 "la Caixa" Foundation
Junior Leader
Fellowship
LCF/BQ/PI23/11970034

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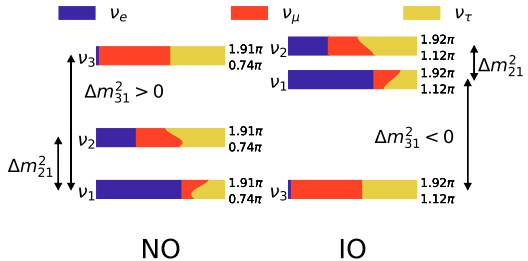
`stefano.gariazzo@ift.csic.es`

Relic neutrinos: decoupling and direct detection perspectives

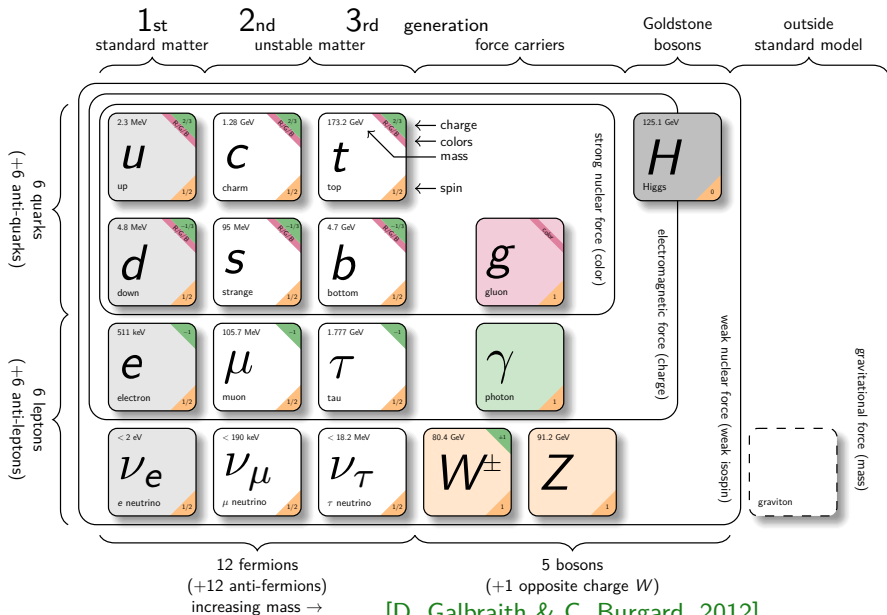
N Neutrinos

Based on:

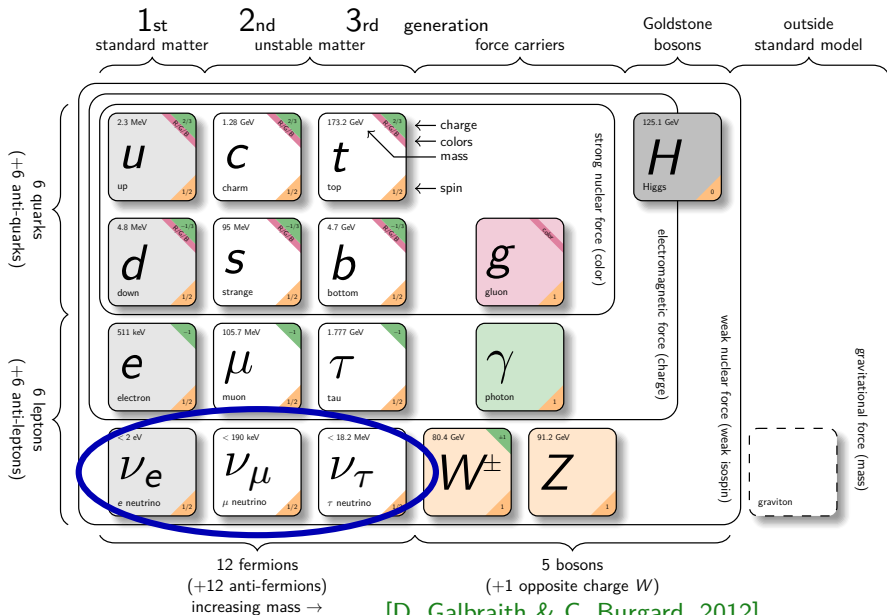
- JHEP 02 (2021) 071 and update

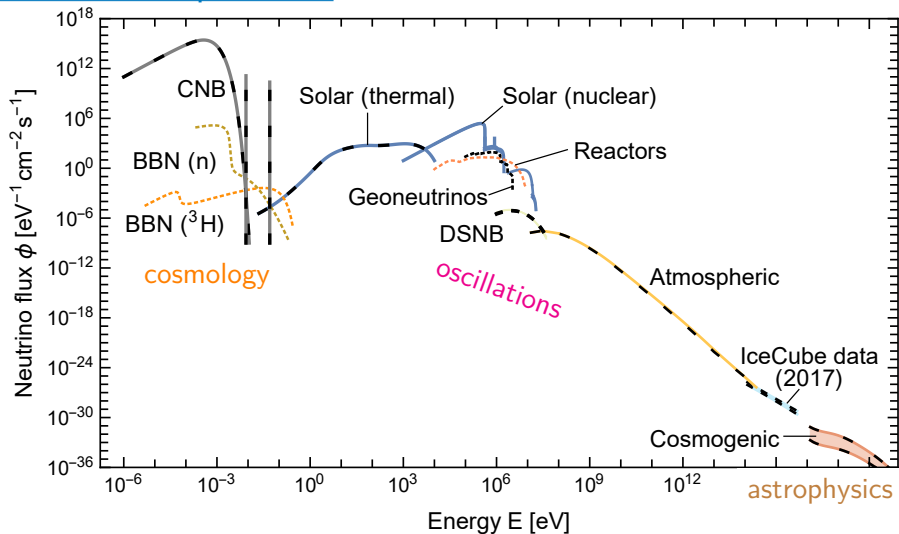


The Standard Model of Particle Physics



The Standard Model of Particle Physics



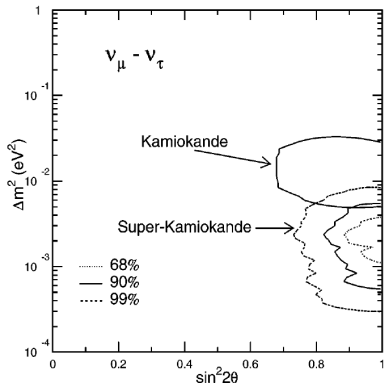


neutrinos at all energies provide valuable information!

Neutrino oscillations

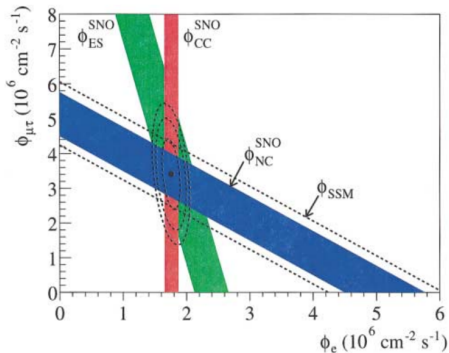
Major discoveries:

[SuperKamiokande, 1998]



first discovery of $\nu_\mu \rightarrow \nu_\tau$
oscillations from atmospheric ν

[SNO, 2001-2002]



first discovery of $\nu_e \rightarrow \nu_\mu, \nu_\tau$
oscillations from solar ν

Nobel prize in 2015

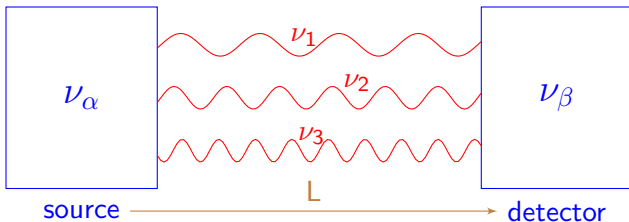
Two neutrino bases

flavor neutrinos ν_α

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k} |\nu_k\rangle$$

massive neutrinos ν_k

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U_{\alpha 1} |\nu_1\rangle + U_{\alpha 2} |\nu_2\rangle + U_{\alpha 3} |\nu_3\rangle$$



$$|\nu(t > 0)\rangle = |\nu_\beta\rangle = U_{\alpha 1} e^{-iE_1 t} |\nu_1\rangle + U_{\alpha 2} e^{-iE_2 t} |\nu_2\rangle + U_{\alpha 3} e^{-iE_3 t} |\nu_3\rangle \neq |\nu_\alpha\rangle$$

$$E_k^2 = p^2 + m_k^2 \longleftarrow \text{define} \longrightarrow t = L$$

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = |\langle \nu_\alpha | \nu(L) \rangle|^2 = \sum_{k,j} U_{\beta k} U_{\alpha k}^* U_{\beta j}^* U_{\alpha j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

The mixing matrix

U can be parameterized using 3 angles (θ_{12} , θ_{13} , θ_{23}) and max 3 (1 Dirac δ , 2 Majorana [\exists only for Majorana ν]) phases

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\substack{\text{mainly atmospheric} \\ \text{and LBL} \\ \text{accelerator} \\ \text{disappearance}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\substack{\text{mainly LBL reactors and} \\ \text{LBL accelerator} \\ \text{appearance}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{mainly solar and} \\ \text{VLBL reactors}}} M$$

Majorana phases irrelevant for oscillation experiments ←

Relevant for example in neutrinoless double-beta decay

$$s_{ij} \equiv \sin \theta_{ij}; \quad c_{ij} \equiv \cos \theta_{ij}$$

LBL = long baseline; VLBL = very long baseline;

Three Neutrino Oscillations

$$\nu_\alpha = \sum_{k=1}^3 U_{\alpha k} \nu_k \quad (\alpha = e, \mu, \tau)$$

$U_{\alpha k}$ described by 3 mixing angles θ_{12} , θ_{13} , θ_{23} and one CP phase δ

Current knowledge of the 3 active ν mixing: [JHEP 02 (2021) update]

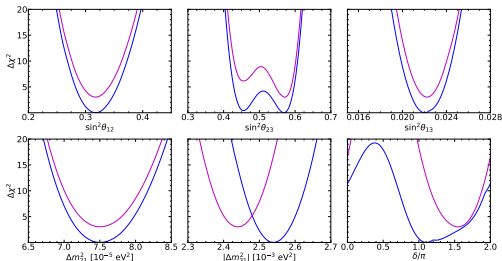
NO/NH: Normal Ordering/Hierarchy, $m_1 < m_2 < m_3$

IO/IH: Inverted O/H, $m_3 < m_1 < m_2$

$$\begin{aligned} \Delta m_{21}^2 &= (7.50^{+0.22}_{-0.20}) \cdot 10^{-5} \text{ eV}^2 \\ |\Delta m_{31}^2| &= (2.54 \pm 0.03) \cdot 10^{-3} \text{ eV}^2 \text{ (NO)} \\ &= (2.44 \pm 0.03) \cdot 10^{-3} \text{ eV}^2 \text{ (IO)} \end{aligned}$$

$$\begin{aligned} 10 \sin^2(\theta_{12}) &= 3.18 \pm 0.16 \\ 10^2 \sin^2(\theta_{13}) &= 2.200^{+0.069}_{-0.062} \text{ (NO)} \\ &= 2.225^{+0.064}_{-0.070} \text{ (IO)} \\ 10 \sin^2(\theta_{23}) &= 4.55 \pm 0.13 \text{ (NO)} \\ &= 5.71^{+0.14}_{-0.17} \text{ (IO)} \end{aligned}$$

$$\begin{aligned} \delta/\pi &= 1.10^{+0.27}_{-0.12} \text{ (NO)} \\ &= 1.54 \pm 0.14 \text{ (IO)} \end{aligned}$$



mass ordering
still unknown

δ still unknown

see also: <http://globalfit.astroparticles.es>

Normal ordering (NO)

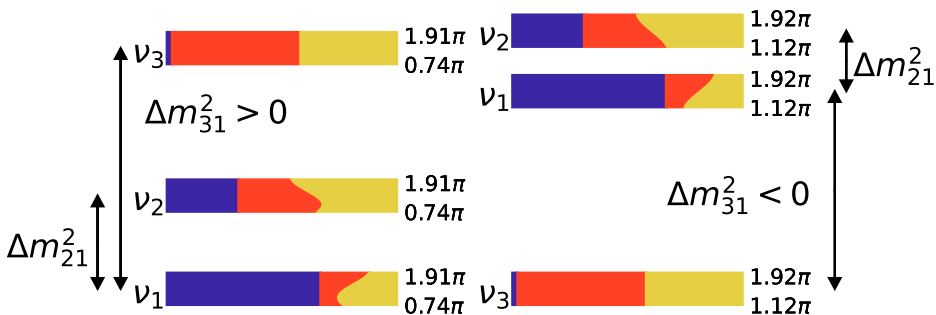
$$m_1 < m_2 < m_3$$

$$\sum m_k \gtrsim 0.06 \text{ eV}$$

Inverted ordering (IO)

$$m_3 < m_1 < m_2$$

$$\sum m_k \gtrsim 0.1 \text{ eV}$$



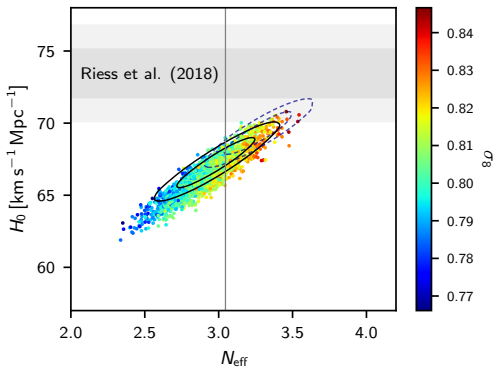
Absolute scale unknown!

Can we constrain the mass ordering using bounds on $\sum m_\nu$?

