Higgs Effective Field Theory

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Combined limit

\[ \frac{d\sigma_{\text{off}}}{d\sigma_{\text{peak}}} = \mu r \frac{d\sigma_{\text{peak}}}{d\sigma_{\text{peak}}} \quad r = \frac{\Gamma_H}{\Gamma_{\text{SM}H}} \qquad \text{assume } \mu = 1 \quad \Rightarrow \quad \text{measure } r \]

\[ -2 \Delta \ln L = 8 \text{ TeV, } L = 19.7 \text{ fb}^{-1} \]

- Combined observed (expected) values
  - \( r = \frac{\Gamma}{\Gamma_{\text{SM}}} < 4.2 (8.5) \) at 95% CL
    (p-value = 0.02)
  - \( r = \frac{\Gamma}{\Gamma_{\text{SM}}} = 0.3^{+1.5}_{-0.3} \)

- equivalent to:
  - \( \Gamma < 17.4 (35.3) \text{ MeV} \) at 95% CL
  - \( \Gamma = (1.4^{+6.1}_{-1.4}) \text{ MeV} \)

BINGO!
The big picture @ 8TeV

- Peak at Z mass due to singly resonant diagrams.
- Interference is an important effect.
- Destructive at large mass, as expected.
- With the standard model width, $\sigma_H$, challenging to see enhancement/deficit due to Higgs channel.

CMS preliminary
$E = 8$ TeV, $L = 19.7$ fb$^{-1}$

$M^2_{VV} \frac{d\sigma}{dM_{VV}}$ [pb]

$\sigma / \sigma_{SM}$

$\sigma = \mu_H^2 - i \mu_H \gamma_H$

CPS required

8 TeV

$g_g g_V$

rising decay

dying line-shape

CPS required

by Gampier
We define an off-shell production cross-section (for all channels) as follows:

\[
\sigma_{ij \rightarrow \text{all}}^{\text{prop}} = \frac{1}{\pi} \sigma_{ij \rightarrow H} \frac{S^2}{|s - s_H|^2} \frac{\Gamma_H^{\text{tot}}}{\sqrt{s}}
\]

When the cross-section \( ij \rightarrow H \) refers to an off-shell Higgs boson the choice of the QCD scales should be made according to the virtuality and not to a fixed value. Therefore, for the PDFs and \( \sigma_{ij \rightarrow H + X} \) one should select \( \mu_F^2 = \mu_R^2 = z s / 4 \) (\( z s \) being the invariant mass of the detectable final state).
We define an off-shell production cross-section (for all channels) as follows:

\[
\sigma_{\text{prop}}^{ij \rightarrow \text{all}} = \frac{1}{2} \frac{1}{\alpha_s} \frac{1}{s - s_0} \frac{\Gamma_i^j}{\sqrt{s}}
\]

\(\sigma_{\text{prop}}^{ij \rightarrow \text{H}}\) When the cross-section \(\sigma_{\text{prop}}^{ij \rightarrow \text{H}}\) refers to an off-shell Higgs boson the choice of the QCD scales should be made according to the virtuality and not to a fixed value. Therefore, for the PDFs and \(\sigma_{\text{prop}}^{ij \rightarrow \text{H}}\), one should select \(\mu_F^2 = \mu_R^2 = \frac{z s}{4}\) (\(z s\) being the invariant mass of the detectable final state).
Let us consider the case of a light Higgs boson; here, the common belief was that

the product of on-shell production cross-section (say in gluon-gluon fusion) and branching ratios reproduces the correct result to great accuracy. The expectation is based on the well-known result

$$\Delta_H = \frac{1}{\left(s - M_H^2\right)^2 + \Gamma_H^2 M_H^2} = \frac{\pi}{M_H \Gamma_H} \delta\left(s - M_H^2\right) + \text{PV} \left[ \frac{1}{\left(s - M_H^2\right)^2} \right]$$

where PV denotes the principal value (understood as a distribution). Furthermore $s$ is the Higgs virtuality and $M_H$ and $\Gamma_H$ should be understood as $M_H = \mu_H$ and $\Gamma_H = \gamma_H$ and not as the corresponding on-shell values. In more simple terms,

the first term puts you on-shell and the second one gives you the off-shell tail

$\Delta_H$ is the Higgs propagator, there is no space for anything else in QFT (e.g. Breit-Wigner distributions).
Let us consider the case of a light Higgs boson; here, the common belief was that the product of on-shell production cross-section (say in gluon-gluon fusion) and branching ratios reproduces the correct result to great accuracy. The expectation is based on the well-known result: 

\[ \Delta H = \frac{1}{\pi} \frac{\int \sigma_{ij} \rightarrow H \left| s \right|^2}{\left| s - M^2_H \right|^2} + \Gamma_{\text{tot}} \sqrt{s} \]

where \(\sigma_{ij} \rightarrow H\) denotes the principal value of the relevant distribution. Furthermore, if the Higgs virtuality and \(M^2_H\) should be understood as \(\mu^2_F\) and \(\mu^2_R\), and not as the corresponding on-shell values, it means that:

- the first term puts you on-shell and the second one gives you the off-shell tail
- \(\Delta_H\) is the Higgs propagator, there is no space for anything else in QFT (e.g. Breit-Wigner distributions).

We define an off-shell production cross-section (for all channels) as follows:

\[ \sigma_{\text{prop}}^{ij \rightarrow all} = \frac{1}{\pi} \frac{\int \sigma_{ij} \rightarrow H + X}{\left| s - s_{\text{ref}} \right|^2} \frac{z}{\sqrt{s}} \]

When the cross-section \(\sigma_{ij} \rightarrow H\) refers to an off-shell Higgs boson, the choice of the PDF scales should be made according to the virtuality and not to a fixed value. Therefore, for the PDFs and \(s_{\text{ref}}\) one should select \(\mu^2_F = \mu^2_R = z s/4\) (\(z\) being the invariant mass of the detectable final state).
A short History of beyond ZWA (don’t try fixing something that is already broken in the first place).

1. There is an enhanced Higgs tail Kauer - Passarino (arXiv:1206.4803):
   away from the narrow peak the propagator and the off-shell H width behave like
   \[ \Delta_H \approx \frac{1}{(M_{VV}^2 - \mu_H^2)^2}, \quad \frac{\Gamma_{H \rightarrow VV}(M_{VV})}{M_{VV}} \sim G_F M_{VV}^2 \]


3. Observe that the enhanced tail is obviously \(\gamma_H\)-independent and that this could be exploited to constrain the Higgs width model-independently

4. Use a matrix element method (MEM) to construct a kinematic discriminant to sharpen the constraint
   Campbell, Ellis and Williams (arXiv:1311.3589)
A short History of beyond ZW A (and by living somewhere that is already broken in the first
place)

1. There is an enhanced Higgs tail [Kauer - Passarino (arXiv:1206.4803)] away from the narrow peak the propagator and the off-shell

H width behave like

\[ \Delta H \approx \frac{1}{(M_{VV} - \mu_H)^2} \]

2. Introduce the notion of \( \infty \) degenerate solutions for the Higgs couplings to SM particles [Dixon - Li (arXiv:1305.3854), Caola - Melnikov (arXiv:1307.4935)]

Observe that the enhanced tail is obviously \( \gamma_H \)-independent and that this could be exploited to constrain the Higgs width model-independently

3. Use a matrix element method (MEM) to construct a kinematic discriminant to sharpen the constraint [Campbell, Ellis and Williams (arXiv:1311.3589)]

We define an off-shell production cross-section (for all channels) as follows:

\[ \sigma_{\text{prop}}^ij \rightarrow \text{all} = \frac{1}{\pi} \sigma_{ij} \rightarrow H \frac{s}{(s - M_{ij})^2} \]

When the cross-section \( \sigma_{ij} \rightarrow H \) refers to an off-shell Higgs boson the choice of the \( QCD \) scales should be made according to the virtuality and not to a fixed value. Therefore, for the PDFs and \( \sigma_{ij} \rightarrow H \rightarrow X \) one should select

\[ \mu_H^2 = \frac{1}{2} \mu_{ij}^2 = \mu_R^2 = \frac{z s}{4} \]

(\( z \) being the invariant mass of the detectable final state).

