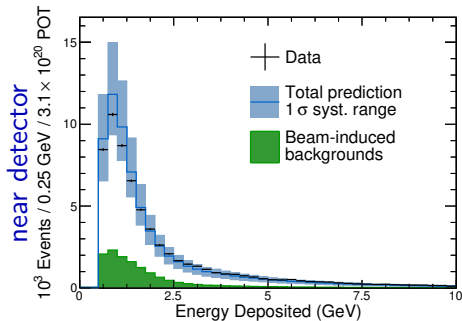
see later for
IceCube 8 yr!





first indication in favor of sterile from ν_μ DIS!

although rather weak: $\log_{10} BF \simeq 1$ (weak preference)
 or compatible with no oscillations at p -value of 8%



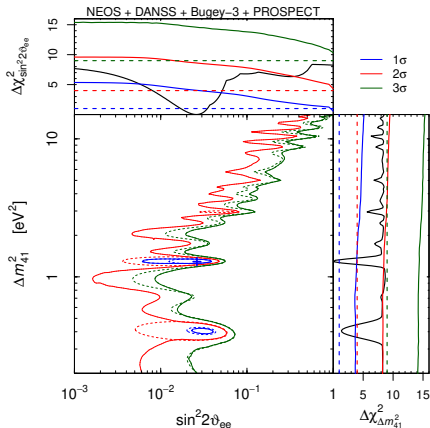
E

disappearance (Electron channel)

reactor and Gallium experiments

Based on:

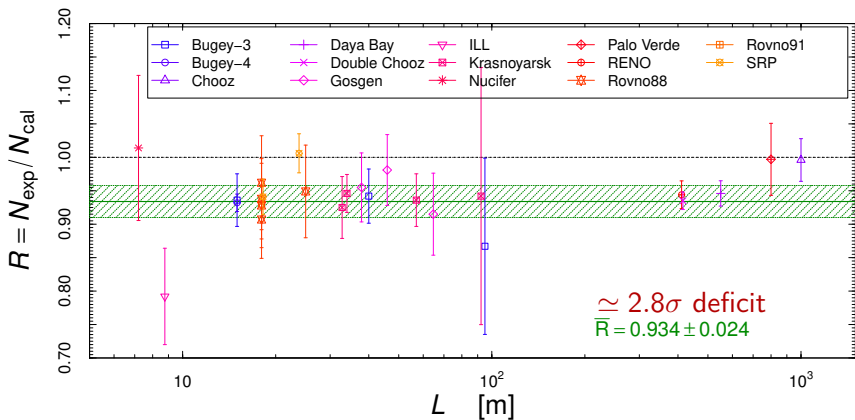
- JPG 43 (2016) 033001
- Neutrino4
- Giunti+ 2020/2021
- Kostensalo+ 2019
- RENO
- DayaBay
- PLB 782 (2018)



2011: new reactor $\bar{\nu}_e$ fluxes by Huber and Mueller+ (HM)

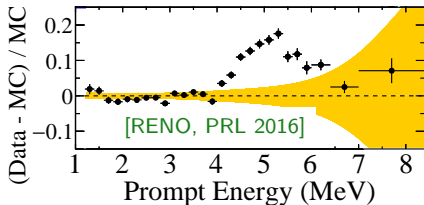
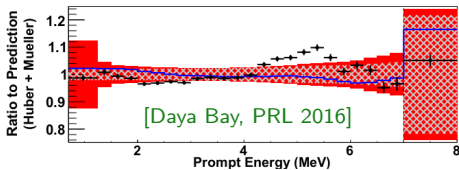
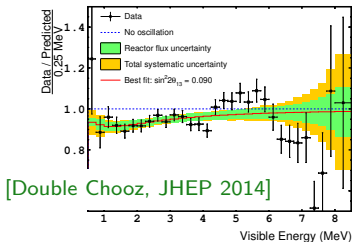
[Huber, PRC 84 (2011) 024617] [Mueller et al., PRC 83 (2011) 054615]

Previous reactor rates evaluated with new fluxes \Rightarrow deficit



Suppression at detector due to active-sterile oscillations?

Can we trust the HM fluxes?



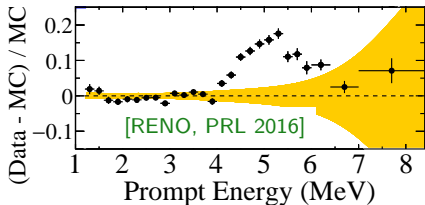
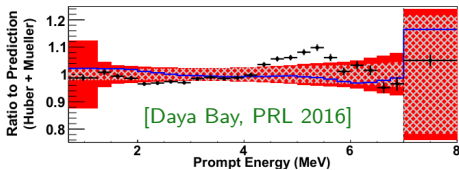
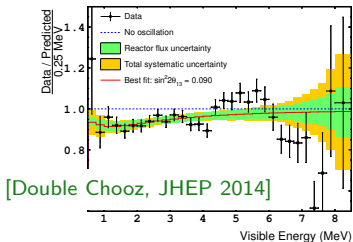
known since 2014:
bump in the spectrum
around 5 MeV!

cannot be explained
by SBL oscillations

(averaged at the ob-
served distances)

many attempts of
possible explanations,
how to clarify the issue?

Can we trust the HM fluxes?



known since 2014:
bump in the spectrum
around 5 MeV!

cannot be explained
by SBL oscillations

(averaged at the ob-
served distances)

many attempts of
possible explanations,
how to clarify the issue?

Model independent information!

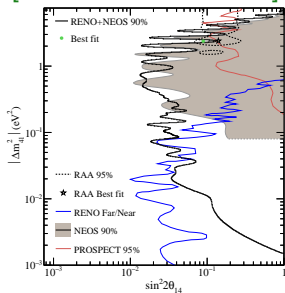
(i.e. take ratio of spectra
at different distances)

$$\Phi_1 = \Phi_0(E)f(L_1, E) \quad \Phi_2 = \Phi_0(E)f(L_2, E)$$

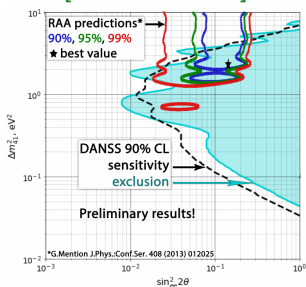
$$\Phi_1/\Phi_2 = f(L_1, E)/f(L_2, E)$$

ν_s at reactors in 2020

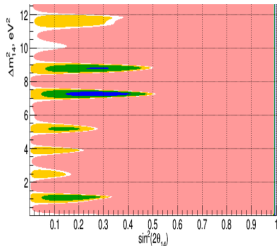
[RENO+NEOS, 2020]



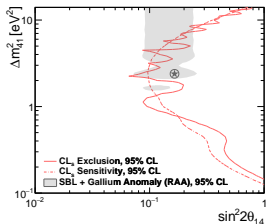
[DANSS, 2020]



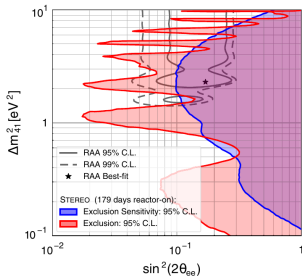
[Neutrino-4, PZETF 2020]



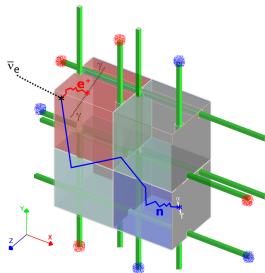
[PROSPECT, PRD 2020]