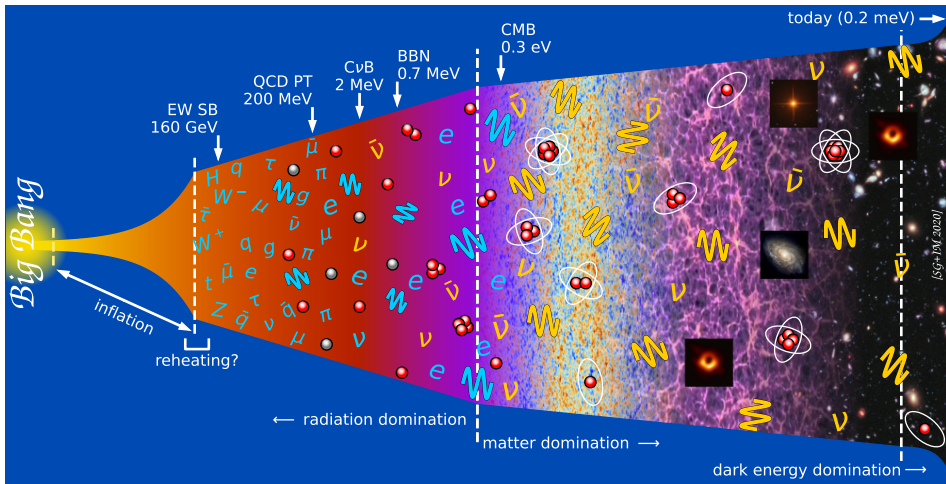
E Neutrinos in the Early Universe

Based on:

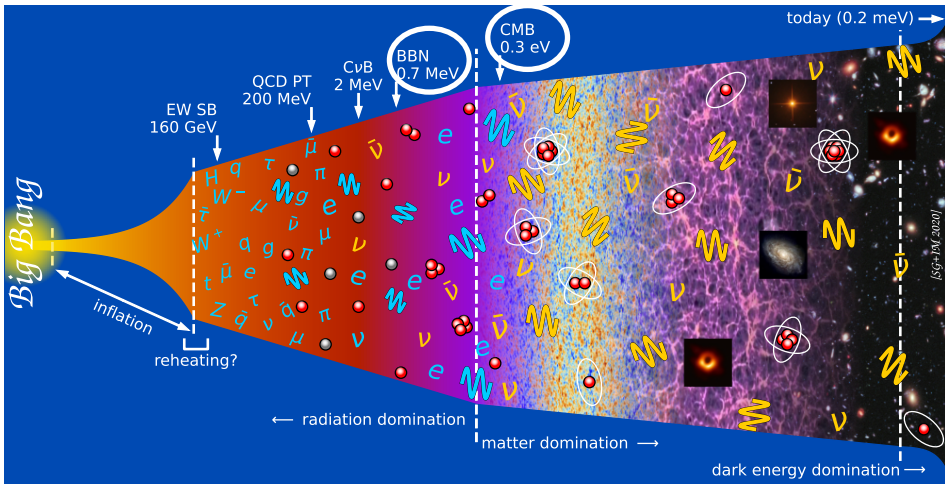
- Planck 2018
- JCAP 04 (2021) 073
- PRD 106 (2022) 043540



History of the universe



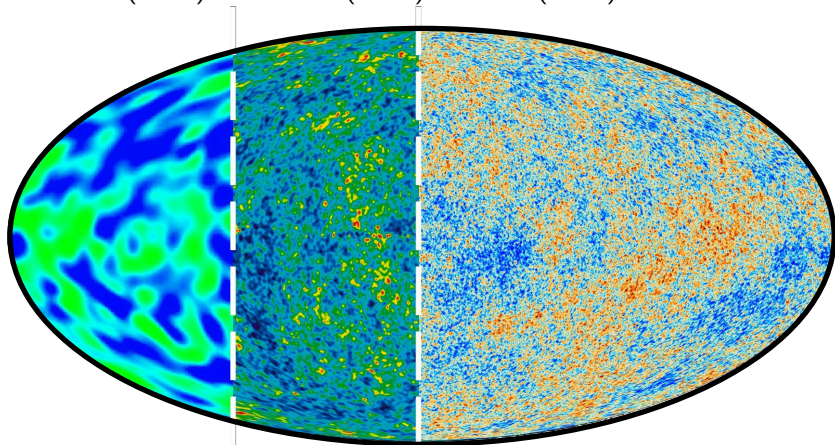
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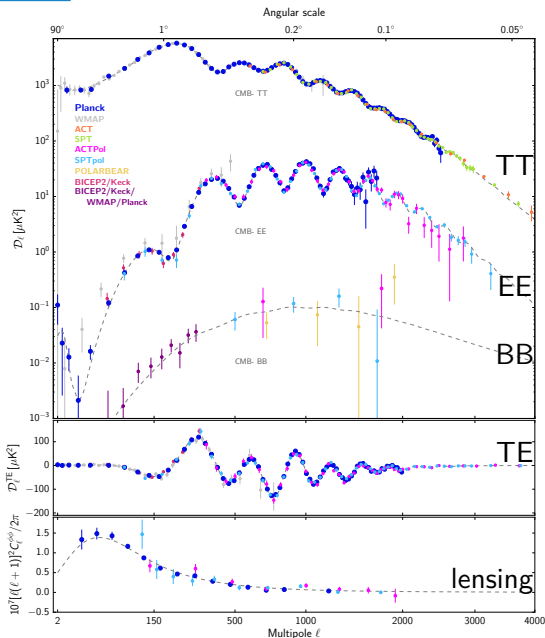
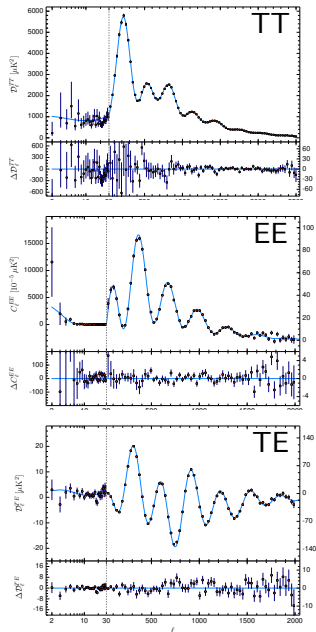


The oldest picture of the Universe

The Cosmic Microwave Background, generated at $t \simeq 4 \times 10^5$ years

COBE (1992) WMAP (2003) Planck (2013)





Big Bang Nucleosynthesis (BBN)

BBN: production of light nuclei at $t \sim 1\text{s}$ to $t \sim \mathcal{O}(10^2)\text{s}$

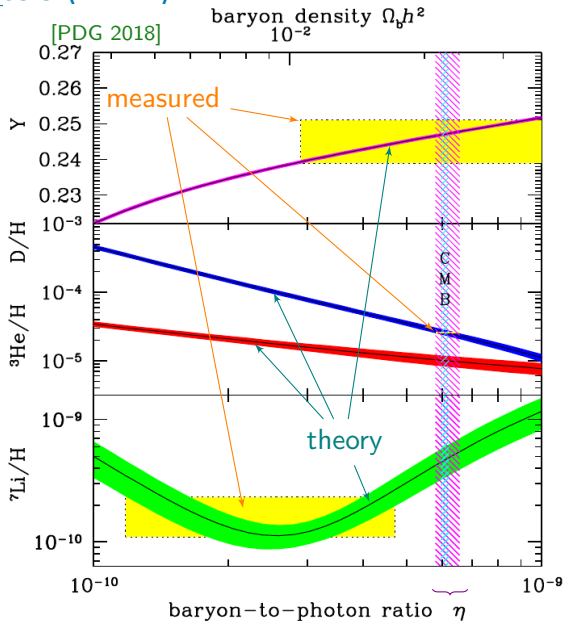
temperature $T_{fr} \simeq 1\text{ MeV}$
from nucleon freeze-out

much earlier than CMB!

strong probe for physics
before the CMB

e.g. neutrinos!

ν affect
universe expansion
and
reaction rates ($\nu_e/\bar{\nu}_e$)
at BBN time...



BBN concordance

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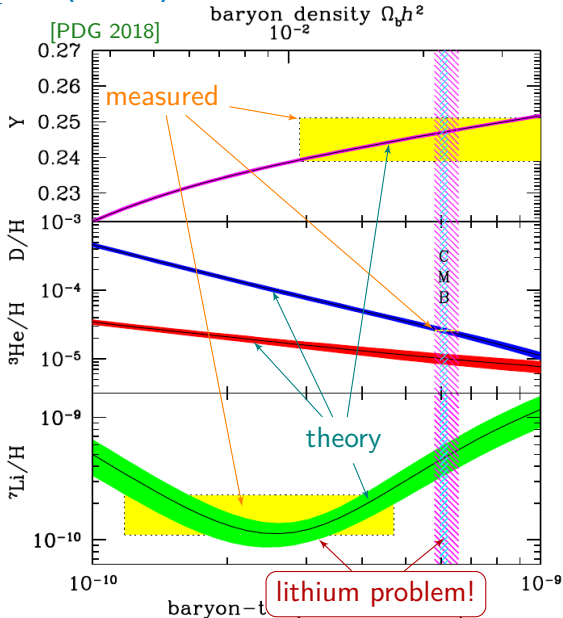
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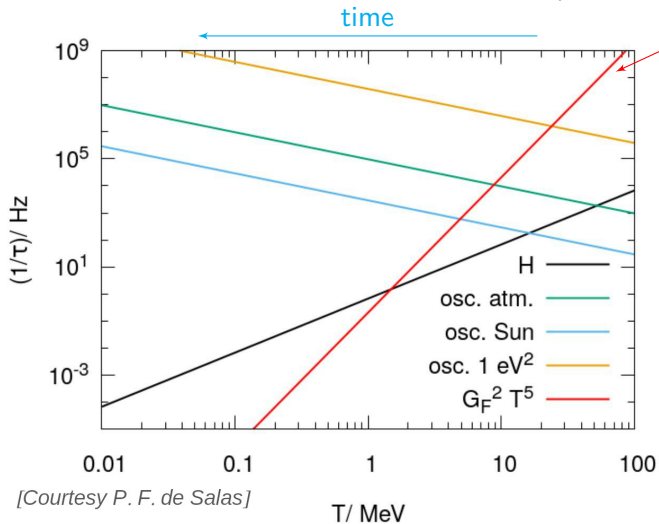
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Neutrinos in the early Universe

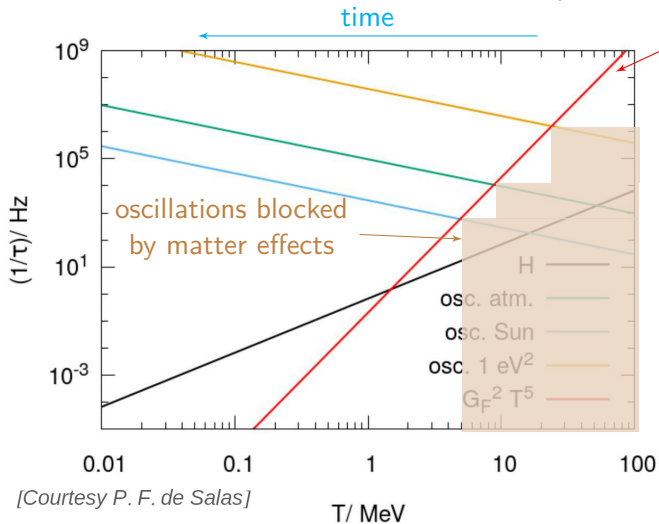
before BBN: neutrinos coupled to plasma ($\nu_\alpha \bar{\nu}_\alpha \leftrightarrow e^+ e^-$, $\nu e \leftrightarrow \nu e$)



[Courtesy P. F. de Salas]

Neutrinos in the early Universe

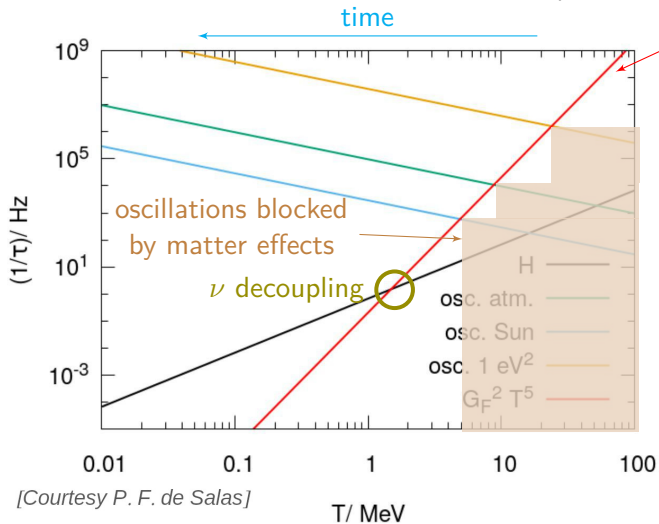
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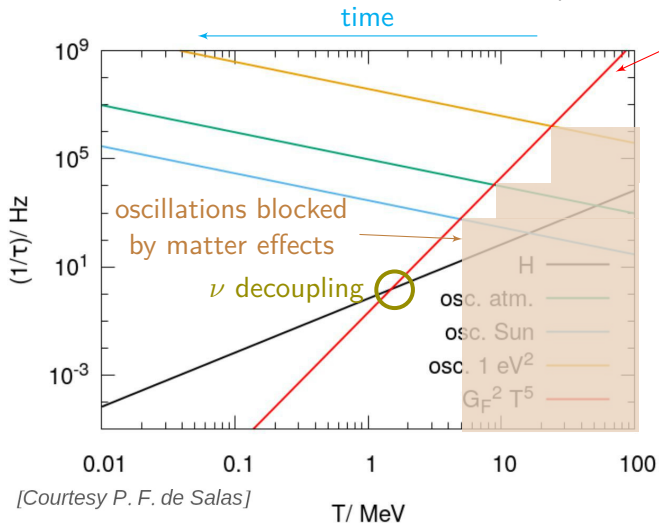
[Courtesy P. F. de Salas]

T/ MeV

ν decouple mostly before $e^+ e^- \rightarrow \gamma\gamma$ annihilation!

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$$T_\nu \simeq (4/11)^{1/3} T_\gamma$$

after $e^+ e^- \rightarrow \gamma\gamma$

f_ν : frozen Fermi-Dirac distribution

Today:

$$T_{\nu,0} = 1.945 \text{ K} \simeq 1.676 \times 10^{-4} \text{ eV}$$

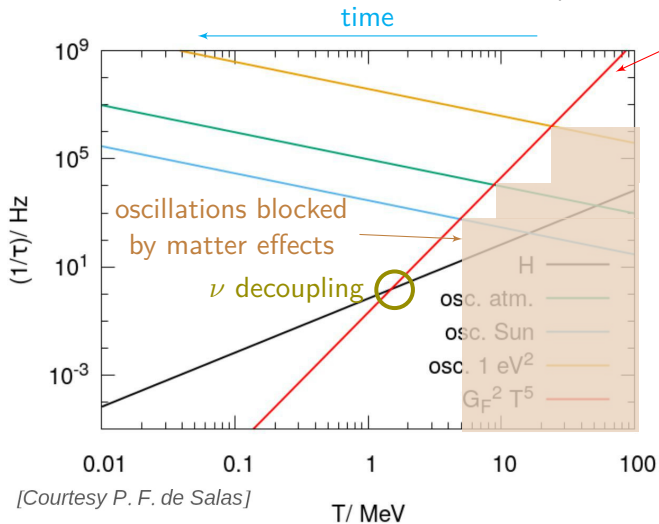
$$\langle E_\nu \rangle \simeq 3.1 T_{\nu,0} \simeq 5 \times 10^{-4} \text{ eV}$$

$$n_0 = n_{\nu,0} = n_{\bar{\nu},0} \simeq 56 \text{ cm}^{-3} \text{ per family}$$

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ν decouple mostly before $e^+ e^- \rightarrow \gamma\gamma$ annihilation!
 actually, the decoupling T is momentum dependent!

distortions to equilibrium f_ν !