Let us consider the case of a light Higgs boson; here, the common belief was that the product of on-shell production cross-section (say in gluon-gluon fusion) and branching ratios reproduces the correct result to great accuracy. The expectation is based on the well-known result

\[ \Delta H = \frac{1}{(s - M_H^2)^2} \]

\[ \Gamma_H^2 \sim \frac{\pi}{M_H^2} \delta(s - M_{ij}^2) + \text{PV} \left( \frac{1}{(s - M_{ij}^2)} \right) \]

where \( \text{PV} \) denotes the principal value understood as a distribution. Furthermore as the Higgs virtuality \( \mu_H^2 \) and \( \mu_{ij}^2 \) should be understood as \( \mu_R^2 \) and \( \mu_{ij}^2 \) are not for the corresponding on-shell values, in more

\[ \text{hence, terms.} \]

\[ \Delta H \] is the Higgs propagator, there is no space for anything else in QFT (e.g. Breit-Wigner distributions).
**OFF – SHELL IV**

\[ pp \rightarrow gg \rightarrow H \rightarrow e^+e^-\gamma \]

\[ M_{e^+e^-\gamma} > 0.1 M(e^+e^-\gamma) \]

\[ M_{e^+e^-} > 0.1 M(e^+e^-\gamma) \]

\[ M_{e^+e^-} > 0.1 M(e^+e^-\gamma) \]

\[ \sigma [fb] \]
Let us consider the case of a light Higgs boson; here, the common belief was that the product of on-shell production ... tail

\[ \Delta H \text{ is the Higgs propagator, there is no space for anything else in QFT (e.g. Breit-Wigner distributions)}. \]

We define an off-shell production cross-section (for all channels) as follows:

\[ \sigma_{\text{prop}}^{ij} = \frac{1}{2} \pi \frac{s^2}{(s - \mu^2)} \frac{\Gamma_{ij}^H}{\Gamma_{ij}^H} \]

\[ R = \frac{1}{2} \pi \frac{s^2}{(s - \mu^2)} \frac{\Gamma_{ij}^H}{\Gamma_{ij}^H} \]

\[ F = \frac{1}{2} \pi \frac{s^2}{(s - \mu^2)} \frac{\Gamma_{ij}^H}{\Gamma_{ij}^H} \]

\[ s \text{ being the invariant mass of the detectable final state).} \]

\[ \Delta \text{ is the Higgs propagator; there is no space for anything else in QFT (e.g. Breit-Wigner distributions).} \]

\[ s - \Delta \text{ is the Higgs virtuality and not as the corresponding on-shell values. In more simple terms,} \]

\[ \Gamma_{ij}^H \text{ refers to an off-shell Higgs} \]

\[ \text{The expectation is based on the well-known result} \]

\[ \text{Campbell, Ellis and Williams (arXiv:1311.3589)} \]

\[ \text{We define an off-shell production cross-section (for all channels) as follows:} \]

\[ \sigma_{\text{prop}}^{ij} = \frac{1}{2} \pi \frac{s^2}{(s - \mu^2)} \frac{\Gamma_{ij}^H}{\Gamma_{ij}^H} \]

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\[ s \text{ being the invariant mass of the detectable final state).} \]

\[ \Delta \text{ is the Higgs propagator; there is no space for anything else in QFT (e.g. Breit-Wigner distributions).} \]
$\sigma_{i \rightarrow H \rightarrow f} = (\sigma \cdot \text{BR}) = \frac{\sigma_{i, \text{prod}}}{\gamma_H}$

$\sigma_{i \rightarrow H \rightarrow f} \propto \frac{g_i^2 g_i^2}{\gamma_H}$

$g_{i,f} = \xi \frac{g_{i,f}^{\text{SM}}}{\gamma_H}$

$\gamma_H = \xi^4 \frac{\gamma_H^{\text{SM}}}{\gamma_H}$

On the whole, we have a constraint in the multidimensional $\kappa$-space

$\kappa_g^2 = \kappa_g^2(\kappa_l, \kappa_b) \quad \kappa_H^2 = \kappa_H^2(\kappa_l, \forall f)$

Only on the assumption of degeneracy one can prove that off-shell effects measure $\gamma_H$

a combination of on-shell effects measuring $g_i^2 g_i^2 / \gamma_H$

and off-shell effects measuring $g_i^2 g_i^2$

gives information on $\gamma_H$

without prejudices
The only limit to our realization of tomorrow will be our doubts of today

Definition:
κ-language is BSM MI approach

Chapter II
Nature of $\phi^d$

Corollary:
κ-language requires insertion of $\phi^d$ operators in SM loops

Chapter III
Ontology of HEFT

Main Theorem:
HEFT is a realization of κ-language

Chapter IV
Renorm. dim. reg. QFT role of $\Lambda$ top? - down

Memo:
Skip meetings

Strategy: How to interpret κXκXκX?

1. measure κ

\[ \frac{\Gamma^{\phi^d}(m_{\phi^d})}{\Gamma^{\phi^d}(m_{\phi^d})} = \frac{\kappa^2 \cdot \Gamma^{\phi^d}(m_{\phi^d}) + \kappa^2 \cdot \Gamma^{\phi^d}(m_{\phi^d}) + \kappa \cdot \Gamma^{\phi^d}(m_{\phi^d})}{\Gamma^{\phi^d}(m_{\phi^d}) + \Gamma^{\phi^d}(m_{\phi^d}) + \Gamma^{\phi^d}(m_{\phi^d})} \]

2. find $\Theta \Leftrightarrow \kappa$ (epistemological stop, true ESM believers stop here)

\[ \mathcal{L}_{\text{ESM}} = \mathcal{L}_{\text{SM}} + \sum_{n>4} \sum_{i=1}^{N_{\phi^d}} \frac{a_n^i}{\Lambda^{d-n}} \phi^{(d-n)} \]

3. find $\{\mathcal{L}_{\text{ESM}}\}$ that produces $\Theta$

is there a QFT behind degeneracy?
**anntated DIAGRAMMATICIA**

Figure 3: Example of one-loop SM diagrams with $\mathcal{O}$-insertions, contributing to the amplitude for $H \rightarrow \gamma \gamma$

\[ \mathcal{O}(g^3 g_6) \]

\[ H \rightarrow t, b \]

\[ H \rightarrow W/\phi/X^\pm \]

\[ H \rightarrow W/\phi \]

\[ \kappa_f, A_{NF} \]

\[ \kappa_W, A_{NF} \]

\[ \text{mix under ren. with } \mathcal{O}(g g_6) \]

\[ \mathcal{O}(g^3 g_6) \]

\[ \mathcal{O}_{\phi W} \]

\[ W \]

Figure 4: Example of one-loop $\mathcal{O}$-diagrams, contributing to the amplitude for $H \rightarrow \gamma \gamma$
\[ A = \sum_{n=N}^{\infty} \sum_{l=0}^{n} \sum_{k=1}^{l} g^k \hat{g}_{l+k} A_{nlk} \]

PTG: \( T \) - generated in at least one extension of SM

\[ A = \sum_{n=N}^{\infty} \sum_{l=0}^{n} \sum_{k=1}^{l} g^k \hat{g}_{l+k} A_{nlk} \]

\[ g_{l+k} = \frac{1}{(\sqrt{2}G_F\Lambda^2)} \]

\[ \frac{1}{(\sqrt{2}G_F\Lambda^2)} \approx \frac{g^2}{4\pi} \]

i.e. the contributions of \( d = 6 \) operators are \( \approx \) loop effects.

For higher scales, loop contributions tend to be more important.

PTG - operators versus LG - operators, cf. Einhorn, Wudka, ...

It can be argued that (at LO) the basis operator should be chosen from among the PTG operators.

A SM vertex with \( \mathcal{O}_{PTG}^{(6)} \) required \( \approx \) same order

\( 1/\Lambda \) expansion \( \rightarrow \) power-counting \( \checkmark \)

LG \( \rightarrow \) low-energy analytic structure \( \times \)

\[ \Lambda \approx 5 \text{ TeV} \]

w...
**PROPOSITION:** There are two ways of formulating HEFT

**a)** mass-dependent scheme(s) or **Wilsonian** HEFT

**b)** mass-independent scheme(s) or **Continuum** HEFT (CHEFT)

- only **a)** is conceptually consistent with the image of an EFT as a low-energy approximation to a high-energy theory

- however, inclusion of NLO corrections is only meaningful in **b)** since we cannot regularize with a cutoff and NLO requires regularization

There is an additional problem, CHEFT requires evolving our theory to lower scales until we get below the “heavy-mass” scale where we use $\mathcal{L} = \mathcal{L}_{\text{SM}} + d\mathcal{L}$, $d\mathcal{L}$ encoding matching corrections at the boundary. Therefore, CHEFT does not integrate out heavy degrees of freedom but removes them compensating for by an appropriate matching calculation.

