Battling with Uncertainties

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May 2013

What is really meant by *scale variation* problem?

- Is it a problem related to a $\mu_{\rm R}, \mu_{\rm F}$ dependence of the results?
- Is it a problem of *Missing Higher Orders* (**MHO**)?
- Is it a problem of large logaritmic terms in fixed-order perturbation theory?

It is a problem of **renormalization**, based on

bare parameters $\longrightarrow$ renormalized parameters $\rightarrow$ input data set

- Observable $\mathcal{O}$, at a given scale s, written in terms of bare quantities

$$\mathcal{O}_{\rm bare} \quad = \quad \mathcal{O}\left(\{p_{\rm bare}\}, s, \text{cutoff}\right)$$

- Same observable written in terms of renormalized quantities in a given **Renormalization Scheme**

$$\mathcal{O}_{\rm ren} \quad = \quad \mathcal{O}_{\rm RS}\left(\{p_{\rm ren}\}, s, \mu_{\rm R}\right)$$

- Same observable written in terms of physical quantities

$$\mathcal{O}_{\rm phys} \quad = \quad \mathcal{O}\left(\{p_{\rm phys}\}, s, s_0\right)$$

where s_0 is the subtraction scale, i.e. $p_{\rm phys}(s_0)$. Typical example is QED where $p_{\rm bare} = e$, $p_{\rm ren} = e_{\rm R}$, $p_{\rm phys} = \alpha(0)$, i.e. $s_0 = 0$.

To be more precise

$$\mathcal{O}_{\rm ren} = \mathcal{O}\left(\ln\frac{\mu_{\rm R}}{s}\right) \qquad \mathcal{O}_{\rm phys} = \mathcal{O}\left(\ln\frac{s_0}{s}\right)$$

To Summarize:

- Do we have a scale problem in QED (EW)? At the renormalized level *Yes*
- Do we have a *large-logs* problem in QED (EW)? It depends on the ratio s_0/s. For instance, it is not a good idea to use the $\alpha(0)$-scheme for large values of s. Usuallly the $G_{\rm F}$,-scheme works much better because $G_{\rm F}$ has a milder running with the scale.

Additional Questions:

- What to do if $\ln(s_0/s)$ is large? Re-summation is the answer and calculation where re-summation is included are to be preferred.
- Is re-summation always available? This is a thought question, difficult to answer in multi-scale problems.
- Is the **MHO** problem solved? No, there will always be **MHO**s in fixed-order perturbation theory that have nothing to do with scales!

Why is QCD different?

- Because of $\mu_{\rm R}$? No, because we are missing a physical subtraction point! Hence the introduction of $\Lambda_{\rm QCD}$
- Therefore, how to set $\mu_{\rm R}$ is debatable:
  - Making minimal the effect of N^nLO versus N^{n-1}LO?
  - Looking for a plateau?
  - Other?

Do we have a $\mu_{\rm F}$ problem in QED? In principle yes, because of initial state radiation (ISR) but the solution is clear, inside the QED Structure Functions we have $m_{\rm e}$ not μ_F.

How do we quantify theoretical uncertainty (**THU**)?

- Does it make sense to quantify **THU** (solely) by scale variations? This has been the conventional choice for many years, based on

$$\text{THU} \quad \hookrightarrow \quad \left[\frac{\mu}{\lambda}\,,\lambda\,\mu\right]$$

where the choice of λ is popular wisdom. The choice is hard to embed in a frequentist approach and it is a logical fallacy in which *the reasoner begins with what he or she is trying to end up with*.

Should we talk instead of **MHO**? Yes, this is better when we understand the origins:

- missing a *physical subtraction point*
- having to deal with *large logarithms* and missing *re-summation*
- missing *higher orders* in *fixed-order* perturbation theory

It is hard to have real control when the same single label has be assigned!

Sometimes, people talk about *physically motivated* choice of μ_{F}. This is based on the fact that *explicit* N^{n}LO corrections are small for this choice. However, arguments of convenience lack integrity and inevitably trip you up.

One may argue that the correct scale(s) can be decided by comparing with experimental data. This is a reasonable choice, even though it greatly depresses the predictive power of the theory.

It is difficult to decide which method to use: in principle, intuition and logic are two strategies for prediction and problem solving:

- intuition requires **prior** experience. Intuitions are acquired by learning, and the benefit of learning what happens in a given situation is only available if you encounter a sufficiently similar situation again.
- Lacking prior experience with a identical situation, you have to make a generalization of a previous *precedent* experience in order to guess what the *consequent* event will be. This is an error prone operation; the ability to generalize correctly is intimately tied to the ability to get to the *semantics* of the situation.

Logical formulas (as scale variation) can be manipulated *mechanically*, by following syntax based rules that specify which operations are allowed. In performing these manipulations, Innovation is not introduced (in theory) – that would need using Intuition.

Credible intervals incorporate problem-specific contextual information from the prior distribution whereas confidence intervals are based only on the data: there are no data for **MHO**. Furthermore, credible intervals and confidence intervals treat nuisance parameters in radically different ways.

When there is no prior information available the uninformative prior should be constant (as originally postulated by Laplace).