ν oscillations in the early universe

[Bennett, SG+, JCAP 2021]
[Sigl, Raffelt, 1993]

comoving coordinates: $a = 1/T$ $x \equiv m_e a$ $y \equiv p a$ $z \equiv T_\gamma a$ $w \equiv T_\nu a$

density matrix: $\varrho(x, y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} \end{pmatrix}$
 $\propto \langle a_j^\dagger(p, t) a_i(p, t) \rangle$
off-diagonals to take into account coherency in the neutrino system

$$\varrho \text{ evolution from } x \text{ to } y: \quad x H \frac{d\varrho(y, x)}{dx} = -i a [\mathcal{H}_{\text{eff}}, \varrho] + b \mathcal{I}$$

H Hubble factor \rightarrow expansion (depends on universe content)

$$\text{effective Hamiltonian } \mathcal{H}_{\text{eff}} = \frac{M_{\text{F}}}{2y} - \frac{2\sqrt{2}G_{\text{F}}ym_e^6}{x^6} \left(\frac{E_\ell + P_\ell}{m_W^2} + \frac{4}{3} \frac{E_\nu}{m_Z^2} \right)$$

vacuum oscillations \longleftarrow \longrightarrow matter effects

\mathcal{I} collision integrals

take into account ν -e scattering and pair annihilation, ν - ν interactions

2D integrals over momentum, take most of the computation time

$$\text{solve together with } z \text{ evolution, from } x \frac{d\rho(x)}{dx} = \rho - 3P$$

ρ, P total energy density and pressure, also take into account FTQED corrections

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[Sigl, Raffelt, 1993]

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FORTRAN-EVOLVED PRIMORDIAL NEUTRINO OSCILLATIONS
(FORTePIANO)

https://bitbucket.org/ahep_cosmo/fortepiano_public

vacuum oscillations



matter effects

\mathcal{I} collision integrals

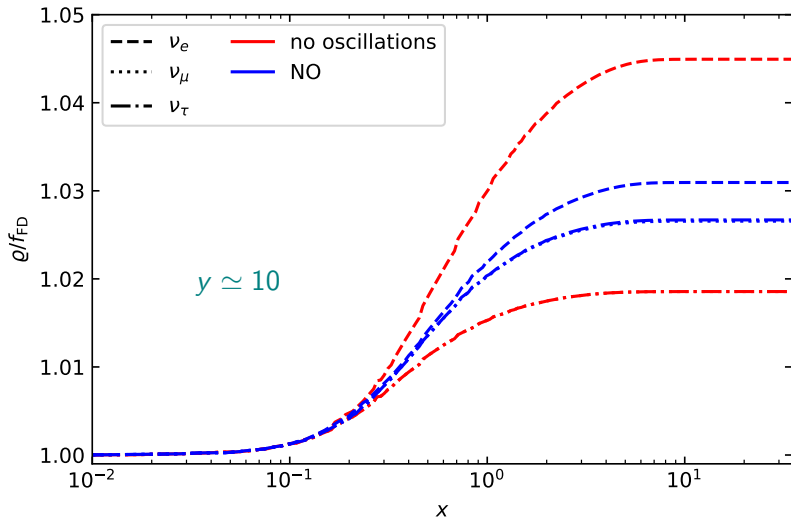
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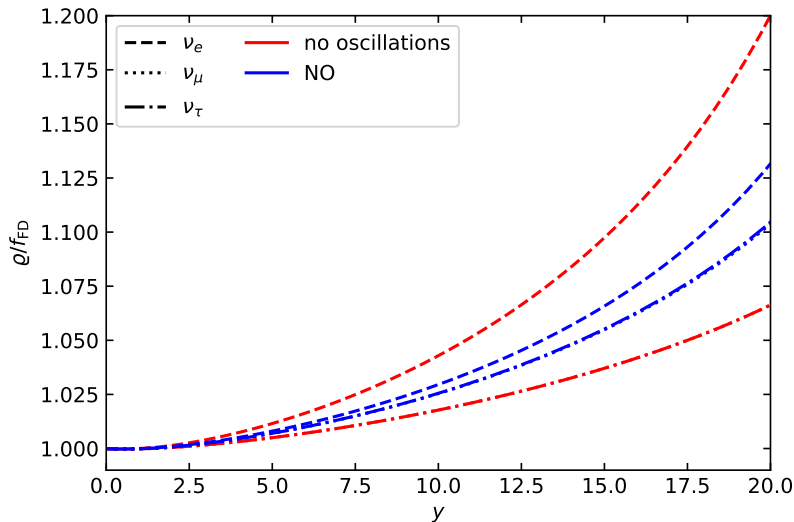
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Distortion of the momentum distribution (f_{FD} : Fermi-Dirac at equilibrium)

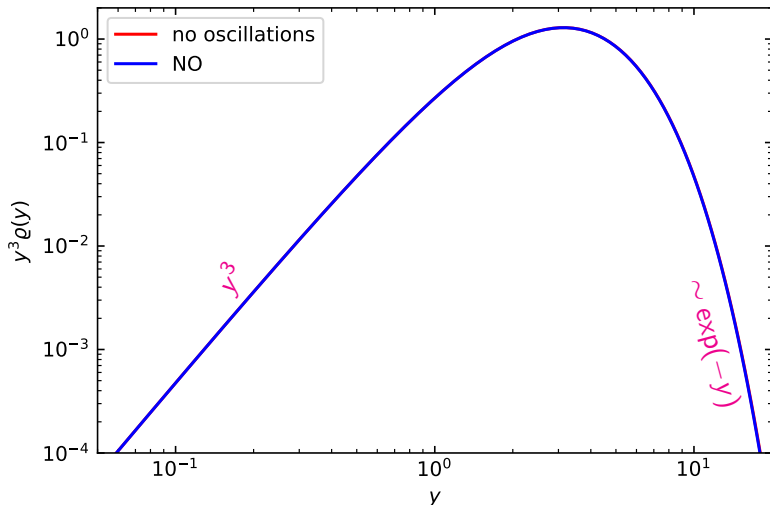


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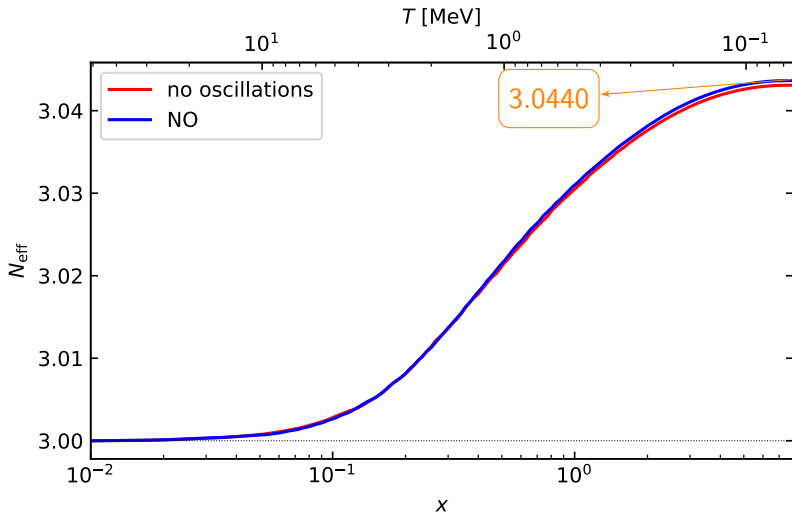


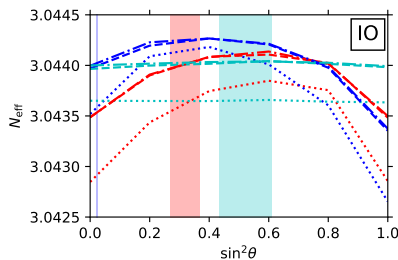
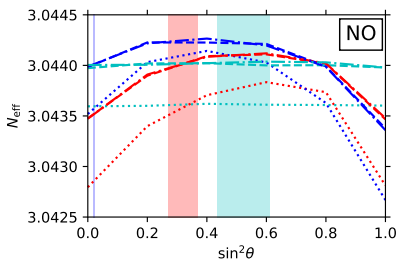
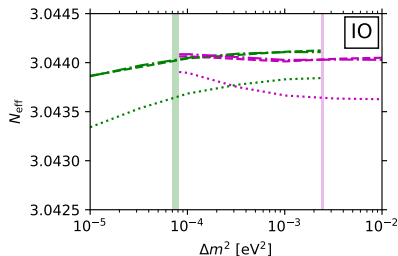
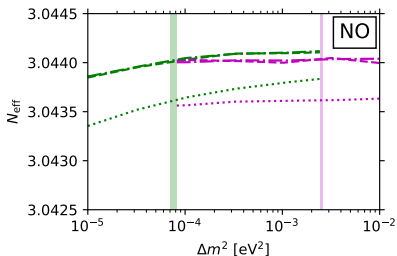
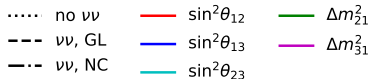
$$N_{\text{eff}}^{\text{final}} = \frac{8}{7} \left(\frac{11}{4}\right)^{4/3} \frac{\rho_\nu}{\rho_\gamma} = \frac{8}{7} \left(\frac{11}{4}\right)^{4/3} \frac{1}{\rho_\gamma} \sum_i g_i \int \frac{d^3 p}{(2\pi)^3} E(p) f_{\nu,i}(p)$$

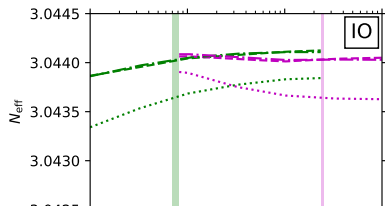
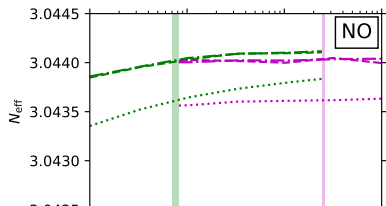
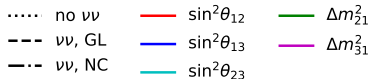
$(11/4)^{1/3} = (T_\gamma/T_\nu)^{\text{fin}}$
 $\hookrightarrow \propto y^3 g_{ii}(y)$



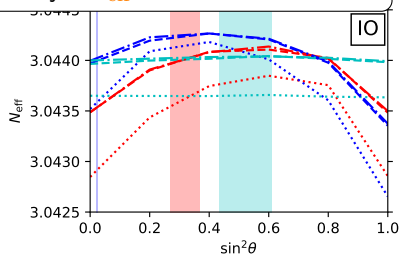
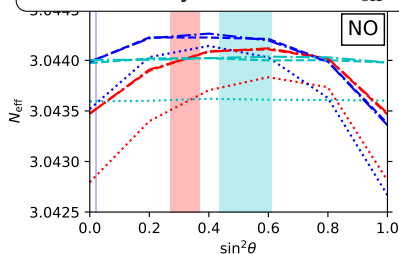
$$N_{\text{eff}}^{\text{any time}} = \frac{8}{7} \left(\frac{T_\gamma}{T_\nu} \right)^4 \frac{\rho_\nu}{\rho_\gamma} = \frac{8}{7} \left(\frac{T_\gamma}{T_\nu} \right)^4 \frac{1}{\rho_\gamma} \sum_i g_i \int \frac{d^3 p}{(2\pi)^3} E(p) f_{\nu,i}(p)$$





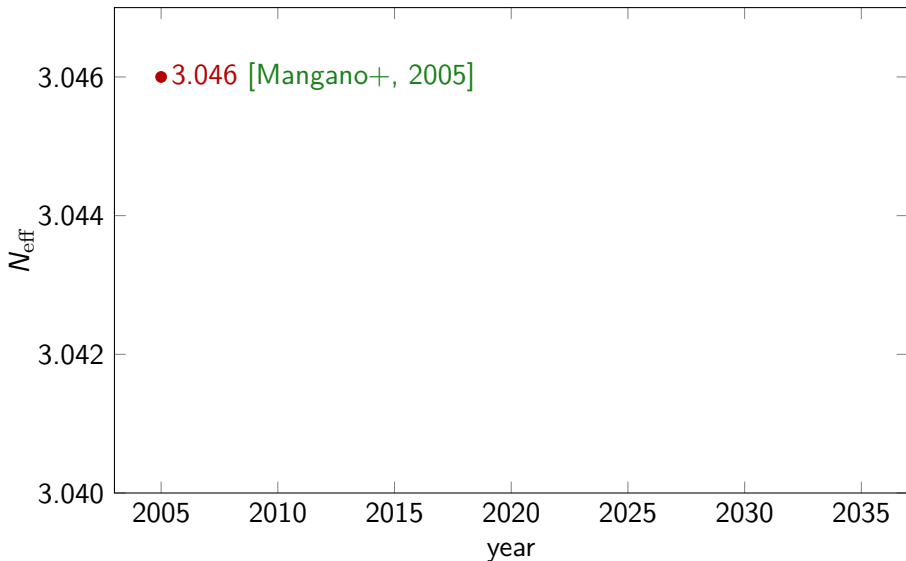


within 3σ ranges allowed by global fits [deSalas, SG+, JHEP 2021]
 only θ_{12} affects N_{eff} , at most by $\delta N_{\text{eff}} \approx 10^{-4}$



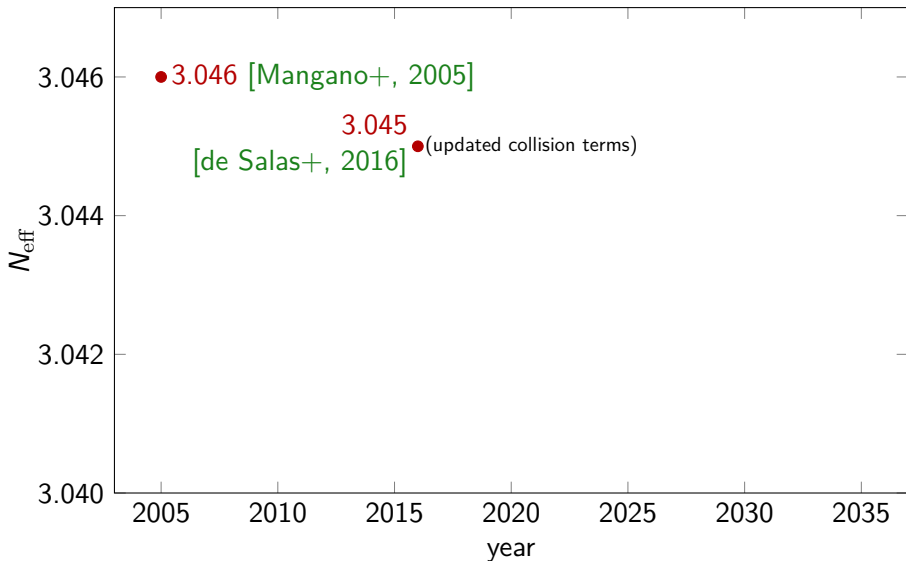
How precise is $N_{\text{eff}} = 3.04\dots?$

Full 3ν mixing results:



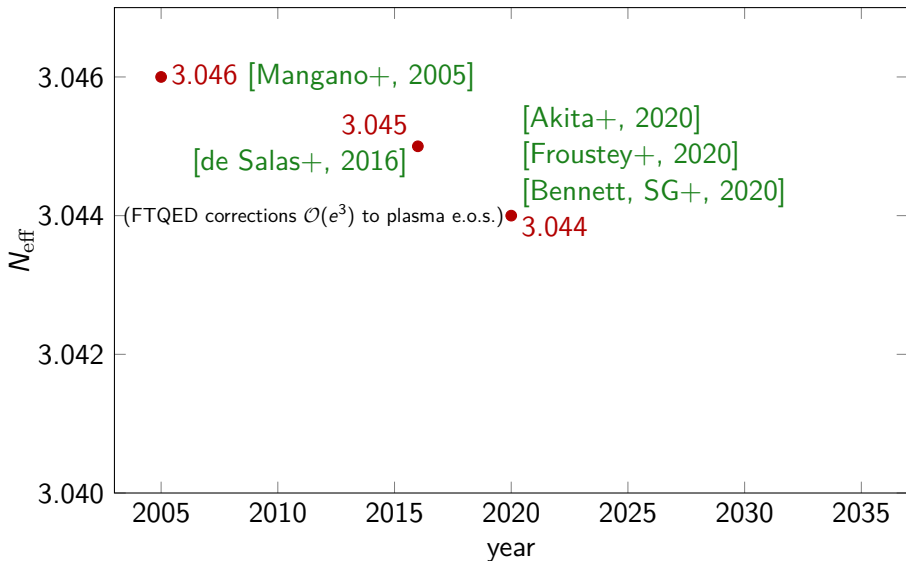
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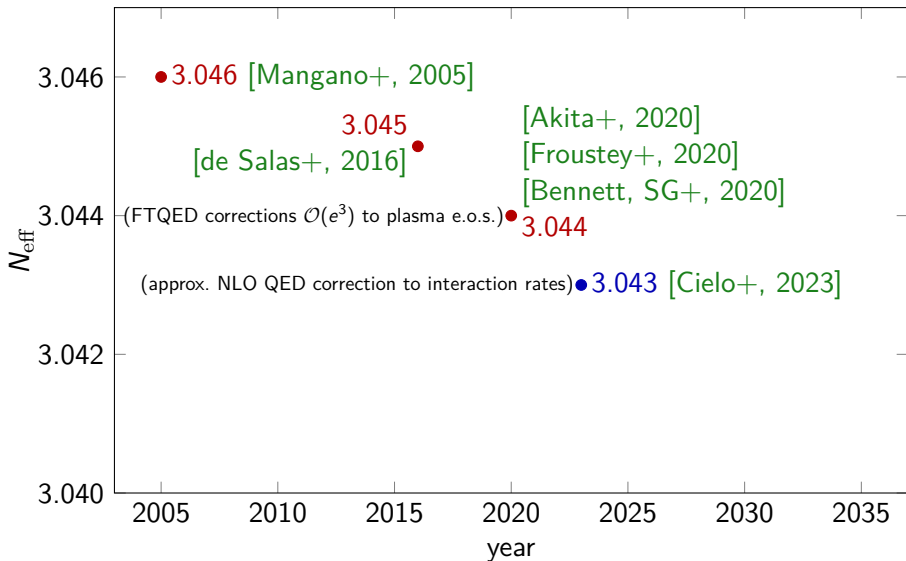
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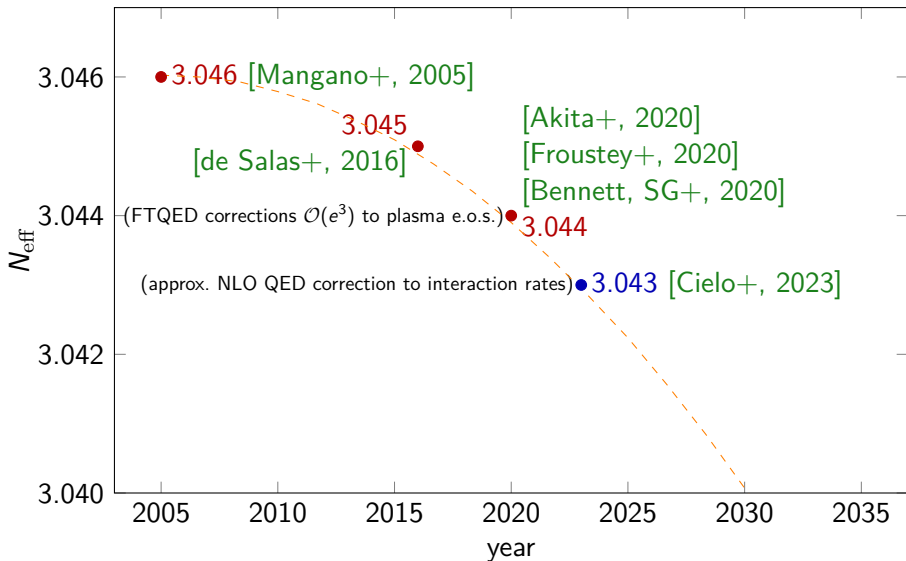
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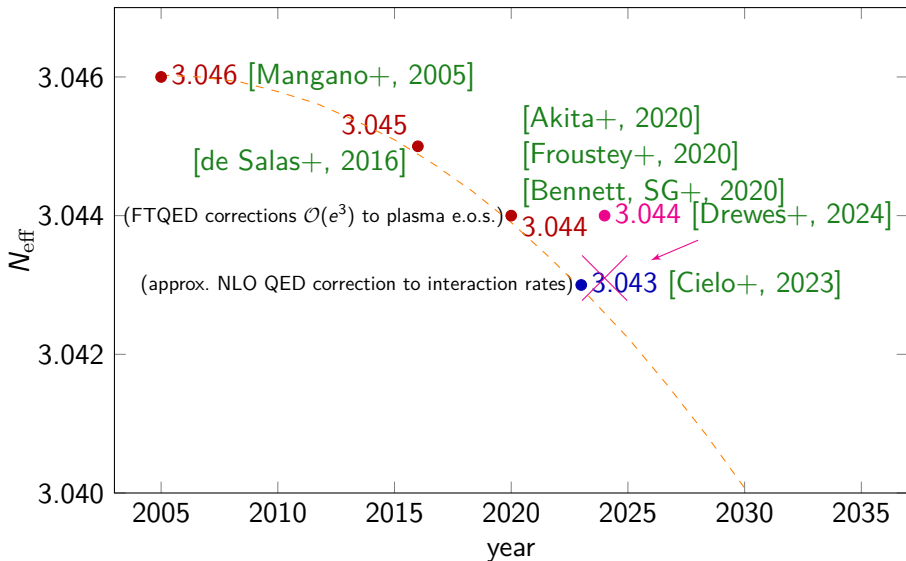
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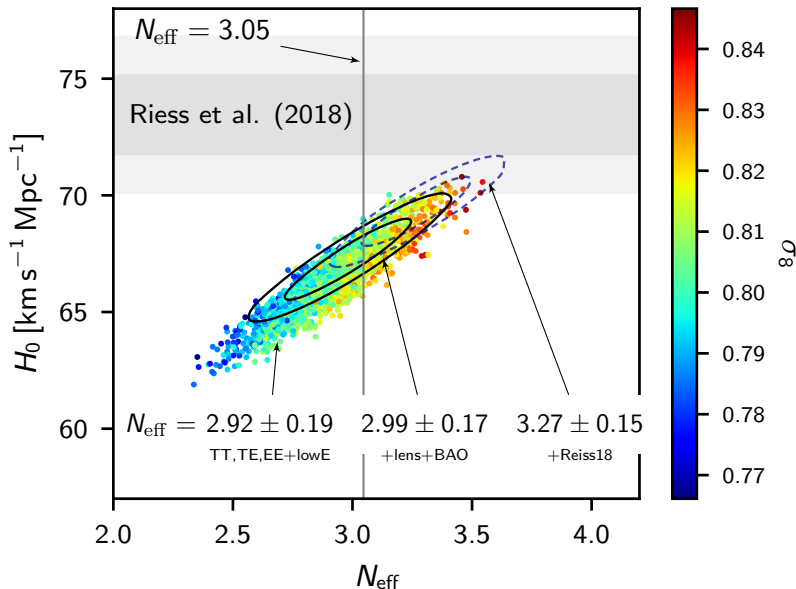
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Full 3ν mixing results:





N_{eff} and BBN

BBN: production of light nuclei
at $t \sim 1\text{s}$ to $t \sim \mathcal{O}(10^2)\text{s}$

temperature $T_{\text{fr}} \simeq 1\text{ MeV}$
from nucleon freeze-out:

$$\Gamma_{n \leftrightarrow p} \sim G_F^2 T^5 = H \sim \sqrt{g_* G_N T^2}$$

$$T_{\text{fr}} \simeq (g_* G_N / G_F^4)^{1/6}$$

enters

$$n/p = \exp(-Q/T_{\text{fr}})$$

which controls element abundances

$$g_* \text{ depends on } N_{\text{eff}}$$

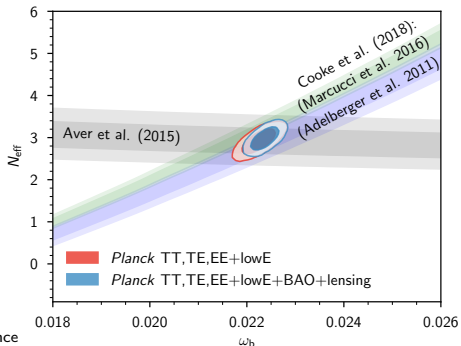
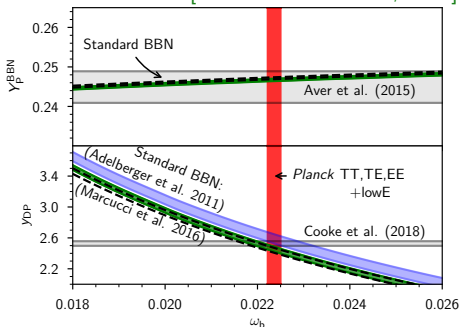
abundances depend on N_{eff}

G_F Fermi constant n, p : neutron, proton density number
 G_N Newton constant $Q = 1.293\text{ MeV}$ neutron-proton mass difference

S. Gariazzo

"Relic neutrinos: decoupling and direct detection perspectives"

[Planck Collaboration, 2018]



UTFSM, 28/03/2024

18/38

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which controls element abundances

$$g_* \text{ depends on } N_{\text{eff}}$$

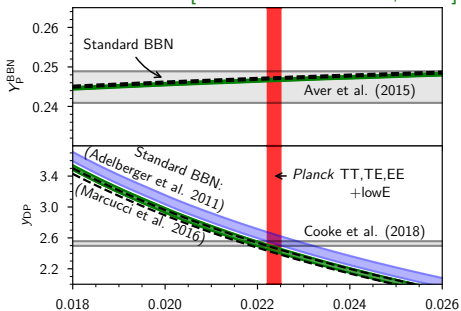
abundances depend on N_{eff}

G_F Fermi constant n, p : neutron, proton density number
 G_N Newton constant $Q = 1.293\text{ MeV}$ neutron-proton mass difference

S. Gariazzo

"Relic neutrinos: decoupling and direct detection perspectives"

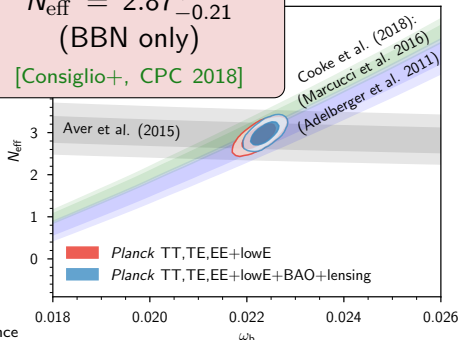
[Planck Collaboration, 2018]



$$N_{\text{eff}} = 2.87^{+0.24}_{-0.21}$$

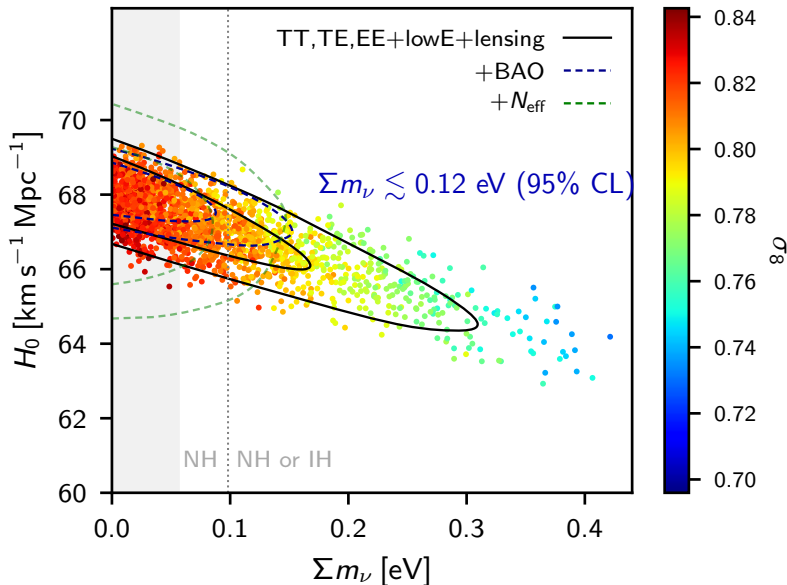
(BBN only)

[Consiglio+, CPC 2018]