*Not quite the same as it is usually discussed (no theory approaching the boundary from above ...)* cf. low-energy SM, weak effects on $g-2$ etc.
\[ \text{dim} \phi = d/2 - 1 \]
\[ \text{dim} \phi^d = N \phi \text{dim} \phi + N_{\text{der}} \]
For \( d \geq 3 \) there is a finite number of relevant + marginal operators
For \( d \geq 1 \) there is a finite number of irrelevant operators
Sounds good for finite dependence on high-energy theory

This assumes that high-energy theory is weakly coupled

Dimensional arguments work for LO HEFT

In NLO HEFT scaling may break down, implying appeal to a particular renormalization scheme

Ren. group should only be applied to EFTs that are nearly massless

**Decoupling theorem fails for CHEFT, but, arguably this does not prevent them from supporting a well defined scheme, but decoupling must be inserted in the form of matching calculations (which we don’t have . . .)**

Match Feynman diagrams \( \in \) HEFT with corresponding \( 1(\text{light})\text{PI} \) diagrams \( \in \) high-energy theory
(and discover that Taylor-expanding is not always a good idea)
Having said that ... no space left for annotations

Renormalization

\[ g = g_{ren} \left[ 1 + \frac{g_{ren}^2}{16 \pi^2} \left( dZ_g + g_6 dZ_g^{(6)} \right) \frac{1}{\varepsilon} \right] \]

\[ M_W = M_{ren} \left[ 1 + \frac{1}{2} \frac{g_{ren}^2}{16 \pi^2} \left( dZ_{M_W} + g_6 dZ_{M_W}^{(6)} \right) \frac{1}{\varepsilon} \right] \]

etc.

Wilson coefficients \[ \rightarrow \] \[ W_i \]

\[ W_i = \sum_j Z_{ij}^{wc} W_j^{ren} \]

\[ Z_{ij}^{wc} = \delta_{ij} + \frac{g_{ren}^2}{16 \pi^2} dZ_{ij}^{wc} \frac{1}{\varepsilon} \]
### Appendix C. Dimension-Six Basis Operators for the SM\textsuperscript{22}.

<table>
<thead>
<tr>
<th>$X^3$ (LG)</th>
<th>$\varphi^6$ and $\varphi^4D^2$ (PTG)</th>
<th>$\psi^2\varphi^3$ (PTG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_G$</td>
<td>$f^{ABC}G^{A\mu}G^{B\rho}G^{C\mu}$</td>
<td>$Q_\varphi$ $(\varphi^\dagger\varphi)^3$</td>
</tr>
<tr>
<td>$Q_{\bar{G}}$</td>
<td>$f^{ABC}\bar{G}^{A\mu}G^{B\rho}G^{C\mu}$</td>
<td>$Q_\varphi\square$ $(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$</td>
</tr>
<tr>
<td>$Q_W$</td>
<td>$\varepsilon^{IJK}W^I_\mu W^J_\nu W^K_\rho$</td>
<td>$Q_\varphi D$ $(\varphi^\dagger D^\mu\varphi)^* (\varphi^\dagger D_\mu\varphi)$</td>
</tr>
<tr>
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</thead>
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<td>$Q_{eW}$ $(\bar{l}<em>p\sigma^{\mu\nu}e_r)^T\varphi W^I</em>{\mu\nu}$</td>
</tr>
<tr>
<td>$Q_\varphi G$</td>
<td>$\varphi^\dagger\varphi \bar{G}^A_{\mu\nu}G^{A\mu\nu}$</td>
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<tr>
<td>$Q_\varphi W$</td>
<td>$\varphi^\dagger\varphi W^I_{\mu\nu}W^I_{\mu\nu}$</td>
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</tr>
<tr>
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<td>$\varphi^\dagger\varphi \bar{W}^I_{\mu\nu}W^I_{\mu\nu}$</td>
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</tr>
<tr>
<td>$Q_\varphi B$</td>
<td>$\varphi^\dagger\varphi B^I_{\mu\nu}B^I_{\mu\nu}$</td>
<td>$Q_{uB}$ $(\bar{q}<em>u\sigma^{\mu\nu}u_r)^T\varphi B</em>{\mu\nu}$</td>
</tr>
<tr>
<td>$Q_{\bar{B}}$</td>
<td>$\varphi^\dagger\varphi \bar{B}^I_{\mu\nu}B^I_{\mu\nu}$</td>
<td>$Q_{dG}$ $(\bar{q}<em>d\sigma^{\mu\nu}d_r)^T\varphi G^A</em>{\mu\nu}$</td>
</tr>
<tr>
<td>$Q_\varphi WB$</td>
<td>$\varphi^\dagger\varphi W^I_{\mu\nu}B^I_{\mu\nu}$</td>
<td>$Q_{dW}$ $(\bar{q}<em>d\sigma^{\mu\nu}d_r)^T\varphi W^I</em>{\mu\nu}$</td>
</tr>
<tr>
<td>$Q_{\bar{W}B}$</td>
<td>$\varphi^\dagger\varphi \bar{W}^I_{\mu\nu}B^I_{\mu\nu}$</td>
<td>$Q_{dB}$ $(\bar{q}<em>d\sigma^{\mu\nu}d_r)^T\varphi B</em>{\mu\nu}$</td>
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Table C.1: Dimension-six operators other than the four-fermion ones.

\textsuperscript{22}These tables are taken from [5], by permission of the authors.

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Effective Lagrangians cannot be blithely used without acknowledging implications of their choice ex: non gauge-invariant, intended to be used in U-gauge ex: $H \rightarrow WW^*$ is virtual $W +$ something else, depending on the operator basis
✓ Tadpoles $\mapsto \beta_H$

$\Phi = \frac{Z_1}{2} \phi \Phi_R$
\[ Tadpoles \mapsto \beta_H \]

\[ \Phi = Z_\phi^{1/2} \Phi_R \text{ etc.} \]
✓ Tadpoles $\mapsto \beta_H$
✓ $\Phi = Z_\phi^{1/2} \Phi_R$ etc.

$$Z_\phi = 1 + \frac{g^2}{16\pi^2} \left( \delta Z^{(4)}_\phi + g_6 \delta Z^{(6)}_\phi \right)$$
✓ Tadpoles $\mapsto \beta_H$
✓ $\Phi = Z^{1/2}_\phi \Phi_R$ etc.

$$ Z_\phi = 1 + \frac{g^2}{16\pi^2} \left( \delta Z_\phi^{(4)} + g_6 \delta Z_\phi^{(6)} \right) $$
✓ Self-energies UV $\mathcal{O}^{(4)}, \mathcal{O}^{(6)}$-finite
✓ Tadpoles $\mapsto \beta_H$
✓ $\Phi = Z_{\phi}^{1/2} \Phi_R$ etc.

$Z_{\phi} = 1 + \frac{g^2}{16\pi^2} \left( \delta Z^{(4)}_{\phi} + g_6 \, \delta Z^{(6)}_{\phi} \right)$

✓ Self-energies UV $O^{(4)}, O^{(6)}$-finite

✍ $\mu$-decay
✓ Tadpoles $\mapsto \beta_H$
✓ $\Phi = Z^{1/2}_\phi \Phi_R$ etc.

$Z_\phi = 1 + \frac{g^2}{16\pi^2} \left( \delta Z^{(4)}_\phi + g_6 \delta Z^{(6)}_\phi \right)$

✓ Self-energies UV
$\mathcal{O}^{(4)}, \mathcal{O}^{(6)}$-finite

✍ $\mu$-decay
✓ $g \mapsto g_R$
\( \checkmark \) Tadpoles \( \mapsto \beta_H \)
\( \checkmark \) \( \Phi = Z_\phi^{1/2} \Phi_R \) etc.

\[ Z_\phi = 1 + \frac{g^2}{16\pi^2} \left( \delta Z_\phi^{(4)} + g_6 \, \delta Z_\phi^{(6)} \right) \]

\( \checkmark \) Self-energies UV
\( \mathcal{O}^{(4)}, \mathcal{O}^{(6)} \)-finite

\( \blacklozenge \) \( \mu \)-decay
\( \checkmark \) \( g \rightarrow g_R \)

\( \checkmark \) Finite ren.

\( \mu \)-decay

Finite ren.
Tadpoles $\mapsto \beta_H$

$\Phi = Z_\phi^{1/2} \Phi_R$ etc.

$Z_\phi = 1 + \frac{g^2}{16\pi^2} \left( \delta Z_\phi^{(4)} + g_6 \delta Z_\phi^{(6)} \right)$

Self-energies UV
$
\mathcal{O}^{(4)}, \mathcal{O}^{(6)}$-finite

$\mu$-decay

g $\mapsto g_R$

Finite ren.