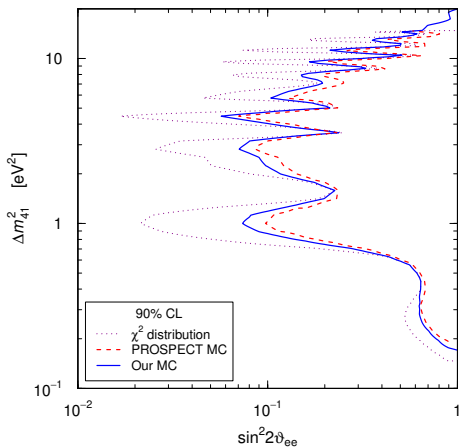
[STEREO, PRD 2020]



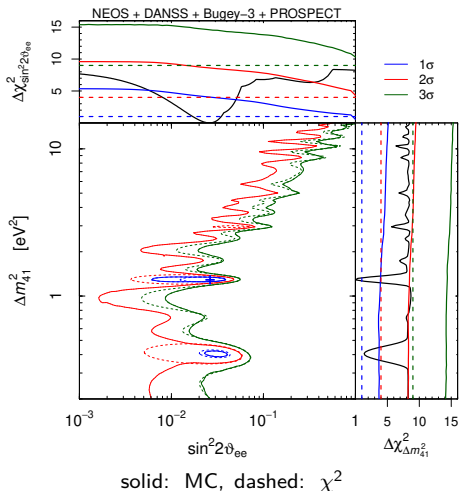
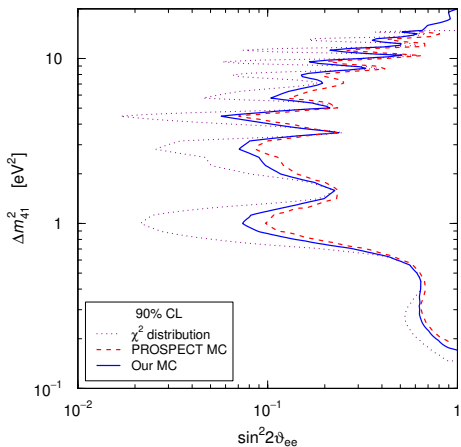
[SoLiD, JINST 2018]



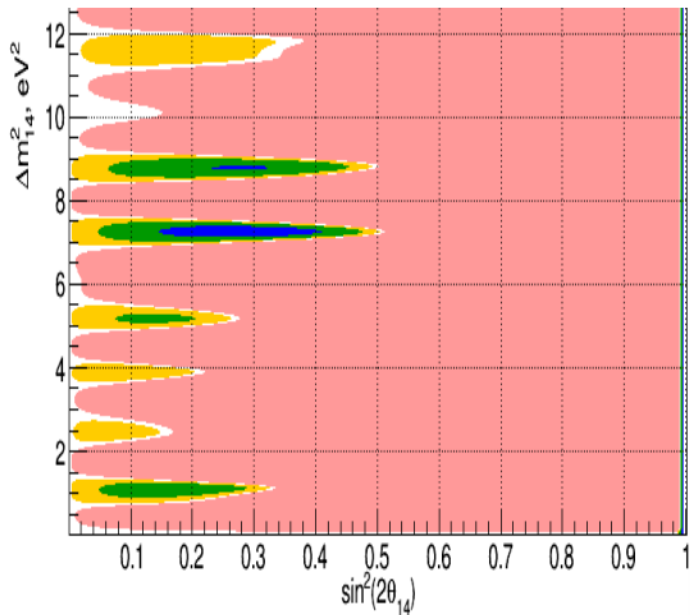
standard χ^2 distribution may be not appropriate to study the significance due to **undercoverage** at angles below the **experiment sensitivity**



standard χ^2 distribution may be not appropriate to study the significance due to undercoverage at angles below the experiment sensitivity

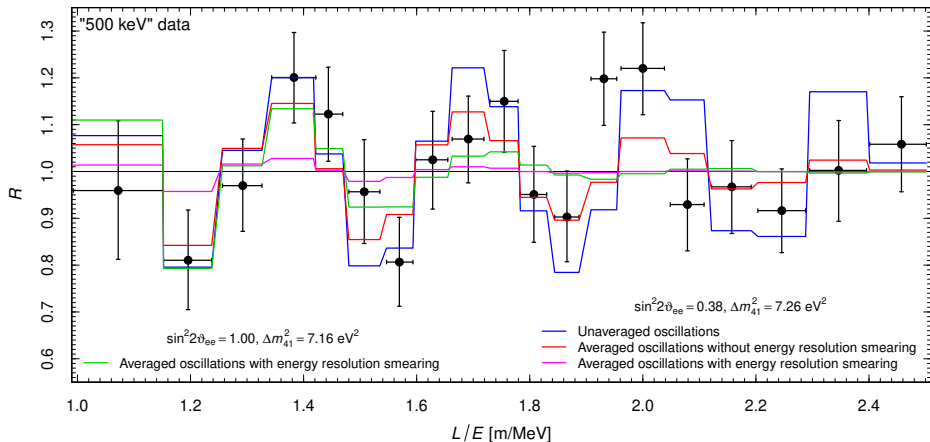


True significance smaller than usually quoted (e.g. $2.4 \rightarrow 1.8\sigma$)

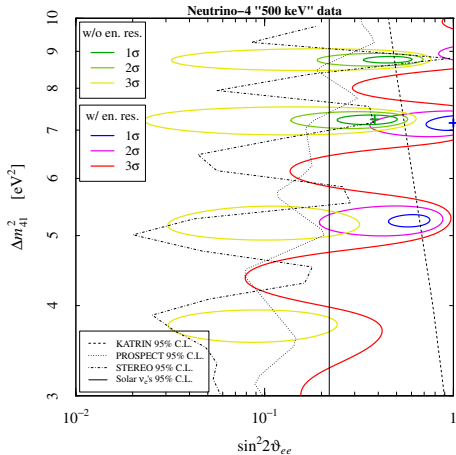


claimed $> 3\sigma$
preference for
 $3+1$ over 3ν case

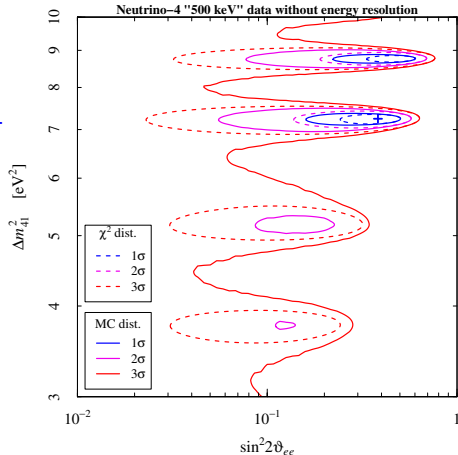
best fit
incompatible
with other
reactor
experiments



energy resolution smearing not properly taken into account?



proper energy resolution treatment
moves best-fit $\rightarrow \sin^2 2\vartheta \simeq 1$



need to take into account
violation of Wilk's theorem

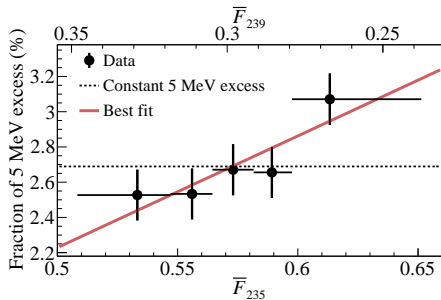
↓
relaxed constraints

Fuel evolution

Reactor fluxes produced
by decay fissions of

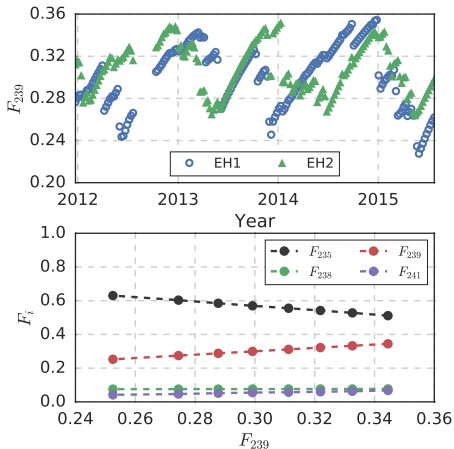
^{235}U ^{238}U ^{239}Pu ^{241}Pu

Can we use time evolution to
identify source of 5 MeV bump?