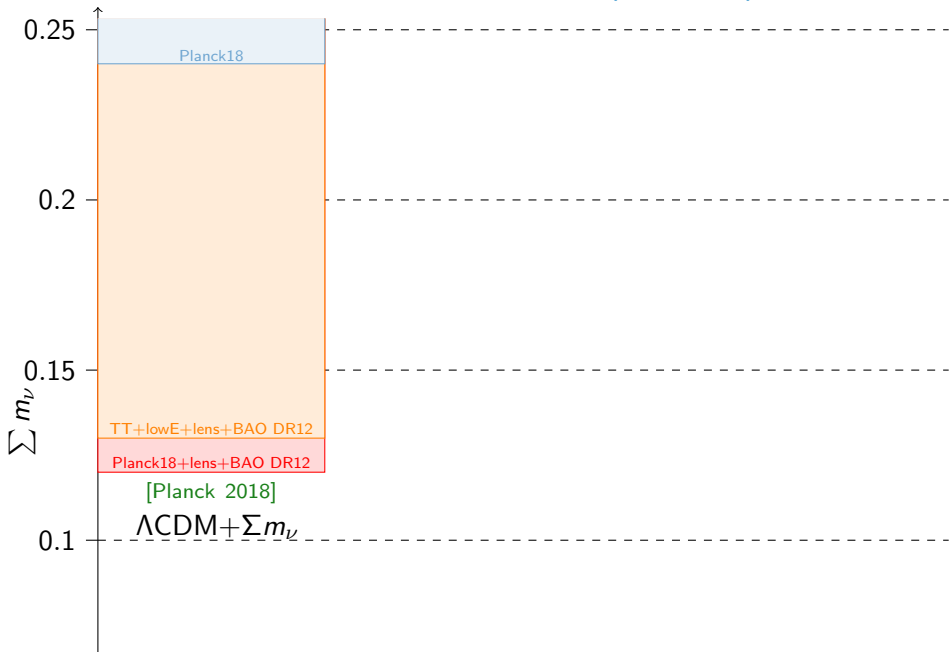


UTFSM, 28/03/2024

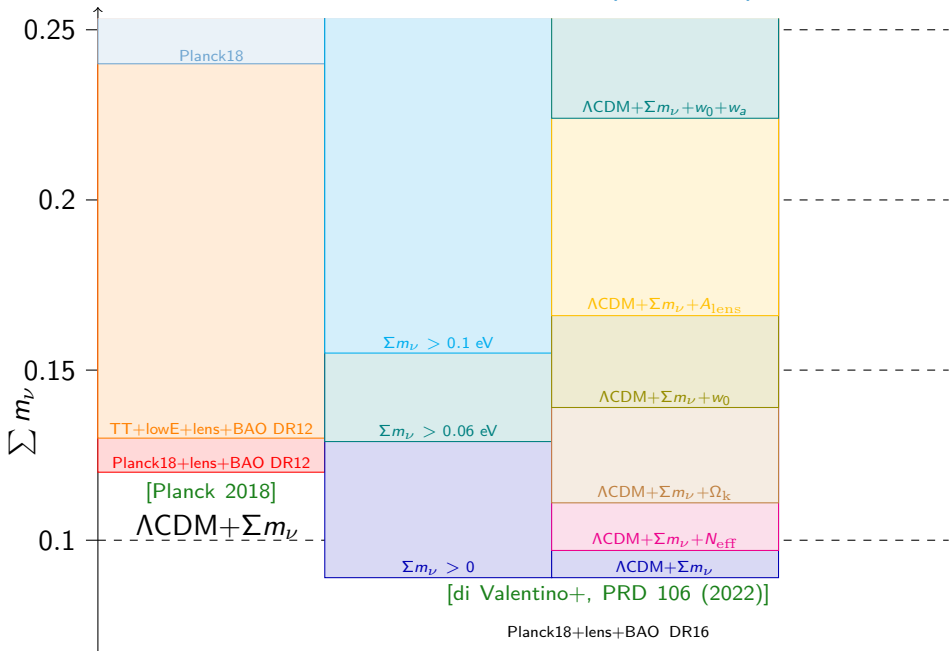
18/38



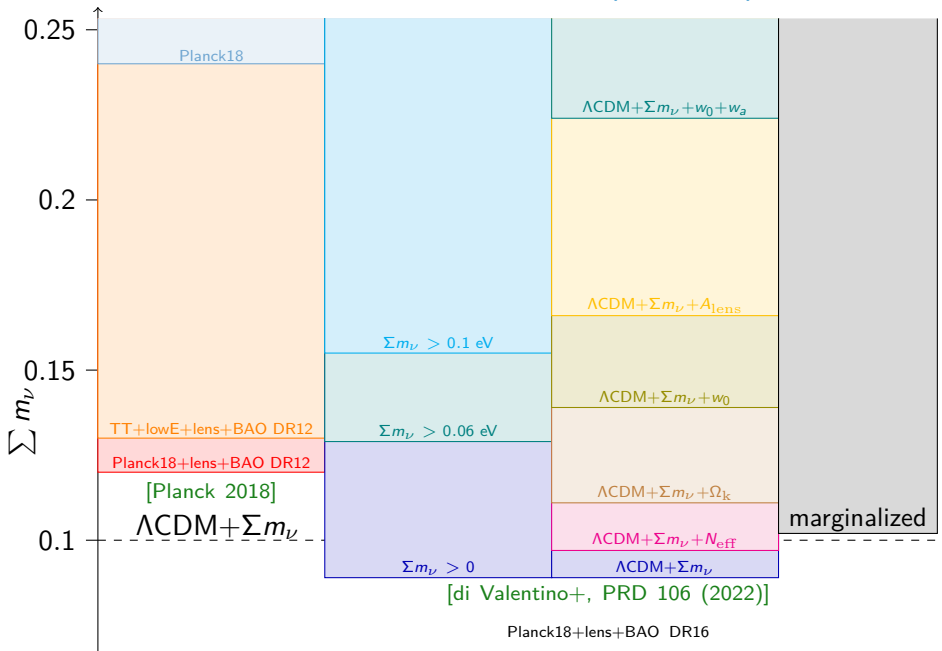
Cosmological neutrino mass bounds (95% CL)



Cosmological neutrino mass bounds (95% CL)



Cosmological neutrino mass bounds (95% CL)



Can a cosmological limit on Σm_ν disfavor IO? [PDU 40 (2023)]

[PDU 40 (2023)]

standard factor

Cosmology measures $\omega_\nu = \Omega_\nu h^2 = \Sigma m_\nu / (94.12 \text{ eV})$

NO: $\Sigma m_\nu \gtrsim 0.06 \text{ eV}$

Current: $\Sigma m_\nu \lesssim 0.1 \text{ eV}$ (95%)

IO: $\Sigma m_\nu \gtrsim 0.1 \text{ eV}$

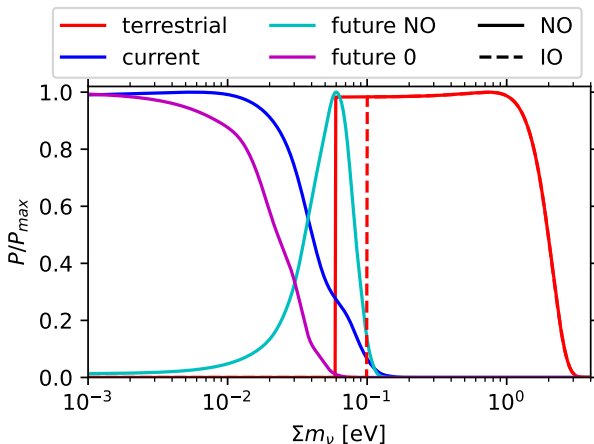
Future sensitivity: $\sigma(\Sigma m_\nu) \simeq 0.02 \text{ eV}$

Still preferring $\Sigma m_\nu = 0$?

Will measure e.g. $\Sigma m_\nu = 0.06 \text{ eV}$?

tension even with NO!

confirm NO, disfavor IO



Can a cosmological limit on Σm_ν disfavor IO? [PDU 40 (2023)]

standard factor

Cosmology measures $\omega_\nu = \Omega_\nu h^2 = \Sigma m_\nu / (94.12 \text{ eV})$

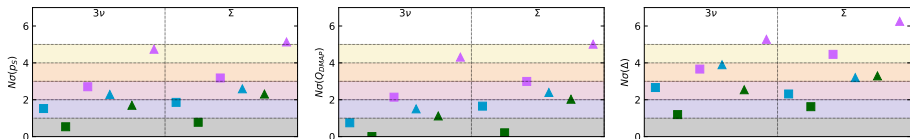
Is there a tension between cosmology and oscillations?

or will there be a tension?

several possible tests can be considered, similar results

$\Sigma m_\nu \lesssim 0.1 \text{ eV}$ (95%)
 $\Sigma m_\nu = 0.06 \pm 0.02 \text{ eV}$ (1σ)
 $\Sigma m_\nu = 0.00 \pm 0.02 \text{ eV}$ (1σ)

● current ■ NO
● future NO ▲ IO
● future 0



currently only mild tension between cosmology and oscillations

future NO can be at $\sim 2\sigma$ tension with IO

future 0 can be at $\sim 2 - 3\sigma$ tension with NO, $\gtrsim 4\sigma$ with IO

Can a cosmological limit on Σm_ν disfavor IO?

[PDU 40 (2023)]

standard factor

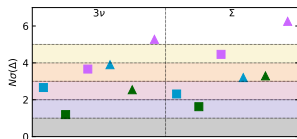
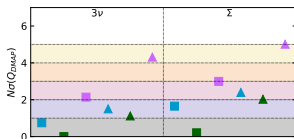
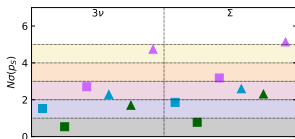
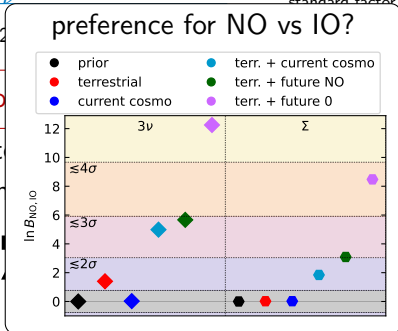
Cosmology measures $\omega_\nu = \Omega_\nu h^2$

Is there a tension between cosmo

or will there be a t

several possible tests can be con

- $\Sigma m_\nu \lesssim 0.1$ eV (95%) ● current
- $\Sigma m_\nu = 0.06 \pm 0.02$ eV (1σ) ● future NO
- $\Sigma m_\nu = 0.00 \pm 0.02$ eV (1σ) ● future 0



currently only mild tension between cosmology and oscillations

future NO can be at $\sim 2\sigma$ tension with IO

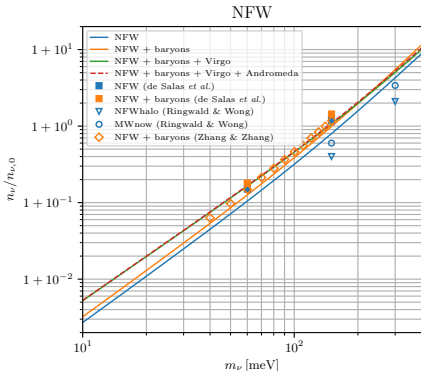
future 0 can be at $\sim 2 - 3\sigma$ tension with NO, $\gtrsim 4\sigma$ with IO

C

Clustering in the local Universe

Based on:

- JCAP 09 (2017) 034
- JCAP 01 (2020) 015



ν clustering with N-one-body simulations

Relic neutrinos are **slow!** [$c_\nu \sim 160(1+z)(1 \text{ eV}/m_\nu) \text{ km s}^{-1}$]

Can be trapped in the gravitational potential of the Milky Way and neighbours

$f_c(m_i) = n_i/n_{i,0}$ clustering factor \rightarrow How to compute it?

Idea from [Ringwald & Wong, 2004] \rightarrow **N-one-body** = $N \times$ single ν simulations

Assumptions:

- ν s are independent
- only gravitational interactions
- ν s do not influence matter evolution
- ($\rho_\nu \ll \rho_{\text{DM}}$)

\rightarrow each ν evolved from initial conditions at $z = 3$

\rightarrow spherical symmetry, coordinates (r, θ, p_r, l)

\rightarrow need $\rho_{\text{matter}}(z) = \rho_{\text{DM}}(z) + \rho_{\text{baryon}}(z)$

how many ν s is "N"?

\rightarrow must sample all possible r, p_r, l

\rightarrow must include all possible ν s that reach the MW

(fastest ones may come from
several (up to $\mathcal{O}(100)$) Mpc!)

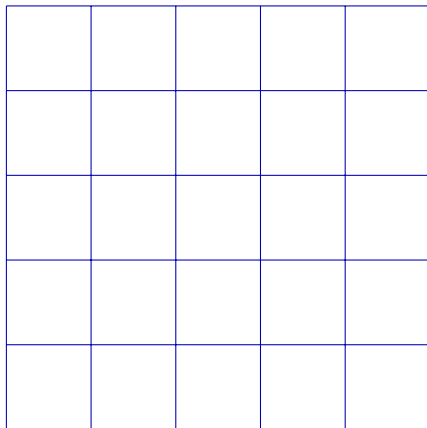
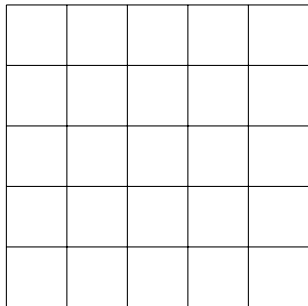
given $N \nu$:

\rightarrow weigh each neutrinos

\rightarrow reconstruct final density profile with kernel method from [Merritt & Tremblay, 1994]

Forward-tracking and back-tracking

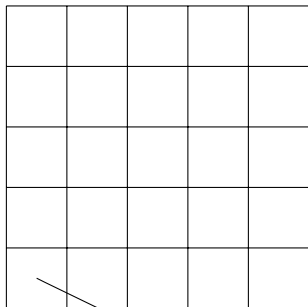
initial phase space, $z = 4$ \longrightarrow homogeneous Fermi-Dirac distribution



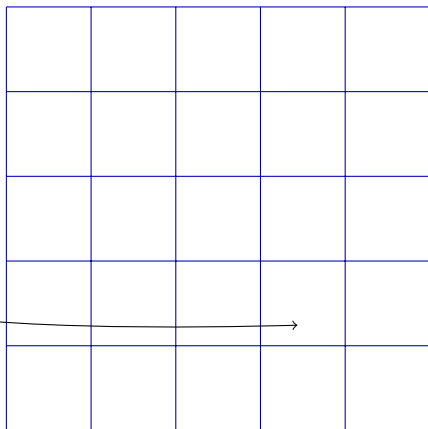
final phase space, $z = 0$

Forward-tracking and back-tracking

initial phase space, $z = 4$ \longrightarrow homogeneous Fermi-Dirac distribution



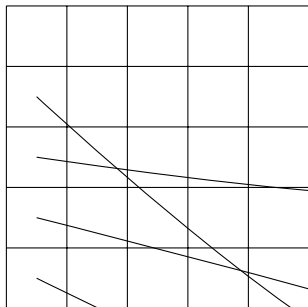
compute final position of each particle



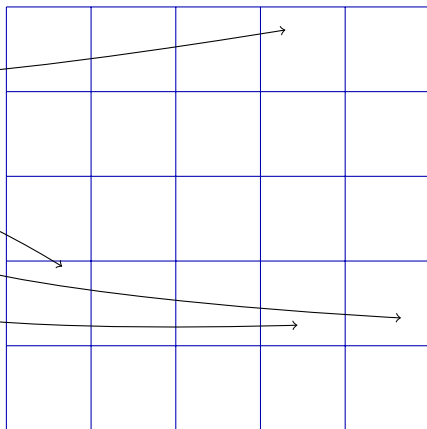
final phase space, $z = 0$

Forward-tracking and back-tracking

initial phase space, $z = 4$ \longrightarrow homogeneous Fermi-Dirac distribution



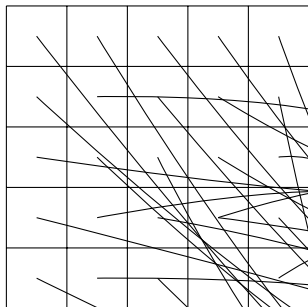
compute final position of each particle



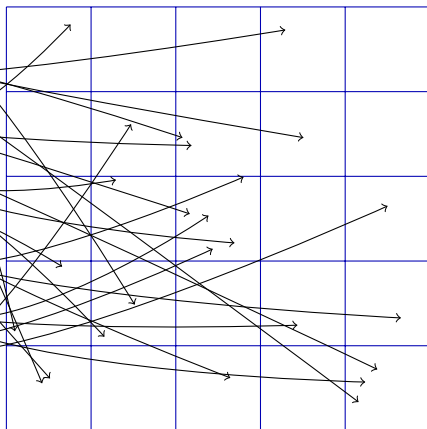
final phase space, $z = 0$

Forward-tracking and back-tracking

initial phase space, $z = 4$ \longrightarrow homogeneous Fermi-Dirac distribution



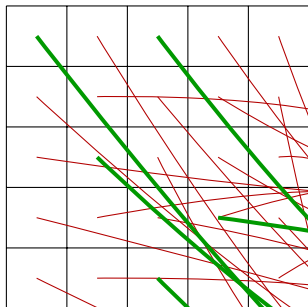
use positions to find neutrino distribution today



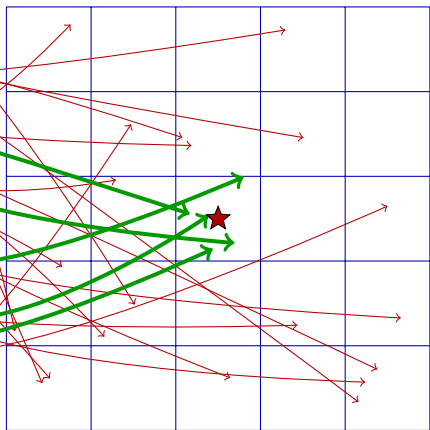
final phase space, $z = 0$

Forward-tracking and back-tracking

initial phase space, $z = 4$ \longrightarrow homogeneous Fermi-Dirac distribution



only interested in overdensity at Earth? ★

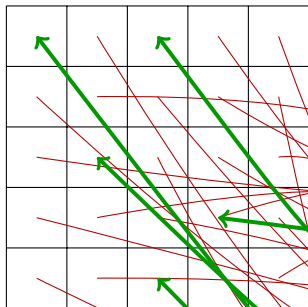


a lot of time is wasted!