$M_R^2 = M_W^2 \left[ 1 + \frac{g_R^2}{16\pi^2} \left( \text{Re} \Sigma_{WW} - \delta Z_M \right) \right]$

etc Propagators finite and $\mu_R$-independent
\[ \begin{align*}
\textbf{H}-\text{propagator} \\
\Delta^{-1}_H &= Z_H \left(-s + Z_{m_H} M_H^2\right) - \frac{1}{(2\pi)^4 i} \Sigma_{HH} \\
Z_H &= 1 + \frac{g_R^2}{16\pi^2} \left( \delta Z_H^{(4)} + g_6 \delta Z_H^{(6)} \right) \frac{1}{\epsilon} \\
\delta Z_H^{(4)} &= 16 \left[ \frac{1}{288} \left( 82 - \frac{16}{c_\theta^2} - 25 \frac{s_\theta}{c_\theta} - 14 s_\theta^2 - 14 s_\theta c_\theta \right) \\
&\quad - \frac{3}{32} \left( m_b^2 + m_t^2 \right) \frac{m_b^2 + m_t^2}{M^2} \right] \\
\delta Z_H^{(6)} &= \frac{1}{6\sqrt{2}} \left[ \frac{5}{c_\theta^2} + 12 - 18 \frac{m_b^2 + m_t^2}{M^2} - 21 \frac{m_H^2}{M^2} \right] a_{\phi \square} \\
&\quad + \text{etc}
\end{align*} \]
EXAMPLE finite ren.

\[ m_H^2 = M_H^2 \left[ 1 + \frac{g_R^2}{16 \pi^2} \left( dM_H^{(4)} + g_6 dM_H^{(6)} \right) \right] \]

\[
\frac{M_H^2}{16} dM_H^{(4)} = \frac{1}{16} M_W^2 \left( \frac{1}{c_\theta^4} + 2 \right) \\
- \frac{3}{32} \frac{M_t^2}{M_W^2} (M_H^2 - 4 M_t^2) B_0 \left( -M_H^2 ; M_t, M_t \right) \\
- \frac{3}{32} \frac{M_b^2}{M_W^2} (M_H^2 - 4 M_b^2) B_0 \left( -M_H^2 ; M_b, M_b \right) \\
- \frac{9}{128} \frac{M_H^4}{M_W^2} B_0 \left( -M_H^2 ; M_H, M_H \right) \\
- \frac{1}{64} \left( \frac{M_H^4}{M_W^2} - 4 M_H^2 - 12 M_W^2 \right) B_0 \left( -M_H^2 ; M_W, M_W \right) \\
- \frac{1}{128} \left( \frac{M_H^4}{M_W^2} - 4 \frac{M_H^2}{c_\theta^2} + 12 \frac{M_W^2}{c_\theta^4} \right) B_0 \left( -M_H^2 ; M_Z, M_Z \right) \]
\( \nu_H = \text{Higgs virtuality} \)
\(v_H = \text{Higgs virtuality}\)

- Requires \(Z_H, Z_g, Z_g, Z_{gs}\)
- It is \(O^4\)-finite but not \(O^6\)-finite
$v_H = \text{Higgs virtuality}$

✓ requires $Z_H, Z_g, Z_g, Z_{gs}$
✓ It is $\mathcal{O}(4)$-finite but not $\mathcal{O}(6)$-finite
✓ involves $a_{\phi D}, a_{\phi \Box}, a_{t\phi}, a_{b\phi}, a_{\phi W}, a_{\phi g}, a_{tg}, a_{bg}$

$a_{tg} = W_1 \quad a_{bg} = W_2 \quad a_{\phi g} = W_3$

$a_{b\phi} + \frac{1}{4} a_{\phi D} - a_{\phi W} - a_{\phi \Box} = W_4$

$a_{t\phi} - \frac{1}{4} a_{\phi D} + a_{\phi W} + a_{\phi \Box} = W_5$
\

\[ \nu_H = \text{Higgs virtuality} \]

\[ a_{tg} = W_1 \quad a_{bg} = W_2 \quad a_{\phi g} = W_3 \]

\[ a_{\phi D} + \frac{1}{4} a_{\phi W} - a_{\phi \Box} = W_4 \]

\[ a_{t\phi} - \frac{1}{4} a_{\phi D} + a_{\Phi W} + a_{\phi \Box} = W_5 \]

\[ a_{b\phi} + \frac{1}{4} a_{\phi D} - a_{\Phi W} - a_{\phi \Box} = W_4 \]

\[ a_{t\phi} - \frac{1}{4} a_{\phi D} + a_{\Phi W} + a_{\phi \Box} = W_5 \]

\[ \text{\checkmark \ requires extra renormalization} \]

\[ W_i = \sum_j Z_{ij}^{\text{mix}} W_j^R (\mu_R) \]

\[ Z_{ij}^{\text{mix}} = \delta_{ij} + \frac{ggS}{16\pi^2} \delta Z_{ij}^{\text{mix}} \frac{1}{\bar{\epsilon}} \]

\[ \delta Z_{31(2)}^{\text{mix}} = -\frac{1}{2\sqrt{2}} \frac{M_t(b)}{M_W} \]
Define building blocks

\[ \frac{8 \pi^2}{i g_s^2} \frac{M_W}{M_q^2} A_q^{\text{LO}} = 2 \left( 4 M_q^2 - v_H \right) C_0 ( -v_H, 0, 0 ; M_q, M_q, M_q ) \]

\[ \frac{32 \pi^2}{i g_s^2} \frac{M_W^2}{M_q} A_q^{\text{nf}} = 8 M_q^4 C_0 ( -v_H, 0, 0 ; M_q, M_q, M_q ) \]

+ \[ v_H \left[ 1 - B_0 ( -v_H ; M_q, M_q ) \right] - 4 M_q^2 \]
Define (process dependent) $\kappa$-factors

$$\kappa_b = 1 + g_6 \left[ \frac{1}{2} \frac{M_b}{M_W} W_2^R - \frac{1}{\sqrt{2}} W_4^R \right]$$

$$\kappa_t = 1 + g_6 \left[ \frac{1}{2} \frac{M_t}{M_W} W_1^R - \frac{1}{\sqrt{2}} W_5^R \right]$$
Define (process dependent) $\kappa$-factors

\[
\kappa_b = 1 + g_6 \left[ \frac{1}{2} \frac{M_b}{M_W} W_2^R - \frac{1}{\sqrt{2}} W_4^R \right]
\]

\[
\kappa_t = 1 + g_6 \left[ \frac{1}{2} \frac{M_t}{M_W} W_1^R - \frac{1}{\sqrt{2}} W_5^R \right]
\]

Obtain the $4+6$ amplitude

\[
A^{(4+6)} = g \sum_{q=b,t} \kappa_q A_q^{\text{LO}} + i g_6 g_s \frac{M_H^2}{M_W} W_3^R
\]

\[
+ g_6 g \left[ W_1^R A_t^{\text{nf}} + W_2^R A_b^{\text{nf}} \right]
\]
Define (process dependent) $\kappa$-factors

\[
\kappa_b = 1 + g_6 \left[ \frac{1}{2} \frac{M_b}{M_W} W_2^R - \frac{1}{\sqrt{2}} W_4^R \right]
\]

\[
\kappa_t = 1 + g_6 \left[ \frac{1}{2} \frac{M_t}{M_W} W_1^R - \frac{1}{\sqrt{2}} W_5^R \right]
\]

Obtain the 4+6 amplitude

\[
A^{(4+6)} = g \sum_{q=b,t} \kappa_q A_{q}^{LO} + ig_6 g_S \frac{M_H^2}{M_W} W_3^R
\]

\[
+ g_6 g \left[ W_1^R A_t^{nf} + W_2^R A_b^{nf} \right]
\]

Derive true relation

\[
A^{(4+6)} (gg \rightarrow H) = g_g \left( v_H \right) A^{(4)} (gg \rightarrow H)
\]
Define (process dependent) $\kappa$-factors

$$
\kappa_b = 1 + g_6 \left[ \frac{1}{2} \frac{M_b}{M_W} W_2^R - \frac{1}{\sqrt{2}} W_4^R \right]
$$

$$
\kappa_t = 1 + g_6 \left[ \frac{1}{2} \frac{M_t}{M_W} W_1^R - \frac{1}{\sqrt{2}} W_5^R \right]
$$

Obtain the 4+6 amplitude

$$
A^{(4+6)} = g \sum_{q=b,t} \kappa_q A_q^{LO} + i g_6 g_S \frac{M_H^2}{M_W} W_3^R
$$

$$
+ g_6 g \left[ W_1^R A_t^{nf} + W_2^R A_b^{nf} \right]
$$

Derive true relation

$$
A^{(4+6)} (gg \rightarrow H) = g_g \left( \nu_H \right) A^{(4)} (gg \rightarrow H)
$$

Effective (running) scaling ($g_i$) is not a $\kappa$ (constant) parameter (unless $\Theta^{(6)} = 0$ and $\kappa_b = \kappa_t$)
Non-factorizable not included

\[ \Gamma_{gg} \]

![Graph showing \( \Gamma_{gg} \) as a function of H virtuality [GeV].]