[RENO, PRL 122 (2019)]

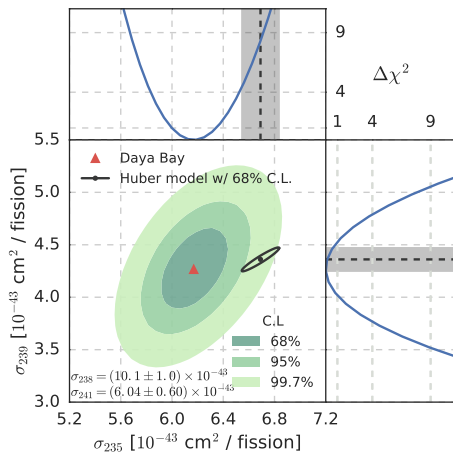
Fuel fractions in reactors
change with time



[Daya Bay, PRL 118 (2017)]

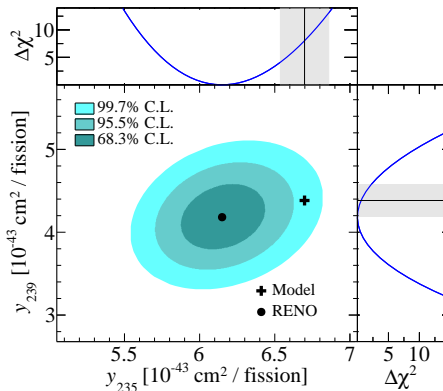
Fuel evolution

Fit bump amplitude as a function of flux normalizations



[Daya Bay, PRL 118 (2017) 251801]

Normalization of ^{235}U flux smaller than prediction!



[RENO, PRL 122 (2019)]

Again, we need **model-independent information!**

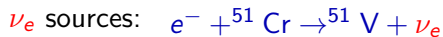
take ratios **at different distances** to avoid normalization dependency

Gallium anomaly

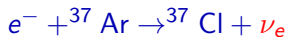
[SAGE, 2006][Laveder, 2007][Giunti&Laveder, 2011]

$L \simeq 1.9 \text{ m}$ $L \simeq 0.6 \text{ m}$

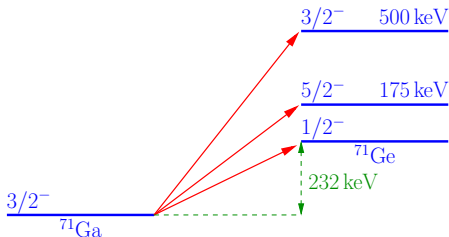
Gallium radioactive source experiments: **GALLEX** and **SAGE**



$E \simeq 0.75 \text{ MeV}$



$E \simeq 0.81 \text{ MeV}$



cross sections of
the transitions from

[Krofcheck et al., PRL 55 (1985) 1051]

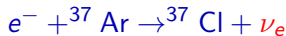
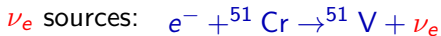
[Frekers et al., PLB 706 (2011) 134]

Gallium anomaly

[SAGE, 2006][Laveder, 2007][Giunti&Laveder, 2011]

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Gallium radioactive source experiments: **GALLEX** and **SAGE**

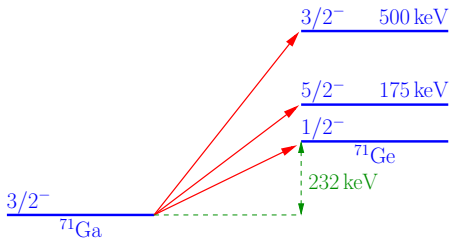
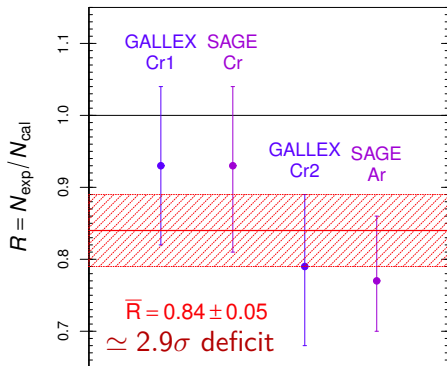


$E \simeq 0.75 \text{ MeV}$

$E \simeq 0.81 \text{ MeV}$



Test detection of solar ν_e

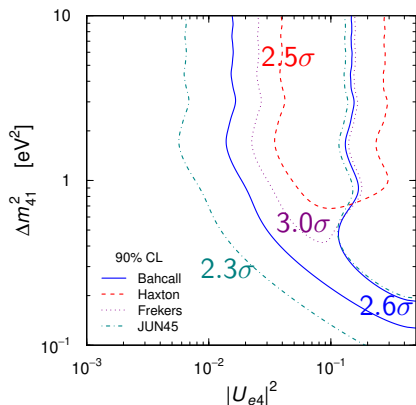


cross sections of the transitions from

[Krofcheck et al., PRL 55 (1985) 1051]

[Frekers et al., PLB 706 (2011) 134]

New cross section calculations:
(interacting nuclear shell model)



original Gallium anomaly: $\sim 2.9\sigma$

[SAGE, 2006]

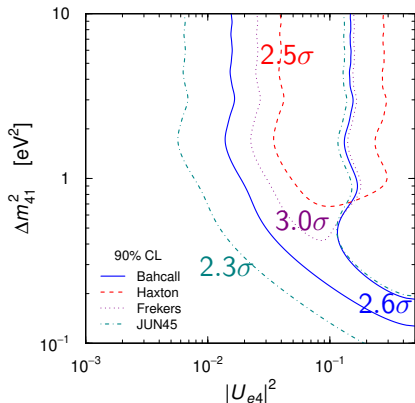
[Laveder, 2007]

[Giunti&Laveder, 2011]

Gallium anomaly revisited

[Kostensalo+, PLB 795 (2019) 542-547]

New cross section calculations:
(interacting nuclear shell model)



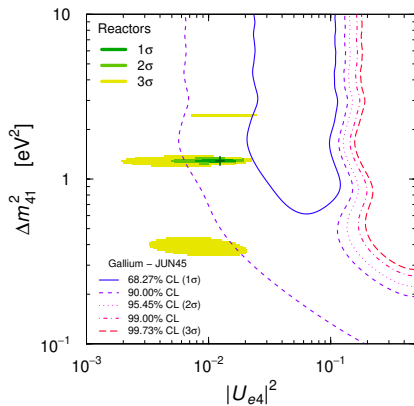
original Gallium anomaly: $\sim 2.9\sigma$

[SAGE, 2006]

[Laveder, 2007]

[Giunti&Laveder, 2011]

Compare with DANSS+NEOS:

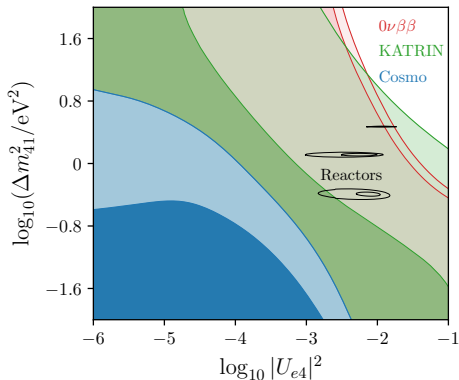


Better compatibility with reactors

F Fit

Based on:

- in preparation
- Dentler+ 2018
- arxiv:2003.02289

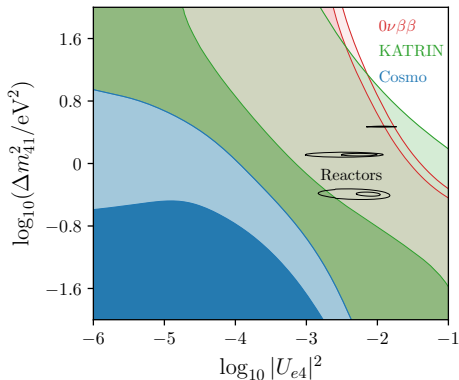


G Fit = Global Fit

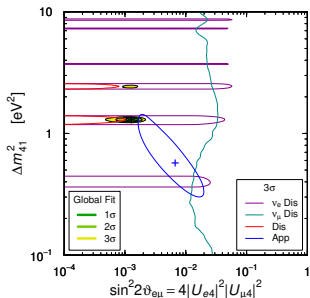
like “Bond, James Bond”

Based on:

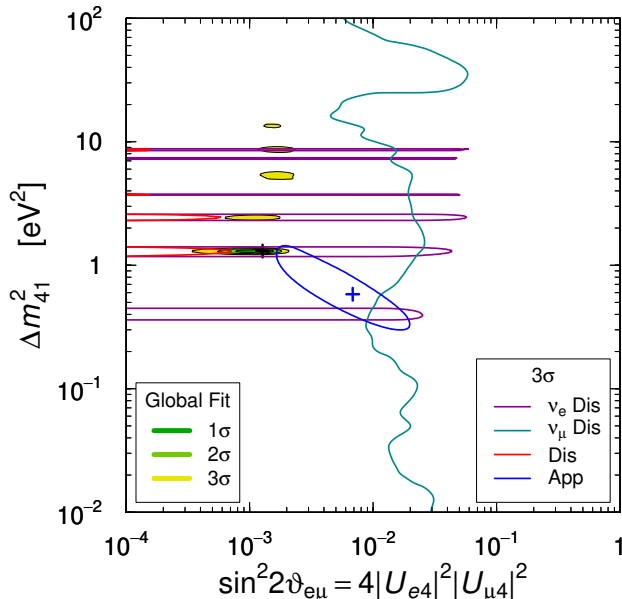
- in preparation
- Dentler+ 2018
- arxiv:2003.02289



Status just after
Neutrino 2018:

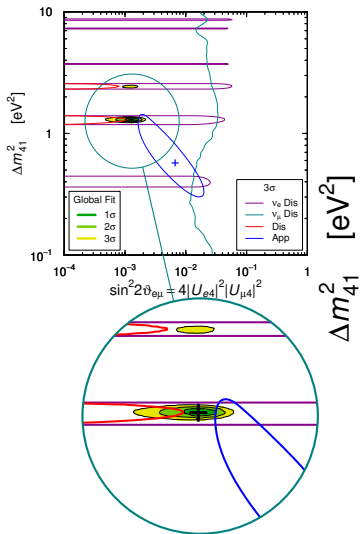


Status in early 2019

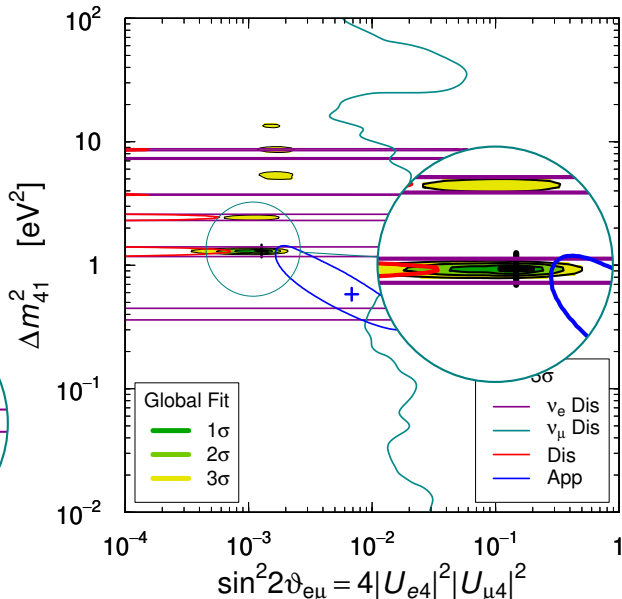


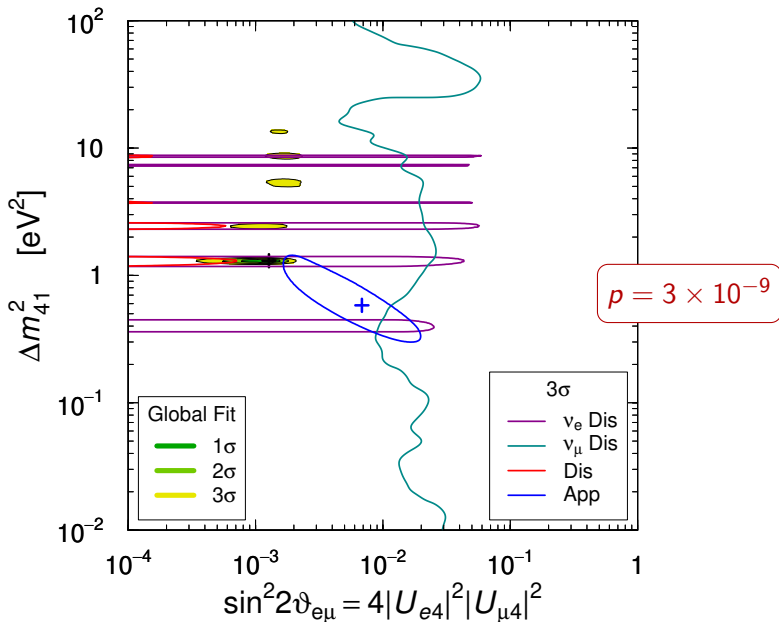
MINOS+ update,
new data
including MiniBooNE
(all bins)

Status just after
Neutrino 2018:



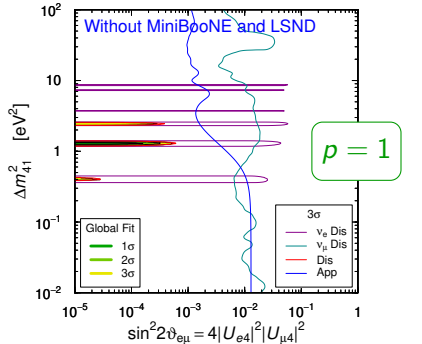
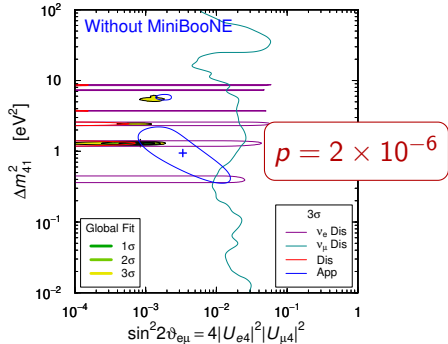
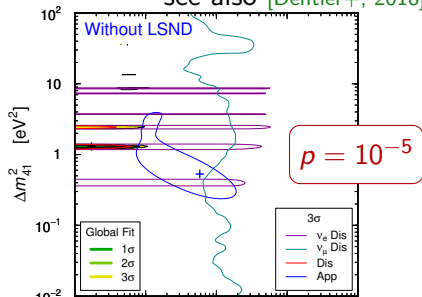
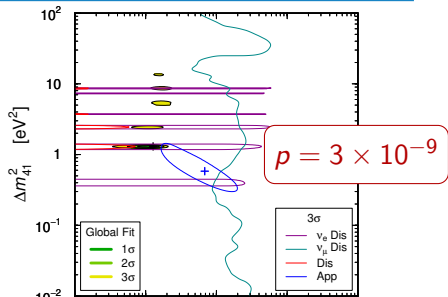
Status in early 2019





APP – DIS tension in 2019

[SG+, in preparation]
see also [Dentler+, 2018]



May something be wrong?