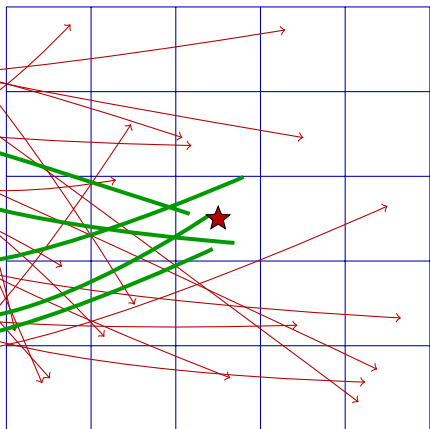
final phase space, $z = 0$

Forward-tracking and back-tracking

initial phase space, $z = 4$ \longrightarrow homogeneous Fermi-Dirac distribution



only interested in overdensity at Earth? ★



a lot of time is wasted!

smarter way: track backwards
only interesting particles!

final phase space, $z = 0$

Advantages of tracking back

First advantage is in computational terms: much less points to compute

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Second advantage: no need to use spherical symmetry!

Forward-tracking

initial conditions need to sample
1D for position + 2D for momentum
when using spherical symmetry

with full grid would re-
quire 3+3 dimensions!

Impossible to relax
spherical symmetry!

Back-tracking

“Initial” conditions only described
by 3D in momentum

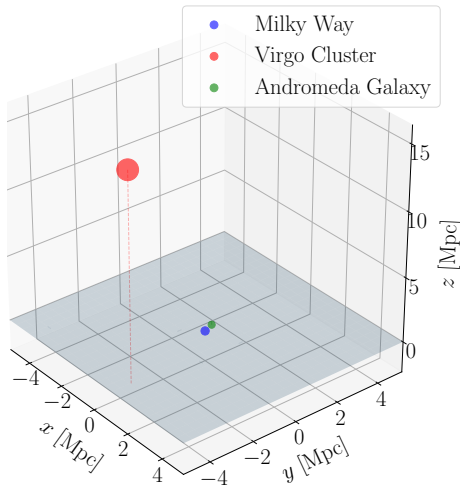
(position is fixed, apart for checks)

can do the calculation with
any astrophysical setup

Advantages of tracking back

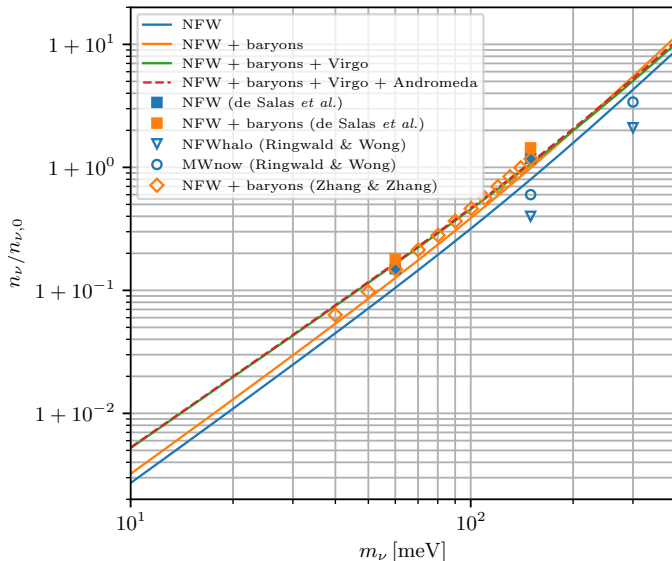
First advantage is in computational terms: much less points to compute

Second advantage: no need to use spherical symmetry!

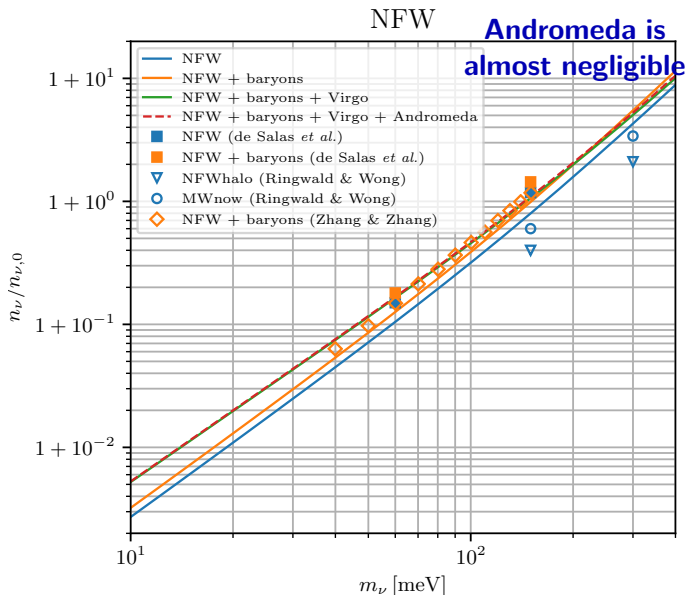


In comparison with previous results:

NFW

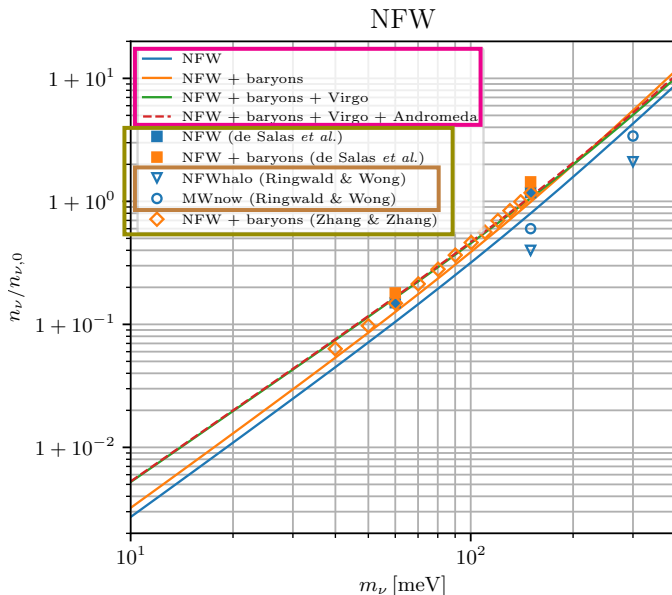


In comparison with previous results:



Clustering results with back-tracking

In comparison with previous results:



Warning: NFW is not the same for all the cases!

[de Salas+, 2017]
and

[Zhang², 2018]

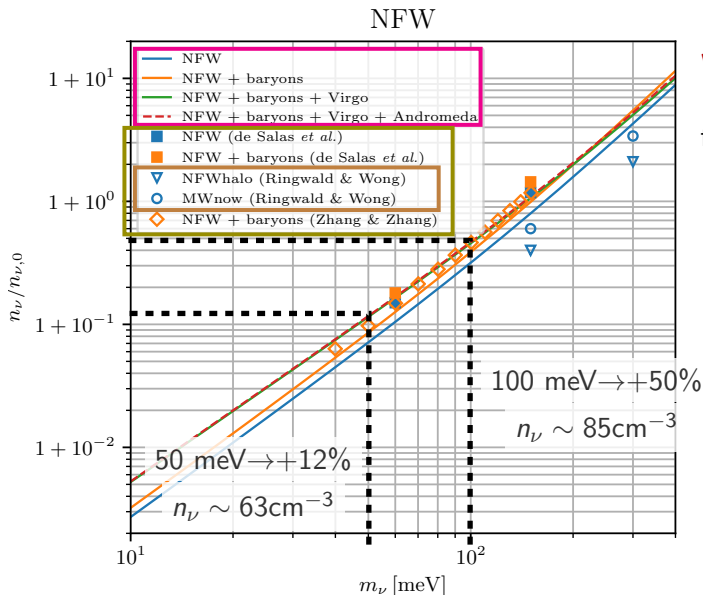
use $\gamma \neq 1$,
now we have

$$\gamma = 1$$

[Ringwald&Wong, 2004] uses old parameters

Clustering results with back-tracking

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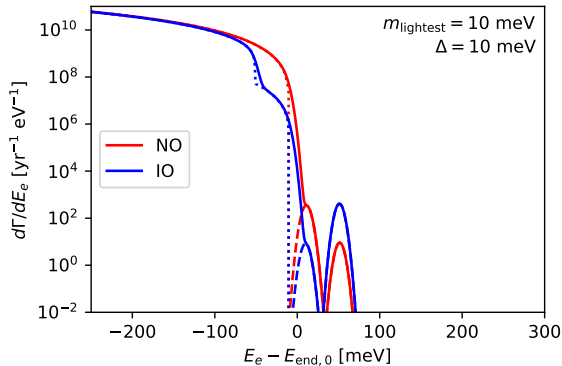
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D Direct detection of relic neutrinos

Based on:

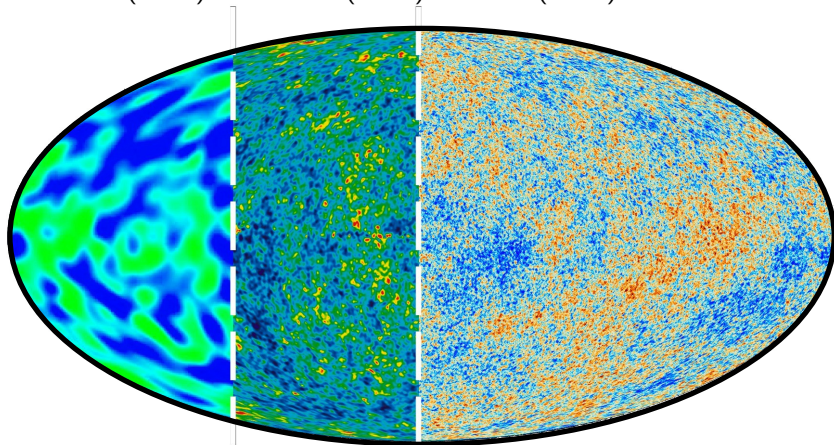
- JCAP 01 (2023) 003
- JCAP 08 (2014) 038
- JCAP 07 (2019) 047



The oldest picture of the Universe

The Cosmic Microwave Background, generated at $t \simeq 4 \times 10^5$ years

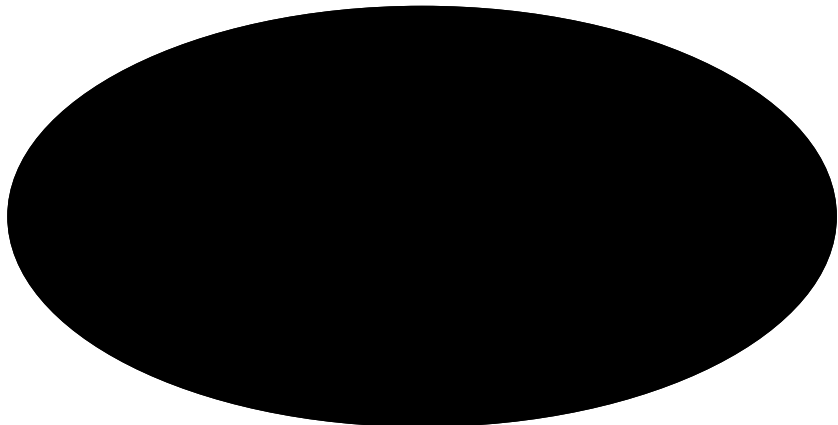
COBE (1992) WMAP (2003) Planck (2013)



The oldest picture of the Universe

The Cosmic Neutrino Background, generated at $t \simeq 1$ s

... → 2024 → ...



$$T_\nu \sim 10^{-4} \text{ eV}, E_\nu \sim 5 \times 10^{-4} \text{ eV today!}$$

We need **thresholdless detection process...** How do we get them?

Stodolsky effect?

How to directly detect non-relativistic neutrinos?

Stodolsky effect

[Stodolsky, 1974][Duda+, 2001]

(only if there is
lepton asymmetry)

energy splitting of e^- spin states due to
coherent scattering with relic neutrinos



torque on e^- in lab rest frame



use a ferromagnet to build detector



measure torque with a torsion balance

Stodolsky effect?

How to directly detect non-relativistic neutrinos?

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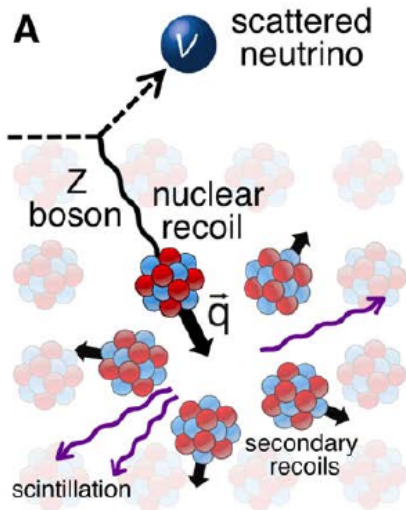
expected $a_\nu \simeq \mathcal{O}(10^{-26}) \text{ cm/s}^2$



$a_{\text{exp}} \simeq \mathcal{O}(10^{-12}) \text{ cm/s}^2$

First of all: what's **Coherent Elastic ν -Nucleous Scattering**?

elastic scattering where ν interacts with **nucleous "as a whole"**



Predicted for $|\vec{q}|R \lesssim 1$
by [Freedman, PRD 1974]

small recoil energies! $\lesssim 10$ keV...
difficult to measure

$$\frac{d\sigma}{dT}(E_\nu, T) \sim \frac{G_F^2 M}{4\pi} N^2$$

[Drukier, Stodolsky, PRD 1984]

enhancement N^2 because
 ν interacts
coherently with all nucleons

may give huge cross
section enhancement

CE ν NS?

First of all: what's **Coherent Elastic ν -Nucleous Scattering**?