- \( \kappa_t = 1.5 \)
- \( \kappa_b = 1.5 \)
- \( \kappa_t = 0.5 \)
Non-factorizable not included
✓ SCALE dependence (no subtraction point)
\checkmark \text{SCALE dependence (no subtraction point)}

\checkmark \text{Consider } H \rightarrow \gamma \gamma

\[ Z_{ij}^{\text{mix}} = \delta_{ij} + \frac{g_R^2}{16 \pi^2} \left[ \delta Z_{ij}^{\text{mix}} \frac{1}{\varepsilon} + \Delta_{ij} \ln \frac{M_H^2}{\mu_R^2} \right] \]

\[ W_1 = a_{\gamma \gamma} = s_\theta c_\theta a_{\phiWB} + c_\theta^2 a_{\phiB} + s_\theta^2 a_{\phiW} \]

\[ M_W^2 \Delta_{11} = \frac{1}{4} \left[ 8 s_\theta^2 \left( 2 s_\theta^2 - c_\theta^2 \right) M_W^2 + \left( 4 s_\theta^2 c_\theta^2 - 5 \right) M_H^2 \right] \]
 SCALE dependence (no subtraction point)

Consider $H \rightarrow \gamma\gamma$

$$Z_{ij}^{\text{mix}} = \delta_{ij} + \frac{g_R^2}{16 \pi^2} \left[ \delta Z_{ij}^{\text{mix}} \frac{1}{\varepsilon} + \Delta_{ij} \ln \frac{M_H^2}{\mu^2_R} \right]$$

$$W_1 = a_{\gamma\gamma} = s_\theta c_\theta a_{\Phi WB} + c^2_{\theta} a_{\Phi B} + s^2_{\theta} a_{\Phi W}$$

$$M_W^2 \Delta_{11} = \frac{1}{4} \left[ 8 s^2_\theta \left( 2 s^2_\theta - c^2_\theta \right) M_W^2 + \left( 4 s^2_\theta c^2_\theta - 5 \right) M_H^2 \right]$$

 etc
toy model: $S$ dark Higgs field

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{2} \partial_\mu S \partial_\mu S - \frac{1}{2} M_S^2 S^2 + \mu_S \Phi^\dagger \Phi S$$

$$I_{\text{DR}}^{\text{eff}} = \frac{3}{4} g \frac{M_H^2}{M_W \Lambda^2} \left[ \left( \frac{1}{2} s - 3 M_H^2 \right) \left( \frac{1}{\varepsilon} - \ln \frac{-s-i0}{\mu_R^2} \right) + \text{finite part} \right]$$

$$I_{\text{full}} = -\frac{3}{2} g \frac{M_H^2 \mu_S^2}{M_W M_S^2} \left[ 1 - \frac{1}{4} \frac{s}{M_S^2} - \left( 1 - \frac{1}{2} \frac{s}{M_S^2} \right) \ln \frac{-s-i0}{M_S^2} \right] + \mathcal{O} \left( \frac{s^2}{M_S^4} \right)$$

*full* starts at $\mathcal{O}(\mu_S^2/M_S^2)$
*eff* starts at $\mathcal{O}(s/\Lambda^2)$

large mass expansion of *full* follows from Mellin-Barnes expansion and not from Taylor expansion
✓ Background? Consider $\bar{u}u \rightarrow ZZ$
The following Wilson coefficients appear:

\[
W_1 = a_{\gamma\gamma} = s_\theta c_\theta \ a_{\Phi WB} + c_\theta^2 \ a_{\Phi B} + s_\theta^2 \ a_{\Phi W} \\
W_2 = a_{ZZ} = -s_\theta c_\theta \ a_{\Phi WB} + s_\theta^2 \ a_{\Phi B} + c_\theta^2 \ a_{\Phi W} \\
W_3 = a_{\gamma Z} = 2 s_\theta c_\theta \left( a_{\Phi W} - a_{\Phi B} \right) + \left( c_\theta^2 - s_\theta^2 \right) \ a_{\Phi WB} \\
W_4 = a_{\phi D} \\
W_5 = a_{\phi q}^{(3)} + a_{\phi q}^{(1)} - a_{\phi u} \\
W_6 = a_{\phi q}^{(3)} + a_{\phi q}^{(1)} + a_{\phi u}
\]
Background? Consider $\bar{u}u \to ZZ$

The following Wilson coefficients appear:

- $W_1 = a_{\gamma\gamma} = s_\theta c_\theta \ a_{\Phi WB} + c_\theta^2 \ a_{\Phi B} + s_\theta^2 \ a_{\Phi W}$
- $W_2 = a_{ZZ} = -s_\theta c_\theta \ a_{\Phi WB} + s_\theta^2 \ a_{\Phi B} + c_\theta^2 \ a_{\Phi W}$
- $W_3 = a_{\gamma Z} = 2 \ s_\theta \ c_\theta \ (a_{\Phi W} - a_{\Phi B}) + \ (c_\theta^2 - s_\theta^2) \ a_{\Phi WB}$
- $W_4 = a_{\Phi D}$
- $W_5 = a_{\phi q}^{(3)} + a_{\phi q}^{(1)} - a_{\phi u}$
- $W_6 = a_{\phi q}^{(3)} + a_{\phi q}^{(1)} + a_{\phi u}$

Define

$$A^{LO} = \frac{M_Z^4}{t^2} + \frac{M_Z^4}{u^2} - \frac{t}{u} - \frac{u}{t} - 4 \frac{M_Z^2 s}{tu}$$
✓ Obtain the result ($\bar{u}u \rightarrow ZZ$)

\[
\sum_{\text{spin}} \left| A^{(4+6)} \right|^2 = g^4 A^{\text{LO}} \left[ F^{\text{LO}}(s_\theta) + \frac{g_6}{\sqrt{2}} \sum_{i=1}^{6} F^i(s_\theta) W_i \right]
\]
✓ Obtain the result $(\bar{u}u \rightarrow ZZ)$

$$\sum_{\text{spin}} \left| A^{(4+6)} \right|^2 = g^4 A^{\text{LO}} \left[ F^{\text{LO}}(s_\theta) + \frac{g_6}{\sqrt{2}} \sum_{i=1}^{6} F^i(s_\theta) W_i \right]$$

✓ Background changes!
Obtain the result ($\bar{u}u \to ZZ$)

$$\sum_{\text{spin}} \left| A^{(4+6)} \right|^2 = g^4 A^{\text{LO}} \left[ F^{\text{LO}}(s_\theta) + \frac{g_6}{\sqrt{2}} \sum_{i=1}^{6} F^i(s_\theta) W_i \right]$$

Background changes!

Note that

$$F^{\text{LO}} \approx -0.57 \quad F^1 \approx +2.18 \quad F^2 \approx -3.31$$

$$F^3 \approx +4.07 \quad F^4 \approx -2.46 \quad F^4 \approx -2.46 \quad F^6 \approx -5.81$$
CONCLUSIONS

FUTURE (Moriod EW 2014)

\[ L = L_4 + \sum_{n>4} \sum_{i=1}^{N_n} \frac{a_n^2}{\Lambda_n^{d-n}} \phi_i (d=n) \]

TH is improving with NLO \( \kappa \)-language

NLO \( \kappa \)-language is NOT a simple scaling
Thanks for your attention
Backup Slides
Large Scale
φ, χ

renormalization group

L_H(χ, φ) + L(φ)

µ = M
particle mass

MATCHING

L(φ) + δL(φ)

renormalization group

Low Energy
φ

Figure 4: The general form of a matching calculation.

terms.