[Dentler+, JHEP 08 (2018) 010]
(2013 data from MiniBooNE, MINOS+ v1!)

Analysis	$\chi^2_{\min, \text{global}}$	$\chi^2_{\min, \text{app}}$	$\Delta\chi^2_{\text{app}}$	$\chi^2_{\min, \text{disapp}}$	$\Delta\chi^2_{\text{disapp}}$	$\chi^2_{\text{PG}}/\text{dof}$	PG
Global	1120.9	79.1	11.9	1012.2	17.7	29.6/2	3.71×10^{-7}
Removing anomalous data sets							
w/o LSND	1099.2	86.8	12.8	1012.2	0.1	12.9/2	1.6×10^{-3}
w/o MiniBooNE	1012.2	40.7	8.3	947.2	16.1	24.4/2	5.2×10^{-6}
w/o reactors	925.1	79.1	12.2	833.8	8.1	20.3/2	3.8×10^{-5}
w/o gallium	1116.0	79.1	13.8	1003.1	20.1	33.9/2	4.4×10^{-8}
Removing constraints							
w/o IceCube	920.8	79.1	11.9	812.4	17.5	29.4/2	4.2×10^{-7}
w/o MINOS(+)	1052.1	79.1	15.6	948.6	8.94	24.5/2	4.7×10^{-6}
w/o MB disapp	1054.9	79.1	14.7	947.2	13.9	28.7/2	6.0×10^{-7}
w/o CDHS	1104.8	79.1	11.9	997.5	16.3	28.2/2	7.5×10^{-7}
Removing classes of data							
$\bar{\nu}_e$ dis vs app	628.6	79.1	0.8	542.9	5.8	6.6/2	3.6×10^{-2}
$\bar{\nu}_\mu$ dis vs app	564.7	79.1	12.0	468.9	4.7	16.7/2	2.3×10^{-4}
$\bar{\nu}_\mu$ dis + solar vs app	884.4	79.1	13.9	781.7	9.7	23.6/2	7.4×10^{-6}

May something be wrong?

[Dentler+, JHEP 08 (2018) 010]
(2013 data from MiniBooNE, MINOS+ v1!)

Analysis	$\chi^2_{\min, \text{global}}$	$\chi^2_{\min, \text{app}}$	$\Delta\chi^2_{\text{app}}$	$\chi^2_{\min, \text{disapp}}$	$\Delta\chi^2_{\text{disapp}}$	$\chi^2_{\text{PG}}/\text{dof}$	PG
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No improvements if MiniBooNE is not considered

May something be wrong?

[Dentler+, JHEP 08 (2018) 010]

(2013 data from MiniBooNE, MINOS+ v1!)

Analysis	$\chi^2_{\min, \text{global}}$	$\chi^2_{\min, \text{app}}$	$\Delta\chi^2_{\text{app}}$	$\chi^2_{\min, \text{disapp}}$	$\Delta\chi^2_{\text{disapp}}$	$\chi^2_{\text{PG}}/\text{dof}$	PG
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$\bar{\nu}_\mu$ DIS also constrain $|U_{e4}|^2$, while $\bar{\nu}_e$ DIS do not constrain $|U_{\mu4}|^2$

May something be wrong?

[Dentler+, JHEP 08 (2018) 010]

(2013 data from MiniBooNE, MINOS+ v1!)

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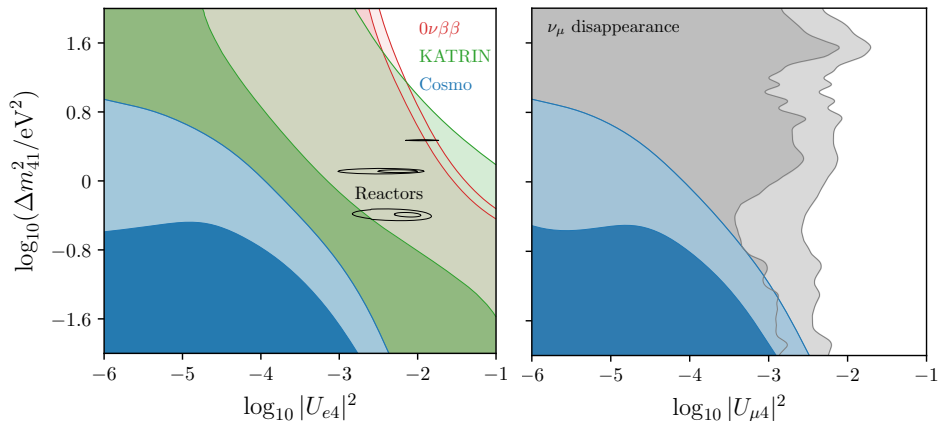
Only removing LSND or all $\bar{\nu}_\mu$ constraints the fit is almost acceptable

No reason to do so!

Comparing constraints

Cosmological constraints are stronger than most other probes

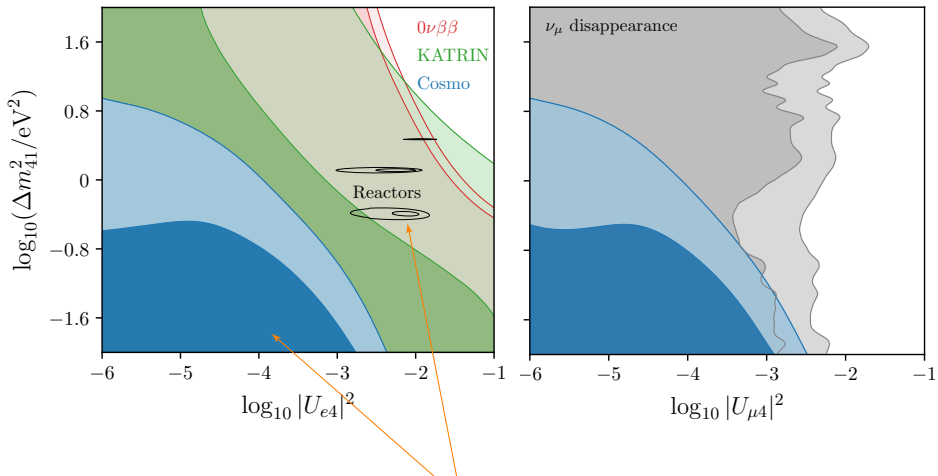
But much more model dependent (as all the cosmological constraints)!



Comparing constraints

Cosmological constraints are stronger than most other probes

But much more model dependent (as all the cosmological constraints)!



Warning: tension between reactor experiments and CMB bounds!