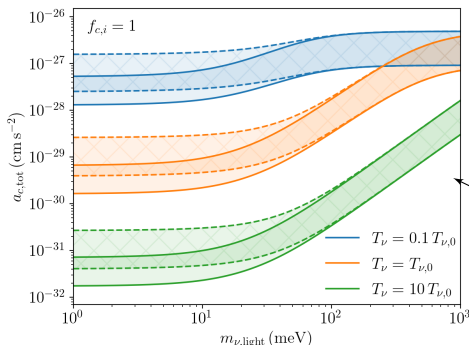
elastic scattering where ν interacts with **nucleous** "as a whole"

Can we detect relic neutrinos with CE ν NS?

relic neutrinos have **de Broglie length** $\lambda \sim 2\pi/p_\nu$



enhancement in interactions due to **coherence** with nuclei in volume λ^3



Acceleration induced by CE ν NS
of relic ν on test mass M :

$$a^N \propto ((A - Z)/A)^2 E_\nu / p_\nu^2 \Delta p_\nu n_\nu \rho$$

A, Z mass, atomic numbers
 p_ν, E_ν neutrino momentum and energy
 Δp_ν net momentum transfer
 n_ν neutrino number density
 ρ target mass density

unclustered relic ν s, $n_\nu = n_0$
 a^N of atoms in silicon target

CE ν NS?

First of all: what's **Coherent Elastic ν -Nucleous Scattering**?

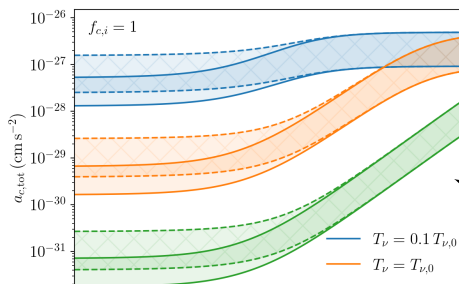
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unclustered relic ν s. $n_\nu = n_0$

proposed torsion balances can most optimistically reach $a \sim 10^{-23} \text{ cm s}^{-2}$

At interferometers?

How to directly detect non-relativistic neutrinos?

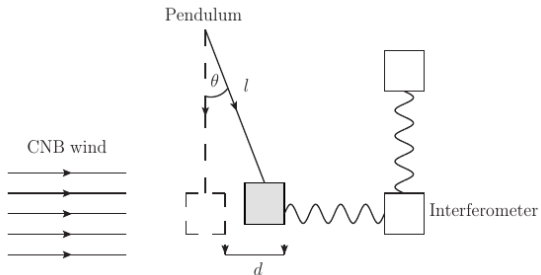
At interferometers

[Domcke+, 2017] [Shergold, 2021]

coherent scattering of relic ν on a pendulum



measure oscillations at interferometers



At interferometers?

How to directly detect non-relativistic neutrinos?

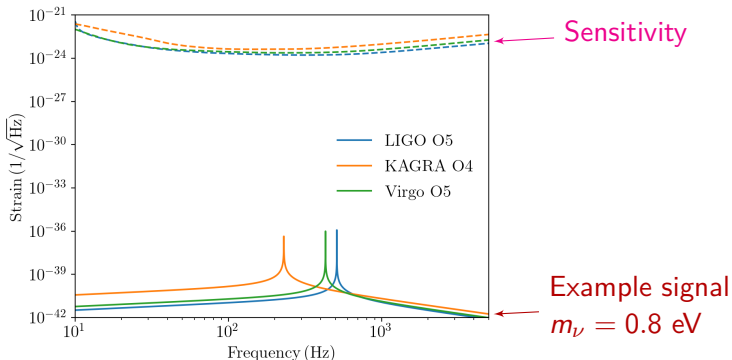
At interferometers

[Domcke+, 2017] [Shergold, 2021]

coherent scattering of relic ν on a pendulum



measure oscillations at interferometers



How to directly detect non-relativistic neutrinos?

Remember that
 $\langle E_\nu \rangle \simeq \mathcal{O}(10^{-4})$ eV today



a process without energy
 threshold is necessary

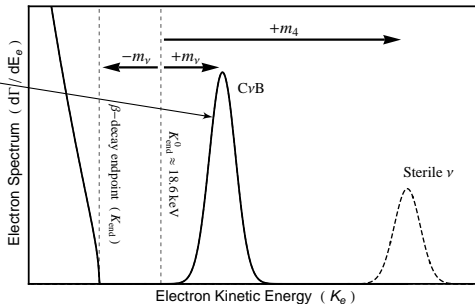
[Weinberg, 1962]: neutrino capture in β -decaying nuclei $\nu + n \rightarrow p + e^-$

Main background: β decay $n \rightarrow p + e^- + \bar{\nu}$!

signal is a peak at $2m_\nu$
 above β -decay endpoint

only with a lot of material

need a very good energy resolution



best element has highest $\sigma_{\text{NCB}}(v_\nu/c) \cdot t_{1/2}$

to minimize contamination from β decay background

Isotope	Decay	Q_β (keV)	Half-life (s)	$\sigma_{\text{NCB}}(v_\nu/c)$ (10^{-41} cm ²)
³ H	β^-	18.591	3.8878×10^8	7.84×10^{-4}
⁶³ Ni	β^-	66.945	3.1588×10^9	1.38×10^{-6}
⁹³ Zr	β^-	60.63	4.952×10^{13}	2.39×10^{-10}
¹⁰⁶ Ru	β^-	39.4	3.2278×10^7	5.88×10^{-4}
¹⁰⁷ Pd	β^-	33	2.0512×10^{14}	2.58×10^{-10}
¹⁸⁷ Re	β^-	2.64	1.3727×10^{18}	4.32×10^{-11}
¹¹ C	β^+	960.2	1.226×10^3	4.66×10^{-3}
¹³ N	β^+	1198.5	5.99×10^2	5.3×10^{-3}
¹⁵ O	β^+	1732	1.224×10^2	9.75×10^{-3}
¹⁸ F	β^+	633.5	6.809×10^3	2.63×10^{-3}
²² Na	β^+	545.6	9.07×10^7	3.04×10^{-7}
⁴⁵ Ti	β^+	1040.4	1.307×10^4	3.87×10^{-4}

best element has highest $\sigma_{\text{NCB}}(v_\nu/c) \cdot t_{1/2}$

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What material?

best element has highest $\sigma_{\text{NCB}}(v_\nu/c) \cdot t_{1/2}$

to minimize contamination from β decay background

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³H better because the cross section (\rightarrow event rate) is higher

What if we consider **accelerated tritium ions**, ${}^3\text{H}^+ + \nu_e \rightarrow {}^3\text{He}^{++} + e^-$?

Large background due to tritium beta decay...

Inverse process ${}^3\text{He}^{++} + \bar{\nu}_e \rightarrow {}^3\text{H}^+ + e^+$ would require **energy threshold**

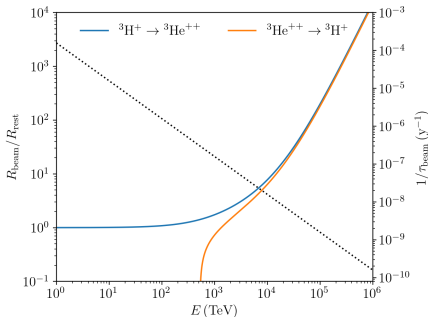
Match threshold in **beam rest frame**: $\tilde{E}_\nu = \frac{m_\nu}{M} E \geq Q$
with M , E ion mass, energy in lab frame

Even better:
resonant



can have many orders of magnitude larger cross-section (which still scales with G_F^2)

but also **large background**...need huge E to overcome it

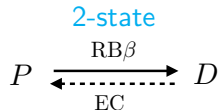


All mentioned cross sections scale with G_F^2

resonant bound beta decay (RB β): ${}^A_Z P + \nu_e \rightarrow {}^A_{Z+1} D + e^-$ (bound)

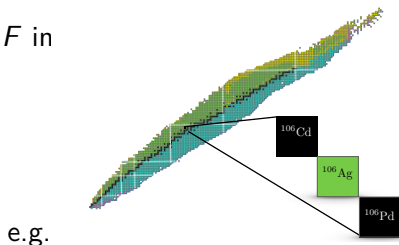
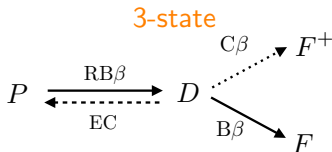
at resonance, G_F^2 suppression is lost in favor of Q^2 suppression!

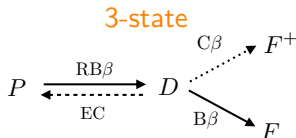
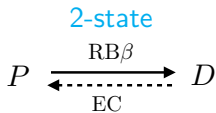
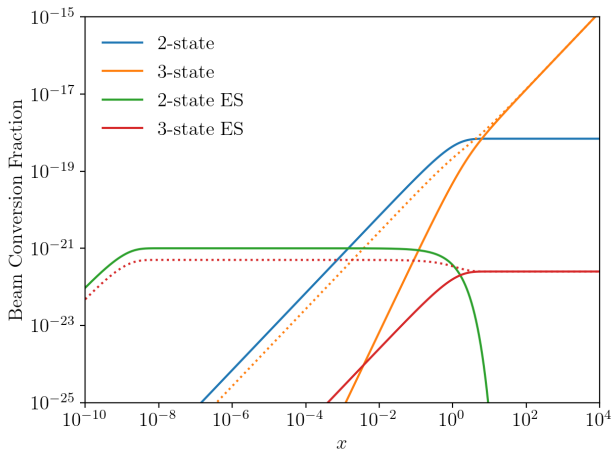
problem: final state D is converted back to P through electron capture (EC)!



Max event rate at equilibrium limits
information on RB β rate when running a long experiment

better: try to measure final stable state F in





excited states
also possible

$$\frac{d\tilde{\Gamma}_{\text{CNB}}}{dE_e}(E_e) = \frac{1}{\sqrt{2\pi\sigma}} \sum_{i=1}^{N_\nu} \bar{\sigma} N_T |U_{ei}|^2 n_0 f_c(m_i) \times e^{-\frac{[E_e - (E_{\text{end}} + m_i + m_{\text{lightest}})]^2}{2\sigma^2}}$$

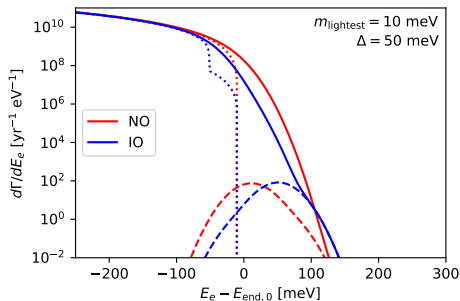
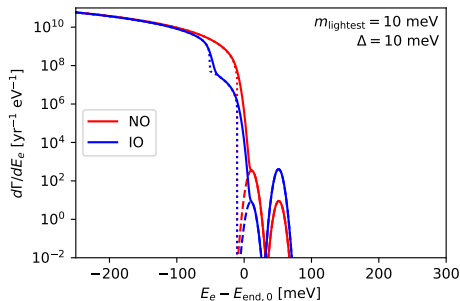
$$\frac{d\Gamma_\beta}{dE_e} = \frac{\bar{\sigma}}{\pi^2} N_T \sum_{i=1}^{N_\nu} |U_{ei}|^2 H(E_e, m_i)$$

$$\frac{d\tilde{\Gamma}_\beta}{dE_e}(E_e) = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{+\infty} dx \frac{d\Gamma_\beta}{dE_e}(x) \exp\left[-\frac{(E_e - x)^2}{2\sigma^2}\right]$$

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Pontecorvo Tritium Observatory for Light, Early-universe, Massive-neutrino Yield (PTOLEMY)

expected resolution $\Delta \simeq 0.1 \text{ eV?}$
 0.05 eV?

can probe $m_\nu \simeq 1.4\Delta \simeq 0.1 \text{ eV}$

built mainly for CNB

$M_T = 100 \text{ g}$ of atomic ^3H

$$\Gamma_{\text{CNB}} = \sum_{i=1}^3 |U_{ei}|^2 [n_i(\nu_{hR}) + n_i(\nu_{hL})] N_T \bar{\sigma} \sim \mathcal{O}(10) \text{ yr}^{-1}$$

N_T number of ^3H nuclei in a sample of mass M_T $\bar{\sigma} \simeq 3.834 \times 10^{-45} \text{ cm}^2$ n_i number density of neutrino i

(without clustering)

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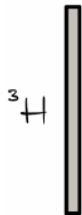
enhancement from
 ν clustering in the galaxy?

enhancement from
 other effects?

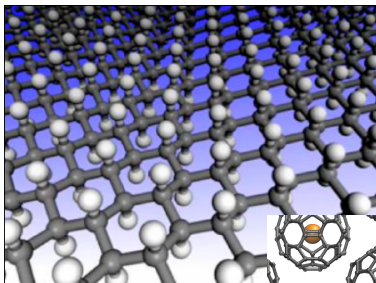
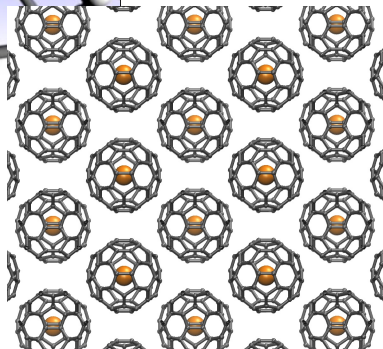
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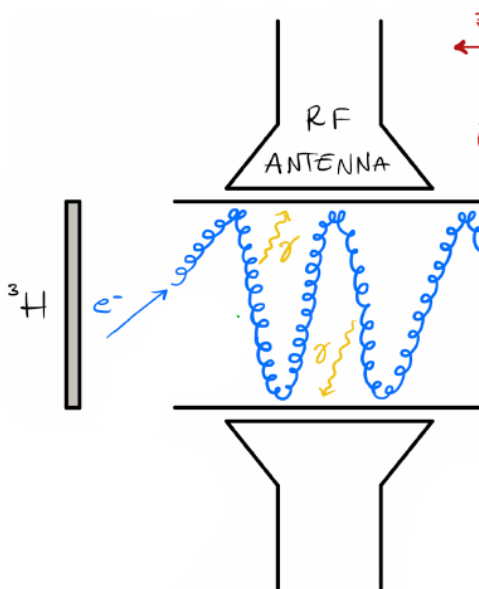


[Courtesy A. Esposito]

3
Ttritium on
graphene?tritium on
fullerene?

[Courtesy A. Esposito]