In this region, the physics is described by a set fields, χ, describing the heaviest particles, of mass M, and a set of light particle fields, φ, describing all the lighter particles. The Lagrangian has the form

\[ L_H(χ, φ) + L(φ) , \]  

where \( L(φ) \) contains all the terms that depend only on the light fields, and \( L_H(χ, φ) \) is everything else. As long as no particle masses are encountered, this evolution is described by the renormalization group. However, when µ goes below the mass, M, of the heavy particles, you should change the effective theory to a new theory without the heavy particles. In the process, the parameters of the theory change, and new, nonrenormalizable interactions may be introduced. Thus the Lagrangian of the effective theory below M has the form

\[ L(φ) + δL(φ) , \]  

(3.16)
Increasing COMPLEXITY

✓ $H \rightarrow \gamma\gamma$

① 3 LO amplitudes $A_t^{LO}, A_b^{LO}, A_W^{LO}$, 3 $\kappa$-factors

② 6 Wilson coefficients & non-factorizable amplitudes
Increasing COMPLEXITY

✓ $H \rightarrow \gamma\gamma$

1. 3 LO amplitudes $A^\text{LO}_t, A^\text{LO}_b, A^\text{LO}_W$, 3 $\kappa$-factors
2. 6 Wilson coefficients & non-factorizable amplitudes

✓ $H \rightarrow ZZ$

1. 1 LO amplitude
2. 6 NLO amplitudes, 6 $\kappa$-factors

\[
\delta^{\mu\nu} \sum_{i=t,b,B} A^\text{NLO}_{i,D} + p_2^\mu p_1^\nu \sum_{i=t,b,B} A^\text{NLO}_{i,P}
\]

2. 16 Wilson coefficients & non-factorizable amplitudes
Increasing COMPLEXITY

✓ $H \rightarrow \gamma\gamma$

1. 3 LO amplitudes $A_t^{\text{LO}}, A_b^{\text{LO}}, A_W^{\text{LO}}$, 3 $\kappa$-factors

2. 6 Wilson coefficients & non-factorizable amplitudes

✓ $H \rightarrow ZZ$

1. 1 LO amplitude

2. 6 NLO amplitudes, 6 $\kappa$-factors

\[ \delta^{\mu\nu} \sum_{i=t,b,B} A_{i,D}^{\text{NLO}} + p_2^{\mu} p_1^{\nu} \sum_{i=t,b,B} A_{i,P}^{\text{NLO}} \]

2. 16 Wilson coefficients & non-factorizable amplitudes

✓ etc.
finite renormalization

\[ g^{2}_{\text{exp}} = G^{2} \left[ 1 + 2 \frac{G^{2}}{16 \pi^2} \left( dG^{(4)} + g_{6} dG^{(6)} \right) \right] \]

\[ G^{2} = 4 \sqrt{2} \, G_{F} \, M_{W}^{2} \]

\[ \checkmark \, \text{d}G^{(4,6)} \, \text{from} \, \mu \,-\text{decay} \]
\[ g^2 \text{ finite renormalization} \]

\[ g_{\text{exp}}^2 = G^2 \left[ 1 + 2 \frac{G^2}{16 \pi^2} \left( dG^{(4)} + g_6 dG^{(6)} \right) \right] \quad G^2 = 4 \sqrt{2} G_F M_W^2 \]

- $dG^{(4,6)}$ from $\mu$-decay
- Involving $\Sigma_{WW}(0)$ (easy)
$g_\text{finite renormalization}$

\[ g_\text{exp}^2 = G^2 \left[ 1 + 2 \frac{G^2}{16 \pi^2} \left( dG^{(4)} + g_6 dG^{(6)} \right) \right] \quad G^2 = 4 \sqrt{2} G_F M_W^2 \]

- $dG^{(4,6)}$ from $\mu$-decay
- Involving $\Sigma_{WW}(0)$ (easy)
- $\not\!X$ and vertices & boxes (not easy with $O(6)$-insertions)
H wave function renormalization \(1 - \frac{1}{2} \frac{g_{\exp}^2}{16 \pi^2} \delta \mathcal{Z}_H\)

\[
\delta \mathcal{Z}^{(4)}_H = \left. \frac{3}{2} \frac{M_t^2}{M_W^2} B_0^f \left( -M_H^2 ; M_t, M_t \right) \right. + \left. \frac{3}{2} \frac{M_b^2}{M_W^2} B_0^f \left( -M_H^2 ; M_b, M_b \right) \right.
\]

\[
- B_0^f \left( -M_H^2 ; M_W, M_W \right) - 1/2 \frac{1}{c_\theta^2} B_0^f \left( -M_H^2 ; M_Z, M_Z \right)
\]

\[
+ \left. \frac{3}{2} \left( M_H^2 - 4 M_t^2 \right) \frac{M_t^2}{M_W^2} B_0^p \left( -M_H^2 ; M_t, M_t \right) \right. + \left. \frac{3}{2} \left( M_H^2 - 4 M_b^2 \right) \frac{M_b^2}{M_W^2} B_0^p \left( -M_H^2 ; M_b, M_b \right) \right.
\]

\[
+ \left. \frac{1}{4} \left( \frac{M_H^4}{M_W^2} - 4 M_H^2 + 12 M_W^2 \right) B_0^p \left( -M_H^2 ; M_W, M_W \right) + \frac{1}{8} \left( \frac{M_H^4}{M_W^2} - 4 \frac{M_H^2}{c_\theta^2} + 12 \frac{M_Z^2}{c_\theta^2} \right) B_0^p \left( -M_H^2 ; M_Z, M_Z \right) \right.
\]

\[
+ \left. \frac{9}{8} \frac{M_H^4}{M_W^2} B_0^p \left( -M_H^2 ; M_H, M_H \right) \right.
\]

etc.
Fine points on PTG versus LG $\mathcal{O}^{(6)}$ operators

✓ Proposition: if we assume that the high-energy theory is

1. weakly-coupled and
2. renormalizable
Fine points on PTG versus LG $\mathcal{O}^{(6)}$ operators

✓ Proposition: if we assume that the high-energy theory is

① weakly-coupled and

② renormalizable

✓ it follows that the PTG/LG classification of arXiv:1307.0478 (used here) is correct.
Fine points on PTG versus LG $\mathcal{O}^{(6)}$ operators

✓ Proposition: if we assume that the high-energy theory is

  ① weakly-coupled and
  ② renormalizable

✓ it follows that the PTG/LG classification of arXiv:1307.0478 (used here) is correct.

✓ If we do not assume the above but work always in some EFT context (i.e., also the next high-energy theory is EFT, possibly involving some strongly interacting theory) then classification changes, see Eqs. (A1-A2) of arXiv:1305.0017v2
### Wilson Coefficients

<table>
<thead>
<tr>
<th>( W )</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_1 )</td>
<td>( a_{\gamma \gamma} = s_\theta c_\theta \ a_{\phi WB} + c_\theta^2 \ a_{\phi B} + s_\theta^2 \ a_{\phi W} )</td>
</tr>
<tr>
<td>( W_2 )</td>
<td>( a_{ZZ} = -s_\theta c_\theta \ a_{\phi WB} + s_\theta^2 \ a_{\phi B} + c_\theta^2 \ a_{\phi W} )</td>
</tr>
<tr>
<td>( W_3 )</td>
<td>( a_{\gamma Z} = 2 s_\theta c_\theta \left( a_{\phi W} - a_{\phi B} \right) + \left( c_\theta^2 - s_\theta^2 \right) \ a_{\phi WB} )</td>
</tr>
</tbody>
</table>

**Subscripts:**

- \( \phi \) for \( \phi \)-type
- \( D \) for \( D \)-type
- \( b \) for \( b \)-type
- \( t \) for \( t \)-type

### Additional Equations

- \( a_{\gamma W} = s_\theta \ a_{\gamma WB} + c_\theta \ a_{\gamma BW} \)
- \( a_{q W} = s_\theta \ a_{q WB} - c_\theta \ a_{q BW} \)
\[ W_{12} = a_{\phi bA} \]
\[ W_{14} = a_{\phi tA} \]

\[ W_{13} = a_{\phi bV} \]
\[ W_{15} = a_{\phi tV} \]

\[ a_{\phi bV} = a_{\phi q}^{(3)} - a_{\phi b} - a_{\phi q}^{(1)} \]

\[ a_{\phi tV} = a_{\phi q}^{(3)} - a_{\phi t} - a_{\phi q}^{(1)} \]

\[ a_{\phi bA} = a_{\phi q}^{(3)} + a_{\phi b} - a_{\phi q}^{(1)} \]

\[ a_{\phi tA} = a_{\phi q}^{(3)} + a_{\phi t} - a_{\phi q}^{(1)} \]
$\Sigma_{\gamma\gamma}(s) = \Pi_{\gamma\gamma}(s) s$