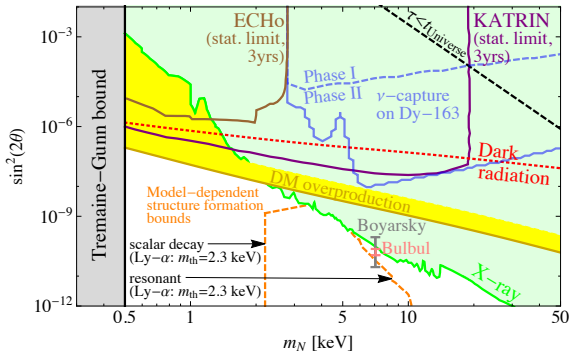
H

Heavier sterile neutrinos

beyond eV: other types of sterile neutrinos

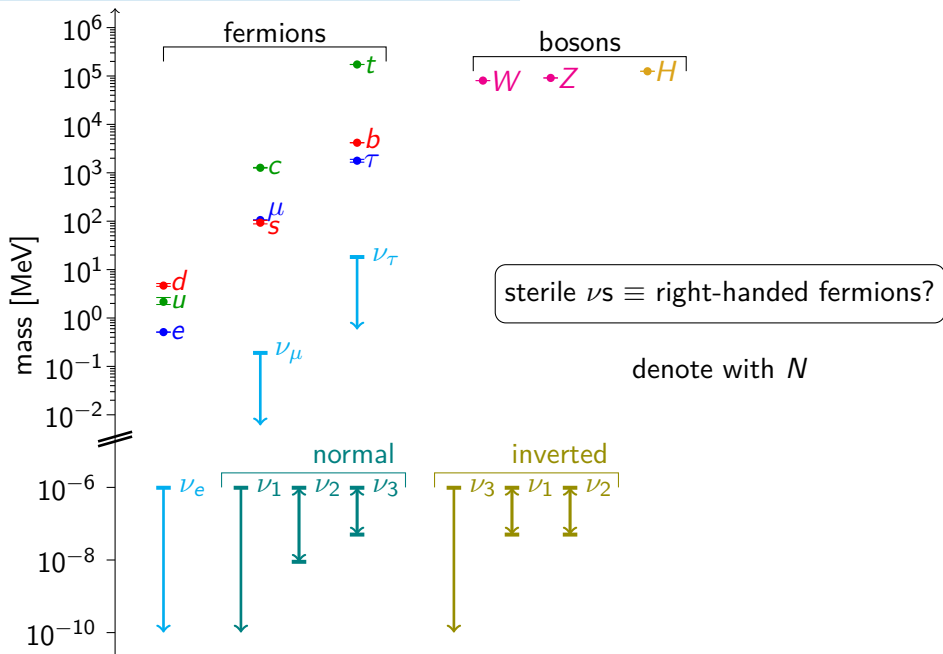
Based on:

- JCAP 01 (2017) 025



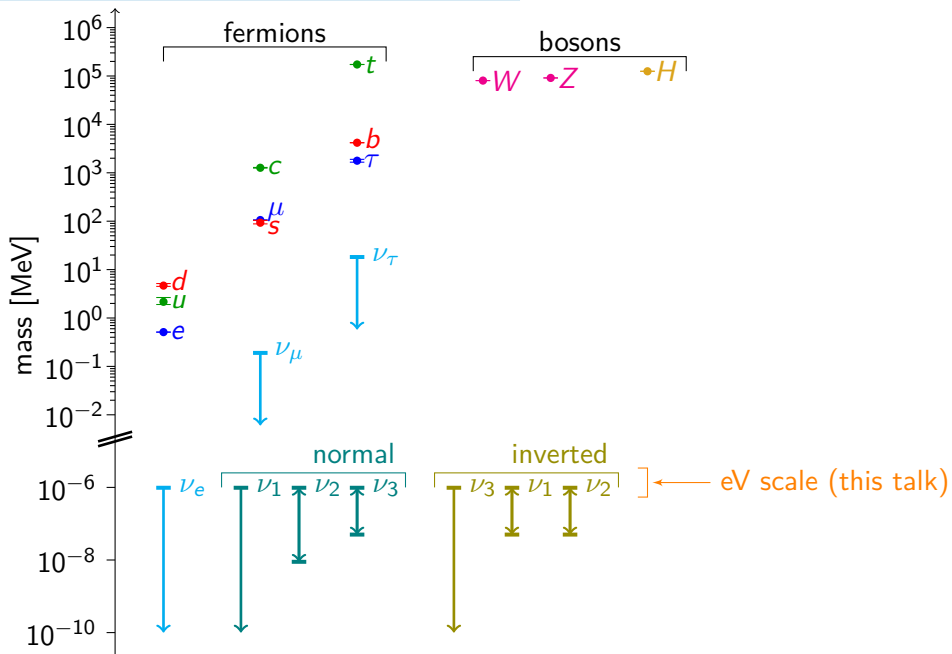
Masses in the Standard Model

[masses from PDG 2020]



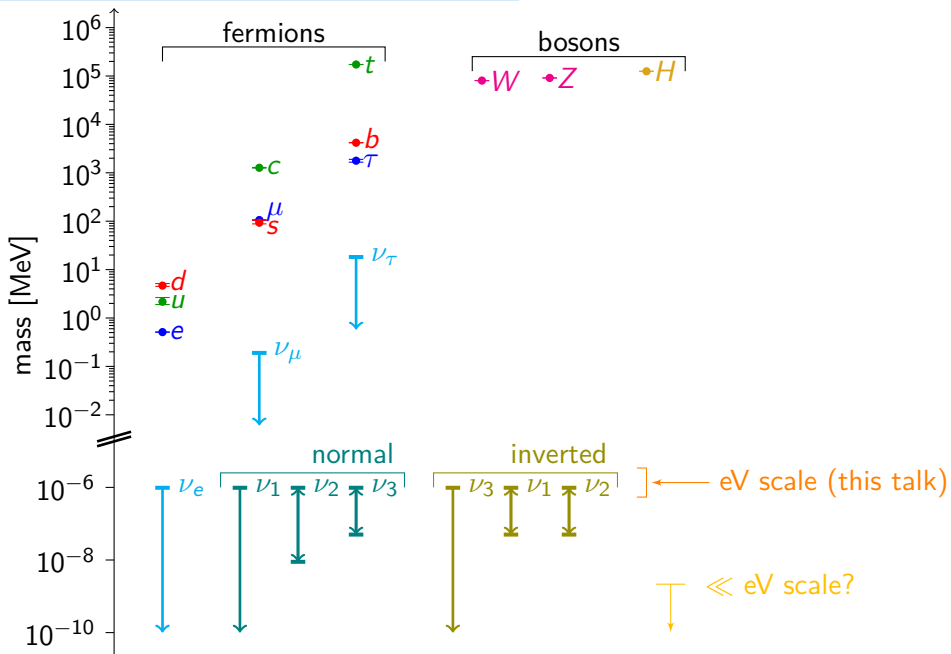
Masses in the Standard Model

[masses from PDG 2020]



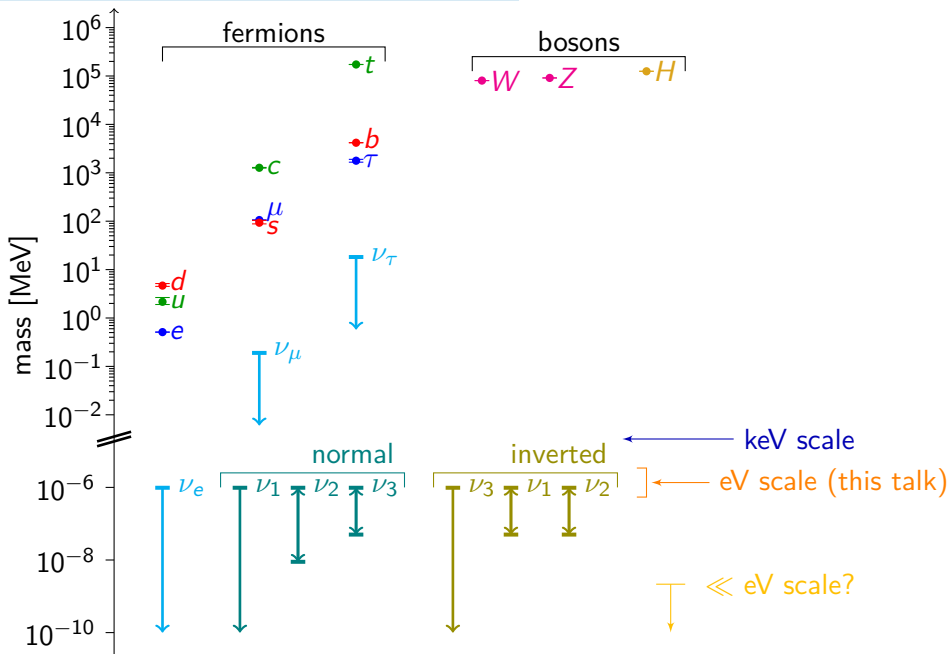
Masses in the Standard Model

[masses from PDG 2020]