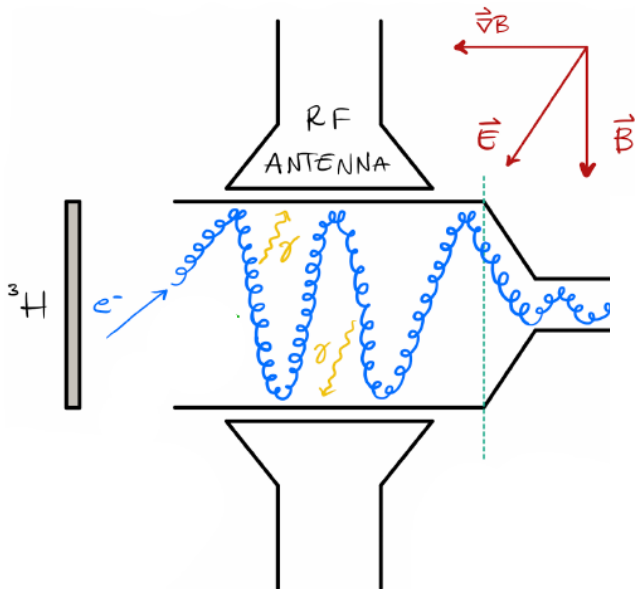
[Courtesy V. Tozzini]



need to reduce
amount of **electrons**
at final energy sensors:
EM filter

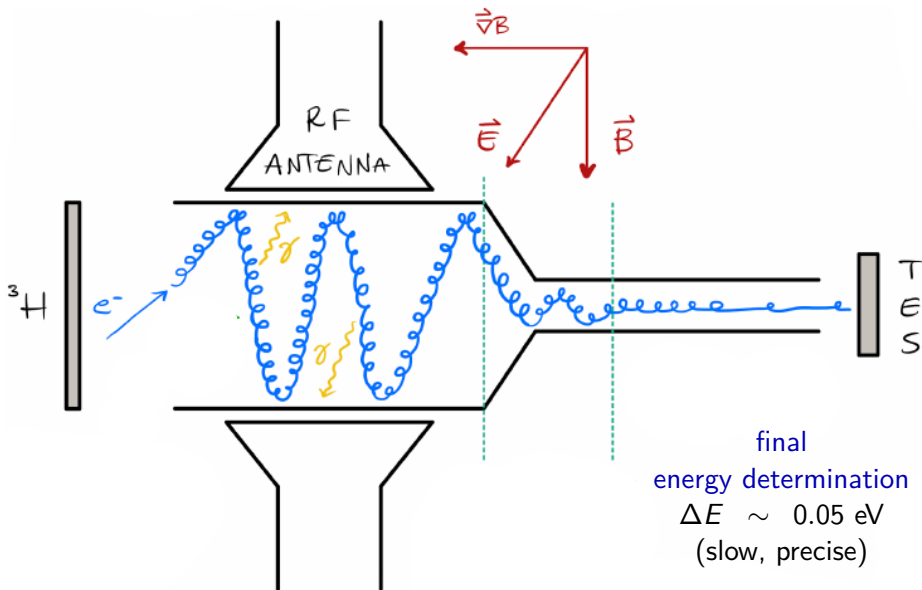
additional benefit:
first (**fast**) energy determination
thanks to **radio-frequency antenna**

[Courtesy A. Esposito]



filter only events
close to endpoint
($E \gtrsim E_0 - 10 \text{ eV}$)

[Courtesy A. Esposito]



[Courtesy A. Esposito]

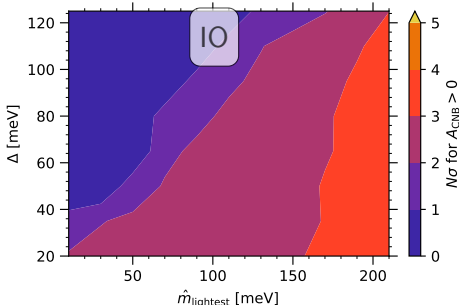
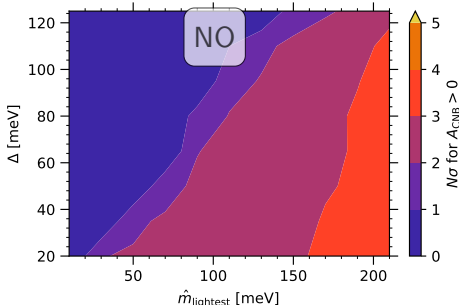
using the definition:

$$N_{\text{th}}^i(\theta) = A_\beta N_\beta^i(\hat{E}_{\text{end}} + \Delta E_{\text{end}}, m_i, U) + \mathbf{A}_{\text{CNB}} N_{\text{CNB}}^i(\hat{E}_{\text{end}} + \Delta E_{\text{end}}, m_i, U) + N_b$$

if $\mathbf{A}_{\text{CNB}} > 0$ at $N\sigma$, direct detection of CNB accomplished at $N\sigma$

statistical only!

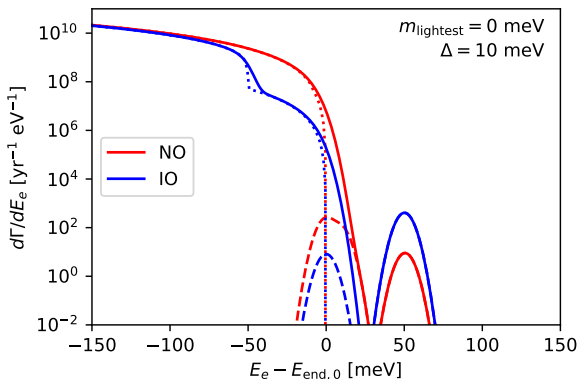
significance on $A_{\text{CNB}} > 0$
as a function of $\hat{m}_{\text{lightest}}$, Δ



What if the lightest neutrino is massless
and Δ cannot be small enough?

single NC events cannot be distinguished by the background (β -decay)!

$$\frac{\nu \text{ capture rate}}{\beta \text{ decay rate}} = \frac{\Gamma_{\text{NC}}}{\Gamma_{\beta}} \approx \frac{n_{\nu}}{56 \text{ cm}^{-3}} \frac{2.54 \times 10^{-11}}{(\Delta/\text{eV})^3} \quad \text{rates in the bin } \Delta \text{ on the endpoint}$$

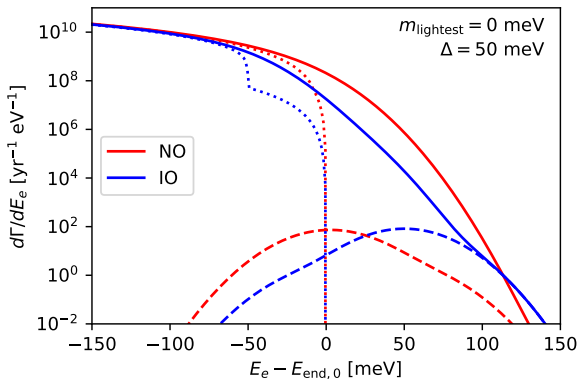


Time variations of ν capture rates

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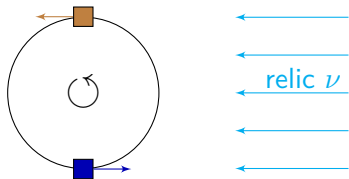
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rates in the bin Δ
on the endpoint



can be **daily** or annual modulation!

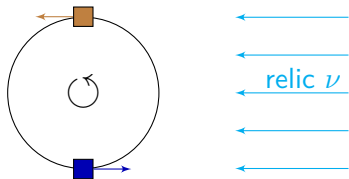
only for ν capture (no β -decay)

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can be **daily** or annual modulation!

only for ν capture (no β -decay)

Problem:

Expected **daily modulation**
is $\sim 1\%$ of the signal!!

Must use powerful technique
for signal/noise separation

**Fourier analysis and frequency
filtering may be sufficient**

no m_{ν} information in this way!



Z

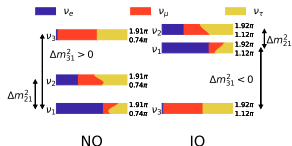
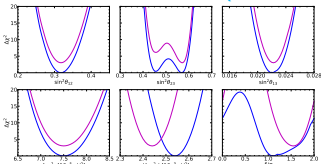
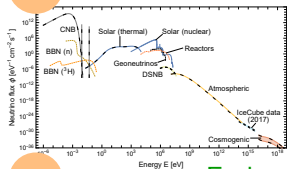
Conclusions

almost there!

What do we learn from relic neutrinos?

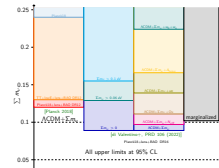
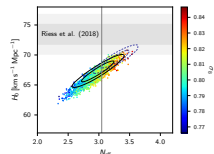
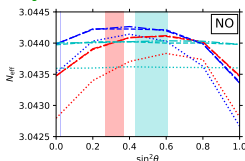
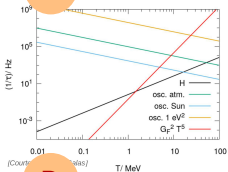
N

Neutrinos: precision era (many sources)



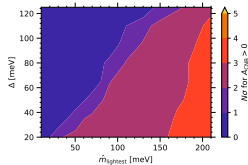
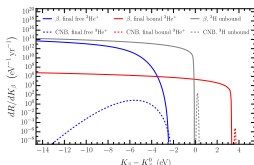
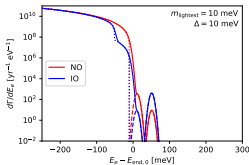
E

Early universe effects: indirect indications



D

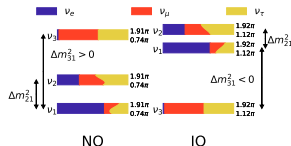
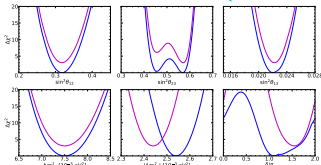
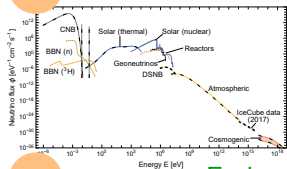
Direct detection: still long to go



What do we learn from relic neutrinos?

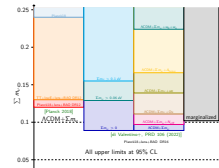
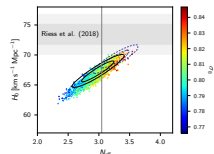
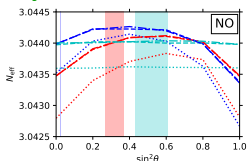
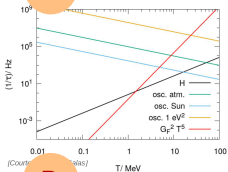
N

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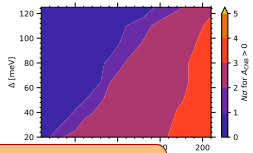
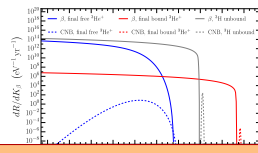
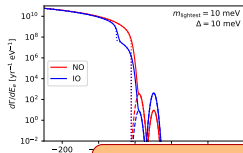
E

Early universe effects: indirect indications



D

Direct detection: still long to go



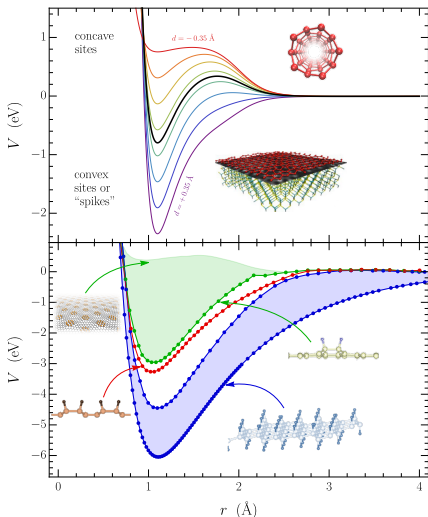
Thanks for your attention!

6 Appendix

Tritium atoms will be attached to graphene sheets



Quantum uncertainty on electron energy due to condensed matter state



Binding potential depends on distance graphene-tritium and graphene sheet configuration

Concave sites

behave differently than

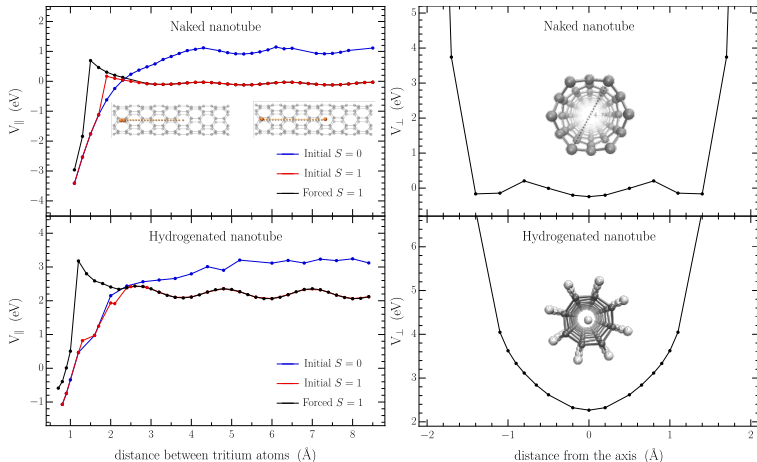
convex ones

Hydrogen coverage (amount of H atoms in the graphene sheet) is also important

Tritium atoms will be attached to graphene sheets



Quantum uncertainty on electron energy due to condensed matter state

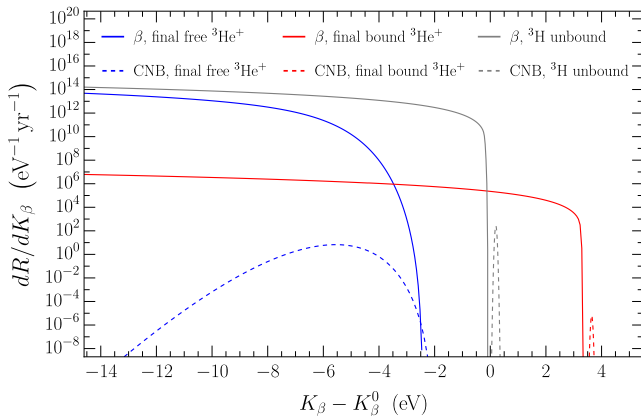


Nanotube configurations have flat potential parallel to nanotube axis

Tritium atoms will be attached to graphene sheets



Quantum uncertainty on electron energy due to condensed matter state



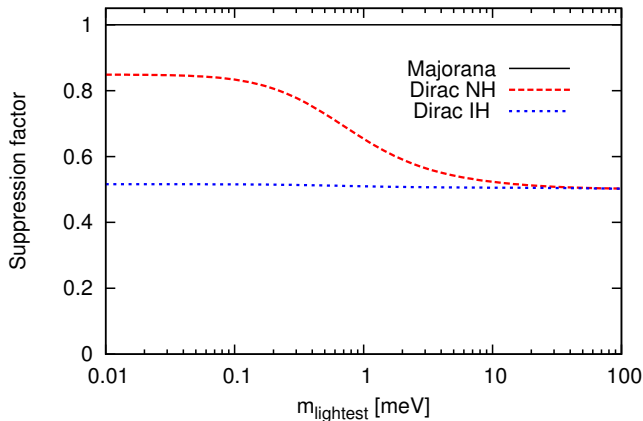
Graphene configuration can make observation range from
completely impossible or **suppressed but possible**

direct detection through $\nu_e + {}^3\text{H} \rightarrow e^- + {}^3\text{He}$

only neutrinos with correct chirality can be detected!

non-relativistic **Majorana** case: ν and $\bar{\nu}$ cannot be distinguished!

expect **more events** for the **Majorana** than for **Dirac** case



Dirac **normal**
or **inverted**
ordering differ
because lighter
 ν_1 and ν_2 in **NH**
are **relativistic**
↓
almost
indistinguishable
from **Majorana**