$\Pi_{\gamma\gamma}(s) = \frac{g^2 s_\theta^2}{16 \pi^2} \Pi_{\gamma\gamma}^{(4)}(s) + \frac{g^2 g_6}{16 \sqrt{2} \pi^2} \sum_{i=1}^{11} \Pi_{\gamma\gamma i}^{(6)}(s) W_i$

$\Pi_{\gamma\gamma}^{(4)}(0) = 3 a_0^f(M_W) + \frac{1}{9} \left[ 1 - 4 a_0^f(M_b) - 16 a_0^f(M_t) \right]$
\[ \Pi^{(6)}_{\gamma\gamma 1}(0) = -\left(1 - 8s^2 + 2s^4 \right)a^f_0(M_W) \]
\[ - \frac{1}{2}\frac{M_H^2}{M_W^2}a^f_0(M_H) - \frac{1}{2}\frac{1}{c^2_\theta}a^f_0(M_Z) \]
\[ - \frac{4}{9}s^2_\theta \left[16\left(1 - \frac{1}{2}s^2_\theta\right)a^f_0(M_t) + 4\left(1 - \frac{1}{2}s^2_\theta\right)a^f_0(M_b) + 17\left(1 - \frac{35}{34}s^2_\theta\right)\right] \]
\[ \Pi^{(6)}_{\gamma\gamma 2}(0) = s_\theta c_\theta \left\{ \frac{2}{9}\left[35 + 16a^f_0(M_t) + 4a^f_0(M_b)\right] - 2a^f_0(M_W) \right\} \]
\[ \Pi^{(6)}_{\gamma\gamma 3}(0) = s_\theta c_\theta \left\{ 4\left(1 - \frac{35}{18}c^2_\theta\right) + 4\left(1 - \frac{1}{2}s^2_\theta\right)a^f_0(M_W) \right. \]
\[ - \frac{8}{9}c^2_\theta \left[4a^f_0(M_t) + a^f_0(M_b)\right] \} \]
\[ \Pi^{(6)}_{\gamma\gamma 4}(0) = c^2_\theta \left\{ -\frac{3}{2}a^f_0(M_W) + \frac{1}{18}\left[16a^f_0(M_t) + 4a^f_0(M_b) - 1\right] \right\} \]
\[ \Pi^{(6)}_{\gamma\gamma 6}(0) = -2\frac{M_b^2}{M_W^2}s_\theta \left[a^f_0(M_b) + 1\right] \]
\[ \Pi^{(6)}_{\gamma\gamma 8}(0) = -4\left(c^2_\theta - s^2_\theta\right)s_\theta \frac{M_b^2}{M_W^2}\left[a^f_0(M_t) + 1\right] \]
\[ \Pi^{(6)}_{\gamma\gamma 9}(0) = 8s^2_\theta c_\theta \frac{M_t^2}{M_W^2}\left[a^f_0(M_t) + 1\right] \]
\[ \Sigma_{Z\gamma}(s) = \Pi_{Z\gamma}(s) s \]

\[ \Pi_{Z\gamma}(s) = \frac{g^2}{16 \pi^2} \frac{s_{\theta}}{c_{\theta}} \Pi_{Z\gamma}^{(4)}(s) + \frac{g^2 g_6}{16 \sqrt{2} \pi^2} \sum_{i=1}^{15} \Pi_{Z\gamma i}^{(6)}(s) W_i - \frac{g_6}{\sqrt{2}} W_3 \]

\[ \Pi_{Z\gamma}^{(4)}(0) = \frac{1}{6} \left( 19 - 18 s_{\theta}^2 \right) a_0^f(M_W) - \frac{2}{9} \left( 3 - 8 s_{\theta}^2 \right) a_0^f(M_t) \]

\[ - \frac{1}{9} \left( 3 - 4 s_{\theta}^2 \right) a_0^f(M_b) + \frac{1}{18} \left( 21 - 2 s_{\theta}^2 \right) \]
\[
\begin{align*}
\Pi^{(6)}_{Z\gamma 1}(0) &= \frac{s_{\theta}}{c_{\theta}} \left[ \frac{1}{3} \left( 1 + 6 c_{\theta}^4 \right) a_0^f (M_W) + \frac{4}{9} \left( 5 - 8 c_{\theta}^4 \right) a_0^f (M_t) \\ + \frac{2}{9} \left( 1 - 4 c_{\theta}^4 \right) a_0^f (M_b) - \frac{1}{9} \left( 33 - 122 s_{\theta}^2 + 70 s_{\theta}^4 \right) \right] \\
\Pi^{(6)}_{Z\gamma 2}(0) &= s_{\theta} c_{\theta} \left[ +2 \left( 3 - c_{\theta}^2 \right) a_0^f (M_W) - \frac{32}{9} s_{\theta}^2 a_0^f (M_t) \\ - \frac{8}{9} s_{\theta}^2 a_0^f (M_b) - \frac{2}{9} \left( 8 - 35 c_{\theta}^2 \right) \right] \\
\Pi^{(6)}_{Z\gamma 3}(0) &= -\frac{1}{18} \left( 33 - 174 s_{\theta}^2 + 140 s_{\theta}^4 \right) + \frac{1}{3} \left( 2 - 9 s_{\theta}^2 + 6 s_{\theta}^4 \right) a_0^f (M_W) \\ - \frac{1}{4} \frac{M_H^2}{M_W^2} a_0^f (M_H) - \frac{1}{4} \frac{1}{c_{\theta}^2} a_0^f (M_Z) - \frac{2}{9} \left( 3 - 24 s_{\theta}^2 + 16 s_{\theta}^4 \right) a_0^f (M_t) - \frac{1}{9} \left( 3 - 12 s_{\theta}^2 + 8 s_{\theta}^4 \right) a_0^f (M_b) \\
\Pi^{(6)}_{Z\gamma 4}(0) &= \frac{1}{s_{\theta} c_{\theta}} \left[ -\frac{1}{24} \left( 19 - 56 s_{\theta}^2 + 36 s_{\theta}^4 \right) a_0^f (M_W) + \frac{1}{18} \left( 3 - 24 s_{\theta}^2 + 16 s_{\theta}^4 \right) a_0^f (M_t) \\ + \frac{1}{36} \left( 3 - 12 s_{\theta}^2 + 8 s_{\theta}^4 \right) a_0^f (M_b) - \frac{1}{72} \left( 21 + 4 s_{\theta}^4 \right) \right] \\
\Pi^{(6)}_{Z\gamma 6}(0) &= \frac{1}{4 c_{\theta}} \frac{M_b^2}{M_W^2} \left( 1 - 4 c_{\theta}^2 \right) \left[ a_0^f (M_b) - 1 \right] \\
\Pi^{(6)}_{Z\gamma 7}(0) &= -\frac{M_b^2}{4 c_{\theta}} \frac{M_W^2}{s_{\theta}^2} \left[ a_0^f (M_b) + 1 \right] \\
\Pi^{(6)}_{Z\gamma 8}(0) &= -\frac{1}{4 c_{\theta}} \frac{M_t^2}{M_W^2} \left( 5 - 34 c_{\theta}^2 + 32 c_{\theta}^4 \right) \left[ a_0^f (M_t) - 1 \right] \\
\Pi^{(6)}_{Z\gamma 9}(0) &= \frac{1}{2} \frac{s_{\theta}}{c_{\theta}} \frac{M_t^2}{M_W^2} \left( 7 - 16 s_{\theta}^2 \right) \left[ a_0^f (M_t) + 1 \right]
\end{align*}
\]
\[ \Pi^{(6)}_{Z\gamma 13}(0) = -\frac{2}{3} \frac{s_\theta}{c_\theta} \frac{M_b^2}{M_W^2} \left[ a_0^f(M_b) + 1 \right] \]

\[ \Pi^{(6)}_{Z\gamma 15}(0) = -\frac{4}{3} \frac{s_\theta}{c_\theta} \frac{M_t^2}{M_W^2} \left[ a_0^f(M_t) + 1 \right] \]
STU: building blocks $Z-Z$

$$\Sigma_{ZZ}(s) = S_{ZZ} + \Pi_{ZZ} s + O(s^2)$$

$$S_{ZZ} = \frac{g^2}{16 \pi^2 c_\theta^2} S_{ZZ}^{(4)} + \frac{g^2 g_6}{16 \sqrt{2} \pi^2} \sum_{i=1}^{15} S_{ZZ_i}^{(6)} W_i$$