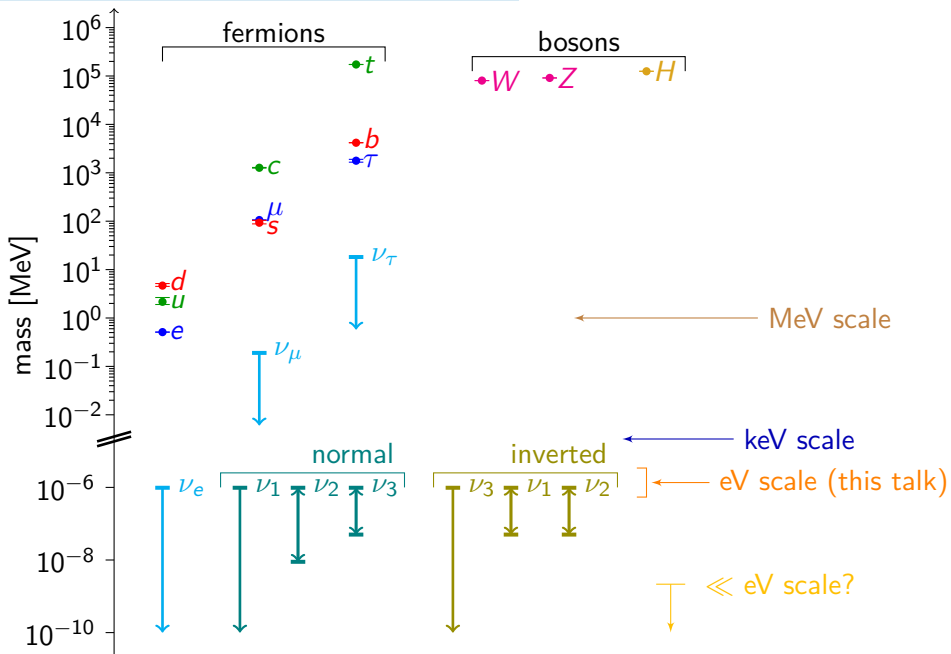
Masses in the Standard Model

[masses from PDG 2020]



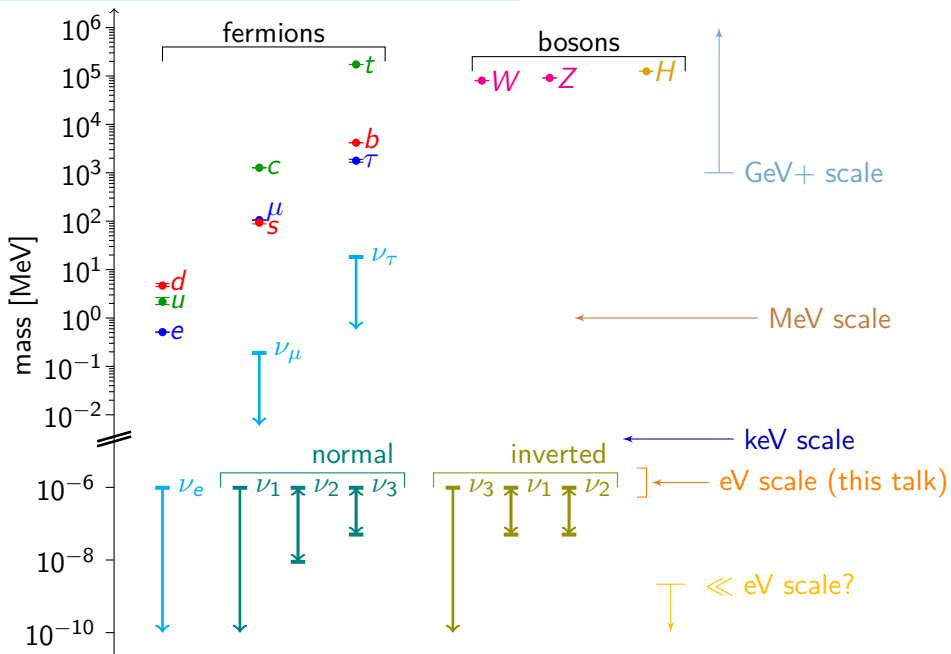
Masses in the Standard Model

[masses from PDG 2020]



Masses in the Standard Model

[masses from PDG 2020]



Heavier neutrino states at oscillation/mass experiments

Oscillation probability:

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = |\langle \nu_\alpha | \nu(L) \rangle|^2 = \sum_{k,j} U_{\beta k} U_{\alpha k}^* U_{\beta j}^* U_{\alpha j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

oscillation length **decreases with increasing Δm_{kj}^2 !**

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Concerning the mixing matrix (3+1 scenario):

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$$\Rightarrow |U|^2 = \begin{pmatrix} c_{14}^2 c_{13}^2 c_{12}^2 & c_{14}^2 c_{13}^2 s_{12}^2 & c_{14}^2 s_{13}^2 & s_{14}^2 \\ \dots & \dots & \dots & c_{14}^2 s_{24}^2 \\ \dots & \dots & \dots & c_{14}^2 c_{24}^2 s_{34}^2 \\ \dots & \dots & \dots & c_{14}^2 c_{24}^2 c_{34}^2 \end{pmatrix}, \quad s_{i4} \simeq 0, \quad c_{i4} \simeq 1$$

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Effect of neutrino masses in β and $0\nu\beta\beta$ decays:

$$K(T) = \left[(Q_\beta - T) \sum_{i=1}^{N_\nu} |U_{ei}|^2 \sqrt{(Q_\beta - T)^2 - m_i^2} \right]^{1/2} \quad \text{and} \quad m_{\beta\beta} = \left| \sum_k e^{i\alpha_k} \mu_k \right|, \quad \text{with } \mu_k \equiv U_{ek}^2 m_k$$

N production in the early Universe?

Thermal

Non thermal

cannot be only through neutrino oscillations if they are copiously produced in the early Universe



given mean number of active neutrinos n_0 , $\rho_N = m_N n_0 > \rho_C$



OK if early decoupling



dilution of energy density ρ_N to acceptable values during expansion

decay of heavier particles

including e.g. decoupling of inflatons or generic scalar fields

OK also if N is not produced in the early Universe



produced through oscillations, but never reaches equilibrium thanks to small mixing angle

$m_{\text{sn}} \simeq \mathcal{O}(\text{keV}) \longrightarrow$ non-relativistic at CMB decoupling

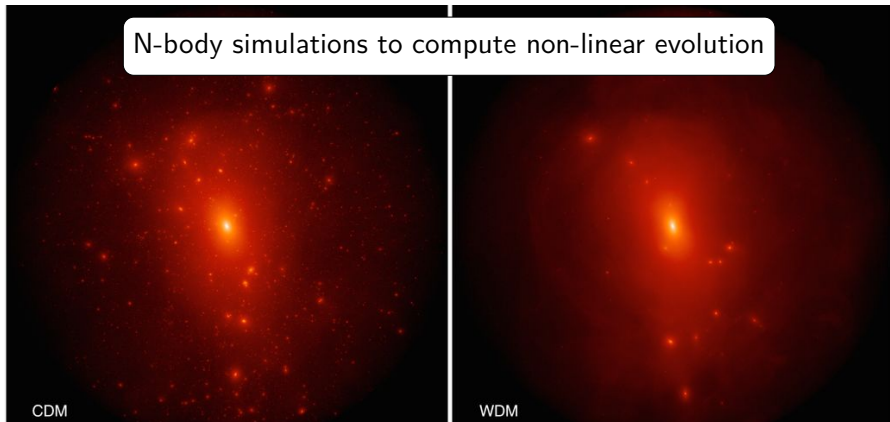
indistinguishable from Cold Dark Matter at the CMB level

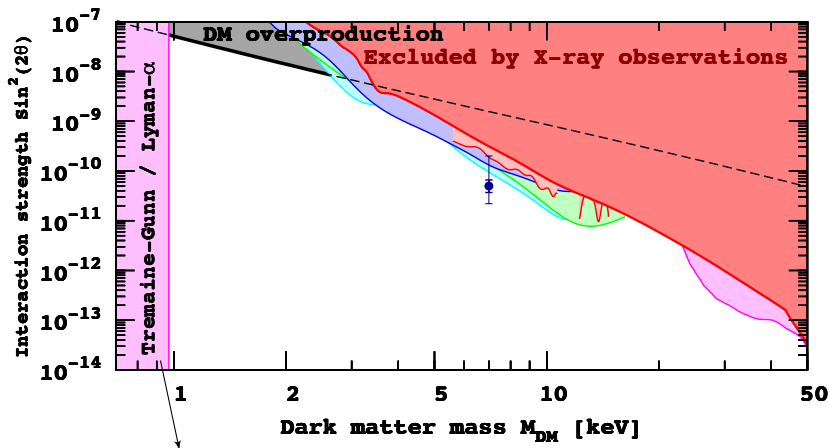
but

its free-streaming affects small scales!

Warm Dark Matter (WDM)

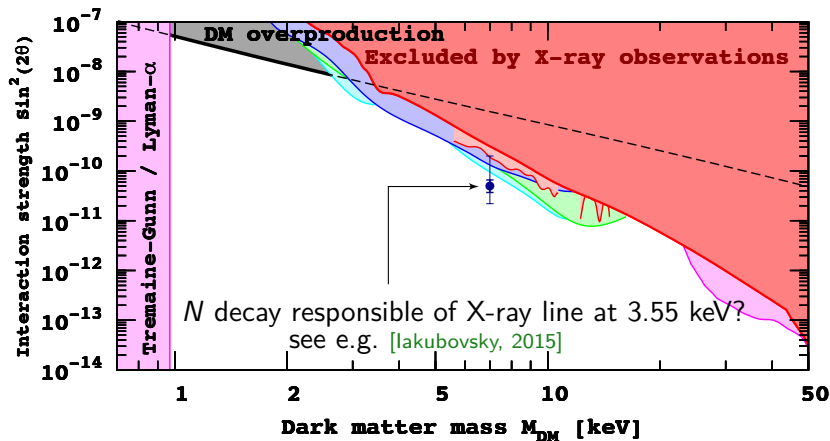
N-body simulations to compute non-linear evolution





[Tremaine-Gunn 1979]

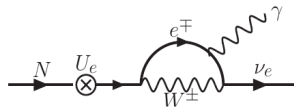
phase space distribution
in galaxy cannot exceed
degenerate Fermi gas

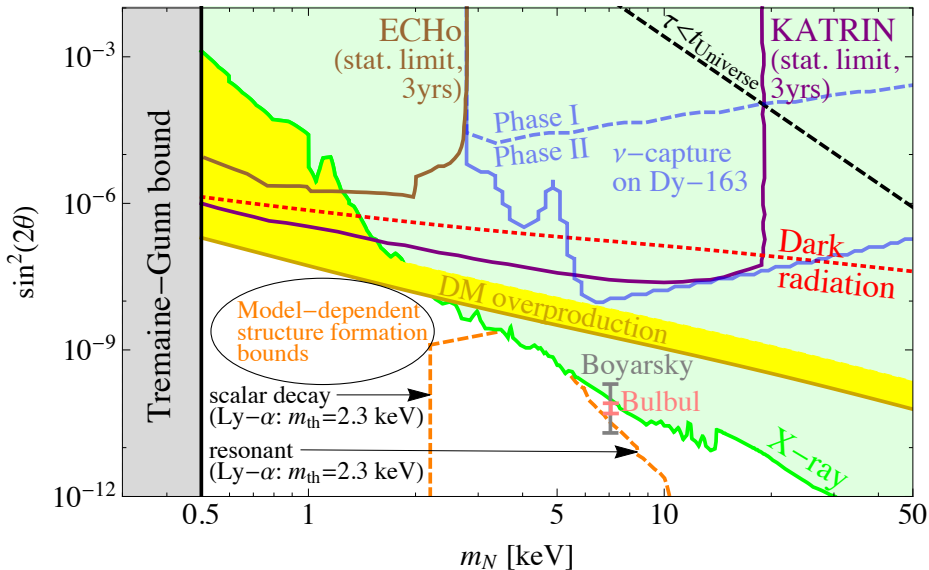


radiative decay $N \rightarrow \nu + \gamma$

with $E_\gamma = E_\nu = m_{\text{sn}}/2 \longrightarrow$ X-rays

$$\Gamma_{N \rightarrow \gamma \nu} \simeq 1.38 \times 10^{-22} \sin^2 2\theta \left(\frac{m_{\text{sn}}}{\text{keV}} \right)^5 \text{s}^{-1}$$





Astrophysics bounds stronger than those at terrestrial experiments!

Z

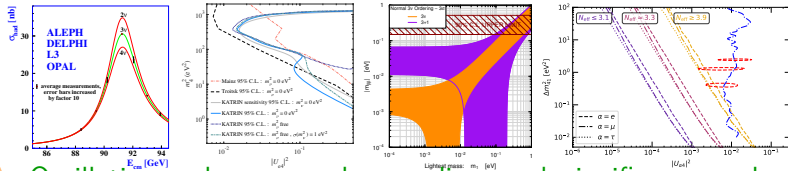
Conclusions

The situation is NOT favorable
for the light sterile neutrino...

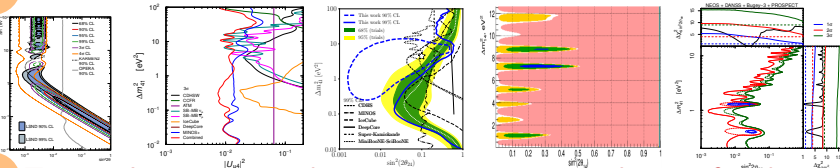


What do we learn on sterile neutrinos?

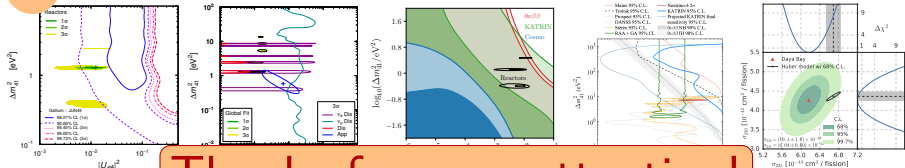
N Non-oscillation probes: no signal, possibly strong constraints



O Oscillation probes: several anomalies, weak significance each



T Tensions! even strong, between experiments or classes of probes



Thanks for your attention!