$$\Pi_{ZZ} = \frac{g^2}{16 \pi^2 c_\theta^2} \Pi_{ZZ}^{(4)} + \frac{g^2 g_6}{16 \sqrt{2} \pi^2} \sum_{i=1}^{15} \Pi_{ZZ_i}^{(6)} W_i$$
\[
S_{ZZ}^{(4)} = \left( M_Z^2 - \frac{1}{3} M_H^2 + \frac{1}{12} \frac{M_W^4}{M_Z^2} \right) B_0^f \left( -M_Z^2 ; M_H, M_Z \right) \\
+ \frac{1}{18} \left[ \left( 7 - 16 c_\theta^2 - 64 c_\theta^2 s_\theta^2 \right) M_t^2 + \left( 17 - 8 c_\theta^2 - 32 c_\theta^2 s_\theta^2 \right) M_Z^2 \right] B_0^f \left( -M_Z^2 ; M_t, M_t \right) \\
+ \frac{1}{18} \left[ \left( 5 + 4 c_\theta^2 - 8 c_\theta^2 s_\theta^2 \right) M_b^2 - \left( 17 - 8 c_\theta^2 + 16 c_\theta^2 s_\theta^2 \right) M_b^2 \right] B_0^f \left( -M_Z^2 ; M_b, M_b \right) \\
+ \frac{1}{12} \left( M_Z^4 - 2 M_W^2 M_Z^2 + M_H^4 \right) B_0^f \left( 0 ; M_H, M_Z \right) + \frac{2}{3} \left( M_Z^2 + \frac{M_Z^4}{M_H^2 - M_Z^2} - \frac{3}{8} M_H^2 + \frac{1}{8} \frac{M_W^4}{M_Z^2} \right) a_0^f \left( M_H \right) \\
+ \frac{1}{4} \left( \frac{M_t^2}{M_H^2 - M_Z^2} - \frac{1}{3} M_H^2 \right) a_0^f \left( M_Z \right) - \frac{4}{27} \left( 2 + c_\theta^2 - 5 c_\theta^2 s_\theta^2 \right) M_Z^2 \\

\Pi_{ZZ}^{(4)} = \frac{5}{6} \left( M_Z^2 - \frac{1}{5} M_H^2 \right) B_0^p \left( 0 ; M_H, M_Z \right) + \frac{1}{18} \left( 7 - 16 c_\theta^2 - 64 c_\theta^2 s_\theta^2 \right) M_t^2 B_0^p \left( 0 ; M_t, M_t \right) \\
- \frac{1}{18} \left( 17 - 8 c_\theta^2 + 16 c_\theta^2 s_\theta^2 \right) M_b^2 B_0^p \left( 0 ; M_b, M_b \right) + \frac{1}{3} \left[ 5 M_Z^2 c_\theta^2 - 4 \left( 5 - 3 s_\theta^2 \right) M_Z^2 c_\theta^6 \right] B_0^p \left( 0 ; M_W, M_W \right) \\
- \frac{1}{24} \left( M_Z^4 - 2 M_W^2 M_Z^2 + M_H^4 \right) B_8^f \left( 0 ; M_H, M_Z \right) + \frac{1}{12} \left( 1 + \frac{M_Z^2}{M_H^2 - M_Z^2} \right) a_0^f \left( M_H \right) \\
- \frac{1}{12} \frac{M_Z^2}{M_H^2 - M_Z^2} a_0^f \left( M_Z \right) + \frac{4}{27} \left( 2 + c_\theta^2 - 5 c_\theta^2 s_\theta^2 \right)
KEEP CALM TO BE CONTINUED
The life and death of $\mu_R$

✓ $\gamma$ bare propagator

$$\Delta^{-1}_\gamma = -s - \frac{g^2}{16 \pi^2} \Sigma_{\gamma\gamma}(s)$$

$$\Sigma_{\gamma\gamma}(s) = \left(D^{(4)} + g_6 D^{(6)}\right) \frac{1}{\epsilon} + \sum_{x \in \mathcal{X}} \left(L_i^{(4)} + g_6 L_i^{(6)}\right) \ln \frac{x}{\mu_R^2} + \Sigma_{\gamma\gamma}^{\text{rest}}$$

$$\{ \mathcal{X} \} = \{ s, m^2, m_0^2, m_H^2, m_t^2, m_b^2 \}$$
The life and death of $\mu_{\mathbf{R}}$

✓ $\gamma$ bare propagator

$$\Delta^{-1}_{\gamma} = -s - \frac{g^2}{16\pi^2} \Sigma_{\gamma\gamma}(s)$$

$$\Sigma_{\gamma\gamma}(s) = \left(D^{(4)} + g_6 D^{(6)}\right) \frac{1}{\varepsilon} + \sum_{x \in \mathcal{X}} \left(L_i^{(4)} + g_6 L_i^{(6)}\right) \ln \frac{x}{\mu_R^2} + \Sigma_{\gamma\gamma}^{\text{rest}}$$

$$\mathcal{X} = \{s, m^2, m_0^2, m_H^2, m_t^2, m_b^2\}$$

✓ $\gamma$ renormalized propagator

$$\left.\Delta^{-1}_{\gamma}\right|_{\text{ren}} = -Z_\gamma s - \frac{g^2}{16\pi^2} \Sigma_{\gamma\gamma}(s)$$

$$= -s - \frac{g^2}{16\pi^2} \Sigma_{\gamma\gamma}^{\text{ren}}(s)$$
The life and death of $\mu_R$

$$\Sigma^{\text{ren}}_{\gamma\gamma}(s) = \sum_{x \in \mathcal{X}} \left( L^{(4)}_i + g_6 L^{(6)}_i \right) \ln \frac{x}{\mu_R^2} + \Sigma^{\text{rest}}_{\gamma\gamma}$$

✓ finite renormalization

$$\Sigma^{\text{ren}}_{\gamma\gamma}(s) = \Pi^{\text{ren}}_{\gamma\gamma}(s) s$$

$$\frac{\partial}{\partial \mu_R} \left[ \Pi^{\text{ren}}_{\gamma\gamma}(s) - \Pi^{\text{ren}}_{\gamma\gamma}(0) \right] = 0$$
The life and death of $\mu_R$

$$\Sigma_{\gamma\gamma}^{\text{ren}}(s) = \sum_{x \in X} \left( L_i^{(4)} + g_6 L_i^{(6)} \right) \ln \frac{x}{\mu_R^2} + \Sigma_{\gamma\gamma}^{\text{rest}}$$

✓ finite renormalization

$$\Sigma_{\gamma\gamma}^{\text{ren}}(s) = \Pi_{\gamma\gamma}^{\text{ren}}(s) s$$

$$\frac{\partial}{\partial \mu_R} \left[ \Pi_{\gamma\gamma}^{\text{ren}}(s) - \Pi_{\gamma\gamma}^{\text{ren}}(0) \right] = 0$$

✓ including $\mathcal{O}^{(6)}$ contribution. There is no $\mu_R$ problem when a subtraction point is available.
\( \mathcal{O}^{(6)} \rightarrow \mathcal{O}^{(4)} \rightarrow \text{field(parameter) redefinition} \)

\[
\mathcal{L} = -\partial_\mu K^\dagger \partial^\mu K - \mu^2 K^\dagger K
- \frac{1}{2} \lambda (K^\dagger K)^2
- \frac{1}{2} \frac{M_0^2 \phi_0^2 - M^2 \phi^+ \phi^- + g^2 a_\phi \Lambda^2}{\Lambda^2} (K^\dagger K)^3
- g \frac{a_\phi \square}{\Lambda^2} K^\dagger K \Box K^\dagger K
- g \frac{a_{\phi D}}{\Lambda^2} \left| K^\dagger \partial^\mu K \right|^2
\]

\[
\sqrt{2} K_1 = H + 2 \frac{M}{g} + i \phi_0 \quad K_2 = i \phi^-
\]
Requires

\[ \mu^2 = \beta_H - 2 \frac{\lambda}{g^2} M^2 \quad \lambda = \frac{1}{4} g^2 \frac{M_H^2}{M^2} \]

\[ H \rightarrow \left[ 1 - (a_{\phi D} - 4 a_{\phi \Box}) \frac{M_H^2}{g^2 \Lambda^2} \right] H \]

\[ M_H \rightarrow \left[ 1 + (a_{\phi D} - 4 a_{\phi \Box} + 24 a_{\phi}) \frac{M_H^2}{g^2 \Lambda^2} \right] M_H \]

e.tc. with non-trivial effects on the S -matrix